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## MECHANICS OF MATERIALS: STRENGTH, RESOURCE, AND SAFETY

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# Methods for the Analysis of Residual Stress Fields in Spatial Details

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**Abstract**—A brief review of the methods for the study of residual process stresses (RSs) in the elements of structures and specimens is given. The possibilities and the fields of the application of nondestructive testing of RSs are indicated. Destructive experimental and calculation methods for the study of two-dimensional and three-dimensional nonuniform fields of RSs in spatial structures, which are based on the interpretation of experimental data as an inverse problem of elasticity theory, are particularly noted. It is recommended to use optical-digital methods for the recording of displacement fields in order to obtain significant collections of experimental information necessary in this case, which are caused by the formation of the cuts of various configurations in the object under study.

**Keywords:** residual stresses, stress–strain state, finite element method, electron digital speckle interferometry, mechanics of deformed solid

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An important part of integrated study that is related to the evaluation of durability and residual life of high-load state-of-the-art structures, which operate under extreme conditions, is the analysis of residual process stresses [1, 3]. Moreover, the solution of this problem is one of the most difficult problems of the mechanics of a deformed solid, which considers the analysis of physicomachanical processes and structural transformations occurring in the material at various mechanical and heat exposures. At present, various methodological approaches providing the solution of typical problems of the study of residual stresses (RSs) in most practical cases are developed. The feasibility of a particular method depends on the type of stress–strain state (SSS), the shape of the object under study, the RS pattern, characteristics of the material, and some other factors.

The methods for the study of RSs can be divided into two main groups, namely, calculated (analytical and numerical) and experimental.

Experimental methods for the investigation of RSs are also divided into two groups, which are nondestructive and destructive.

### NONDESTRUCTIVE METHODS

Nondestructive methods, which are based on the measurement of the changes of various physical parameters under the action of mechanical stresses, are superior to destructive ones, because they do not require the cutting of the object under study. However, these methods either evaluate the degree of RSs in the

thin surface layer of the material (X-ray method, the method of Rayleigh surface acoustic waves, eddy-current resistive electrical contact, magnetic, and some other methods [4–8]), or give some integral estimates of the thickness-weighted average RSs of the detail (ultrasonic method and acoustoelasticity method [4, 9, 10]).

In recent years, the method of neutron diffraction has been extensively developed. In contrast to X-ray diffraction [4, 5], which provides the measurement of RSs only in the thin surface layer of metal (5 to 20  $\mu\text{m}$  in metals), high thermal neutron permeability allows for the measurement of residual stresses of up to several centimeters. The procedure is based on the measurement of the displacement of the shift of Bragg diffraction peaks from the positions corresponding to the lattice parameters of nondeformed material [11], which gives the change in planar distances caused by internal stresses. In recent years, the method has been employed in practice in order to study RSs in 3D structural elements. Because the degree of deformations caused by RSs are usually  $10^{-3}$  to  $10^{-4}$ , a high-resolution neutron diffractometer is used for their measurement. The absolute measurement error of RSs in steel elements of structures is 20–40 MPa.

In Fig. 1, the results of the study of RSs in a specimen cut from the full-scale bimetallic material of a VVER-1000 reactor vessel using neutron diffraction are given [12]. The size of the prismatic specimen is  $45 \times 100 \times 100$  mm and its thickness is 45 mm (including the size of weld deposit of 8 mm). A good accordance of the experimental results with the data of

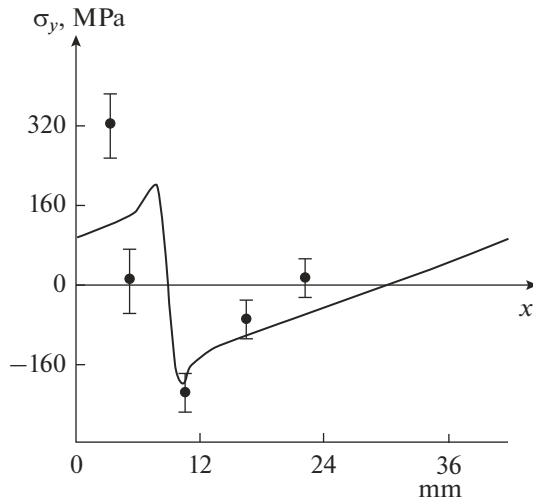


Fig. 1. Residual stress distribution across the thickness of the specimen: (●) neutron diffraction method and (—) FEM calculation.

calculations using the finite element method (FEM) follows from Fig. 1. This example shows that neutron diffraction gives broad opportunities for the experimental evaluation of internal stresses in 3D structural elements.

On the other hand, the neutron diffraction method, which is almost a unique nondestructive testing method for the determination of inhomogeneous RSs which are localized remotely from the surface of details is restricted in practical application. This is caused not only by the uniqueness of the equipment (primarily the source of neutron radiation) and, accordingly, extremely high cost, but also by the possibility of investigations only under “laboratory” conditions and in specimens with quite limited sizes.

As computational engineering developed and modern software packages for the calculation of SSS based on the finite element method (FEM) appeared, numerical methods were successfully used for modeling of complex (in view of RS initiation) fabrication techniques of structural elements. At present, the approaches and corresponding computer programs have been developed which simulate RS initiation during casting, welding, heat treatment, and other fabrication techniques of details [13–16, etc.]. In addition, in order to employ numerical methods successfully, on one hand, it is necessary to develop a mathematical model which provides a reasonable description of mechanical, physicochemical, heat, and other processes in the material, and, on the other hand, it is necessary to know the principles of the change in the parameters which describe the characteristics and the structure of the material during the RS initiation. With consideration of this fact, experimental verification of the exactness of the results is always necessary in addition to the evaluation of the validity of the statement of

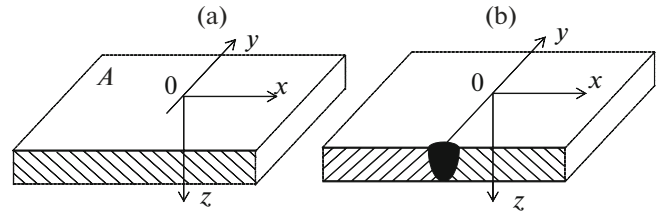


Fig. 2. Residual stress fields in plates: (a)  $s_{xy}(z)$  and (b)  $s_{xy}(xz)$ .

corresponding problems of mechanics and stability of the algorithms for their solution.

## DESTRUCTIVE METHODS

In the practice of the investigation of RSs in the structural elements of modern equipment, so-called destructive methods are most widespread. A procedure for the study of residual stresses using destructive methods consists of following stages: (a) experimental recording of the initial SSS of the detail under study; (b) formation of a cut as an indicator of RSs (or cutting the detail), which leads to the release of RSs on the “released” surfaces; (c) recording of the parameters characterizing the SSS of the object under study after cutting; and (d) processing of the experimental results with the aim to determine RSs.

The characteristics of the RS distribution and the configuration of the object determine the particular features of a destructive method, which are due to the procedure for the calculation of RSs (stage d) on the basis of experimental information of the strain state of the object after cutting (stage c).

## TYPES OF RESIDUAL STRESS FIELDS

Let us introduce Cartesian coordinates  $x$ ,  $y$ , and  $z$ , where the plane  $z = 0$  is the free surface of the detail under study  $A$  ( $A \in z \leq 0$ ) (Fig. 2a). Residual stresses in the general case of their three-dimensional distribution are characterized by the tensor  $s_{ij}(x, y, z)$ , where  $i, j = x, y, z$ . Let us indicate several main types of RS fields.

1.  $s_{ij} = s_{ij}(z)$ ; i.e., RSs do not depend on coordinates  $x$  and  $y$  and vary only across the depth of the detail; in this case, usually  $s_{xx}(z) \neq s_{yy}(z)$ . (It should be noted that this problem belongs to the class of planar problems.) This type is related to the problems of the investigation of RSs in rods and sheets (plates).

2.  $s_{ij}(r, \theta, z) = s_{ij}(r)$ , where  $i, j = r, \theta, z$  are cylindrical coordinates, corresponds to a *one-dimensional axially symmetric RS distribution*. This type includes the problems of the study of RSs in thin-wall rings and tubes, thick-wall tubes, and disks.

3.  $s_{ij}(x, y) = s_{ij}(x, y)$  is the RS distribution on the surface of the details of planar and 3D details, when they do

not change (or change little) across the thickness of the detail, and the changes in RSs along the  $x$  and  $y$  axes can be neglected. This class usually includes the problems of the study of residual process strains at points of the surface of massive details located at a distance from the stress concentration zones.

4.  $s_{ij}(x, y, z) = s_{ij}(x, z)$  is a two-dimensional RS distribution which does not depend on the  $y$  axis. An example of this field is the RS distribution in the butt weld between two plates (Fig. 2b).

5. *Three-dimensional RS distribution.* The solution of the problems of this type in the general case of SSS gives rise to significant and sometimes insurmountable methodological challenges. On the contrary, most of the real practical problems of the analysis of RSs are related to simpler RS distribution, which are usually related to the symmetry sections of the detail, where intrinsic features of the stress state are a priori known.

#### METHODS FOR STUDY OF ONE-DIMENSIONAL RS DISTRIBUTIONS

In order to solve “one-dimensional” problems of the study of RSs corresponding to types 1 and 2, “sequential sewing-up” methods (dissection method) are usually used. Features of their practical application to the analysis of RSs in various structural elements (rods, plates, rings, tubes, and others) are described in the monographs [3, 4, 17]. One important fact should be noted. In order to define RS as the function  $z$  for problem 1 or function  $r$  for problem 2 according to the experimental changes in the deformation response (in terms of deformations  $\varepsilon_{ij}$  or displacements  $u_i$ ) at each step  $\Delta z$  (or  $\Delta r$ ), it is necessary to calculate the derivative  $\Delta\varepsilon_{ij}/\Delta z$  ( $\Delta u_i/\Delta z$ ). Assuming that the changes of deformations (or displacements) are determined with a particular error (primarily the experimental error), one should use the experimental conditions which would provide the measurement of the parameters with the exactness necessary for the subsequent calculation of residual stresses with the required reliability. It should be noted that the increment of the measured parameters depends on three factors, namely, the degree of required residual stresses, the sewing-up step, and the coefficient of the effect characterizing the relationship of RSs to the change in the measured deformations or displacements.

These facts should be considered when using the dissection methods in order to solve practical problems. The error of the results should be evaluated during the study with the assumption of the results. In many cases, special techniques of mathematical processing of the results could be required in order to provide the exactness necessary in practice.

In order to solve the problems of type 3 in planar details on the surface of 3D structural elements, a

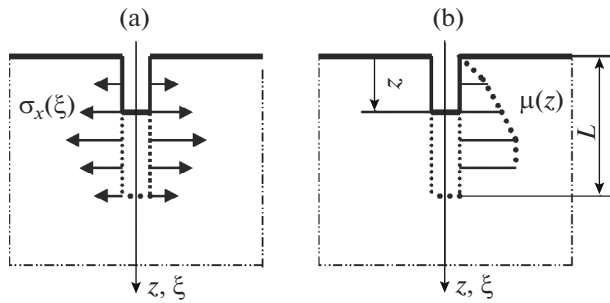
small hole drilling method is widely used [18–22]. It should be noted that the surface of the object must not be planar; it is sufficient that RSs vary insignificantly at  $0 \leq z \leq d$  ( $d$  is the hole diameter). At present, the hole drilling method is usually used when the following limitations on the form of the RS distribution in the zone of the detail under study are fulfilled:  $r \leq 2.5d$  and  $0 \leq z \leq d$  (Fig. 2a) and the residual stress field is homogeneous, that is,  $s_{ij}(x, y, z) = \text{const}$ . It is clear that the method can be used for the study of two-dimensional RS distributions on the surfaces of details  $s_{ij}(x, y, z) = s_{ij}(x, y)$  by the determination of RSs at a series of points of the surface of the detail at distances of larger than  $5d$  from each other.

In order to record the strain response arising in the drilling hole zone as an RS indicator, various methods and instrumentation are used: electrical stress sensors, including those fabricated in the form of special sockets [18]; photoelastic coatings; and holographic [20] and electronic speckle pattern interferometry (ESPI) [21, 22]. The ESPI method, which records a significant array of experimental information in the form of displacement fields directly in digital form, should be considered the most promising. It is extensively employed only for the studies of RSs both under laboratory and full-scale conditions [23]. In the latter case, the correlation of digital images is widely employed in order to obtain information on the strain response [24].

#### METHODS OF STUDY OF TWO-DIMENSIONAL RS FIELDS

The methods for the investigation of two-dimensional RS distributions of  $s_{ij}(x, y, z)$  type were developed relatively recently. It should be noted that these methods of solution are applicable only for the analysis of a biaxial stress field (when the size of the detail on the  $z$  axis is significantly less than its sizes in the other two directions).

When there is a plane strain in the object under study, investigations are also carried out on a planar specimen with small thickness cut perpendicularly to the  $z$  axis (Fig. 2b). In this case, the ratio of RSs on the  $x$  and  $z$  axes are firstly evaluated ( $s_{xx}^{\text{RS}}$  and  $s_{zz}^{\text{RS}}$ ). In order to solve this problem, the hole drilling method can be used as an example. If  $s_{xx}^{\text{RS}}$  and  $s_{zz}^{\text{RS}}$  are the values of the same order, the RSs in a planar specimen will be determined by two components, namely, the required field of residual stresses and the “additional” SSS caused by the release of RSs occurring on the released surfaces of the specimen  $z = \text{const}$ . One of the possible approaches to the solution of the problem based on the calculation of an “additional” RS field using the FEM (with consideration of the results of the studies on a planar specimen) was described in [25].



**Fig. 3.** Diagram of Volterra integral equation method: (a) stresses on the “released profile”  $\sigma_{xx}(\xi) = -s_{xx}(\xi)$  and (b) measured parameters of stress state  $\mu(\xi)$ .

In the general case, the problem of the determination of RSs  $s_{ij}(x, y, z) = s_{ij}(x, z)$  can be stated as follows. The object under study is cut along a particular line, which leads to the additional SSS in it. This (additional) stress, strain, or displacement field is recorded using experimental methods. On the basis of the mathematical processing of the experimental data, RSs are determined using analytical or numerical methods, which were recorded on the profile of a cut. The mentioned stages laid the foundation for all destructive methods of RS testing. A significant difference of the procedures considered below is the assumption of an arbitrary RS distribution. In this statement, which was considered for the first time in [26], the problem of the determination of RS is related to inverse problems whose intrinsic feature is ill-conditioning, which considers that close (within the experimental error) distributions of the parameters recorded using experimental methods can correspond to significantly different RS distributions along the line of section [27, 28]. Corresponding approaches to the investigation of RSs are based on the statement of the analysis of RSs as an inverse problem of elasticity theory and are reduced to the solutions of Volterra or Fredholm integral equations. Practical implementation of these approaches is related to the mathematical processing of a significant volume of experimental information, which requires the employment of optical interference methods as a means to record the strain response caused by the cutting of the detail.

When using the method of Volterra integral equation [26] in a planar detail with arbitrary shape where there is an RS field, the incision is incremented using discrete steps; a particular parameter of stress state is measured at each step at a particular point, which allows controlling the propagation of the incision point (Fig. 3a). The dependence of residual stresses  $s_j(\xi)$  on the experimental change in the SSS parameter

along the line of section  $\mu(z)$  can be represented in the form of an integral Volterra equation of first kind:

$$\int_0^z B(z, \xi) \sigma_{xx}(\xi) d\xi = \mu(z), \quad 0 \leq \xi \leq z \leq L, \quad (1)$$

where  $L$  is the maximum length of the incision. The kernel  $B(z, \xi)$  corresponds to the values of  $\mu(z)$  at point  $z$  from the  $\delta$ -effect at point  $\xi$ , and the range of its definition is a triangle; i.e., at  $\xi > z$ , we have  $B(z, \xi) = 0$ .

During the practical use of the method of the integral Volterra equation, the incision increment step is determined during the preparation of the experiment on the basis of a numerical simulation. The demand in the preliminary processing of initial experimental information and the generation of some continuous and differentiable counterparts according to discrete data within the errors comparable to experimental errors follows from here. In the choice of the initial data processing schemes, Tikhonov regularization methods are superior. However, in spite of the opportunities to solve the problems of the determination of RSs of general form, the difficulty and labor intensity of the described approach restrict its use to solve practical problems.

The method of extension of a crack, which was suggested in [29] to study two-dimensional RS fields in internal regions of thin plates, can be interpreted as a particular case of the approach considered above. Here, a mathematical cut (crack) is considered as an incision, on the banks of which residual stresses are released, while the experimental parameters of the SSS are  $\mu(x)$ , corresponding to the distribution of stress intensity coefficients of normal fracture and transversal shear,  $K_I(x)$  and  $K_{II}(x)$ . In this statement, the main equation is reduced to the integral Volterra equation of first kind with Abel core in the case of some boundary value problems, which gives their closed analytical solutions. As a result, a stable procedure for the calculation of RSs based on experimental data becomes sufficiently simple. At the same time, the experimental investigation becomes more labor consuming, because the values of  $K_I$  and  $K_{II}$  should be determined at each stage in order to obtain the function of the parameter of stress state. In order to determine the SICs in the cracks as RS indicators, it is reasonable to use optical interference methods combined with the modern methods for mathematical processing of experimental information [30]. In [31], the relationships were obtained which extend the method of extended crack to study RSs in the near-surface zones of planar details, including piecewise uniform bands (method of boundary crack). The relationship between residual stresses  $s_{xx}(z)$  and  $s_z(z)$  and the experimental values obtained during a successive increase in the length of the boundary crack (Fig. 4a) can be determined using the dependences of  $K_I$  and  $K_{II}$  on the basis of the analytical solution of the problem of SIC

in the boundary crack in a semi-infinite plate, on the banks of which there are randomly distributed loads  $\sigma_{xx}(z) = -s_{xx}(z)$  and  $\tau_{xy}(x) = -s_{xy}(x)$ :

$$\begin{aligned} \sigma_{xx} = -s_y(z) &= -\frac{d}{dz} \int_0^z \left[ \frac{\tau \psi_1(\xi)}{\sqrt{(z^2 - \xi^2)}} \right] d\tau \\ &\quad - \int_0^a [\psi_1(u) m(z, u)] du, \\ m(z, u) &= \int_0^\infty \alpha u (1 - az) e^{-\alpha x} n(\alpha u) d\alpha, \\ n(\alpha u) &= \frac{2}{\pi} \int_0^{\pi/2} (1 - \alpha u \sin \varphi) e^{-\alpha u \sin \varphi} d\varphi. \end{aligned} \quad (2)$$

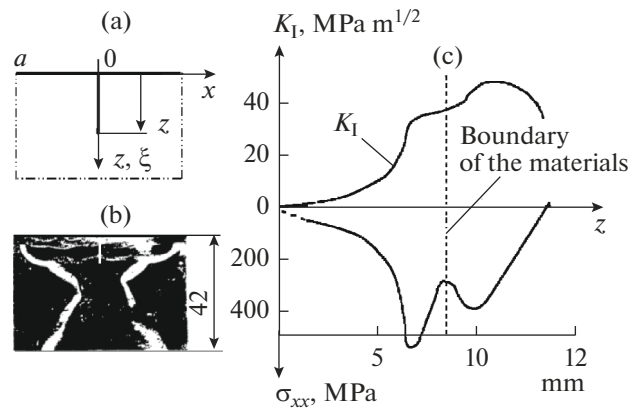
These equations are the main calculated relationships of the method of extended crack. The validity of the statement of the problem of the calculation of residual stresses based on experimental data follows from them, which in turn provides a stable calculation of residual stresses with respect to the error of experimental results.

The method was effectively employed for the study of RSs in modern equipment designs [25, 27, 32]. An example of its use for the study of RSs in the butt weld zone in a full-scale specimen of bimetallic material of the AST reactor vessel is given in Figs. 4b and 4c. Cladding (from 08Kh18N10T material) was deposited onto the main material (10KhN1M) by welding using explosion according to in-house technology. The technology of fabrication of large elements from this material considers the butt welding of bimetallic elements. RSs in the welding zone (butt section) were investigated.

Because the crack as an RS indicator is a very high stress “concentrator,” the method intrinsically has a higher sensitivity as compared to the hole drilling method (without mentioning other approaches, which consider cutting of the detail). The crack extension steps can be very small (less than 1.0 mm), which provides the determination of residual stresses at their high gradients. Thus, various modifications of the method of sequentially extended crack along with optical interference digital methods of experimental recording of strain responses and modern programs to solve inverse problems should be considered the most promising approach to the solution of two-dimensional problems of the analysis of inhomogeneous RS fields [27, 28, 33].

#### METHODS OF STUDY OF RS FIELDS IN 3D DETAILS

Let us consider the methods for the study of inhomogeneous RS fields in spatial objects, which represent a definite development of hole drilling and successively extended crack methods. They are based on a



**Fig. 4.** (a) Diagram of the method of study of RSs in the butt weld zone in full-scale specimen of bimetallic material of the AST reactor vessel, (b) image of the specimen surface in the butt weld zone, and (c) the dependence of  $K_I(z)$  and the RS diagram  $s_{xx}(z)$ .

methodological approach to the determination of SSS parameters (residual stresses) based on the evaluation of the correspondence of a large array of experimental data and the results of numerical solution of a series of model problems of FEM for the current values of the mentioned parameters (required RSs) [34, 35]. Considering the possibilities of the solution of boundary value problems of the mechanics of deformed solid based on FEM, these approaches do not have fundamental limitations on the geometry and mechanical characteristics of the detail under study in contrast to conventional methods or on the distribution of  $s_{ij}(x, y, z)$ . The required values of residual stresses  $P_q$  at points  $k$  are determined from the solution of the problem of minimization of the target function, which shows in a complex manner the difference between the experimental data  $\bar{e}_k$  and their calculated values  $e_k$ . In this case,  $e_k$  are determined by the solution of a series of direct problems at current values of required parameters  $P_q$  using the ANSYS program, while the procedure of the search for the minimum of target function is based on the method of deformed simplex elements (Nelder–Mead method), which is implemented in the MATLAB program. As a target function  $I$ , root-mean-square and maximum deviations of the calculated displacements  $e_k$  from the experimental data  $\bar{e}_k$ , as well as a function of a special form, are considered [35]. One of the most essential elements of the method is the development of the so-called response database, which represents the set of the data reflecting the relationship between  $e_k$  and  $P_q$ , which is determined from the results of preliminarily and multiply solved boundary problem at various values of parameters  $P_q$ . This approach provides a relatively rapid convergence of the procedure for the search for the parameters of state based on the minimization of the target function  $I$ .

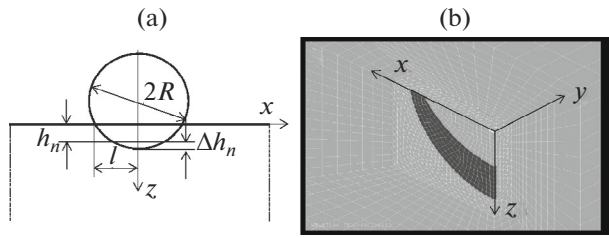


Fig. 5. (a) Diagram of the method of successively extended disk crack as an indicator of residual stresses and (b) an example of finite element splitting of the cut zone.

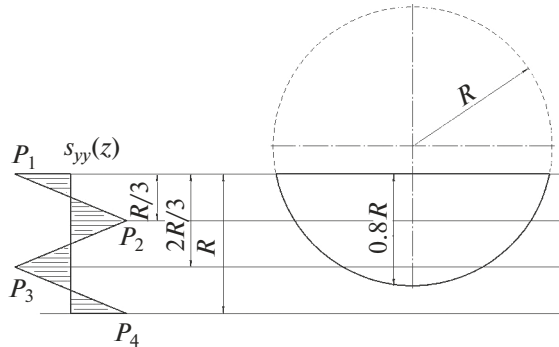


Fig. 6. Diagram of the RS distribution in the model problem.

#### Successively extended round hole as an RS indicator.

A promising direction [34, 35] is the development of the hole drilling method to the analysis of the RS fields that are inhomogeneous across the thickness of the detail. Opportunities of the hole drilling method in its conventional variant were considered in the review [36], where the statement on the prospect of the method of integral Eq. (1) for the interpretation of the data obtained during the drilling of a successively extended round hole is made from the analysis of numerous papers. Moreover, this method has significant limitations related to the fact that the function which relates RSs and the parameters of strain response recorded on the surface of the detail is rapidly damping. Therefore, this approach can be used only for the evaluation of the RS distribution in a quite low depth of  $L < 2\rho$  ( $\rho$  is the radius of hole as an RS indicator). In many papers (see, for example, [28]), it was shown that, at  $L/\rho > 2$ , further extension of the hole does not cause any change in the strain field on the surface of the detail. The possibilities of the hole drilling method for the analysis of inhomogeneous RS fields across the depth of the detail are given in [37], where its radius  $\rho$  increases with high exactness at each new hole extension stage  $\Delta L$  with the aim to obtain the strain response on the surface. It was shown from the results of numerical experiments that this approach along with the methodology and the program indicated in [34, 35] provides a high-gradient RS distribu-

tion (until that appearing at the boundary of different materials) across the depth of the spatial object. In this case, the relative RS error at a sufficient volume of experimental information (which is obtained using optical interference methods) lies within the range of the dispersion of experimental data (usually about 10%).

#### Successively extended disk cut as an RS indicator.

The procedure represents the development of the method of successively extended cut relative to the studies of spatial objects whose geometry has two symmetry planes, namely,  $xz$  and  $yz$ . A practically important problem is considered where residual stresses  $s_{ij}(i, j = x, y, z)$  randomly depend on the  $z$  axis and do not change along the  $x$  and  $y$  axes. In this case, the RS distributions along the  $x$  and  $y$  axes can differ from each other. In the object under study, a disk cut-crack is formed in  $xz$  plane, whose width is  $t \rightarrow 0$ , while the depth  $h$  sequentially increases, which leads to the release of residual stresses  $s_{yy}(z)$  on the cut surfaces ( $s_{xy} = 0$ ) (Fig. 5).

At each stage of the extension of the cut  $\Delta h_n$ , the strain response is recorded, namely, the displacement fields of the surface of the half-space caused by the release of residual RSs. Derived from this information, the required RS field is calculated. In order to record displacement fields during full-scale experiments, optical interference methods (ESPI methods, digital holography, or correlation of digital images [30]) are suggested.

The required RS distribution is approximated by a piecewise-linear function; that is, residual stresses at the  $n$ th increment of the depth of the section  $\Delta h_n$  at the depth of  $h_n$  are determined using the following relation:

$$s_{yy}(z) = s_{yy}(h_{n-1}) + b_n z. \quad (1)$$

In [38], on the basis of numerical simulation, examples of the solution of typical model problems of the study of RSs are given, which make it possible to evaluate the effect of the errors of experiments as well as the volume and the choice of the localization area of experimental data on the exactness of the results.

A complex piecewise-linear residual stress distribution  $s_y(z)$ ,  $0 \leq z \leq 0.8R$  (Fig. 6) was considered as a model, which can be described using four parameters  $P_j, j = 1, 2, 3, 4$ . The shape of the cut corresponded to a thin disk; in this case, the width of the cut to its radius ratio was  $t/R = 0.05$ .

The initial information used for the evaluation of the exactness of the procedure corresponded to the values of two components of the displacement vector of the surface of the detail ( $z = 0$ ): on the  $z$  axis,  $w(x, y)$ , and on the  $y$  axis,  $v(x, y)$ . The points used for the calculation of mentioned displacements of the required parameters were uniformly arranged near the cut on the lines of equivalent displacements and the number of points was  $\sim 60$ . During the calculation of model

problems, it was considered that the arbitrary relative error of the measured values  $\delta_e = 10\%$ .

Because it is impossible to have reliable information on the position of the bend points of the RS diagram in practice, the condition was fulfilled in all calculations of model problems that the depth of the disk cut does not coincide with the depth of the bend point of the distribution diagram  $s_{yy}(z)$ .

The results of numerical experiments show that the mentioned approach and the corresponding program allow the investigation of complex nonuniform distributions across the depth of the detail with the error at least lower than the relative measurement error of the changes in strain responses.

After the determination of RS distribution  $s_{yy}(z)$ ,  $s_{xx}(z)$  is determined analogously. It should be noted that this study can be carried out on the same detail, where there is a disk cut in the  $zx$  plane performed during the investigation of stresses  $s_{yy}(z)$ .

On the basis of the analysis of the publications concerning the methods of study of residual stresses, the following statements can be made.

1. The most promising approach to the solution of the problems of the analysis of residual stresses can be considered the methods and corresponding programs for numerical simulations of complex nonlinear physico-mechanical formation of RS fields arising during the fabrication of the structural element, as well as their change under the action of force and temperature loads which occur during the operation of the construction (as well as other types of exposures).

On the other hand, in order to evaluate the validity of the statement of corresponding problems of mechanics and stability of the algorithms for their solution, as well as the effect of possible errors of physico-mechanical characteristics of the materials used in calculations, experimental verification of the exactness of the results is always required.

2. Among nondestructive testing methods of RS, neutron diffraction should be emphasized, which allows the study of inhomogeneous stress distributions across the depth of the detail. However, the field of its practical application is quite limited owing to the uniqueness of necessary equipment and extremely high costs.

Other nondestructive methods are extensively employed in practice year after year; however, their use is limited by the study of only surface RSs, as well as the evaluation of the integral estimates of mean stresses over the thickness.

3. Destructive methods of study of RSs provide the solution of a broad range of problems for constructions differing in dimensional characteristics and the form of RS distribution. In addition to the known procedures of the study of RSs in typical structural elements (in rods, plates, tubes, and disks and on the surfaces of spatial elements), methodological approaches provid-

ing the opportunity to solve three-dimensional RS distributions in details of random configuration were developed in recent years. These approaches are based on the solution of the problem of analysis of RSs as an inverse problem of mechanics of deformed solid. In this case, the most promising method is the determination of the required parameters of the problem (RS values) on the basis of the analysis of the correspondence of a large collection of experimental data (which characterize strain responses during the cut of the detail) and the results of numerical solution of a series of model problems of FEM for the current values of the mentioned parameters (that is, the required RSs).

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