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PLASMA INVESTIGATIONS

The Longitudinal Electric Current in Maxwellian Collisional Plasma Generated by a Transverse Electromagnetic Wave

A. V. Latyshev† and A. A. Yushkanov*

Moscow State Regional University, Moscow, Russia

**e-mail: yushkanov@inbox.ru*

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Abstract—Allowance for nonlinearity leads to the appearance of the longitudinal electric current directed along a wave vector. This longitudinal current is orthogonal to the known transverse classical current at linear analysis. The kinetic Vlasov equation for collisional Maxwellian plasma is used upon the determination of the longitudinal electric current. The Bhatnagar–Gross–Krook collision integral is applied. The electron distribution function is taken from the Vlasov equation in the approximation quadratic over an electromagnetic field. The formula for the calculation of the electric current is derived. When the collision frequency tends to zero, all results for collisional plasma transfer into a corresponding known formula for collisionless plasma. The case of small wave numbers is considered. The value of the longitudinal current when the collision frequency tends to zero also transfers into the known expression for the current in collisionless plasma. The dependence of the dimensionless current on the wave number, frequency of electromagnetic field oscillations, and the collision frequency of electrons with plasma particles is studied.

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INTRODUCTION

In this work, we derive formulas for the calculation of the electric current in Maxwellian collisional plasma. The Vlasov equation with the Bhatnagar–Gross–Krook (BGK) collision integral is applied [1–4]. Quantities proportional to the square of the external electric field strength are taken into account when solving the kinetic Vlasov equation describing the plasma behavior, and quadratic expansions of the distribution function and collision integral are used.

It occurred at such a nonlinear approach that the electric current has two nonzero components. One component is directed along the electric field strength and is proportional to it. It is exactly the same as in the linear analysis and is described by the known expression for the transverse electric current.

The second nonzero component of the electric current has the second order of smallness with respect to the electric field strength and is directed along the wave vector perpendicularly to the electric field. This is the "longitudinal" current orthogonal to the first component. The generation of the longitudinal current by the transverse electromagnetic field in plasma is revealed by the nonlinear analysis of the interaction between the electromagnetic field and plasma.

The analytical solution of the Vlasov equation with the BGK collision integral is found in [4] for the problem of plasma oscillations. Nonlinear effects in plasma have been studied for a long time [5–15] (see also review [16]). We note that nonlinear effects manifested in the generation of the second harmonic were studied in [17]. In [6] the linear current was considered, in particular, in connection with probabilities of decay processes. The existence of the nonlinear current along the wave vector was noted in [7] (formula (2.9) in [7]). The generation of the longitudinal current by the transverse electromagnetic field in classical and quantum Fermi–Dirac plasma was studied in [18], and that in collisionless Maxwellian plasma was studied in [19].

In this work, we derived formulas for the calculation of the longitudinal current generated by the transverse electromagnetic field in Maxwellian collisional plasma.

SOLUTION OF THE VLASOV EQUATION

We show that, in Maxwellian plasma described by the Vlasov equation, the longitudinal current is generated by the transverse electric field, and we calculate its density. The existence of this current was indicated more than half a century ago [7].

Further we consider the case of weakly ionized plasma. It is possible to ignore electron-electron collisions [20–22]. The way in which to account for electron-electron collisions on the plasma kinetics was considered in [23, 24]. The Vlasov equation describing the behavior of collisional plasma with the BGK collision integral has the form

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + e\left(\mathbf{E} + \frac{1}{c}[\mathbf{v},\mathbf{H}]\right)\frac{\partial f}{\partial \mathbf{p}} = \nu\left(f^{(0)} - f\right). \quad (1)$$

† Deceased.

Here, f is the electron distribution function of plasma, **E**, **H** are components of the electromagnetic field, c is the velocity of light, $\mathbf{p} = m\mathbf{v}$ is the momentum of electrons, **v** is their velocity, ν is the effective collision frequency of electrons with plasma particles, and $f^{(0)} = f_{\rm eq}(\mathbf{r}, \mathbf{v})$ (eq—equilibrium) is the locally equilibrium Maxwellian distribution function:

$$f_{\rm eq}(\mathbf{r}, v) = v_{\rm T}^{-3}\pi^{-\frac{3}{2}}N(\mathbf{r}, t)\exp\left(-\frac{\varepsilon}{k_{\rm B}}\right),$$

where $\varepsilon = \frac{mv^2}{2}$ is the energy of electrons, $k_{\rm B}$ is the Boltzmann constant, $\mathbf{P} = \mathbf{p}/p_{\rm T} = \mathbf{v}/v_{\rm T}$ is the dimensionless momentum (velocity) of electrons, $p_{\rm T} = mv_{\rm T}$, $v_{\rm T}$ is the thermal velocity of electrons, and $v_{\rm T} = \sqrt{2k_{\rm B}T/m}$, $k_{\rm B}T = \varepsilon_{\rm T} = mv_{\rm T}^2/2$ is the thermal energy of electrons, T is the plasma temperature. The equilibrium distribution function is normalized to the numerical density:

$$\int f_{\rm eq}(\mathbf{r}, v)d^3v = N(\mathbf{r}, t).$$

We consider that there is an electromagnetic field in plasma, which is a traveling harmonic wave: $\mathbf{E} = \mathbf{E_0}e^{i(\mathbf{kr}-\omega t)}$, $\mathbf{H} = \mathbf{H_0}e^{i(\mathbf{kr}-\omega t)}$. The electric and magnetic fields can be expressed in terms of the vector potential **A** by the following equalities:

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{H} = \mathrm{rot}\mathbf{A}.$$

For certainty, we consider that the wave vector is directed along the x axis, and, the electric field, along the y axis, i.e., $\mathbf{k} = k(1, 0, 0)$, $\mathbf{E} = E_y(x, t)(0, 1, 0)$. Consequently,

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} = \frac{i\omega}{c}\mathbf{A}, \quad \mathbf{H} = \frac{ck}{\omega}E_y(0,0,1),$$

$$[\mathbf{v}, \mathbf{H}] = \frac{ck}{\omega}E_y(v_y, -v_x, 0), \quad e\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}, \mathbf{H}]\right)\frac{\partial f}{\partial \mathbf{p}}$$
$$= \frac{e}{\omega}E_y\left[kv_y\frac{\partial f}{\partial p_x} + (\omega - kv_x)\frac{\partial f}{\partial p_y}\right],$$

and also $[\mathbf{v}, \mathbf{H}] = \partial f/\partial \mathbf{p} = 0$, since $\partial f/\partial \mathbf{p} \sim v$.

We consider the linearization of the locally equilibrium distribution function

$$f_{\rm eq}(P, x) = f_0(P) + f_0(P)\frac{\delta N}{N},$$

where

$$f_0(P) = v_{\rm T}^{-3}\pi^{-\frac{3}{2}}N\exp\left(-\frac{\varepsilon}{k_{\rm B}}\right) = \frac{N}{v_{\rm T}^3\pi^{3/2}}e^{-p^2},$$
$$N(x, t) = N + \delta N(x, t), \quad N = \mathrm{const}.$$

Equation (1) can be rewritten in the form

$$\frac{\partial f}{\partial t} + v_x\frac{\partial f}{\partial x} + \frac{eE_y}{\omega}\left[kv_x\frac{\partial f}{\partial p_x} + (\omega - kv_x)\frac{\partial f}{\partial p_y}\right]$$
$$+ \nu f = \nu f_0(P) + \nu\frac{\delta N}{N}f_0(P), \tag{2}$$

and the value $\delta N/N$ is found from the law of conservation of the number of particles

$$\int(f_{\rm eq} - f)d^3v = 0.$$

From this conservation law, we find

$$\frac{\delta N}{N}\int f_0(P)d^3v = \int(f - f_0(P))d^3v,$$

and we find

$$\delta N = \int(f - f_0(P))d^3v = v_{\rm T}^3\int(f - f_0(P))d^3P.$$

Equation (2) can be transformed now to the integral equation

$$\frac{\partial f}{\partial t} + v_x\frac{\partial f}{\partial x} + \nu f = \nu f_0(P)$$
$$-\frac{eE_y}{\omega}\left[kv_y\frac{\partial f}{\partial p_x} + (\omega - kv_x)\frac{\partial f}{\partial p_y}\right] \tag{3}$$
$$+ \nu f_0(P)\frac{v_{\rm T}^3}{N}\int[f - f_0(P)]d^3P.$$

We search for solution to (3) in the form

$$f = f_0(P) + f_1 + f_2, \tag{4}$$

where

$$f_1 \sim E_y \sim e^{i(kx-\omega t)}, \quad f_2 \sim E_y^2 \sim e^{2i(kx-\omega t)}.$$

We shall operate by the method of successive approximations considering the value of the electric field strength as a small parameter. It is possible to rewrite Eq. (3) using (4) in the form

$$\frac{\partial(f_1 + f_2)}{\partial t} + v_x\frac{\partial(f_1 + f_2)}{\partial x} + \nu(f_1 + f_2) = -\frac{eE_y}{\omega}$$
$$\times\left[kv_y\frac{\partial(f_0(P) + f_1)}{\partial p_x} + (\omega - kv_x)\frac{\partial(f_0(P) + f_1)}{\partial p_y}\right] \tag{5}$$
$$+ \nu f_0(P)\frac{v_{\rm T}^3}{N}\int[f_1 + f_2]d^3P.$$

Equation (5) is equivalent to the following two equations:

$$\frac{\partial f_1}{\partial t} + v_x\frac{\partial f_1}{\partial x} + \nu f_1$$
$$= -\frac{eE_y}{\omega}\left[kv_y\frac{\partial f_0(P)}{\partial p_x} + (\omega - kv_x)\frac{\partial f_0(P)}{\partial p_y}\right] \tag{6}$$
$$+ \nu f_0(P)\frac{v_{\rm T}^3}{N}\int f_1 d^3P,$$

$$\frac{\partial f_2}{\partial t}+v_x\frac{\partial f_2}{\partial x}+\nu f_2$$
$$=-\frac{eE_y}{\omega}\left[kv_y\frac{\partial f_1(P)}{\partial p_x}+(\omega-kv_x)\frac{\partial f_1(P)}{\partial p_y}\right] \quad (7)$$
$$+\nu f_0(P)\frac{v_T^3}{N}\int f_2 d^3P.$$

From Eq. (6), we find

$$(\nu-i\omega+ikv_x)f_1=-\frac{eE_y}{\omega}$$
$$\times\left[kv_y\frac{\partial f_0(P)}{\partial p_x}+(\omega-kv_x)\frac{\partial f_0(P)}{\partial p_y}\right] \quad (8)$$
$$+\nu f_0(P)\frac{v_T^3}{N}A_1,$$

where

$$A_1=\int f_1 d^3P.$$

We introduce dimensionless parameters

$$\Omega=\frac{\omega}{k_Tv_T},\quad y=\frac{\nu}{k_Tv_T},\quad q=\frac{k}{k_T}.$$

Here, q is the dimensionless wave number, $k_T = mv_T/\hbar$ is the thermal wave number, and Ω is the dimensionless frequency of the electromagnetic field oscillations.

We transfer to dimensionless parameters in Eq. (8)

$$i(qP_x-z)f_1=-\frac{eE_y}{\Omega k_Tp_Tv_T}$$
$$\times\left[qP_y\frac{\partial f_0(P)}{\partial P_x}+(\Omega-qP_x)\frac{\partial f_0(P)}{\partial P_y}\right]+yf_0(P)\frac{v_T^3}{N}A_1, \quad (9)$$

where

$$z=\Omega+iy=\frac{\omega+iy}{k_Tv_T}.$$

We note that

$$\frac{\partial f_0(P)}{\partial P_x}\sim P_x,\quad \frac{\partial f_0(P)}{\partial P_y}\sim P_y.$$

Consequently,

$$\left[qP_y\frac{\partial f_0(P)}{\partial P_x}+(\Omega-qP_x)\frac{\partial f_0(P)}{\partial P_y}\right]=\Omega\frac{\partial f_0(P)}{\partial P_y}.$$

Now, from Eq. (9), we find

$$f_1=\frac{ieE_y}{k_Tp_Tv_T}\frac{\partial f_0/\partial P_y}{qP_x-z}-iy\frac{v_m^3}{N}\frac{f_0(P)}{qP_x-z}A_1. \quad (10)$$

Substituting (10) in Eq. (9), we find the equality

$$A_1\left(1+iy\frac{v_T^3}{N}\int\frac{f_0(P)d^3P}{qP_x-z}\right)$$
$$=\frac{ieE_y}{k_Tp_Tv_T}\int\frac{\partial f_0(P)/\partial P_y}{qP_x-z}d^3P.$$

It is easy to see that the integral in the right-hand side of this equality is zero. Consequently, $A_1 = 0$. Then, according to (10), the function f_1 is determined as

$$f_1=\frac{ieE_y}{k_Tp_Tv_T}\frac{\partial f_0/\partial P_y}{qP_x-z}. \quad (11)$$

We substitute (11) into (7) and find

$$(\nu-2i\omega+2ikv_x)f_2=\frac{ie^2E_y^2}{k_Tp_Tv_T\omega}$$
$$\times\left[kv_y\frac{\partial}{\partial p_x}\left(\frac{\partial f_0(P)/\partial P_y}{qP_x-z}\right)\right.$$
$$\left.+(\omega-kv_x)\frac{\partial}{\partial p_y}\left(\frac{\partial f_0(P)/\partial P_y}{qP_x-z}\right)\right]$$
$$+\nu f_0(P)\frac{v_T^3}{N}A_2,$$

where

$$A_2=\int f_2 d^3P. \quad (12)$$

We transfer to dimensionless variables in this equation

$$2i\left(qP_x-x-\frac{iy}{2}\right)f_2=-\frac{ie^2E_y^2}{\Omega k_T^2p_T^2v_T^2}$$
$$\times\left[qP_x\frac{\partial}{\partial y}\left(\frac{\partial f_0(P)/\partial P_y}{qP_x-z}\right)+(\Omega-qP_x)\right. \quad (13)$$
$$\left.\times\frac{\partial}{\partial y}\left(\frac{\partial f_0(P)/\partial P_y}{qP_x-z}\right)\right]+yf_0(P)\frac{v_T^3}{N}A_2.$$

We denote

$$z'=\Omega+\frac{iy}{2}=\frac{\omega}{k_Tv_T}+i\frac{\nu}{2k_Tv_T}=\frac{\omega+i\nu/2}{k_Tv_T}.$$

We find from (13)

$$f_2=-\frac{e^2E_y^2}{k_T^2p_T^2v_T^2\Omega}\left[qP_y\frac{\partial}{\partial P_x}\left(\frac{\partial f_0(P)/\partial P_y}{qP_x-z}\right)\right.$$
$$\left.+\frac{\Omega-qP_x}{qP_x-z}\frac{\partial^2 f_0}{\partial P_y^2}\right]\frac{1}{qP_x-z'}-\frac{iy}{2}\frac{v_T^3}{N}\frac{f_0(P)}{qP_x-z'}A_2. \quad (14)$$

Substituting (14) into (12), we find

$$A_2=-\frac{e^2E_y^2}{k_T^2p_T^2v_T^2\Omega}\frac{J_1}{1+\frac{iy}{2}\frac{v_T^3}{N}J_0}.$$

Here,

$$J_0 = \int \frac{f_0(P)\,d^3P}{qP_x - z'}, \quad J_1 = \int \left[qP_y \frac{\partial}{\partial P_x} \left(\frac{\partial f_0(P)/\partial P_y}{qP_x - z} \right) + \frac{\Omega - qP_x}{qP_x - z} \frac{\partial^2 f_0}{\partial P_y^2} \right] \frac{d^3P}{qP_x - z'}.$$

We substitute A_2 into (14) and find f_2 in the explicit form

$$f_2 = -\frac{e^2E_y^2}{k_T^2p_T^2v_T^2\Omega} \left[qP_y \frac{\partial}{\partial P_x} \left(\frac{\partial f_0(P)/\partial P_y}{qP_x - z} \right) + \frac{\Omega - qP_x}{qP_x - z} \frac{\partial^2 f_0}{\partial P_y^2} \right] \frac{1}{qP_x - z'} + \gamma \frac{e^2E_y^2}{k_T^2p_T^2v_T^2\Omega} \frac{v_T^3}{N} \frac{f_0(P)}{qP_x - z'}. \tag{15}$$

Here,

$$\gamma = \frac{(iy/2)\,J_1}{1 + (iy/2)\left(v_T^3/N\right) J_0}. \tag{16}$$

ELECTRIC CURRENT DENSITY

We find the electric current density

$$\mathbf{j} = e\int \mathbf{v} f d^3v. \tag{17}$$

It is seen from (4) and (17) that the density current vector has two nonzero components $\mathbf{j} = (j_x, j_y, 0)$. Here, j_x, j_y are the density of longitudinal and transverse currents, respectively,

$$j_x = e\int v_x f d^3v = e\int v_x f_2 d^3v = ev_T^4 \int P_x f_1 d^3P. \tag{18}$$

$$j_y = e\int v_y f d^3v = e\int v_y f_1 d^3v = ev_T^4 \int P_y f_1 d^3P. \tag{19}$$

The transverse current is directed along the electric field, its density j_y is determined only by the first approximation of the distribution function, and the second approximation does not contribute. According to (19) and (12), we have

$$j_y = \frac{ie^2v_T^3}{k_Tp_T} E_y(x, y) \int \frac{(\partial f_0(P)/\partial P_y)\,P_y}{qP_x - z} d^3P.$$

This current is proportional to the first power of the electric field strength.

We find, for the longitudinal current density substituting (15) into (18),

$$j_x = \frac{e^3E_y^2v_T^2}{2k_T^2p_T^2\Omega} \left\{ -\int \left[qP_y \frac{\partial}{\partial P_x} \left(\frac{\partial f_0(P)/\partial P_y}{qP_x - z} \right) + \frac{\Omega - qP_x}{qP_x - z} \frac{\partial^2 f_0}{\partial P_y^2} \right] \frac{P_x d^3P}{qP_x - z'} + \gamma \frac{v_T^3}{N} \int \frac{P_x f_0(P)\,d^3P}{qP_x - z'} \right\}. \tag{20}$$

In the integral of the second term in square brackets in (20), the internal integral over P_y is zero:

$$\int_{-\infty}^{\infty} \frac{\partial^2 f_0}{\partial P_y^2} dP_y = \left. \frac{\partial f_0}{\partial P_y} \right|_{P_y=-\infty}^{P_y=+\infty} = 0.$$

In the first integral in square brackets, the internal integral over P_x is calculated in parts:

$$\int_{-\infty}^{\infty} \frac{\partial}{\partial P_y} \left(\frac{\partial f_0(P)/\partial P_y}{qP_x - z} \right) \frac{P_x dP_x}{qP_x - z'} = z' \int_{-\infty}^{\infty} \frac{\partial f_0(P)/\partial P_y}{(qP_x - z)(qP_x - z')^2} dP_x.$$

As a result, Eq. (20) is simplified:

$$j_x = \frac{e^3E_y^2v_T^2}{2k_T^2p_T^2\Omega} \times \left[-z'q \int \frac{P_y\,(\partial f_0(P)/\partial P_y)\,d^3P}{(qP_x - z)(qP_x - z')^2} + \gamma \frac{v_T^3}{N} \int \frac{P_x f_0(P)\,d^3P}{qP_x - z'} \right].$$

The internal integral over P_y is calculated in parts

$$\int_{-\infty}^{\infty} P_y \frac{\partial f_0}{\partial P_y} dP_y = P_y f_0 \Big|_{P_y=-\infty}^{P_y=+\infty} - \int_{-\infty}^{\infty} f_0(P)\,dP_y = -\int_{-\infty}^{\infty} f_0(P)\,dP_y$$

and we find the expression for the longitudinal current

$$j_x = \frac{e^3E_y^2v_T^2}{2k_T^2p_T^2\Omega} \left[z'q \int \frac{f_0(P)\,d^3P}{(qP_x - z)(qP_x - z')^2} \gamma \frac{v_T^3}{N} \int \frac{P_x f_0(P)\,d^3P}{qP_x - z'} \right]. \tag{21}$$

The first integral from (21) is

$$\int \frac{f_0(P)\,d^3P}{(qP_x - z)(qP_x - z')^2} = \int_{-\infty}^{\infty} \frac{dP_x}{(qP_x - z)(qP_x - z')^2} \times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_0(P)\,dP_y dP_z = \frac{N}{v_T^3\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-P_x^2} dP_x}{(qP_x - z)(qP_x - z')^2} = \frac{N}{v_T^3\sqrt{\pi}} J_{12},$$

where

$$J_{12} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-P_x^2} dP_x}{(qP_x - z)(qP_x - z')^2};$$

the second integral

$$\frac{v_T^3}{N}\int\frac{P_x f_0(P)\,d^3P}{qP_x - z'} = \frac{v_T^3}{N}\pi\int\frac{P_x f_0(P_x)\,dP_x}{qP_x - z'}$$
$$= \frac{1}{\sqrt{\pi}}\int\limits_{-\infty}^{\infty}\frac{e^{-\tau^2}d\tau}{q\tau - z'}.$$

We denote

$$J_{02} = \frac{1}{\sqrt{\pi}}\int\limits_{-\infty}^{\infty}\frac{e^{-\tau^2}d\tau}{q\tau - z'}.$$

Then, it is possible to present (21) in the following form:

$$j_x = \frac{e^3E_y^2v_T^2}{2k_T^2p_T^2\Omega}\left[z'q\frac{N}{v_T^3}J_{12} + \gamma J_{02}\right]. \qquad (22)$$

We return to the consideration of the quantity γ. We calculate integrals J_1 and J_0, entering (16). It was already indicated that the integral from the second term of J_1 is zero. The second integral, the same as earlier, is calculated in parts. As a result, we find

$$J_1 = q\int qP_y\frac{\partial}{\partial P_x}\left(\frac{\partial f_0(P)/\partial P_y}{qP_x - z}\right)\frac{d^3P}{qP_x - z'}$$
$$= q^2\int\frac{P_y\left(\partial f_0(P)/\partial P_y\right)d^3P}{(qP_x - z)(qP_x - z')^2}.$$

The internal integral over the variable P_y is also calculated in parts. As a result, we find the integral

$$J_1 = -q^2\int\frac{f_0(P)\,d^3P}{(qP_x - z)(qP_x - z')^2},$$

which was already calculated. Consequently,

$$J_1 = -q^2\frac{N}{v_T^3\sqrt{\pi}}\int\limits_{-\infty}^{\infty}\frac{e^{-\tau^2}d\tau}{(q\tau - z)(q\tau - z')^2} = -q^2\frac{N}{v_T^3\sqrt{\pi}}J_{12}.$$

For the second integral in (16) we have

$$J_0 = \int\frac{f_0(P)\,d^3P}{qP_x - z'} = \frac{N}{v_T^3\sqrt{\pi}}\int\limits_{-\infty}^{\infty}\frac{e^{-P_x^2}dP_x}{qP_x - z'}.$$

Hence

$$1 + \frac{iy}{2}\frac{v_T^3}{N}J_0 = 1 + \frac{iy}{2}\frac{1}{\sqrt{\pi}}\int\limits_{-\infty}^{\infty}\frac{e^{-\tau^2}d\tau}{q\tau - z'}$$
$$= \frac{1}{\sqrt{\pi}}\int\limits_{-\infty}^{\infty}e^{-\tau^2}d\tau + \frac{iy}{2}\frac{1}{\sqrt{\pi}}\int\limits_{-\infty}^{\infty}\frac{e^{-\tau^2}d\tau}{q\tau - z'}$$
$$= \frac{1}{\sqrt{\pi}}\int\limits_{-\infty}^{\infty}\frac{q\tau - \Omega}{q\tau - z'}e^{-\tau^2}d\tau.$$

Thus,

$$\gamma = -\frac{iy}{2}q^2\frac{N}{v_T^3}\frac{J_{12}}{J_{01}},$$

where

$$J_{01} = \frac{1}{\sqrt{\pi}}\int\limits_{-\infty}^{\infty}\frac{q\tau - \Omega}{q\tau - z'}e^{-\tau^2}d\tau.$$

Now formula (22) can be presented in the form

$$j_x = \frac{Ne^3E_y^2q}{2k_T^2p_T^2v_T\Omega}\left[\Omega + \frac{iy}{2} - \frac{iy}{2}q\frac{J_{02}}{J_{01}}\right]J_{12},$$

or expressing in terms of the plasma (Langmuir) frequency $\omega_p = \sqrt{4\pi e^2N/m}$:

$$j_x^{\text{long}} = \left(\frac{e\Omega_p^2}{k_T p_T}\right)\frac{kE_y^2}{8\pi\Omega}\left[\Omega + \frac{iy}{2} - \frac{iy}{2}q\frac{J_{02}}{J_{01}}\right]J_{12}, \qquad (23)$$

where

$$\Omega_p = \frac{\omega_p}{k_T v_T} = \frac{\hbar\omega_p}{mv_T^2}$$

is the dimensionless plasma frequency.

We rewrite equality (23) in the form

$$j_x^{\text{long}} = J(\Omega, y, q)\,\sigma_{l,tr}kE_y^2, \qquad (24)$$

where $\sigma_{l,tr}$ is the longitudinal-transverse conductivity, $J(\Omega, y, q)$ is the dimensionless current:

$$\sigma_{l,tr} = \frac{e\Omega_p^2}{p_T k_T} = \frac{e\hbar}{p_T^2}\left(\frac{\hbar\omega_p}{mv_T^2}\right)^2 = \frac{e}{k_T p_T}\left(\frac{\omega_p}{k_T v_T}\right)^2,$$

$$J(\Omega, y, q) = \frac{1}{8\pi\Omega}\left[\Omega + \frac{iy}{2} - \frac{iy}{2}q\frac{J_{02}}{J_{01}}\right]J_{12}.$$

We introduce the transverse electric field

$$\mathbf{E}_{tr} = \mathbf{E} - \frac{\mathbf{k}(\mathbf{kE})}{k^2} = \mathbf{E} - \frac{\mathbf{q}(\mathbf{qE})}{q^2},\ \mathbf{kE}_{tr} = \frac{\omega}{c}[\mathbf{E},\mathbf{H}].$$

Then (24) can be presented in the invariant form

$$\mathbf{j}^{\text{long}} = J(\Omega, y, q)\,\sigma_{l,tr}\mathbf{kE}_{\text{tr}}^2 = J(\Omega, y, q)\,\sigma_{l,tr}\frac{\omega}{c}[\mathbf{E},\mathbf{H}].$$

It is seen from (24) that, at $y = 0$ (or $\nu = 0$), plasma becomes collisionless ($z \to \Omega$, $z' \to \Omega$) and formula (22) coincides with the corresponding formula from [19] for collisionless plasma:

$$j_x^{\text{long}} = \sigma_{l,tr}kE_y^2\frac{1}{8\pi\sqrt{\pi}}\int\limits_{-\infty}^{\infty}\frac{e^{-\tau^2}d\tau}{(q\tau - \Omega)^3}.$$

In the case of the small wave numbers, we find from (23)

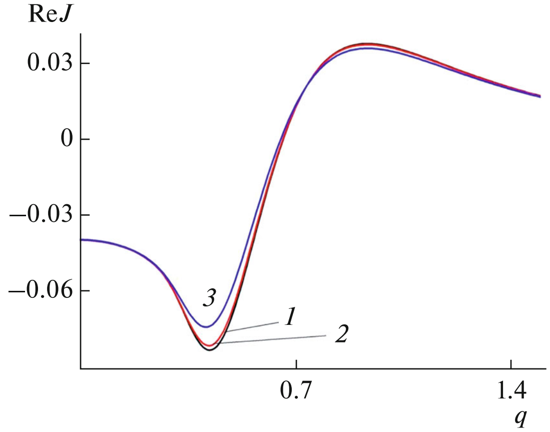

Fig. 1. Real part of the density of the dimensionless longitudinal current as a function of dimensionless wave number q at $\Omega = 1$ and three values of the dimensionless collision frequency: *1*—$y = 0.001$, *2*—0.01, *3*—0.05.

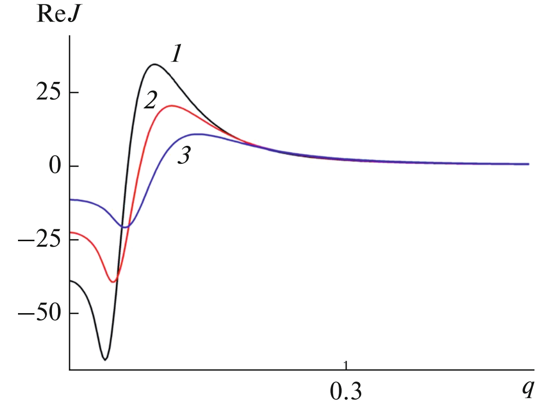

Fig. 3. Real part of the density of the dimensionless longitudinal current as a function of q at $y = 0.01$ and three values of the dimensionless electromagnetic field frequency: *1*—$\Omega = 0.1$, *2*—0.12, *3*—0.15.

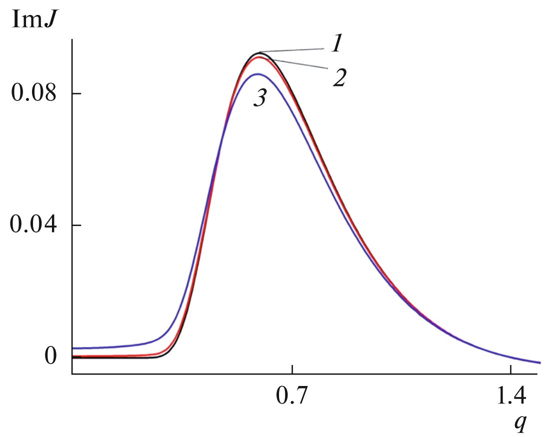

Fig. 2. Imaginary part of the density of the dimensionless longitudinal current as a function of q at Ω =1 and dimensionless collision frequencies: notations as in Fig. 1.

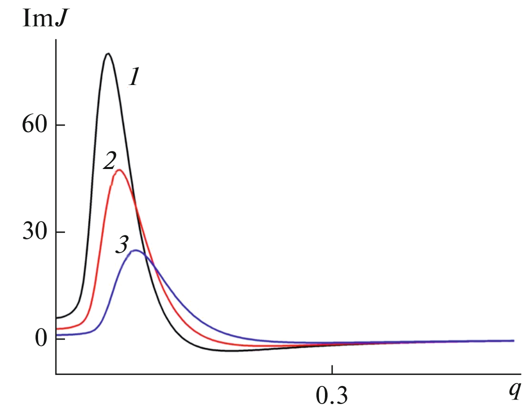

Fig. 4. Imaginary part of the density of the dimensionless longitudinal current as a function of q at $y = 0.01$: notations as in Fig. 3.

$$j_x^{\text{long}} = -\frac{\sigma_{l,tr} k E_y^2}{8\pi\Omega z z'} = -\frac{\sigma_{l,tr} k E_y^2}{8\pi\Omega(\Omega + iy)\left(\Omega + \dfrac{iy}{2}\right)}$$
$$= -\frac{e}{8\pi m\omega}\left(\frac{\omega_p}{\omega}\right)^2 \frac{k E_y^2}{\left(1 - i\dfrac{\nu}{\omega}\right)\left(1 - i\dfrac{\nu}{2\omega}\right)}.$$

At $\nu = 0$, we transfer from this formula to formula from [18] for the longitudinal current for small wave numbers in collisionless plasma.

Figures 1 and 2 show the behavior of the real and imaginary parts of the density of the dimensionless longitudinal current at $\Omega = 1$, respectively, as a function of the dimensionless wave number q at different values of the dimensionless collision frequency. At small and large values of parameter q, the real part almost terminates to depend on the frequency. First it has a minimum, then a maximum. The imaginary part of the density current has one maximum, and its frequency dependence disappears at an increase in q.

Figures 3 and 4 show dependences of the real and imaginary parts of the density of the longitudinal current on the dimensionless wave number, q, respectively, at $\Omega = 1$, $y = 0.01$ and different frequencies of the electromagnetic field. At large values of the dimensionless wave number, the frequency dependence disappears.

Figures 5 and 6 show dependences of the real and imaginary parts of the density of the longitudinal current on q at different field frequencies Ω and $y = 0.01$. At an increase in the dimensionless wave number, q, the real parts of the current density almost terminate to depend on the field frequency. For the imaginary parts, the dependence on the field frequency disappears at small and large wave numbers.

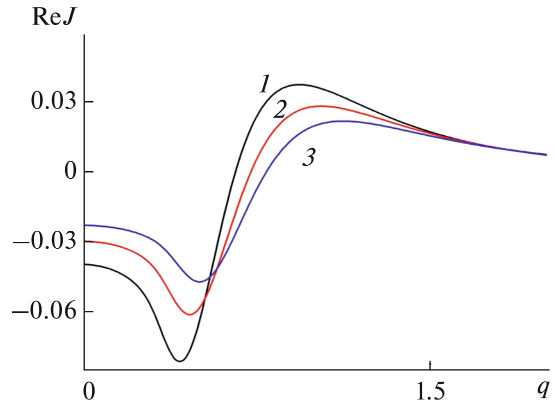

Fig. 5. Real part of the density of the dimensionless longitudinal current at $y = 0.01$: *1*—$\Omega = 1$, *2*—1.1, *3*—1.2.

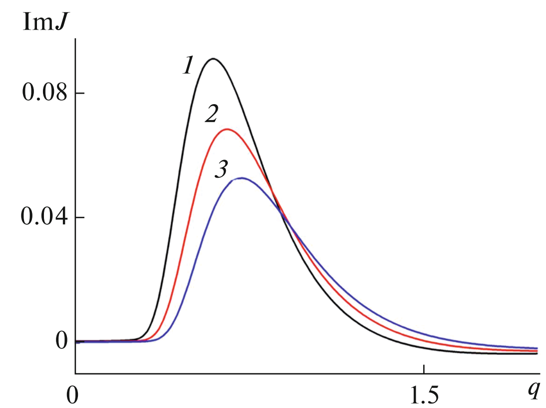

Fig. 6. Imaginary part of the density of the dimensionless longitudinal current at $y = 0.01$: notations as in Fig. 5.

CONCLUSIONS

In the work, the problem of the generation of the longitudinal electric current by the transverse electromagnetic field in Maxwellian collisional plasma was solved. The expansion of the electron distribution function over the electric field strength with the accuracy to quadratic terms was found from the solution of the kinetic Vlasov equation with the BGK collision integral. It was shown that the component of the electric current in the direction of the electric field (longitudinal current) is proportional to its strength, while the perpendicular component (transverse current) is proportional to the square of the field strength. A particular case of collisionless plasma was considered as well as the case of small wave numbers. The dependences of the current on the wave number, the electromagnetic field frequency, and the collision frequency of electrons with plasma particles were determined.

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