

Influence of the Scale Factor on the Young's Modulus of Steel Determined by the Tensile Strength of the Specimens

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Abstract—The modulus of elasticity is considered a fairly stable physico-mechanical characteristic of materials, which is little dependent on their composition and structure. Among the factors influencing the modulus of elasticity, temperature and anisotropy are distinguished. Information about the influence of scale factor on the modulus of elasticity is quite limited and sometimes contradictory. The aim of this study is to investigate the influence of the scale factor on the elastic modulus of steel 45 determined by tension geometrically similar specimens with different initial diameters. The specimens were tested on an Instron 8801 universal testing machine at a deformation rate of 0.1 mm/min at room temperature. Elastic deformations during tension were measured using two methods—with the help of a strain gauge and the digital image correlation method. Both methods showed fairly close results when testing specimens of the same diameter. However, the digital image correlation method allowed for measurements of elastic deformations on specimens with small diameters, where it was not possible to attach a strain gauge. A decrease in the modulus of elasticity with an increase in the initial diameter of the specimen was established. Graphical dependences of the modulus of elasticity on the diameter of the specimen and the area of its cross section were obtained. Possible reasons for the decrease in the modulus of elasticity under the influence of the scale factor are outlined. A decrease in specific surface area and specific surface energy, an increase in the deformable volume, and a decrease in the deformation rate at a constant deformation speed are among the main reasons. The decrease in the modulus of elasticity under the influence of the scale factor should be taken into account in strength calculations and when assessing the residual life of parts and structures with relatively large cross sections and wall thicknesses.

Keywords: modulus of normal elasticity, tension of specimens, extensometer, digital image correlation method, scale effect

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INTRODUCTION

The modulus of normal elasticity (Young's modulus) E_m is a fundamental physical and mechanical property of a metal that characterizes the rigidity of its crystal lattice, which depends on interatomic attraction. The higher the modulus of normal elasticity, the less elastic deformation ε is caused by the same acting stress, which follows from the Hooke's law:

$$\sigma = E_m \varepsilon. \quad (1)$$

The modulus of normal elasticity is a parameter widely used in mechanics of materials and structures, physics, materials science, crystallography, and other disciplines. For example, E_m is used for strength calculations of parts and structures, assessment of the service life of metal and products under the influence of operating factors, establishment of relationships between various mechanical characteristics, and other purposes.

It is commonly believed that the modulus of normal elasticity is a stable physico-mechanical character-

istic of metals and alloys, which is not significantly affected by factors such as chemical composition, heat treatment conditions, and loading speed. Among the factors that can influence the modulus of normal elasticity, temperature [1, 2], anisotropy [3, 4], and structural damage are considered to be the most significant. However, the influence of the scale factor determined by the specimen size has been poorly studied. Similar information can be found in the technical literature, but it is limited and ambiguous. In some cases, the diameter or specific surface area of the specimen are used as scale parameters. For example, in [5], a significant increase in E_m was observed when reducing the diameters of specimens from 70 to 30 mm for composite materials. It should be noted that, with decreasing diameters, the specific surface area of such specimens increases. However, according to the results of experiments presented in [6], no significant changes in E_m were observed with an increase in the specific surface area of the specimen. The authors of [7], while study-

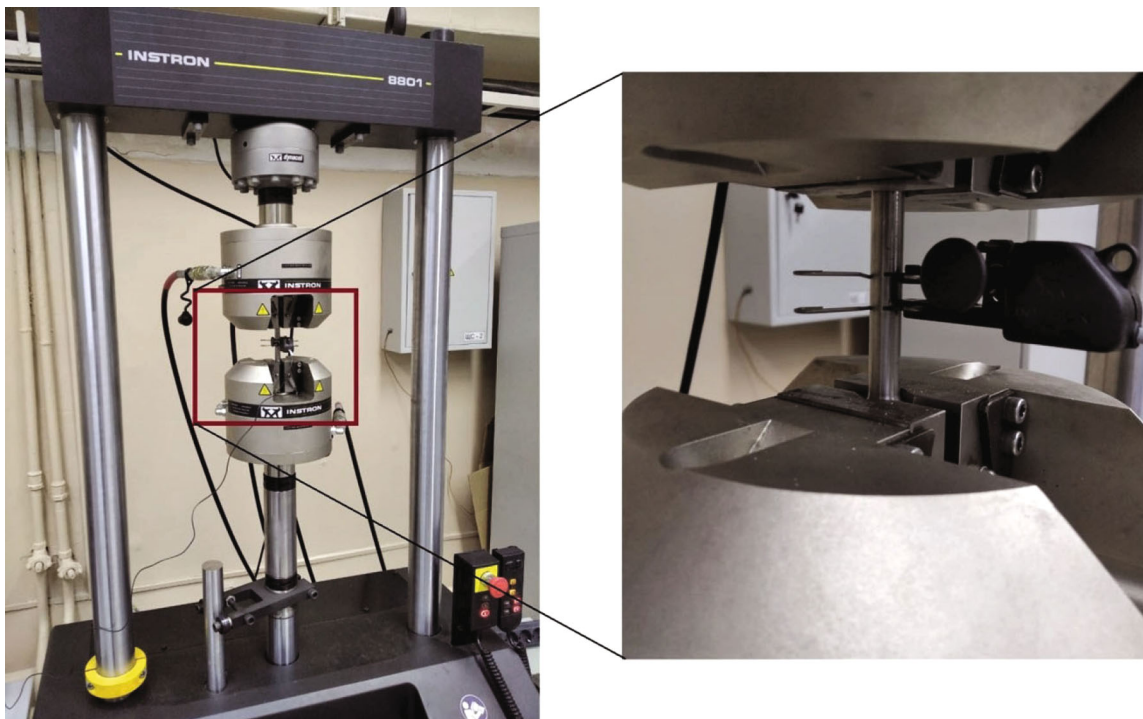


Fig. 1. Instron 8801 testing machine with extensometer for Young's modulus determination.

ing E_m using the resonance method, found a significant increase in E_m in the transition from the micro- to nanoscale range.

In light of the above, the aim of this study is to investigate the influence of specimen size on the modulus of normal elasticity during its determination by the tensile testing method.

EXPERIMENTAL

To establish the influence of the scale factor on the value of the modulus of normal elasticity, a series of tensile tests were conducted on specimens with different sizes. Proportional cylindrical specimens were used, varying in diameter and gauge length.

The material for the specimens was a rod of 20 mm in diameter made of medium carbon structural steel 45. Similar proportional cylindrical specimens with working part diameters d_0 of 19.00, 13.01, 10.79, 4.99, 3.05, 1.85, and 0.99 mm were machined from this rod. In the final stage of preparation of samples, their surfaces were treated electrolytically to remove any surface roughness caused by mechanical processing.

Tensile tests on the specimens were performed using an Instron 8801 machine at a deformation rate of 0.1 mm/min. To determine the modulus of normal elasticity E_m in the elastic region of the stress–strain diagram, an attached Instron GL10 extensometer with a gauge length of 10 mm was used (Fig. 1).

The attached extensometer accurately recorded the change in longitudinal strain of the gauge length of the specimen (ϵ_{ext}) and allowed for the construction of a diagram in the coordinates of stress σ and strain ϵ_{ext} . Each specimen was loaded in the elastic region until a stress of at least 200 MPa was reached, after which the specimen was unloaded to ensure that its deformation was fully elastic. Such loading–unloading cycles were repeated at least five times for each specimen. Then, in the stress range from 0 to 200 MPa, the obtained stress–strain diagram was approximated by a straight line using the least squares method, and the modulus of normal elasticity was determined from the slope of the obtained elastic region relative to the horizontal axis using the equation

$$E_m = \sigma / \epsilon_{\text{ext}}. \quad (2)$$

The extensometer could not be securely attached to the gauge length of the thinnest specimens ($d_0 = 0.99$ mm). Therefore, to determine the modulus of normal elasticity on these specimens, as well as on some specimens with a larger diameter, we used an optical method of deformation measurement—Digital Image Correlation (DIC). It is based on obtaining images of the specimen surface during deformation. Cross-correlation processing of images allows tracking changes occurring on the surface during testing and calculating the deformation field with high spatial resolution. The LaVision StrainMaster system (Fig. 2) was used as the measuring setup.

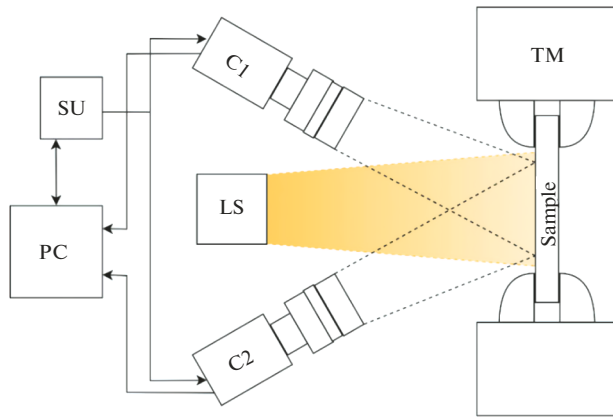


Fig. 2. Static tensile test scheme for a steel specimen using a LaVision StrainMaster optical measuring system: PC—personal computer; TM—testing machine; LS—lighting system; C1 and C2—cameras; SU—synchronization unit.

The LaVision StrainMaster system consists of two Imager SX cameras (C1 and C2), an image capture synchronization device PTU (SU), and a personal computer (PC) with DaVis 8.4 software. The Imager SX digital cameras are based on a monochrome CCD matrix with a size of 2/3" and a resolution of 2456×2058 pixels, pixel size of $3.45 \times 3.45 \mu\text{m}$, and a bit depth of the obtained images of 12 bits. The field of view for registering experimental images for different sizes of specimens varied in the range from 3.5×3.0 to $17.5 \times 14.0 \text{ mm}^2$. An additional external light source (LS) was used to obtain high-quality contrast images.

Before the tensile testing of the specimen, a random speckle pattern was applied to the lateral surface of its gauge length using white and black paints to enhance the image contrast when using the digital image correlation method. Then the specimen was fixed in the grips of the machine, and the lighting and equipment for video recording and deformation measurement were set up. Figure 3 shows a photo of the experiment using the optical method.

During the tension of the specimen, periodic video recording of the surface was performed at a frequency of 2 frames/s with a resolution of 2400×2000 pixels. The sequential automated analysis of the obtained image series using the software allows reconstructing the displacement field of speckle points on the working part of the specimen during tension, the magnitude of which can be used to calculate the strain tensor ϵ_{opt} at any point at any time [8]:

$$\epsilon_{\text{opt},j} = \frac{dV_i}{dr_j}, \quad (3)$$

where $i, j = 1, 2, 3$ are the ordinal numbers of the tensor elements corresponding to the x, y, z axes; r_j are the components of the spatial vector along the axes; and V_i

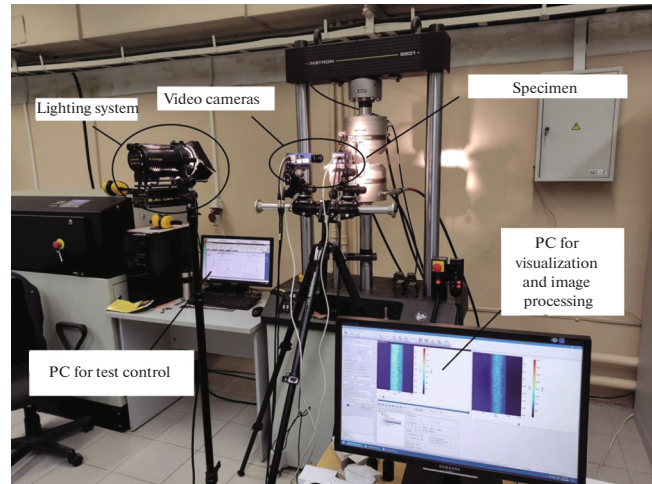


Fig. 3. Photo of the experimental set for Young's modulus determination using the digital image correlation method on an Instron 8801 testing machine.

are the components calculated during the processing of the displacement along the axes.

By tracking the kinetics of changes in the deformation field over time, taking into account machine data on the applied force, it is possible to construct an elastic section of the tension diagram and estimate the value of the modulus of normal elasticity at any time interval at any stage of the test:

$$E_m = \sigma / \epsilon_{\text{opt}}. \quad (4)$$

The arithmetic mean of all E_m values recorded in the initial section of tension to a stress σ of 200 MPa was taken as the value of the normal modulus of elasticity determined by the method of digital image correlation.

RESULTS AND DISCUSSION

Figure 4a shows the elastic sections of the tension diagram σ — ϵ for specimens with diameters d_0 equal to 10.79, 3.05, and 1.85 mm determined using an attached extensometer. It is clearly seen that the straight elastic sections of the tension diagrams for specimens with different initial diameters d_0 are stratified. The smaller the diameter of the specimen, the steeper the slope of the line, and therefore the higher the normal modulus of elasticity. Similar results were obtained using measurements performed with the digital image correlation method (Fig. 4b). Table 1 presents the results of determining the normal modulus of elasticity E_m for specimens tested using both measurement methods.

It can be seen that the values of E_m determined using the extensometer and the digital image correlation method are quite close for the same specimen diameters. A more clear representation of the change

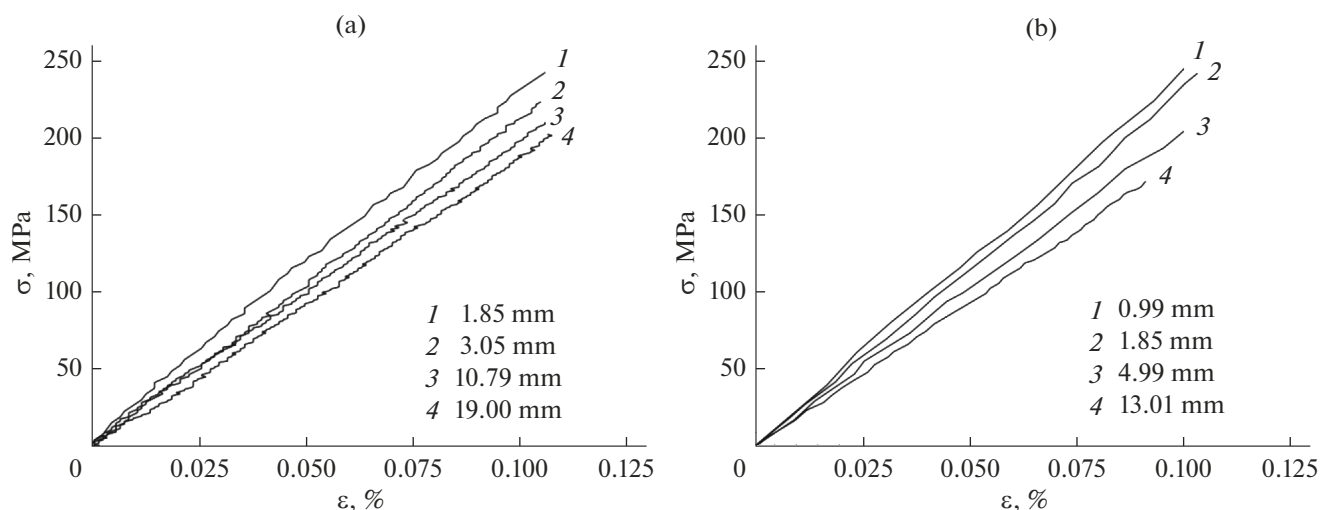


Fig. 4. Elastic sections of tension diagrams σ – ϵ obtained using an attached extensometer (a) and the digital image correlation method (b) for specimens with different diameters d_0 ; material: steel 45.

in E_m with different d_0 can be obtained from Fig. 5. Since d_0 is uniquely related to the cross-sectional area A_0 of the cylindrical specimen ($A_0 = \pi d_0^2/4$), the dependences of E_m on A_0 and E_m on d_0 are similar (see Fig. 5). However, the shape of the cross section of the specimens may vary, so it is advisable to assess the influence of the scale factor by comparing E_m values for specimens with various A_0 . In this case, a more general relationship between E_m and A_0 can be obtained, including for specimens with a different cross-sectional shape, such as rectangular.

Analysis of the results of determining the modulus of normal elasticity E_m (see Table 1) allows the following conclusions to be made. As the initial diameter of the specimen d_0 decreases, E_m increases. However, if the decrease in d_0 from 19 to 5 mm results in a rela-

tively small increase in E_m (approximately 10%), then at smaller d_0 it occurs more intensely. For a specimen with $d_0 = 1.85$ mm, this increase is already about 20%, and for a specimen with $d_0 = 0.99$ mm, it is about 25%. It can be assumed that, at $d_0 \leq 1$ mm, an even more intense increase in E_m will occur.

According to the authors of this work, the main reason for the increase in E_m with a decrease in specimen diameter is the influence of the scale factor. It should be noted that the scale factor has a similar effect on other mechanical properties of materials determined on specimens of different diameters. For example, significant increases in the tensile strength of glass fibers with decreasing diameter were observed in studies by Griffiths [9] and Aleksandrov and Zhurkov [10]. Similarly, an increase in the true tensile strength and true fracture strength of steel specimens with

Table 1. Young's modulus E_m of steel 45 determined by tension tests with an extensometer and the digital image correlation method (average values)

d_0 , mm	A_0 , mm ²	E_m			
		measurements by extensometer		digital image correlation method	
		N/mm ²	kg/mm ²	N/mm ²	kg/mm ²
19.00	283.4	187528	19116	–	–
13.01	132.9	197229	20105	191768	19555
10.79	91.4	200217	20409	–	–
8.00	50.2	209578	21364	–	–
4.99	19.6	212897	21702	206187	21025
3.05	7.3	219556	22381	–	–
1.85	2.7	235463	24001	235803	24045
0.99	0.8	–	–	246320	25118

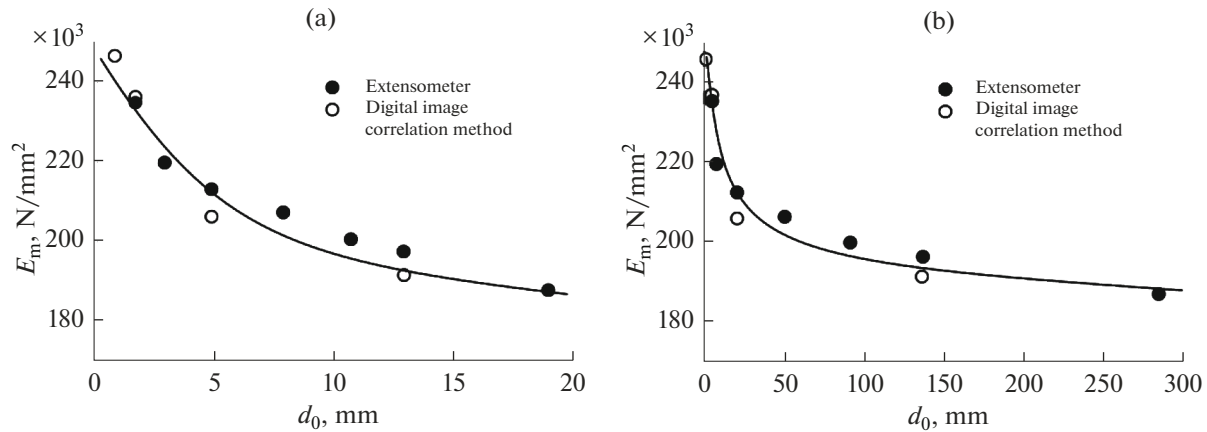


Fig. 5. Young's modulus E_m as a function of initial diameter d_0 (a) and cross section A_0 (b) of specimen; material: steel 45.

decreasing diameter was noted in [11, 12]. According to [11], the true fracture strength of a 30KhN3 steel specimen with a diameter of 1.5 mm increased by approximately 30% compared to this characteristic for a specimen with a diameter of 15 mm. Similar increases in yield strength and ultimate tensile strength have been established upon reduction of the diameter of the specimen from 10 to 1.5 mm [8]. Scale dependences of material hardness are well known [13–16]. When determining hardness by microindentation, its values are significantly increased compared to the macrolevel. And when transitioning to the nanolevel of indentation, hardness can increase by an order of magnitude or more [14]. The strong influence of the scale factor is also manifested in cyclic tests of specimens of different diameters at a cycle amplitude below the elastic limit [17]. Thus, it can be assumed that the scale factor also influences the modulus of normal elasticity.

There are various approaches to explanation of the reasons for the influence of the scale factor on the mechanical properties of materials [10]. Among these approaches, the statistical one (a greater number of defects of various origins in the volume of a large specimen), the structural-mechanical one (the heterogeneity of the structure, properties of materials, stress and strain distributions across the specimen cross-section), and the energetic one (differences in the elastic energy storage) can be mentioned.

In the monograph by Chechulin [12], a large amount of experimental data on the influence of specimen sizes on mechanical properties was systematized for the first time, and theories of the scale effect of mechanical strength for ductile and brittle fracture of materials were considered.

Some of the positions of the aforementioned approaches to explaining the influence of the scale factor can also be applied to the modulus of normal elasticity. However, in small specimens (microspecimens), the number of initial defects and imperfections

in the crystal structure of the metal at the atomic level, including dislocations, is significantly smaller, which leads to an increase in the stiffness of the crystal lattice and, consequently, to an increase in the modulus of normal elasticity.

CONCLUSIONS

The modulus of normal elasticity was determined for specimens of different diameters made of steel 45 using tensile testing with an extensometer and the digital image correlation method for strain measurement. It was shown that the values of the modulus of normal elasticity obtained using the extensometer and the digital image correlation method are fairly close. This allows the use of the digital image correlation method to determine the modulus of normal elasticity on small specimens where it is not possible to reliably attach an extensometer. A decrease in the modulus of normal elasticity with increasing specimen diameter was observed. For example, as the specimen diameter increased from 0.99 to 19 mm, the modulus of normal elasticity decreased by approximately 25%. Graphical dependences of the modulus of normal elasticity on the specimen diameter and cross-sectional area were obtained. The main reason for the increase or decrease in the modulus of normal elasticity determined on specimens of different diameters is the influence of the scale factor. This influence should be taken into account in strength calculations for components and structures of various sizes, especially for large-scale products.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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