Dynamics of Plasma in a Plasma-Microwave Amplifier under the Action of the Miller's Force

I. N. Kartashov[a, *] **and M. V. Kuzelev**[a, **]

[a] *Department of Physics, Lomonosov Moscow State University, Moscow, 119991 Russia*
*e-mail: igorkartashov@mail.ru
**e-mail: kuzelev@mail.ru

Abstract—In this paper, we consider the dynamics of plasma in a high-power plasma-microwave amplifier with a submicrosecond pulse duration. It is shown that, in the case of a linear mode (a short system length or a small input signal level), a discontinuity in the plasma density can form near the output boundary due to the escape of particles towards the amplifier input, which can lead to radiation breakdown. When the amplifier operates in the saturation mode, the plasma displacement has a multidirectional character, and a density discontinuity is not formed. At an increase in the initial plasma density, the effect of its pushing out weakens.

INTRODUCTION

According to the experimental data, the frequency of the amplified signal in the Cherenkov plasma-microwave amplifier is somewhat different from the frequency of the signal generated at the input of the amplifier by the master oscillator [1]. In the same work, it was shown that this can be explained by the instability of the plasma parameters in the microwave amplifier. Plasma is produced in such an amplifier via ionization of the residual gas with an auxiliary, nonrelativistic, low-density electron beam. Since such a plasma is highly stable (the characteristic decay time is about hundreds of microseconds), its change during the microwave pulse of the amplifier (no more than 1 μs) can only be due to the effect of a relativistic high-current beam or microwave electromagnetic field on the plasma. Below, we discuss the possibility of affecting the plasma in the amplifier with the electromagnetic field of the amplified signal.

The amplitude of the amplified electromagnetic wave[1] increases from the input of the amplifier ($z = 0$) to its output ($z = L$). From the side of the field of an inhomogeneous electromagnetic wave, an average force (Miller's force) acts on a charged particle [2, 3], pushing the particle into the region of a weak field. The effect of the Miller force on the beam-plasma instability in continuous mode was studied in [4]. This effect leads to the establishment of spatial-density modulation as a balance of high-frequency pressure (due to the Miller's force) and additional gas-kinetic pressure in the plasma. In view of the pronounced resonant nature of the beam instability, this can lead to a

violation of resonance conditions and suppression of the instability. In this paper, we consider the effect of the Miller's force on the beam-plasma instability in pulsed mode when stationary states with modulated plasma density are not established, while plasma particles are impulsively affected by a microwave field that is inhomogeneous in the longitudinal direction. The expulsion of electrons from the region of a strong alternating field also has important practical applications. In the optical frequency range, the expulsion of electrons from the region of the propagation of a short laser pulse leads to the formation of strong electric fields and electron acceleration [5–7].

EXPULSION OF PLASMA BY AN AMPLIFIED MICROWAVE FIELD

We calculate the average force acting on a plasma electron in the field of an inhomogeneous microwave wave. Due to the presence of the strong, longitudinal, external magnetic field used in plasma microwave electronics systems, electrons can only move in the longitudinal direction, i.e., parallel to the z axis. Thus, we write the following motion equation for plasma electrons:

$$m\frac{d^2 z_e}{dt^2} = e\frac{1}{2}[E_z(z_e, r_p)\exp(-i\omega t + ikz_e) + C.C.]. \quad (1)$$

Here, r_p is the radius of the plasma tube, ω is the frequency of the microwave field, $k \approx \omega/u$ is the longitudinal wavenumber (u is the beam velocity), $E_z(z_e, r_p)$ is the slow complex amplitude of the amplified wave, and symbols $C.C.$ here and below denote the complex conjugate to the preceding term. The slow

---

[1] In a plasma amplifier with a thin tubular magnetized plasma, this is the plasma cable wave.

amplitude change $E_z(z_e, r_p)$ is due to the amplification of the wave.

We represent the solution of equation (1) in the form

$$z_e(t) = \hat{z}_e(t) + \tilde{z}_e(t), \tag{2}$$

where $\hat{z}_e(t)$ is a slow function of time, and the function $\tilde{z}_e(t)$ describes fast oscillations occurring over a time of $\sim 2\pi/\omega$. It satisfies the inequality

$$|k\tilde{z}_e| \ll 1. \tag{3}$$

Substituting (2) into (1), taking into account inequality (3), and separating fast and slow processes, we have

$$m\frac{d^2\tilde{z}_e}{dt^2} = e\frac{1}{2}[E_z(\hat{z}_e, r_p)\exp(-i\omega t + ik\hat{z}_e) + C.C.],$$

$$m\frac{d^2\hat{z}_e}{dt^2} = e\frac{1}{2} \tag{4}$$

$$\times \left\langle \tilde{z}_e\left[\left(ik + \frac{\partial}{\partial \hat{z}_e}\right)E_z(\hat{z}_e, r_p)\exp(-i\omega t + ik\hat{z}_e) + C.C.\right]\right\rangle,$$

where the angle brackets indicate averaging over the oscillation period $2\pi/\omega$, and the derivative $\partial/\partial\hat{z}_e$ acts only on the amplitude. With allowance for the slowness of the functions $E_z(\hat{z}_e, r_p)\exp(ik\hat{z}_e)$, the solution of the first equation (4) can be written in the form

$$\tilde{z}_e = -\frac{1}{2}\frac{e}{m\omega^2}[E_z(\hat{z}_e, r_p)\exp(-i\omega t + ik\hat{z}_e) + C.C.]. \tag{5}$$

Further substituting expression (5) into the second equation (4) and discarding the rapidly oscillating terms, we obtain the following expression for the slow component of the electron coordinate:

$$m\frac{d^2\hat{z}_e}{dt^2} = -\frac{1}{4}\frac{e^2}{m\omega^2}\frac{\partial}{\partial\hat{z}_e}|E_z(\hat{z}_e, r_p)|^2. \tag{6}$$

The right-hand side of equation (6) is the desired Miller's force acting on an electron in an inhomogeneous field.

In a plasma amplifier at the linear stage of amplification, the amplitude is $E_z(z, r_p) = E_0\exp(\delta kz)$, where E_0 is the amplitude of the microwave field at the input, and $\delta k > 0$ is the gain coefficient known from the linear theory [8]. In this case, we have from equation (6) for the Miller's force

$$F_e(z) = -\frac{1}{2}\frac{e^2}{m\omega^2}\delta k|E_0|^2\exp(2\delta kz). \tag{7}$$

Similar calculations for the Miller's force acting directly on ions produce the following result:

$$F_i(z) = -\frac{1}{2}\frac{e^2}{M\omega^2}\delta k|E_0|^2\exp(2\delta kz). \tag{8}$$

In equation (8), M is the mass of the ion.

As can be seen, the Miller's force pushes the plasma particles into the weak field region, i.e., at the linear stage of the amplifier operation towards its input. Thus, the Miller's force facilitates the escape of the plasma beyond the amplification region. Let us consider the dynamics of this process. Miller's force acting on an ion is M/m times less than Miller's force acting on an electron. Therefore, electrons and ions are displaced by the Miller's force in different ways.

The displacement of the ion turns out to be $(M/m)^2$ times less than the displacement of the electron (for the same period of time). As a result, the plasma electrons are displaced relative to the ions, the quasineutrality of the plasma is violated, and a longitudinal quasi-static electric field, an ambipolar field, appears. The force of the ambipolar field counteracts the Miller force acting on the electrons but pulls the plasma ions towards the entrance boundary. Plasma begins to gradually leave the amplification area, i.e., there is an ambipolar displacement of the plasma in an inhomogeneous microwave field. Note that collisions of electrons in systems of plasma microwave electronics are usually disregarded, since the time between collisions is long compared to the pulse duration and the time of flight of electrons in the interaction space of the electrodynamic system. The thermal effects in the plasma and beam can be taken into account, but their effect will be reduced to a certain decrease in the instability growth rate calculated in [9–11], including their effect within the framework of the kinetic model. Thus, the displacement of the plasma as a whole due to the ambipolar field is mechanical rather than diffusional.

For a qualitative description of the ambipolar displacement, we consider a homogeneous, one-dimensional layer of a quasi-neutral plasma $0 < z < L$, in which constant external forces F_e and F_i act on electrons and ions. The displacements of electrons and ions relative to the equilibrium position are denoted by $\hat{z}_e(t)$ and $\hat{z}_i(t)$, respectively. These quantities obviously satisfy the following equations:

$$m\hat{z}_e'' + 4\pi e^2 n_0\hat{z}_e - 4\pi e^2 n_0\hat{z}_i = F_e,$$

$$M\hat{z}_i'' + 4\pi e^2 n_0\hat{z}_i - 4\pi e^2 n_0\hat{z}_e = F_i. \tag{9}$$

The last two terms on the left-hand sides of both equations (9) describe the effect on charged particles of a self-consistent field arising from charge separation, and n_0 is the unperturbed plasma density in this case. The solutions of equations (9) with zero initial conditions have the form

$$\hat{z}_e = \frac{1}{2}\frac{F_e + F_i}{M + m}t^2 + \frac{F_e M - F_i m}{\omega_{Li}^2(M + m)^2}$$

$$\times \left(1 - \cos\sqrt{\omega_{Le}^2 + \omega_{Li}^2}\,t\right), \quad \hat{z}_i = \frac{1}{2}\frac{F_e + F_i}{M + m}t^2 \tag{10}$$

$$- \frac{F_e M - F_i m}{\omega_{Le}^2(M + m)^2}\left(1 - \cos\sqrt{\omega_{Le}^2 + \omega_{Li}^2}\,t\right),$$

where ω_{Le} and ω_{Li} are the electron and ion Langmuir frequencies. When the inequalities $F_e \gg F_i$ and $M \gg m$ are satisfied, solutions (10) are simplified:

$$\hat{z}_e = \frac{1}{2}\frac{F_e}{M}t^2 + \frac{F_e}{M\omega_{Li}^2}(1 - \cos\omega_{Le}t),$$

$$\hat{z}_i = \frac{1}{2}\frac{F_e}{M}t^2 - \frac{m}{M}\frac{F_e}{M\omega_{Li}^2}(1 - \cos\omega_{Le}t). \quad (11)$$

As can be seen from solutions (10) and (11), the displacement of electrons and ions has both a constant component and an oscillatory component with an electron Langmuir frequency. Oscillations are associated with the excitation of plasma oscillations in the system during the separation of charges due to various accelerations imparted to electrons and ions. At times of $t \gg \omega_{Li}^{-1} = \omega_{Le}^{-1}\sqrt{M/m}$, the role of these oscillations weakens and the same first terms in (11) become dominant. Thus, the plasma as a whole moves uniformly with acceleration F_e/M. From equations (9), it is also easy to see that the center of mass of the electron-ion system moves in accordance with the equatioÅn

$$\hat{z}'' = \frac{F_e + F_i}{M + m} \approx \frac{F_e}{M}. \quad (12)$$

In the case of a plasma amplifier operating in a linear mode, the force F_e is determined by formula (7) and depends on the z coordinate. However, if this dependence is slow, then formula (12) is applicable in this case. Substituting force (7) into (12), we obtain the following equation for the center of mass of the electron-ion system at ambipolar displacement:

$$\frac{d^2\hat{z}}{dt^2} = -\frac{1}{2}\delta k\frac{e^2}{mM\omega^2}|E_0|^2\exp(2\delta k\hat{z}). \quad (13)$$

Solving equation (13) with the initial conditions

$$\hat{z}(t = 0) = z_0, \quad \frac{d\hat{z}}{dt}(t = 0) = 0, \quad (14)$$

we determine the function $\hat{z}(t, z_0)$ and, from this function, using the method of integration over the initial data [12], we find the expression for the plasma density in the amplifier:

$$n(t, z) = n_0 \int_0^L \delta[z - \hat{z}(t, z_0)]dz_0. \quad (15)$$

If the amplitude of the input signal is large enough, then saturation of the beam-plasma instability and amplifier output to a nonlinear operation mode are possible for a given length of the plasma-microwave amplifier. In this case, the spatial dynamics of amplification is described by a system of nonlinear equations [8]:

$$(1 - \alpha_p\hat{L})j = \theta\alpha_p\alpha_b\hat{L}\rho, \quad \frac{dy}{d\xi} = \eta,$$

$$\frac{d\eta}{d\xi} = \frac{i}{2}\left(1 + 2\gamma^2\frac{u^2}{c^2}\eta\right)^{3/2} \quad (16)$$

$$\times\left[\exp(-iy)\hat{L}(j + \alpha_b\rho) - C.C.\right],$$

$$j(\xi = 0) = j_0, \quad y(\xi = 0) = y_0, \quad \eta(\xi = 0) = 0.$$

Here, j and $\rho = \frac{1}{\pi}\int_0^{2\pi}\exp(iy)dy_0$ are the dimensionless amplitudes of the current in the plasma and the density of the electron beam; $\eta = 1 - v_z/u$, $y = \omega t - \xi$, and $\xi = \omega z/u$ are the dimensionless velocity of the beam electrons, time, and coordinate; $\hat{L} = 1 - 2i\gamma^2 d/d\xi$ is the differential operator; α_p and α_b are the density parameters of the plasma and beam; and θ is the coupling coefficient of beam and plasma waves. The last three quantities are determined by the geometric parameters of the system [8]. The amplitude of the electric-field strength in plasma is related to the amplitude of the current with the relation

$$E_z(z, r_p) = -i\frac{m}{e}\omega u\gamma^{3/2}\frac{\omega_b}{\omega_p}\frac{j(z)}{\sqrt{\alpha_p\alpha_b\theta}}. \quad (17)$$

By resolving system (16), one can obtain the spatial distribution of the field in the plasma-microwave amplifier. Using the expression on the right-hand side of (6) for the Miller's force and substituting it into the motion equation (12), we obtain a nonlinear generalization of equation (13). Its solution, together with the initial conditions (14) after calculation of the integral (15), gives the dynamics of the spatial density of the plasma in the plasma-microwave amplifier with allowance for the nonlinear saturation of the beam instability.

RESULTS AND DISCUSSION

For the calculations, we will focus on the following values of the beam-plasma system used in experimental studies [1]: the velocity of the electron beam is $u = 2.6 \times 10^{10}$ cm/s, its current is 2 kA, the plasma density is $n_0 = 10^{12}$ cm^{-3}, the radius of the waveguide is $R = 4.9$ cm, the average radii of the plasma and beam are $r_p = 2$ cm and $r_b = 1.25$ cm, their thickness are $\Delta_p = 0.2$ cm and $\Delta_b = 0.5$ cm, the length of the system is $L = 50$ cm, and the frequency and power of the master oscillator are $\omega/2\pi = 2.71$ GHz and $P_0 = 50$ kW. The operating gas is xenon.

Figure 1 shows the spatial distribution of the strength of the longitudinal component of the electric field in the amplifier, which is calculated with (16) and (17) for two values of the initial plasma density $n_0 = 10^{12}$ cm^{-3} (curve *1*) and $n_0 = 2 \times 10^{12}$ cm^{-3}
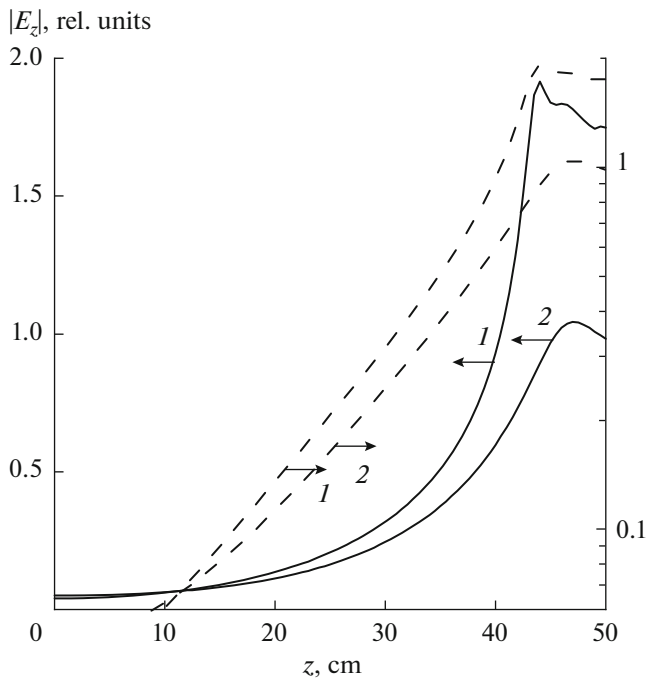
$|E_z|$, rel. units

Fig. 1. Spatial distribution of the electric-field strength of the wave for the plasma densities of (*1*) $n_0 = 10^{12}$ cm^{-3} and (*2*) $n_0 = 2 \times 10^{12}$ cm^{-3}; the dashed lines are the same dependences on a logarithmic scale.

n/n_0

Fig. 2. Spatial distribution of the plasma density in the amplifier in linear operation mode for various times: (*1*) $t = 100$ ns, (*2*) 200, (*3*) 300, and (*4*) 400.

(curve *2*). These values were selected because they correspond to the boundary values of the characteristic range of variation of the plasma density in experiments. The input signal level for both curves is approximately the same. At a lower plasma density, the spatial increment of instability is greater and the signal level at which saturation is reached is approximately two times higher. In the same plot, dashed lines show the same dependences but on a logarithmic scale. Comparison of the curves shows that the saturation of the instability occurs at a low plasma density through the intermediate region of faster (than predicted by the linear theory) signal growth. In Fig. 1, the length of the system and the level of the input signal are chosen such that the instability saturates near the output of the amplifier and the amplitude of the amplified wave stops increasing, i.e., a nonlinear operating mode is realized. By reducing the length or level of the input signal, we come to a situation in which nonlinear effects do not have time to manifest themselves and, thus, the amplifier operates in linear mode.

A microwave field that is inhomogeneous in the longitudinal direction with various parts of the spatial dependence leads to the appearance of the Miller's force and the expulsion of the plasma. The action of the Miller's force differs depending on the operating mode of the amplifier (linear mode or saturation mode). We consider a plasma with a density of $n_0 = 10^{12}$ cm^{-3} and start with a linear amplifier operation mode. It is clear

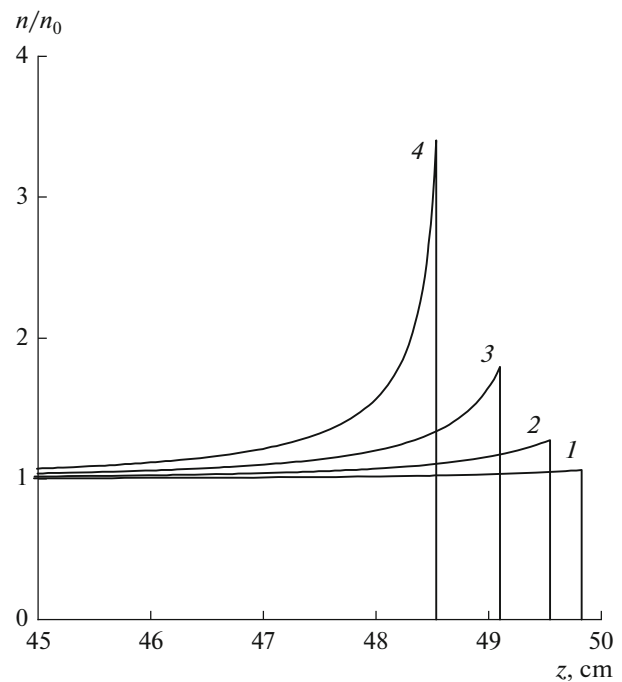in advance that, at a sufficiently low level of the input signal, the amplified signal at the output will also be weak, the Miller's force is small, and the plasma will not have time to shift significantly. When the signal level at the amplifier input is two times lower than that in Fig. 1, nonlinear effects do not appear at a specified length, but the action of the Miller's force turns out to be significant. Figure 2 shows the spatial dependences of the plasma density for various times. Due the growing (exponentially) dependence of the field on the coordinate, the Miller's force is directed to the left (towards the weak field), which leads to a unidirectional displacement of the plasma; the stronger they are, the closer the particles are to the exit. This leads to the formation of a density discontinuity with a length of up to 1.5 cm near the exit boundary and can provoke radiation blocking. The plasma densification near the discontinuity is insignificant and only leads to an insignificant local change in the spatial increment of instability. Note that the formation of a discontinuity in the plasma density was reported in [13, 14], which established based on direct numerical simulation with the particle-in-cell method that the generation pulse in a relativistic plasma microwave generator of nanosecond duration was shortened. These studies indicated that the escape of plasma due to the negative potential arising at the electron collector was the main cause of the formation of the discontinuity. Perhaps the effect of the Miller's force occurred in these cases as well, but it is difficult to single out individual mechanisms based on the simulation of particle motion. Besides, as will be indicated
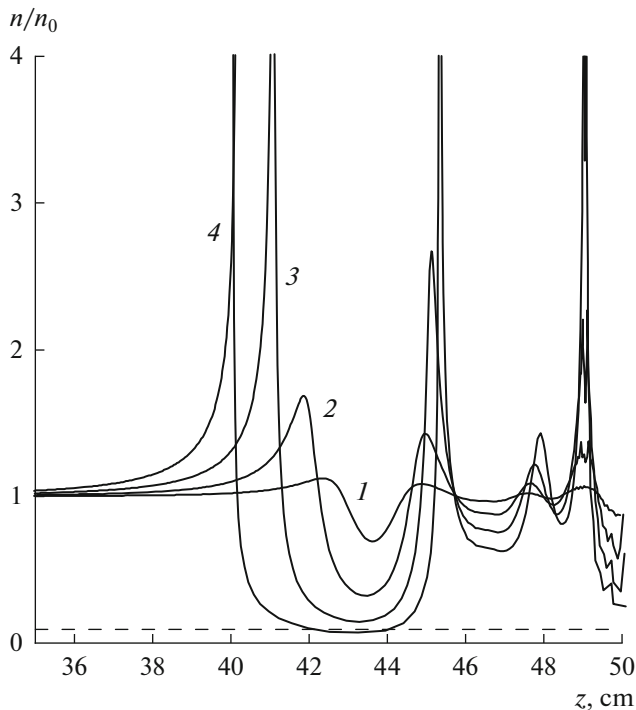
Fig. 3. Spatial distribution of the plasma density in the amplifier in saturation operation mode for various times: (*1–4*) same as in Fig. 2; the dashed line is the critical plasma density for the considered frequency.

below, the Miller's force does not have a significant effect on the plasma-density dynamics in all cases. In particular, as the electron concentration increases, its contribution weakens.

A different situation occurs when the amplifier operates in the saturation mode. At the input signal level given in Fig. 1, the effect of the Miller's force and plasma displacement have different directions for different sections of the plasma column. Near the input boundary of the amplifier, the field strength is still small and the Miller's force does not lead to a significant displacement of the plasma. The Miller's force pushes the plasma to the left in the section near the saturation point and to the right in the section near the output boundary. As a result, a complex spatial structure forms at a distance of about 10 cm from the amplifier output (Fig. 3). One of the elements of this structure is a region with a reduced plasma density. However, up to the end of the electron beam current pulse ($t = 400$ ns), the plasma density does not fall below the critical value; the conditions for the propagation of the plasma wave are maintained, i.e., a discontinuity is not formed, and the radiation pulse is not broken.

CONCLUSIONS

The plasma dynamics in a microwave amplifier at an initial density of $n_0 = 10^{12}$ cm^{-3} is considered. The character of plasma displacement differs depending on the operating mode of the amplifier (linear or satura-

tion mode). In linear mode (a short system length or a small input signal level), a plasma discontinuity can form near the output boundary due to the escape of particles towards the amplifier input, and radiation breakdown occurs. When the amplifier is operating in saturation mode, the plasma displacement has a multidirectional character and a density discontinuity is not formed. The region of reduced plasma density does not prevent wave propagation at density values above the critical value. When increasing n_0 to 2×10^{12} cm^{-3}, as seen in Fig. 1, the saturation level of the longitudinal component of the electric field decreases by about half. Due to the quadratic dependence of the Miller force on the field, this leads to a fourfold decrease. A decrease in the Miller's force is additionally caused by a decrease in the instability increment δk. Thus, the plasma displacement occurs much more slowly, and this does not lead to a significant modulation of the plasma density.

REFERENCES

1. Kartashov, I.N., Kuzelev, M.V., Strelkov, P.S., and Tarakanov, V.P., *Plasma Phys. Rep.*, 2018, vol. 44, no. 2, p. 289.
2. Miller, M.A., *Izv. Vyssh. Uchebn. Zaved., Radiofiz.*, 1958, vol. 1, no. 3, p. 110.
3. Gaponov, A.V. and Miller, M.A., *Zh. Eksp. Teor. Fiz.*, 1958, vol. 34, no. 1, p. 242.
4. Bliokh, Yu.P., Lyubarsky, M.G., Zemlyansky, N.M., et al., *Plasma Phys. Rep.*, 2003, vol. 29, no. 4, p. 307.
5. Tajima, T. and Dawson, J.M., *Phys. Rev. Lett.*, 1979, vol. 43, p. 267.
6. Andreev, N.E. and Gorbunov, L.M., *Phys.—Usp.*, 1999, vol. 42, no. 1, p. 49.
7. Kostyukov, I.Yu. and Pukhov, A.M., *Phys.—Usp.*, 2015, vol. 58, no. 1, p. 81.
8. Kuzelev, M.V., Rukhadze, A.A., and Strelkov, P.S., *Plazmennaya relyativistskaya SVCh-elektronika* (Plasma Relativistic Microwave Electronics), Moscow: Lenand, 2018.
9. Kartashov, I.N. and Kuzelev, M.V., *High Temp.*, 2018, vol. 56, no. 2, p. 334.
10. Kartashov, I.N. and Kuzelev, M.V., *Moscow Univ. Phys. Bull. (Engl. Transl.)*, 2016, vol. 71, no. 6, p. 537.
11. Kartashov, I.N. and Kuzelev, M.V., *Phys. Wave Phenom.*, 2017, vol. 25, no. 1, p. 43.
12. Kuzelev, M.V. and Rukhadze, A.A., *Elektrodinamika plotnyh elektronnyh puchkov v plazme* (Electrodynamics of Dense Electron Beams in Plasma), Moscow: URSS, 2018.
13. Bogdankevich, I.L., Loza, O.T., and Pavlov, D.A., *Kratk. Soobshch. Fiz.*, 2010, no. 2, p. 16.
14. Ernyleva, S.E., Bogdankevich, I.L., and Loza, O.T., *Kratk. Soobshch. Fiz.*, 2013, no. 7, p. 10.

Translated by A. Ivanov