ISSN 0020-1685, Inorganic Materials, 2017, Vol. 53, No. 15, pp. 1578–1584. © Pleiades Publishing, Ltd., 2017.
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MECHANICS OF MATERIALS: STRENGTH, LIFETIME, SAFETY

Evaluation of the Size of the Inelastic Deformation Zone at a Crack Tip Based on the Analysis of Displacement Fields

A. S. Chernyatin[a, b, *], I. A. Razumovskii[b], and Yu. G. Matvienko[b]
[a]*Bauman Moscow State Technical University, Moscow, Russia*
[b]*Blagonravov Mechanical Engineering Research Institute, Russian Academy of Sciences, Moscow, Russia*
**e-mail: cas@inbox.ru*
Received August 10, 2016

Abstract—The damaged zone (the inelastic deformation zone) near a crack tip is the region where the stress–strain state (SSS) cannot be described by fundamental functions in the solution of the elastic problem of a crack (the Williams solution). For the description of SSS outside the damaged zone, the Williams expansion is used, which requires a number of regular terms to be considered. It is proposed to use digital optical techniques for measuring the SSS parameters in the crack zone to provide a large amount of experimental information in the form of displacement fields on the surface of the studied object directly in digital form.

Keywords: crack, fracture mechanics, evaluation techniques for singular and nonsingular components of displacement fields, modeling errors, plasticity zone

DOI: 10.1134/S0020168517150055

Plastic zones that occur in the crack tip region reflect the distinctive features of the material behavior under specific loading conditions of the considered element of a structure. The information on the sizes of these zones can be used for the diagnostics of the damage accumulation and failure process [1–5].

The fundamentals of the calculation analysis of the nonlinear behavior of material in the crack tip zone are rather well explored at present. The calculation and analytical methods for solving the corresponding problems of deformable solid mechanics are described in [5, 6]. The finite element analysis of the elastoplastic SSS in crack zones is discussed in [3, 4, 7–11, etc.].

Another important component of the current approaches to the strength, fracture toughness, and lifetime evaluation of structures is experimental analysis of the material behavior in crack defect regions (plasticity, prefracture, and damage accumulation zones). To solve this problem, diverse techniques are developed and successfully used: optical [12] and X-ray [10] fractography, speckle interferometry [13], and electromagnetic acoustics [12, 14].

For the analysis of the deformed state in crack zones in actual structures, the most promising are digital optical techniques: digital image correlation (DIC) [15–17] and electronic digital speckle interferometry (EDSI) [18, 19]. These techniques, which are widely used in both laboratory and actual studies, provide significant (almost unlimited) amounts of experimental information in the form of displacement fields on the surface of the studied object directly in digital form. In addition, the advantages of these methods are their high sensitivity and contactlessness.

The aim of this study is to develop a methodological approach and a corresponding program for the evaluation of the size of the plastic deformation region (and other types of damage) in crack defect zones on the basis of the mathematical processing of the results of the experimental registration of displacement fields. Here, a "damage zone" is understood as a region where the stress–strain state (SSS) is different from the elastic one.

The technique is based on the fact that the SSS in the local crack tip zone cannot be described by relations corresponding to the elastic SSS in the crack region [1–4]. The necessary condition for the practical application of the proposed technique is its use for the subsequent mathematical processing of large experimental information arrays in the form of tangential displacement fields in the considered zone.

The proposed approach is the development of a technique for the determination of the stress intensity coefficients (K_I, K_{II}) under combined loading on the basis of processing the interference patterns, which are the maximum tangential stress τ_{max} fields [20]. The problem solution is reduced to finding the coefficients of the Williams functions describing the stress fields in the crack zone from the condition of the minimum standard deviation (maximum tangential stress) τ_{max} in a set of points located in the vicinity of a crack tip (Fig. 1a).

Unlike the approach in [20], the proposed technique is based on processing tangential displacement fields u and v (see Fig. 1a), whose asymptotic behavior (unlike that of deformations and stresses) is nonsingular. Apparently, this circumstance has a significant negative effect on the accuracy of the determination of the desired parameters. However, note that both approaches are in fact reduced to the construction of the analytical solution of the elasticity theory problem in the form of an expansion in terms of fundamental functions (Williams functions) corresponding to the initial experimental information on SSS in the crack zone.

Because the geometry of the small vicinity of a real crack tip (both in an actual object and in a sample) cannot possibly be considered as a "mathematical cut," a zone of the order $r \le (3-4)h$ (h is the crack width in the vicinity of a crack tip) is excluded from the localization region of the initial experimental data (Fig. 1b). Another reason for excluding from consideration the mentioned zone r is the three-dimensional stress state that occurs in the place of emergence of a crack on the free surface in the vicinity of the point O (see Fig. 1) and the SSS asymptotics, which is determined by equations that are different from the Williams relations [21].

The proposed procedure for the evaluation of the damaged material zone size, which is based on processing the experimentally constructed displacement fields in "undamaged" zones and in the vicinity of a crack tip, is substantiated by the following circumstance. Apparently, in the case where the SSS in the crack tip zone, which is used for the determination of the Williams function coefficients, is close to the elastic one, the choice of the localization zone of experimentally registered displacement fields $\rho_{\min} \le \rho \le \rho_{\max}$, which are used for the problem solution, does not noticeably affect the results. On the other hand, if significant plastic deformations, damaged metal, etc., occur in the vicinity of a crack tip, the procedure becomes unstable. It can be expected that, starting from some value $\rho > \rho^*$ and under its further increase, the values of the desired parameters do not change. Then, the value $\rho = \rho^*$ can be considered approximately equal to the radius of the zone where there are significant problem modeling errors, which is caused by the presence of plastic deformations or other types of damaged material.

TECHNIQUE AND PROGRAM OF THE DETERMINATION OF THE WILLIAMS EXPANSION COEFFICIENTS BASED ON PROCESSING THE TANGENTIAL DISPLACEMENT FIELDS

The displacement fields u, v in the vicinity of crack tips of types I and II can be presented in the form of the known Williams expansion:

$$
\begin{aligned}
u^{\mathrm{I}}(r,\theta) &= \sum_{n=1}^{\infty} \frac{r^{n/2}}{2G} a_n \left\{ \left[\kappa + \frac{n}{2} + (-1)^n \right] \cos\frac{n\theta}{2} - \frac{n}{2}\cos\frac{(n-4)\theta}{2} \right\},\\
v^{\mathrm{I}}(r,\theta) &= \sum_{n=1}^{\infty} \frac{r^{n/2}}{2G} a_n \left\{ \left[\kappa - \frac{n}{2} - (-1)^n \right] \sin\frac{n\theta}{2} + \frac{n}{2}\sin\frac{(n-4)\theta}{2} \right\},\\
u^{\mathrm{II}}(r,\theta) &= -\sum_{n=1}^{\infty} \frac{r^{n/2}}{2G} b_n \left\{ \left[\kappa + \frac{n}{2} - (-1)^n \right] \sin\frac{n\theta}{2} - \frac{n}{2}\cos\frac{(n-4)\theta}{2} \right\},\\
v^{\mathrm{II}}(r,\theta) &= \sum_{n=1}^{\infty} \frac{r^{n/2}}{2G} b_n \left\{ \left[\kappa - \frac{n}{2} + (-1)^n \right] \cos\frac{n\theta}{2} + \frac{n}{2}\cos\frac{(n-4)\theta}{2} \right\},
\end{aligned}
\tag{1}
$$

where r, θ are the polar coordinates associated with a crack tip (see Fig. 1); κ is the parameter of the stress state type ($\kappa = \frac{3-\nu}{1+\nu}$ is for the flat stress state, and $\kappa = 3 - 4\nu$) is for the flat deformed state); and G and ν are the shear modulus and the Poisson ratio of the material. Note that $a_1 = K_{\mathrm{I}}/\sqrt{2\pi}$, $b_{\mathrm{I}} = K_{\mathrm{II}}/\sqrt{2\pi}$, $a_2 = T/4$, and T are the nonsingular T stresses acting in the xOy plane [22].

The coefficients a_n and b_n from relations (1) can be calculated using the following approach. On the basis of the experiments (or the calculations of the corresponding boundary value problem), the tangential displacements (the displacements that occur on the body's surface) at M points are determined:

$$
u^* = \{u_1^*, u_2^*, \ldots, u_m^*, \ldots, u_M^*\}^T,
$$
$$
v^* = \{v_1^*, v_2^*, \ldots, v_m^*, \ldots, v_M^*\}^T.
$$

The positions of the measurement points are characterized by the polar coordinates $r_m = r_m^*$, $\theta_m = \theta_m^*$, i.e.,

$$
u^* = u^*(r^*, \theta^*), \quad v^* = v^*(r^*, \theta^*).
$$

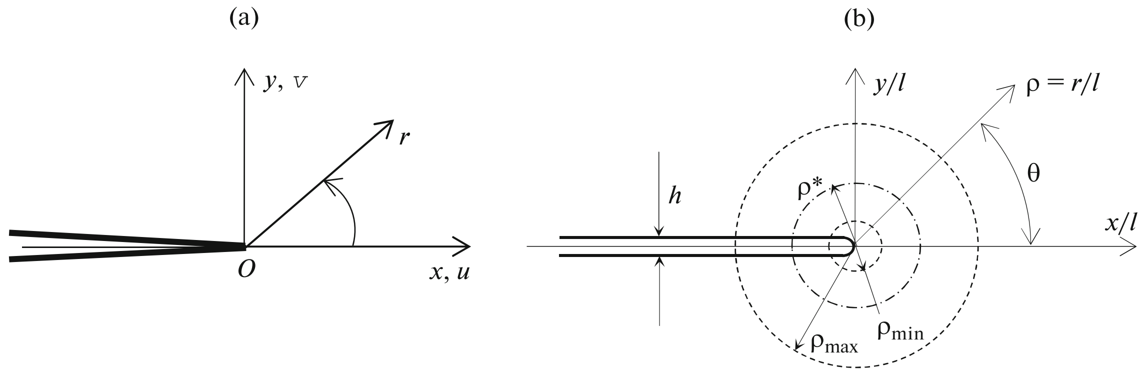

Fig. 1. Vicinity of a crack tip (a) and the localization region of points used for the construction of the analytical representation of displacement fields in the crack zone (b).

On the other hand, the above displacement values can be matched to the displacements calculated for the same points on the basis of the analytical relations (1):

$$u = u(r^*, \theta^*), \quad v = v(r^*, \theta^*).$$

We rewrite expression (1) in a form that is more convenient for the subsequent calculations:

$$\begin{aligned} u^{\mathrm{I}}(r,\theta) &= \sum_{n=1}^{N} f_n^u(r,\theta) a_n, \\ v^{\mathrm{I}} = (r,\theta) &= \sum_{n=1}^{N} f_n^v(r,\theta) a_n, \\ u^{\mathrm{II}}(r,\theta) &= \sum_{n=1}^{N} g_n^u(r,\theta) b_n, \\ v^{\mathrm{II}} = (r,\theta) &= \sum_{n=1}^{N} g_n^v(r,\theta) b_n. \end{aligned} \tag{2}$$

The values of the coefficients a_n and b_n should be determined from the condition of the best correspondence of the displacement fields u_m, v_m ($m = 1, \ldots, M$) in the region $\rho_{\min} \le \rho \le \rho_{\max}$, $-\theta_1 \le \theta_2$ described by expressions (1) to the array of the experimentally obtained displacements u_m^*, v_m^*. This problem can be solved on the basis of fulfilling the condition of the minimization of the total deficiency Δ between the displacements u_i^*, u_i and v_i^*, v_i. The standard deviation can be accepted as the measure of divergence of the displacements

$$\Delta = \sqrt{\frac{1}{2M}\left[\sum_{m=1}^{M}(u_i^* - u_i)^2 + \sum_{m=1}^{M}(v_i^* - v_i)\right]}. \tag{3}$$

It is known that the solution of the above minimization problem can be obtained from the solution of the matrix equation

$$\{\mathbf{U}^*\} = [\mathbf{F}]\{\mathbf{A}\}, \tag{4}$$

where $\mathbf{U}^*$ is the vector of the true displacement values u^* and v^*; $\mathbf{A}$ is the vector N of the unknown coefficients a_n and b_n in the Williams expansion; and $\mathbf{F}$ is the matrix of values of functions (2) at the points r_i^*, θ_i^* for the corresponding expansion coefficients (of dimension $2M \times 2N$):

$$\mathbf{U}^* = \begin{Bmatrix} u_1^* \\ \ldots \\ u_M^* \\ v_1^* \\ \ldots \\ v_M^* \end{Bmatrix}, \quad \mathbf{F} = \begin{bmatrix} f_1^u(r_1^*, \theta_1^*) & \ldots & f_N^u(r_1^*, \theta_1^*) & g_1^u(r_1^*, \theta_1^*) & \ldots & g_N^u(r_1^*, \theta_1^*) \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ f_1^u(r_M^*, \theta_M^*) & \ldots & f_N^u(r_M^*, \theta_M^*) & g_1^u(r_M^*, \theta_M^*) & \ldots & g_N^u(r_M^*, \theta_M^*) \\ f_1^v(r_1^*, \theta_1^*) & \ldots & f_N^v(r_1^*, \theta_1^*) & g_1^v(r_1^*, \theta_1^*) & \ldots & g_N^v(r_1^*, \theta_1^*) \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ f_1^v(r_M^*, \theta_M^*) & \ldots & f_N^v(r_M^*, \theta_M^*) & g_1^v(r_M^*, \theta_M^*) & \ldots & g_N^v(r_M^*, \theta_M^*) \end{bmatrix}, \quad \mathbf{A} = \begin{Bmatrix} a_1 \\ \ldots \\ a_N \\ b_1 \\ \ldots \\ b_N \end{Bmatrix}.$$

Note that relations (1) do not include the linear and the angular displacements in the xOy plane of a body with a crack as a rigid whole. Thus, the following components are added to the matrix $\mathbf{F}$ and the vector $\mathbf{A}$:

$$F = \begin{bmatrix} \{1\} & \left[f_n^u(r_m^*, \theta_m^*)\right] & \{1\} & \left[g_n^u(r_m^*, \theta_m^*)\right] & \{1 - \alpha r_m^* \sin\theta_m^*\} \\ \{1\} & \left[f_n^v(r_m^*, \theta_m^*)\right] & \{1\} & \left[g_n^v(r_m^*, \theta_m^*)\right] & \{\alpha r_m^* \cos\theta_m^*\} \end{bmatrix}, \quad A = \begin{Bmatrix} a_0 \\ \{a_n\} \\ b_0 \\ \{b_0\} \\ \alpha \end{Bmatrix}, \tag{5}$$

where a_0 and b_0 are the displacements along the axes x and y, respectively, and α is a small rotation angle of a body as a rigid whole.

We consider the procedure of choosing the optimal number of N terms considered in the expansion. For the current (chosen) number of terms N, equation (4) is solved, where **A** and **F** are used in the form (5). As a result, the corresponding expansion coefficients are determined

$$\mathbf{A}^N = \{a_0\{a_n\}b_0\{b_n\}\alpha\} \ (j = 1, \dots, N).$$

Note that the matrix $\mathbf{F} = \mathbf{F}^N$ has the dimension $2M \times (2N + 3)$. Further, on the basis of relations (1), the vector of the displacement values $\mathbf{U}^N$ is calculated and the deficiency Δ^N is found according to formula (4), both values corresponding to the current number of terms in the expansion N. The above calculation procedure is repeated for sequentially increased N, starting from $N = 1$ (under the increase in N by one, the number of columns in the matrix **F** and the number of elements of the vector **A** increase by two). For each N, the deficiency Δ^N is determined. The condition for termination of the procedure and the choice of the optimal value of N can be fulfilling one or two of the following conditions simultaneously: (1) $\Delta^N \le \varepsilon_\Delta$; (2) $\left|\Delta^{N-1}\right| / \Delta^N \le \varepsilon_\%$, where the parameters ε_Δ and $\varepsilon_\%$ are chosen on the basis of the accuracy of the calculation for the determination of some assigned parameters or on the basis of the allowable deviations between the displacement fields u_m, v_m and u_m^*, v_m^*.

The presented technique for determining the coefficients of the terms in the Williams expansion and, thus, finding an analytical representation for displacement fields in the crack zone was implemented in the MatLab environment in the form of a program with a graphical interface (Fig. 2).

The developed program performs the following operations.

1. Loading the resulting experimental data (or the calculation results based on a numerical experiment), which are the u^*, v^*, values and different components of the stress tensor; visualizing displacement fields and stresses using interpolation of the data.

2. Forming the r^*, θ^* coordinates uniformly distributed within the region of measurement points (for the numerical determination of the displacement value in them using ANSYS).

3. Performing the retrieval of the arrays r^*, θ^* and u^*, v^* to be used for the determination of the coefficient of the Williams expansion from the available arrays of measurement points (this function provides the possibility to evaluate the influence of the localization of measurement points on the accuracy and convergence of the solution process).

4. Determining the expansion coefficients a_n, b_n and a_0, b_0, α in accordance with the chosen criterion of completion of the calculation.

5. Visualizing the calculated displacement fields on the basis of the found coefficients (the graphical comparison of these fields to the initial one helps to check the correctness of the obtained solution in general and in particular, i.e., in specific regions near a crack tip).

DETERMINATION OF THE PARAMETERS OF FRACTURE MECHANICS AND THE SIZE OF THE ZONE OF DAMAGED MATERIAL

As mentioned above, in the case where SSS occurs in the vicinity of a crack tip and, as a result of plastic deformations, damaged metal, and other indicators, it significantly differs from an elastic one, the procedure of determination of the parameters of fracture mechanics is unstable. On the other hand, it can be expected that, by excluding the "damaged zone" from the initial data region for the calculation of the parameters of fracture mechanics, a stable problem solution can be obtained, which helps to evaluate the size of the damaged zone. To determine the possibilities of this approach, the boundary value problem of the SSS of a plate ($b = 20l$) with a tensile loaded central through

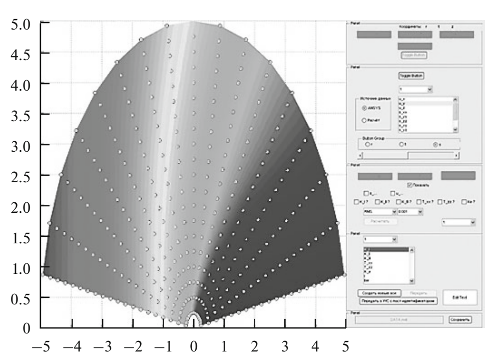

Fig. 2. Interface of the program for the calculation of the Williams expansion coefficients for the SSS visualization in the crack zone.

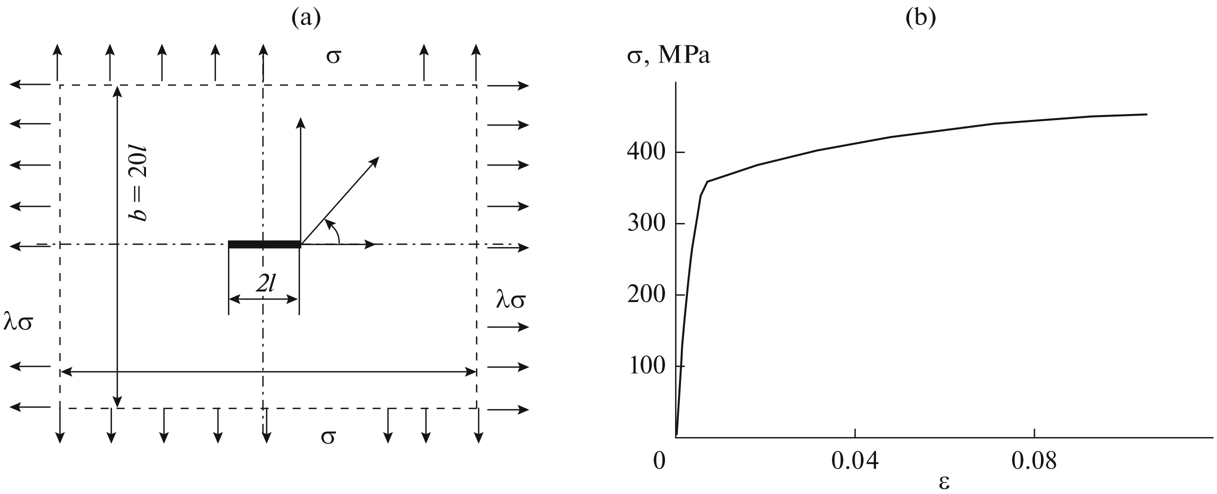

Fig. 3. Scheme of loading a plate with a crack (a) and deformation diagram of plate material (b).

crack with the length of $2l$ was considered (Fig. 3). The problem was solved for the case of a flat stress state.

At the first phase, the influence of the localization zone of the initial data on the accuracy of determination of K_I and the T stresses in the absence of plastic deformations was evaluated. Some results of these calculations are presented in Table 1. They show that excluding zones of large size from the initial data ($\rho_{max} = 0.3$ and more) hardly affects the results of the evaluation of the desired parameters of fracture mechanics. In addition, for the analytical representation of displacement fields, multiple Williams expansion terms ($N > 15$) can be used. Thus, relations (1) can be used for the analytical representation of large-sized regions ($r > l$).

Table 1. Values of K_I and the T stresses depending on the localization zones of initial data and the number of expansion terms N

ρ_{min}	ρ_{max}	N	$K_I/\sigma\sqrt{\pi l}$	T/σ
0.1	0.6	2	0.992	–0.58
		3	0.997	–0.97
		5	1.014	–1.04
		10	1.014	–1.04
		15	1.014	–1.04
0.2	0.6	2	1.014	–0.59
		3	0.978	–0.96
		5	1.014	–1.04
		10	1.014	–1.04
		15	1.014	–1.04
0.3	0.6	2	1.014	–0.60
		3	0.961	–0.96
		5	1.015	–1.04
		10	1.014	–1.04

For the evaluation of the influence of the random error of the initial data, the calculations of the same problem modeling the error of the experimental results were performed. After the SSS calculations, the error was added to the found "exact" displacement values u, v using a random number generator (with preset variance of the relative errors δu, δv). The calculations showed that, even if the variance of the relative errors is δu_{max}, $\delta v_{max} \le 15\%$, they do not significantly affect the accuracy of the results (the relative error of the K_I determination is at least two times less than δu_{max}, δv_{max}).

The considered elastoplastic problem of the calculation of the SSS in the crack zone was a boundary value problem (see Fig. 3a) for a plate made of D16T material, whose deformation diagram is shown in Fig. 3b. The SSS was calculated using ANSYS software. The resulting distributions of displacements and equivalent (von Mises) stresses that occur at $\sigma = 2/5\sigma_t = 150$ MPa, which correspond to the elastoplastic SSS, are shown in Fig. 4. Note that, in the image of the equivalent stress field, the plasticity region, whose size does not exceed 0.15–0.2l, is marked in gray.

The calculation results for the parameters of fracture mechanics of the K_I and the T stresses obtained using the developed program for displacement fields u and v using different localization regions of the initial data are shown in Fig. 5. (Note that it follows from the solution of the elastic problem at the accepted load values that the nominal "elastic" values are K_I = 42.6 MPa m$^{1/2}$ and $T = -77.9$ MPa.) For the calculations, the "measurement" points were located uniformly in a circular sector $\rho_{min} \le \rho \le \rho_{max}$, $0 \le \theta \le \pi$.

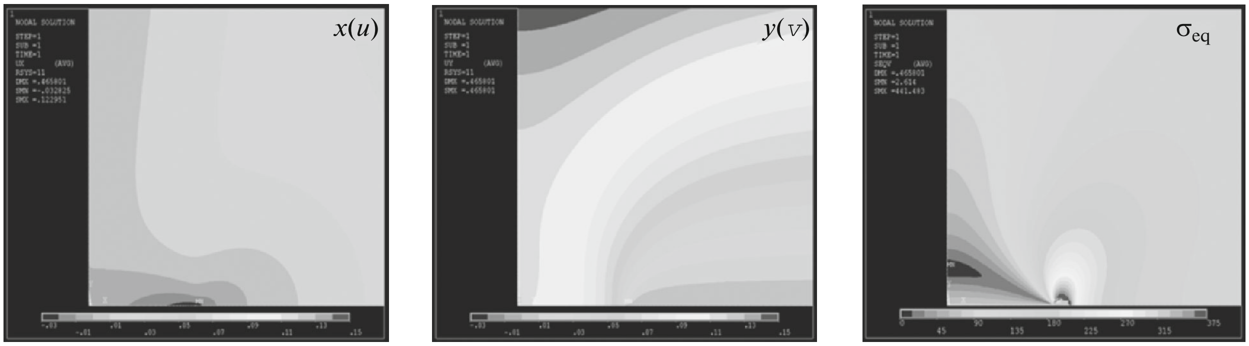

Fig. 4. Distributions of displacements along the axes $x(u)$ and $y(v)$ and the equivalent stresses (σ_{eq}) in the crack zone.

The number of considered Williams expansion terms N was taken equal to 15 because in some cases a zone located at a significant distance from a crack tip, where the influence of the singular component of a stress field is less pronounced than the influence of its regular component, was used as the localization zone of "measurement" points.

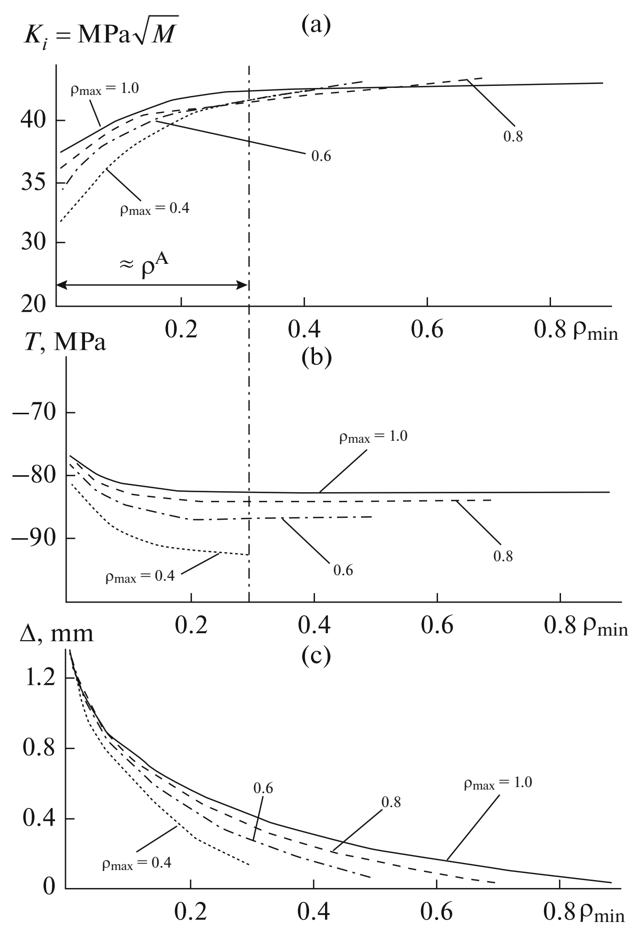

Fig. 5. Calculated dependences of K_I , the T stresses, and the standard deviation Δ on the ρ_{min} value at different ρ_{max} values.

Similar calculations were also performed for modeling the error of experimental data. The results were close to the estimates of the influence of the errors obtained for the elastic problem (for δu_{max}, $\delta v_{max} \leq$ 15%, the relative error of the K_I determination is at least two times less than δu_{max}, δv_{max}, while that of the determination of the T stresses is somewhat higher, but also lower than the error of the initial data).

The main results of the calculation of K_I and the T stresses (see Fig. 5) were obtained as follows. Under the constant value of the "upper" limit of the range of initial data (ρ_{max} = const), a series of calculations of the Williams coefficients were performed under the sequential increase in the ρ_{min} values on the basis of the processing of the initial data localized in the region $\rho_{min} \leq \rho \leq \rho_{max}$. For each subsequent calculation, the ρ_{max} value was increased until stable values of the desired parameters (K_I and the T stresses) were reached. The value of the dimensionless radius corresponding to this state is denoted by ρ^*_{max}. The point $\rho = \rho^*$ of the curve $K_I = K_I(\rho_{min})$ after which the K_I value becomes stable ($\mathrm{d}\,K_1/\mathrm{d}\rho_{min} \to 0$) can be considered the boundary of the zone where the inelastic behavior of material occurs (see Fig. 5a). Apparently, the analytical representation of displacement fields in the form of the expansion in terms of Williams functions found using the initial data in the range $\rho^* \leq \rho \leq \rho^*_{max}$ as the localization zone describes the SSS in this region correctly.

Note that, using this representation, the values of the parameters of fracture mechanics, K_I and the T stresses, which would correspond to the state of the studied object with a crack in the absence of plastic deformations and other types of damage in the vicinity of a crack tip, can be determined with high accuracy.

Hence, the region $\rho < \rho^*$ is considered the inelastic deformation zone (the damaged material zone). This is a conventional concept, which essentially means that the SSS in this zone does not correspond to the asymptotic solution of the crack problem. Along with that, this approach can give rather useful information for the analysis of the crack behavior based on the experimental information on displacement fields registered by the advanced optical correlation techniques.

ACKNOWLEDGMENTS

This work was supported by the Russian Science Foundation, project no. 14-19-00776.

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Translated by E. Petrova