

Nonlinear Dynamics in Strongly Coupled Quantum Plasma

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Abstract—The properties of cylindrical and spherical modified ion-acoustic waves in a strongly coupled plasma (containing strongly correlated non-relativistic ions, weakly correlated relativistic (both non-relativistic and ultra-relativistic) electron and positron fluids, and positively charged static heavy ions) are investigated theoretically. The restoring force is provided by the degenerate pressure of the electron and positron fluids, whereas the inertia is provided by the mass of ions. The positively charged static heavy ions participate only in maintaining the quasi-neutrality condition at equilibrium. By using reductive perturbation method, we have derived modified Burgers and Korteweg–de Vries equations. Their shock and solitary wave solutions are also numerically analyzed to understand the localized electrostatic disturbances. The basic features of modified ion-acoustic shock and solitary waves are found to be significantly modified by the effects of degenerate pressure of electrons, positrons, and ion fluids, their number densities, and various charge states of heavy ions. It is also observed that the amplitude of these shock and solitary profiles are maximum for spherical geometry, intermediate for cylindrical geometry, and minimum for planar geometry. The present analysis can be helpful for understanding different degenerate and relativistic phenomena in dense astrophysical environments as well as laboratory plasma systems.

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INTRODUCTION

Nowadays, the study of electron-positron (EP) plasmas has received a great deal of attention because of its large applications range in both space and laboratory plasmas. The latter are found in some interstellar compact objects (e.g., in non-rotating neutron stars, white dwarfs, active galactic nuclei, etc.) and in the solar atmosphere [1–7]. In case of EP plasmas, electron and positron fluids have the same masses but opposite charges. Generally, the EP plasma symmetry is broken in the presence of ions, and both fast and slow time scales can occur in the dynamics of electron-positron-ion plasmas. As a result of evaporation or seismic process on the surface of an astrophysical compact objects, the ions are originated [8]. Therefore, under certain conditions of massive white dwarfs, the plasma number densities is of the order of 10^{30} cm^{-3} or even more [9–11] (Table 1). As the thermal energy of a white dwarf is slowly lost to the space and the stellar material cools, the ions behavior can be understood in the classical manner, but under certain conditions, this behavior is modified by quantum effects. The EP pairs in white dwarfs can be produced during the collapse of white dwarfs to neutron stars [12, 13]. Therefore, it is important to study the nonlinear dynamics of ion oscillations in the presence of electron and positron fluids.

In a relativistically strongly coupled astrophysical plasma, electron and positron fluids are degenerate and ions are strongly coupled because of the ion Coulomb coupling parameter $\Gamma = Z_i^2 q_i^2 / (a_i k_B T_i)$, where q_i is the charge of ions, a_i is the inter-ion spacing, T_i is the ion temperature, and Z_i is the ion charge state, respectively [14]. The ions are strongly coupled to each other through their mutual Coulomb interaction because of their high charge density. When $\Gamma \gg 1$, the plasma system is said to be strongly coupled. When the mass and charge of ions are very high, then ion plasmas can also show the strongly coupled behavior. Many authors [15–17] have also showed it experimen-

Table 1. Approximate density ρ_0 , corresponding plasma particle (electron, positive ion, and heavy ion) number densities of white dwarfs [13, 23–27]

$\rho_0, \text{ gm/cm}^3$	10^6	10^7	10^8
$n_{i0}, 10^{30} \text{ cm}^{-3}$	0.05	0.5	5
$n_{e0}, 10^{30} \text{ cm}^{-3}$	0.32	3.2	32
$n_{p0}, 10^{30} \text{ cm}^{-3}$	0.71	7.1	71
$Z_h n_{h0}, 10^{30} \text{ cm}^{-3}$	0.51	5.1	51

tally. Generally, the constituents of plasmas are electrons, ions, and atoms or molecules. On the other hand, dusty (or complex) plasmas contain static mesoscopic (multiply charged) particles [18]. The presence of heavier element like iron can be thought in case of some relatively massive white dwarfs. The heavy nuclei are mainly formed into the interiors of massive stars. When these stars contract to very high densities, matter in their interiors cool down and become degenerate under certain conditions. The formation of heavy elements begins in this state of degeneracy. When explosion occurs, part of the heavy elements distribute over the surrounding space and leave one or more stellar remnants in the form of white dwarfs [19]. It is important to note that the degeneracy feature, which is a fundamental aspect of ordinary solids, arise due to exclusion mechanism because of the de Broglie thermal wave length $\lambda_B = h/(2\pi m_e k_B T)^{1/2}$ [20]. The equation of state for degenerate electrons in such space environments and astrophysical objects are explained in [21] for two limits, named as non-relativistic and ultra-relativistic limits. The degenerate electron pressure equation is given in [21, 22] as $P_e \propto n_e^{5/3}$ for non-relativistic limit and $P_e \propto n_e^{4/3}$ for ultra-relativistic limit, where P_e is the degenerate electron pressure and n_e is the degenerate electron number density.

For understanding the localized electrostatic disturbances in such compact astrophysical objects like white dwarfs, a large number of theoretical investigations [28–49] have been made on the nonlinear propagation of ion-acoustic (IA) waves by considering a degenerate dense plasma model that assumes weakly coupled non-degenerate ion fluids and degenerate non-relativistic or ultra-relativistic electron fluids. All of these investigations, which have shown the existence of IA solitary and shock structures, are not valid for strongly coupled non-degenerate or degenerate ion fluids. Strong correlation among ions can be the source of dissipation (dispersion) and can be responsible for the formation of shock (solitary) structures in such compact astrophysical objects. Again, the dense astrophysical quantum plasmas can be confined by stationary heavy ions. Therefore, the effect of the heavy ions has to be taken into account, especially for astrophysical observations, where the degenerate plasma pressure and heavy ions play an important role in the formation and stability of the existing waves.

None of the authors considered the effects of strongly correlated relativistic ions, nonplanar geometry, and various charge states of heavy ions, which can significantly modify the propagation of solitary and shock structures under consideration in such plasma system. As far as we know, the nonplanar modified ion-acoustic (MIA) waves in such plasma system has never been addressed. The aim of this paper is to present a first study for the nonplanar MIA shock and solitary waves where degenerate plasma pressure,

strongly coupled non-relativistic ion fluids, both weakly coupled non-relativistic and ultra-relativistic degenerate electron and positron fluids, and various charge states of heavy ions play a significant role.

THEORETICAL MODEL AND BASIC EQUATIONS

We consider the nonplanar (cylindrical and spherical) geometry of the MIA waves in a strongly coupled, collisionless, homogeneous dense plasma. The plasma is assumed to be composed of non-relativistic degenerate ions, both non-relativistic and ultra-relativistic degenerate electron and positron fluids, and positively charged static heavy ions. Thus, the equilibrium condition is written as $n_{i0} + n_{p0} + Z_h n_{h0} = n_{e0}$, where n_{s0} is the unperturbed number densities of the species s is (here $s = i, e, p$ for positively charged ion, electron, and positron, respectively), Z_h is the number of light ions residing onto the heavy ion surface. The dynamics of low frequency nonlinear MIA waves in such a strongly coupled degenerate plasma system is governed by the well-known generalized viscoelastic hydrodynamic (GH) equations [50, 51], consisting of the continuity and momentum equations, given by

$$\frac{\partial n_s}{\partial t} + \frac{1}{r^\nu} \frac{\partial}{\partial r} (r^\nu n_s u_s) = 0, \quad (1)$$

$$D_\tau \left[n_i \left(D_i u_i + \frac{\partial \phi}{\partial r} \right) + \mu_i k_B T_{ef} \frac{\partial n_i^\alpha}{\partial r} \right] = \frac{\eta_l}{r^\nu} \frac{\partial}{\partial r} \left(r^\nu \frac{\partial u_i}{\partial r} \right), \quad (2)$$

and by the generalized degenerate pressure equations for electron and positron fluids

$$n_e \frac{\partial \phi}{\partial r} - K_2 \frac{\partial n_e^\gamma}{\partial r} = 0, \quad (3)$$

$$n_p \frac{\partial \phi}{\partial r} - K_2 \frac{\partial n_p^\gamma}{\partial r} = 0. \quad (4)$$

The system that is closed by Poisson equation

$$\frac{1}{r^\nu} \frac{\partial}{\partial r} \left(r^\nu \frac{\partial \phi}{\partial r} \right) = -\rho, \quad (5)$$

$$\rho = n_i - \beta n_e + \lambda n_p + Z_h \mu_h,$$

where $\nu = 0$ for one dimensional planar geometry; $\nu = 1(2)$ for nonplanar cylindrical (spherical) geometry; n_s is the plasma number density of the species s ($s = e, i, p$) normalized by its equilibrium value n_{s0} ; u_s is the plasma species fluid speed normalized by $C_{im} = (m_e c^2 / m_i)^{1/2}$ with m_e (m_i) being the electron (ion) rest mass and c being the speed of light in vacuum; ϕ is the electrostatic wave potential normalized by $m_e c^2 / e$ with e being the electron charge; the time variable t is normalized by $\omega_{pi} = (4\pi n_{i0}^2 e / m_i)^{1/2}$, and the space vari-

able r is normalized by $\lambda_m = (m_e c^2 / 4\pi n_{s0})^{1/2}$. In (2), $D_\tau = \frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x}$, where τ_m is the viscoelastic relaxation time and $T_{\text{ef}} = \mu_i T_i + T_*$ is the effective ion temperature. The latter consists of T_* arising from the electrostatic interaction among strongly correlated positive ions and of $\mu_i T_i$ arising from the ion thermal pressure. Again the longitudinal ion viscosity coefficient

$$\eta_l = \left(\frac{\omega_{pi}}{m_i n_{i0} \lambda_{Di}^2} \right) \{ \eta_l + (4/3) \xi_l \}$$

is the normalized longitudinal viscosity, where η_l and ξ_l are transport coefficients of shear and bulk viscosities. The parameter $\beta = n_{e0}/n_{i0}$ is the electron-to-ion number density ratio, $\lambda = n_{p0}/n_{i0}$ is the positron-to-ion number density ratio and $\mu_h = n_{h0}/n_{i0}$ is the heavy ion-to-ion number density ratio. It is needed here to note that $\beta = 1 + \lambda + Z_h \mu_h$ and $\lambda = \beta - Z_h \mu_h - 1$. We have defined

$$K_1 = \frac{n_{i0}^{\alpha-1} K_i}{m_i C_i^2} \text{ and } K_2 = \frac{n_{e0}^{\gamma-1} K_e}{m_i C_i^2} = \frac{n_{p0}^{\gamma-1} K_p}{m_i C_i^2}.$$

There are various approaches to calculating the ion transport coefficients, similar to those of one component of strongly coupled plasmas [50, 52, 53]. The parameter T_* (which arises from the electrostatic interactions among strongly correlated positive ions), viscoelastic ion relaxation time τ_m , and ion compressibility μ_i , for our purposes, are written as in [50]:

$$T_* = \frac{N_{mn} Z_i^2 e^2}{3 a_i K_B} (1 + k) \exp^{-k},$$

$$\tau_m = \eta_l \frac{T_e}{T_{\text{ef}}} \left[1 - \mu_i + \frac{4}{15} u(\Gamma) \right]^{-1}.$$

Here, N_{mn} is determined by the ion structure and corresponds to the number of nearest neighbors (viz., in crystalline state, $N_{mn} = 8$ for a bcc lattice, $N_{mn} = 12$ for fcc lattice, etc.); $k = \frac{a_i}{\lambda_{Di}}$, with λ_{Di} being Thomas–

Fermi screening length and $u(\Gamma)$ is a measure of the excess internal energy of the system and is calculated for weakly coupled plasmas ($\Gamma < 1$) as $u(\Gamma) \approx -\left(\frac{\sqrt{3}}{2}\right) \Gamma^{3/2}$.

We can express $u(\Gamma)$ in terms of Γ for a range of $1 < \Gamma < 100$ [53], deriving an analytical relation

$$u(\Gamma) \approx 0.89\Gamma + 0.95\Gamma^{-4} + 0.19\Gamma^{-4} - 0.81,$$

where a small correction term due to finite number of particles is neglected. The dependence of the other transport coefficient η_l on Γ is somewhat more com-

plex and cannot be expressed in such a closed analytical form. However, tabulated/graphical results of their functional behavior derived from the molecular dynamic simulations and a variety of statistical schemes are also available in literature [50].

FORMULATION OF NONLINEAR EQUATIONS

Derivation of Modified Burgers Equation

Now we derive a dynamical equation for the nonlinear propagation of the MIA shock waves using Eqs. (1)–(5). We employ a reductive perturbation technique to examine electrostatic perturbations propagating in the relativistic degenerate dense plasma due to the effect of dissipation. First, we introduce the stretched coordinates [54]

$$\zeta = \epsilon(r - V_p t), \quad (6)$$

$$\tau = \epsilon^2 t, \quad (7)$$

where V_p is the wave phase speed (ω/k with ω being angular frequency and k being the wave number of the perturbation mode) and ϵ is a smallness parameter measuring the weakness of the dissipation ($0 < \epsilon < 1$). Then we expand n_i , n_e , u_i , and ϕ in power series of ϵ :

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \dots, \quad (8)$$

$$n_e = 1 + \epsilon n_e^{(1)} + \epsilon^2 n_e^{(2)} + \dots, \quad (9)$$

$$n_p = 1 + \epsilon n_p^{(1)} + \epsilon^2 n_p^{(2)} + \dots, \quad (10)$$

$$u_i = \epsilon u_i^{(1)} + \epsilon^2 u_i^{(2)} + \dots, \quad (11)$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \dots \quad (12)$$

and develop equations in various powers of ϵ . For the lowest order of ϵ , using Eqs. (6)–(12) with Eqs. (1)–(5) we get $u_i^{(1)} = V_p \phi^{(1)} / V_p^2 - \mu_i k_B T_{\text{ef}} K_1'$, $n_i^{(1)} = \phi^{(1)} / V_p^2 - \mu_i k_B T_{\text{ef}} K_1'$, $n_e^{(1)} = n_p^{(1)} = \phi^{(1)} / K_2'$, and the

phase speed $V_p = \sqrt{\left(\frac{K_2'}{\lambda - \beta + Z_h \mu_h - 2} + \mu_i k_B T_{\text{ef}} K_1' \right)}$, which is same as we have obtained in case of linear waves.

We are interested in studying the nonlinear propagation of these dissipative MIA type electrostatic waves in a strongly coupled degenerate plasma. For the next order of ϵ , we obtain a set of equations

$$\frac{\partial n_s^{(1)}}{\partial \tau} - V_p \frac{\partial n_s^{(2)}}{\partial \zeta} - \frac{\partial}{\partial \zeta} \left[u_s^{(2)} + n_s^{(1)} u_s^{(1)} - \frac{v u_s^{(1)}}{V_p \tau} \right] = 0, \quad (13)$$

$$\begin{aligned} & \frac{\partial u_i^{(1)}}{\partial \tau} - V_p \frac{\partial u_i^{(2)}}{\partial \zeta} + u_i^{(1)} \frac{\partial u_i^{(1)}}{\partial \zeta} + \frac{\partial \phi^{(2)}}{\partial \zeta} \\ & + \mu_i k_B T_{\text{ef}} K_1' \frac{\partial}{\partial \zeta} \left[n_i^{(2)} + \frac{(\alpha - 2)}{2} (n_i^{(1)})^2 \right] - \eta \frac{\partial^2 u_i^{(1)}}{\partial \zeta^2} = 0, \end{aligned} \quad (14)$$

$$\frac{\partial \phi^{(2)}}{\partial \zeta} - K_2' \frac{\partial}{\partial \zeta} \left[n_e^{(2)} + \frac{(\gamma - 2)}{2} (n_e^{(1)})^2 \right] = 0, \quad (15)$$

$$\frac{\partial \phi^{(2)}}{\partial \zeta} - K_2' \frac{\partial}{\partial \zeta} \left[n_p^{(2)} + \frac{(\gamma - 2)}{2} (n_p^{(1)})^2 \right] = 0, \quad (16)$$

$$\beta n_e^{(2)} - n_i^{(2)} - \lambda n_p^{(2)} = 0. \quad (17)$$

Now combining (13)–(17), we deduce a modified Burgers equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} + \frac{v \phi^{(1)}}{2\tau} = B \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2}, \quad (18)$$

where the values of A and B are given by

$$A = \frac{(V_p^2 - \mu_i k_B T_{ef} K_1')^2}{2V_p} \times \left[\frac{3V_p^2 + \mu_i k_B T_{ef} K_1' (\alpha - 2)}{(V_p^2 - \mu_i k_B T_{ef} K_1')^3} + \frac{(\gamma - 2)(\beta - \lambda)}{K_2'^2} \right],$$

$$B = \frac{\eta_l}{2}.$$

Derivation of Korteweg–de Vries Equation

Introducing the stretched coordinates [54]

$$\zeta = \epsilon^{1/2} (r - V_p t), \quad \tau = \epsilon^{3/2} t,$$

we can expand the perturbed quantities $n_i, n_e, u_i,$ and ϕ about the equilibrium values in power series of ϵ and obtain the $n_i^{(1)}, u_i^{(1)},$ and V_p which are exactly same what we have obtained from Burgers equation.

The same is done for the next order of ϵ . After some algebraic calculations we obtain the nonlinear equation in the form of

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} + G \frac{\partial^3 \phi^{(1)}}{\partial \zeta^3} + \frac{v \phi^{(1)}}{2\tau} = 0. \quad (19)$$

Equation (19) is known as Korteweg–de Vries (KdV) equation where the nonlinear coefficient A has the same value as in Burgers equation and the dispersion coefficient G is given by

$$G = \frac{(V_p^2 - \mu_i k_B T_{ef} K_1')^2}{2V_p}.$$

PARAMETRIC INVESTIGATIONS AND RESULTS

We first briefly discuss the stationary shock wave solution for Eq. (18) at $v = 0$. We should note that for a large value of τ the term $\frac{v \phi^{(1)}}{2\tau}$ is negligible. So, in our

parametric analysis, we start with a large value of τ (–12) and choose the stationary shock wave solution

of equation (18) for $\frac{v \phi^{(1)}}{2\tau} \rightarrow 0$ as our initial pulse. The stationary shock wave solution of standard Burgers equation is obtained by considering a frame $\zeta = \xi - u_0 \tau$ (u_0 is a small increment of wave speed above speed of sound) and imposing the appropriate boundary

conditions: $\phi^{(1)} \rightarrow 0, \frac{d\phi^{(1)}}{d\xi} \rightarrow 0, \frac{d^2\phi^{(1)}}{d\xi^2} \rightarrow 0$ at $\xi \rightarrow \pm\infty$.

Thus, we can express the stationary shock wave solution of (18) as

$$\Phi_{v \rightarrow 0}^{(1)} = \phi_m [1 - \tanh(\xi/\delta)],$$

where the amplitude ϕ_m and the width δ are given by

$$\phi_m = u_0/A,$$

$$\delta = 2B/u_0.$$

And the stationary solitary wave solution of (19) is given by

$$\Phi_{v \rightarrow 0}^{(1)} = \phi_m \operatorname{sech}^2(\xi/\Delta),$$

where $\phi_m = 3u_0/A$ and $\Delta = (4G/u_0)^{1/2}$.

Nonlinear Properties

Figure 1 shows the variation of electron-to-ion number density ratio β on the cylindrical and spherical shock and solitary profiles for both non-relativistic and ultra-relativistic limit. It is observed that the amplitude of the cylindrical and spherical shock and solitary structures increase with increasing values of β . Physically, this happens due to the reason that it decreases the nonlinearity coefficient A . It is also observed that the amplitude of this shock and solitary waves are higher for non-relativistic case than for ultra-relativistic case.

The effect of positron-to-ion number density ratio on the cylindrical and spherical shock and solitary profiles are depicted in Fig. 2 for both non-relativistic and ultra-relativistic limit. It is found that the amplitude of the cylindrical and spherical shock and solitary profiles decrease with increasing of λ . It happens on the basis of the driving force of the MIA wave, as it is provided by ions inertia. Actually, increase in ion concentration (depopulation of electrons) causes decrease in the driving force, which is provided by the ion inertia, and consequently shock and solitary waves enervates. It is found that the amplitude of these shock and solitary structures are distinctly higher for non-relativistic case than for ultra-relativistic one.

The effect of heavy ion-to-ion number density ratio on the amplitude of the cylindrical and spherical MIA shock and solitary structures are shown in Fig. 3. It is found that the amplitude of the cylindrical and spherical shock and solitary structures decrease with

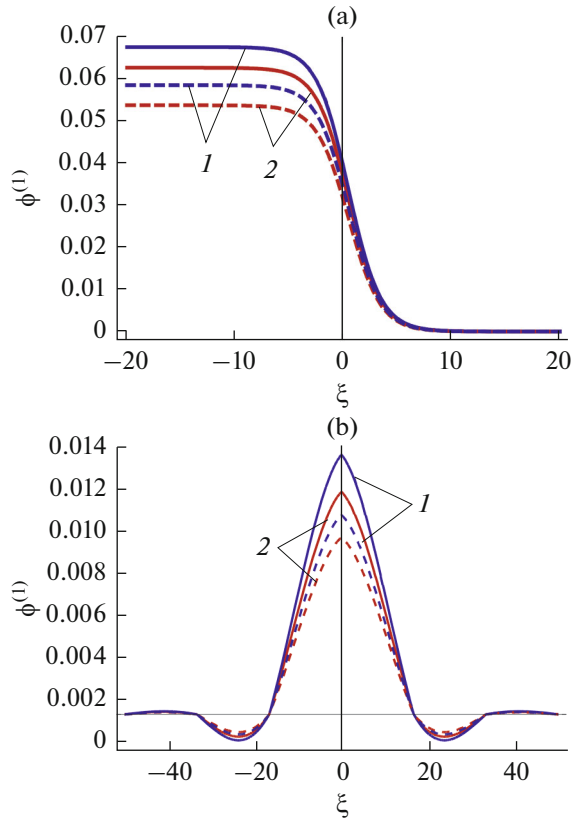


Fig. 1. Variation of cylindrical (a) and spherical (b) shock waves with ξ for different values of β : the solid curves represent the non-relativistic case and the dashed ones represent the ultra-relativistic case; (1) $\beta = 0.5$ and (2) $\beta = 0.1$.

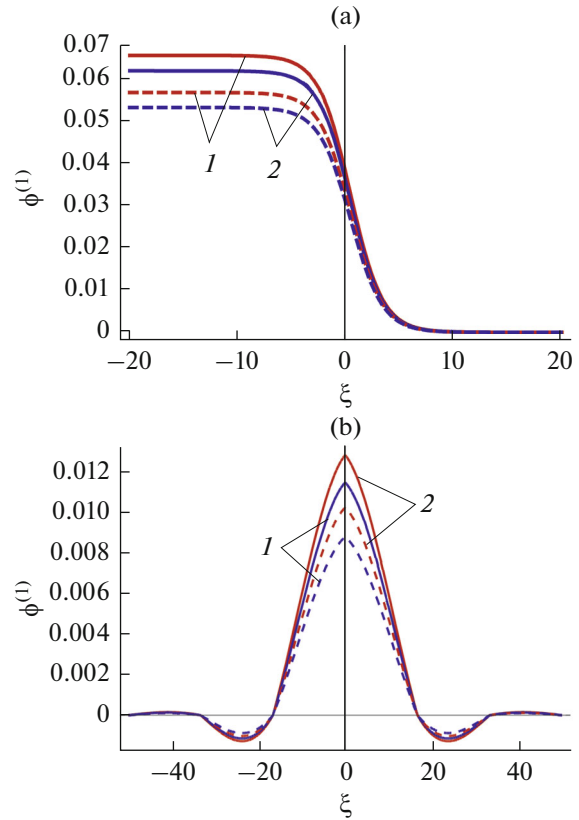


Fig. 2. Variation of cylindrical (a) and spherical (b) shock waves with ξ for different values of λ : the solid curves represent the non-relativistic case and the dashed ones represent the ultra-relativistic case; (1) $\lambda = 0.2$ and (2) $\lambda = 0.5$.

the increasing of μ_h . Physically, this happens due to the reason that it decreases the phase speed of the MIA waves (see expression for V_p).

The amplitude of the shock and solitary structures also depend on the kinematic viscosity coefficient η . It is observed that the amplitude of the cylindrical and spherical shock and solitary structures decrease with the increasing values of η . This happens due to the reason that it increases the dissipative constant B .

The nonplanar shock and solitary wave properties have been significantly affected by the relativistic factors, namely, α and γ . There are two relativistic limits called non-relativistic ($\alpha = \gamma = 5/3$) and ultra-relativistic ($\alpha = \frac{5}{3}, \gamma = 4/3$) which significantly modify the shock and solitary profiles. It is found that the amplitude of the shock and solitary profiles are higher for non-relativistic case than for ultra-relativistic case.

The amplitude of the shock and solitary profiles significantly affected by the nonplanar geometry. The amplitude of the shock and solitary profiles are maximum for spherical ($\nu = 2$) geometry, intermediate for

cylindrical ($\nu = 1$) geometry, and minimum for planar ($\nu = 0$) geometry (Fig. 4).

Figure 5 illustrates the amplitude of cylindrical shock and solitary profiles in time. The amplitude of these shock and solitary profiles decreases with the increasing absolute values of τ .

The amplitude of the cylindrical and spherical solitary and shock structures are significantly modified by the various charge states of heavy ions. Figure 6 shows the variation of the amplitude of the cylindrical shock and solitary structures with ξ for different values of Z_h . The amplitude of the cylindrical shock and solitary structures decrease with the increasing values of Z_h .

It is important to note here that when dispersion (dissipation) effect is much more pronounced than the dissipation (dispersion) effect, and the dissipation (dispersion) effect is neglected, strongly coupled degenerate plasma supports solitary (shock) waves. It is important to mention that the ranges of plasma parameters (Table 2) used in this investigation are correspond to space and laboratory plasma system.

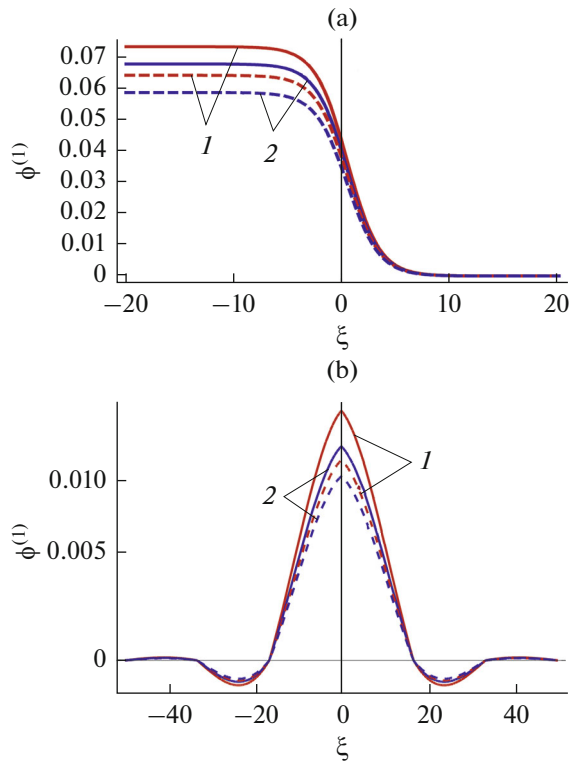


Fig. 3. Variation of cylindrical (a) and spherical (b) shock waves with ξ for different values of μ_h ; the solid curves represent the non-relativistic case and the dashed ones represent the ultra-relativistic case; (1) $\mu_h = 0.2$ and (2) $\mu_h = 0.4$.

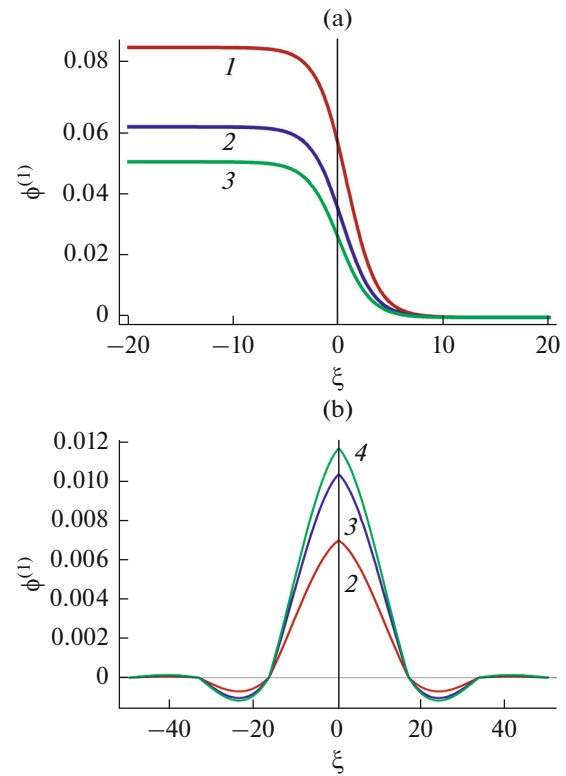


Fig. 5. Variation of shock (a) and solitary (b) waves with ξ for different values of τ : (1) -3 , (2) -6 , (3) -9 , (4) -12 .

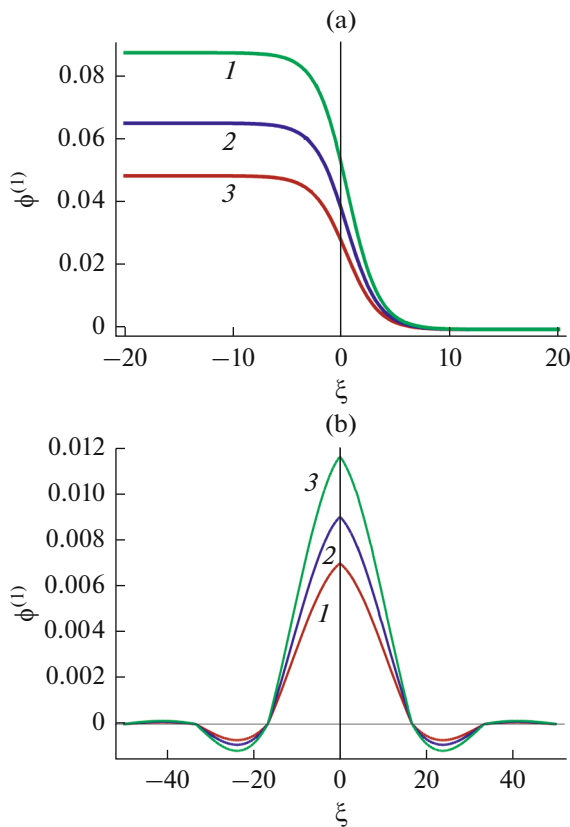


Fig. 4. Variation of shock (a) and solitary (b) waves with ξ for different values of v : (1) 0, (2) 1, (3) 2.

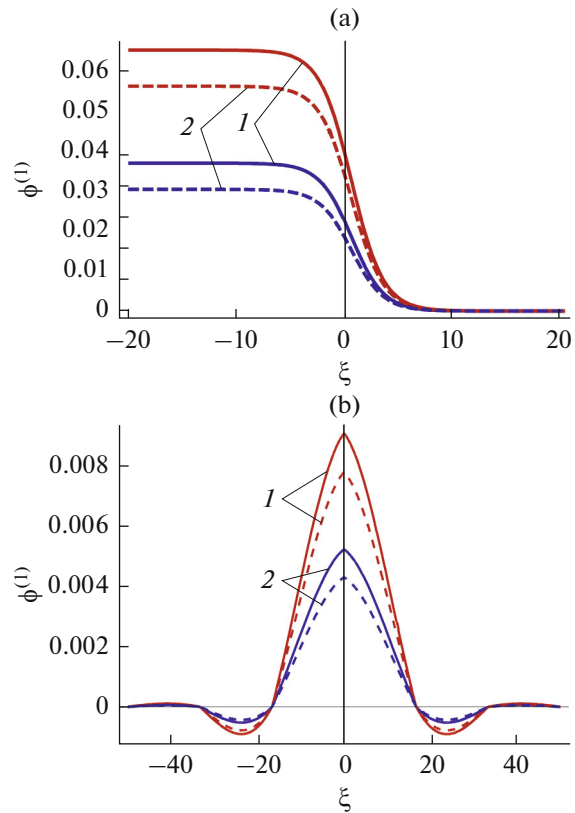


Fig. 6. Variation of cylindrical (a) and spherical (b) solitary waves with ξ for different values of Z_h ; the solid curves represent the non-relativistic case and the dashed curves represent the ultra-relativistic case; (1) $Z_h = 1$ and (2) $Z_h = 5$.

Table 2. Approximate ranges of the plasma parameters used in this investigation [30, 31]

Parameter	Parameter range
β	0.1–0.5
λ	0.1–0.7
μ_h	0.1–0.45
η	0.1–0.8
u_0	0.1–1

CONCLUSIONS

A rigorous theoretical investigation of the nonlinear propagation of nonplanar MIA shock and solitary structures in an unmagnetized, collisionless, strongly coupled, degenerate plasma was conducted. We derived the modified Burgers and KdV equations by using the reductive perturbation method and numerically analyzed their shock and solitary profiles. It was observed that the plasma system under consideration supports MIA shock and solitary structures, whose basic properties were found to be significantly modified due to the plasma particles number densities. Our results also shown how the presence of the ions of the heavier elements (C or O instead of He) can modify the basic features of MIA waves.

The relativistic effects (ultra-relativistic and non-relativistic) of electrons and positron fluids, electron-to-ion number density positron-to-ion number density, and heavy ion-to-ion number density ratios, heavy ions charge states, nonplanar geometry, and degenerate pressure significantly influence the basic properties (amplitude, width) of the MIA waves. Since, in many astrophysical situations there are extremely dense plasma and finite amplitude MIA waves, we propose to develop a theory for the propagation of MIA waves and arbitrary relativistic plasma medium through a generalization of our present work to such kind of waves and modes. We finally hope that our present investigation will be very much helpful for understanding the basic features of the localized electrostatic disturbances in a strongly coupled relativistic degenerate electron-positron-ion plasma, which occurs in some astrophysical compact objects, e.g. non-rotating white dwarf stars, neutron stars, etc. [55–57].

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