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Investigation of Joy's Law According to the Data of the Ussuriisk Astrophysical Observatory

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Abstract—Based on the 66-year series of observations of the Ussuriisk Astrophysical Observatory, the angles of inclination of the bipolar axes of sunspot groups (tilt angles) have been studied. It is found that the average tilt angle is 6.8° ± 0.2°, and the angles of inclination of sunspot groups in the northern hemisphere of the Sun, on average, are 1.0° larger than for the southern one. It was also found that the average tilt angles are practically independent of the maximum area of the sunspot group. The dependence of the tilt angles averaged in 5-degree latitudinal zones T_m from the heliographic latitude F in the first approximation is described by a linear function $T_m = 0.16° + 0.43|F|$. However the dependence $T_m(F)$ also demonstrates significant deviations from the linear function, which look like two almost horizontal sections ("plateaus"). One of them is located near $F = 15°$ and the second is at $F > 25°$. Dependencies $T_m(F)$ calculated for groups of sunspots with small, medium, and large areas do not reveal significant systematic differences. These results are compared with the previously published results of the study of five series of data on the tilt angles of sunspot groups.

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1. INTRODUCTION

It is well known that the preceding parts of bipolar sunspot groups, on average, are located closer to the heliographic equator than the following ones; in this case, the average inclination to the equator of the bipolar axes, the lines connecting the preceding and following parts (poles) of sunspot groups, increases with increasing heliographic latitude. These statements form the content of Joy's law, which was established for sunspot groups more than 100 years ago and then repeatedly confirmed (see, for example, (Howard, 1991; Sivaraman et al., 1999)). Subsequently, Joy's law was also extended to bipolar magnetic regions identified in solar magnetograms (Li and Ulrich, 2012; Stenflo and Kosovichev, 2012; Tlatov et al., 2013; Wang et al. 2015). However, this aspect is not considered in this study. The tilt of the axes of sunspot groups is of interest from different points of view. On the one hand, it is a parameter that carries information about the formation of active regions in the depths of the convective zone of the Sun (see, for example, (Kleeorin et al., 2020)). On the other hand, according to the well-known Babcock–Leighton dynamo theory, inclination to the equator of the axes of sunspot groups and active regions provides regeneration of the poloidal magnetic field of the Sun. At the same time, random variations in tilt angles are probably responsible for variations in the power of the solar cycle (Karak and Miesch, 2017).

To study the tilt angles of the axes of sunspot groups, several data series (catalogs) were created, which in total cover a time interval of more than a century. Thus, the series of data on the tilt angles, which were obtained from the photoheliograms of the Mount Wilson (Howard, 1991) and Kodaikanal (Sivaraman et al., 1999) observatories are well known. Later, the Debrecen observatory catalog (Baranyi, 2015) was added to them, which was created using a similar technique for estimating tilt angles. Another catalog was obtained for Mount Wilson observatory observations (Tlatova et al., 2018), but in a significantly different way than that in the work mentioned above (Howard, 1991). In addition, a series of data on the tilt angles of sunspot groups was created on the basis of the Catalogs of Solar Activity published by the Central Astronomical Observatory of the Russian Academy of Sciences (Ivanov, 2012). There are also measurements of tilt angles obtained from the historical archives of the 19th century (Senthamizh Pavai et al., 2015).

The analysis of several data series, carried out in the works cited above, qualitatively confirms Joy's law. However, there are quantitative discrepancies between the results of the analysis of different data, both in the value of the average tilt angles of sunspot groups and in relation to the type of dependence of the tilt angles on heliographic latitude. These discrepancies are probably associated with differences in the methods for determining the positions of the poles of sunspot groups. At the same time, it is difficult to say which of the methods is “more correct.” Since groups of sunspots often have a complex structure, the algorithm for calculating their poles and axes can hardly be specified in an indisputable and unique way. In addition, most of the available data on the tilt angles of sunspot groups were obtained without taking the measurements of sunspot magnetic polarities into account. It should also be noted that in a number of works a search was made for a relationship between the average tilt angles of sunspot groups and the power of the solar cycle; however, the results obtained in this direction do not always agree with each other (see (Dasi-Espuig et al., 2010; Ivanov, 2012; Jiao et al., 2021)).

Monitoring of solar activity has been carried out at the Ussuriisk Astrophysical Observatory (hereinafter referred to as the UAO) of the IAA of the Russian Academy of Sciences since the mid-1950s. In this case, the positions of sunspots on photoheliograms are measured, which makes it possible, in particular, to determine the angles of inclination of the bipolar axes of sunspot groups. The aim of this work was to study the qualitative and quantitative characteristics of the tilt angles of sunspot groups based on the 66-year UAO data series and to compare the obtained results with the results of the analysis of several other data series described in the literature.

2. DATA

In this paper, we consider the parameters of sunspot groups measured from the photoheliograms of the Ussuriysk Astrophysical Observatory. These data were obtained in the period from 1956 to 2021, which covers six cycles of solar activity. Until 2001, photoheliograms were recorded on photographic plates, and later, on digital cameras. When switching to digital cameras and computer processing of photoheliograms, the algorithms for measuring the parameters of sunspot groups were retained as far as possible so that the homogeneity of the data series was not disturbed.

During the routine processing of photoheliograms, the area of each sunspot group (including sunspot penumbraes and pores) and the area of its largest sunspot were measured daily. In addition, the total number of sunspots in a group was counted and the heliographic (Carrington) coordinates of one or several individual sunspots were measured. When groups with a pronounced bipolar structure were measured coordinates are taken of the two largest sunspots located in the preceding and following parts of the group. In complex groups, the coordinates of a larger number (three or more) sunspots were measured, which were chosen in such a way that their positions outlined the sunspot group in the latitudinal and longitudinal directions.

Spot coordinate measurements were used to determine the positions of the preceding (*p*) and following (*f*) poles of multispot groups. For groups with a pronounced bipolar structure, there were measurements of the coordinates of only two spots, and these measurements were taken as coordinates of the *p*- and *f*-poles of the group. In cases of more complex groups, when there were measurements of the coordinates of three or more sunspots, the positions of the poles were found as follows. First, the central longitude of the group was determined as half the sum of the longitudes of the easternmost and westernmost of the sunspots. Then, the coordinates of the *p*- and *f*-poles of the group were determined by a simple (without weighting) averaging of the measured coordinates of sunspots located respectively to the west and east of the central longitude.

The daily coordinates of the poles of each sunspot group, as obtained by the method described above, were then averaged over the period of passage of this group across the solar disk. At the same time, groups of sunspots for which the correctness of determining the positions of the poles was in serious doubt were excluded from consideration. First, those groups that were observed only near the limb, where perspective distortions are large, were excluded. Second, the data series did not include small groups of sunspots, the maximum areas of which did not reach 20 MH. In addition, some large groups with a complex structure are excluded, for which a somewhat correct selection *p*- and *f*-poles seemed difficult.

As a result of the procedure described above, a list was obtained containing the average for the passage of the coordinates of the poles of 8732 sunspot groups. In addition to the coordinates of the poles, the list included the time of observation of each of the sunspot groups, as well as its maximum measured area A. The parameter A is taken as a characteristic of the power of the magnetic bipole, which is associated with a group of spots. The average latitude F_C was calculated from the coordinates of the poles of each group, as well as the distance between its poles (pole separation) D and tilt angle of the bipolar axis T:

$$D = (\Delta F^2 + \Delta L^2 \cos F_C^2)^{1/2}, \tag{1}$$

$$\tan T = \pm \frac{\Delta F}{\Delta L \cos F_C}, \tag{2}$$

where ΔF is the latitude difference between the *p*- and *f*-poles of the spot group, and ΔL is the difference between their longitudes. The sign on the right side of (2) was chosen according to the following rule: $T > 0$ or $T < 0$ if the *p*- or *f*-pole of a group of spots is located closer to the equator, respectively. We will express parameters D and T in degrees and A, in millionths of hemisphere (MH).

3. RESULTS

We have considered the distributions of sunspot groups according to the angles of inclination of the axes for 5-degree latitude zones. As a result, it was

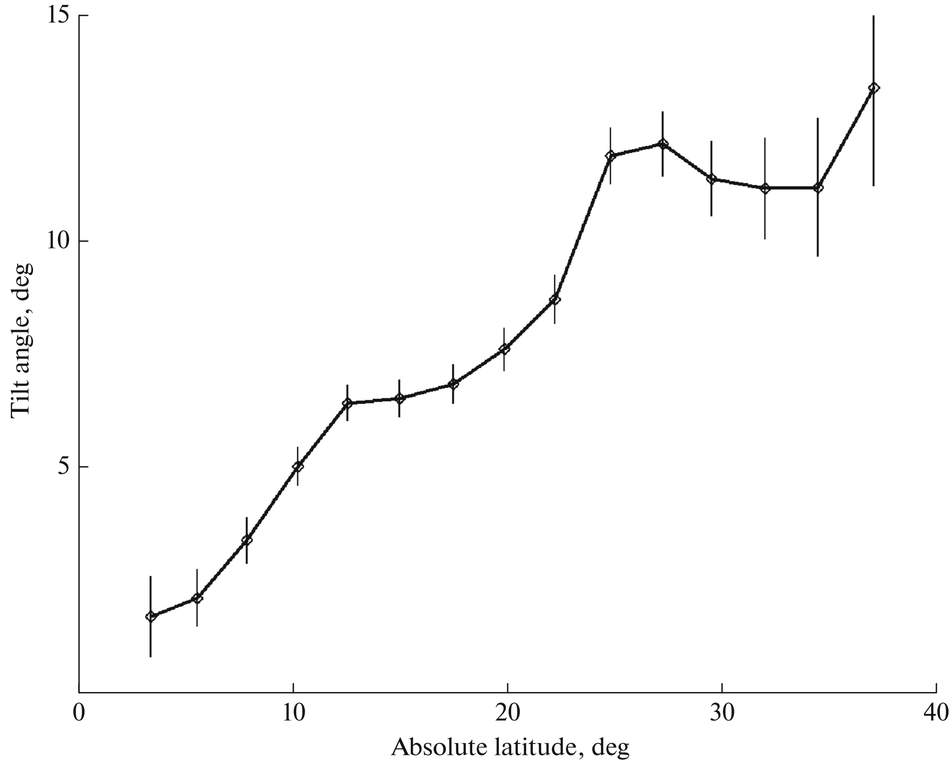

Fig. 1. The dependence of the mean tilt angles of sunspot groups on heliographic latitude. Vertical dashes show probable errors in the means.

found that at least at latitudes below 30° where the majority of sunspot groups are located, these distributions have slight asymmetry and the mean and median values of the tilt angles do not have noticeable systematic differences. Thus, the average values of tilt angles are statistically stable estimates. It should be noted that the tilt angles of the axes of sunspot groups have a large random scatter due to the physical nature of sunspots, and not just random measurement errors (this circumstance has been repeatedly noted by a number of researchers). According to the UAO, the standard deviation of tilt angles is 14° for large groups with $A > 400$ MH and increases to 22° for small groups from $20 < A < 100$ MH.

The tilt angle averaged over the entire sample of sunspot groups is 6.8° ± 0.2°. We also calculated the mean values of the tilt angle separately for the samples of sunspot groups in the northern and southern hemispheres of the Sun. It turned out that in the northern hemisphere the angles of inclination are, on average, 1.0° ± 0.3° higher than in the south.

Figure 1 shows the dependence of the average values of the tilt angle T_m from the heliographic latitude F (in fact, from the absolute value of the latitude, since in this case the groups of sunspots of the northern and southern hemispheres of the Sun are not separated). Estimations of T_m were made for latitude zones of 5°, which were sequentially shifted with a step of 2.5°. The graph in Fig. 1 shows the increase in T_m with latitude, which is in qualitative agreement with Joy's law. At latitudes below 30° the dependence $T_m(F)$ in the first approximation can be expressed by a linear function:

$$T_m = (0.16° \pm 0.50°) + (0.43 \pm 0.03)|F|. \quad (3)$$

On the graph in Fig. 1 are also clearly visible systematic deviations of the dependence $T_m(F)$ from a linear function, which look like two almost horizontal sections ("plateaus"). One of these sites is located at latitudes above 25° and it is difficult to determine the position of its upper boundary, due to the rapid drop in the number of sunspot groups at latitudes above 30° and a corresponding increase in errors in estimates of T_m. The second "plateau" occupies an interval of 8°–10° centered near $F = 15°$. This area contains the maximum distribution of sunspot groups in latitude, so the statistical errors T_m are relatively small and the "plateau" is quite distinct.

Figure 2 shows the dependencies $T_m(F)$ calculated separately for groups of sunspots in the northern and southern hemispheres of the Sun. Their comparison shows that in the northern hemisphere inclination of the axes of sunspot groups are systematically greater than in the southern hemisphere. This agrees with the above difference by 1° between average values of the tilt angles of sunspot groups in the northern and southern hemispheres. We note that the graphs of

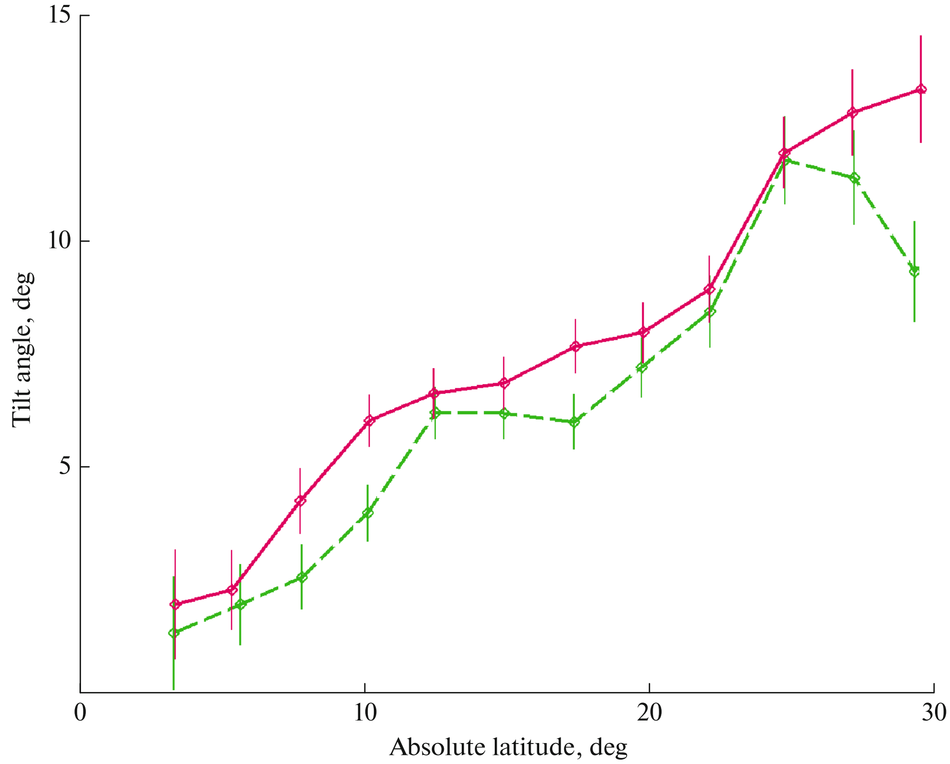

Fig. 2. The dependences of the mean tilt angles of sunspot groups on heliolatitude for the northern (solid red line) and southern (green dashed line) hemispheres of the Sun.

$T_m(F)$ for both hemispheres of the Sun show the presence of a "plateau" in the vicinity of 15° latitude (See Fig. 2).

Figure 3 shows the dependences of the mean inclination angle on latitude for groups of sunspots with small, medium, and large maximum areas (respectively, $20 < A < 100$, $100 < A < 400$ and $A > 400$ MH). As can be concluded from Fig. 3, the $T_m(F)$ dependencies for groups with different areas do not have statistically significant systematic differences, at least at latitudes below ≈27° (the situation at higher latitudes is not clear due to the increase in statistical errors in the T_m estimates). In particular, all $T_m(F)$ dependencies shown in Fig. 3 demonstrate the presence of a "plateau" in the vicinity of 15° latitude, which confirms the real existence of this feature in the latitudinal dependence of the tilt angle. Figure 4 shows the average values of the tilt angle as a function of the maximum area of the sunspot group A. This graph also shows that there is no dependence of the tilt angles on the area of the sunspot group, or it is weak and unreliable. Although the graph in Fig. 4 shows a tendency to some increase in the average tilt angles for the largest sunspot groups this variation does not exceed two standard errors.

In addition to the tilt angle, we also considered the parameter D is the distance between the poles of sunspot groups (pole separation). According to the UAO data, no mutual dependence was found between the angles of inclination of sunspot groups and pole separation. (Earlier, a weak relationship between these parameters was found by Howard (1993) based on Mount Wilson data). At the same time, the average D and T values demonstrate the opposite behavior in some ways. Thus, the average distance between the poles increases significantly with increasing areas of sunspot groups A (see Fig. 4), in contrast to the average tilt angle. The values of D averaged over the 5-degree latitude zones do not show a significant dependence on heliographic latitude, which also differs from the behavior of T. An opposite behavior of the pole separation D and tilts T of the axes of sunspot groups apparently also manifests itself in long-term variations of these parameters. On Fig. 5 showing values D and T, obtained by averaging for each cycle of solar activity, from the 19th to the 24th. As can be seen in Fig. 5, the averages of D show long-term variation with a characteristic time of the order of four solar cycles or more. Although the amplitude of this variation is relatively small (the deviation from the average value over six cycles is about 9%), it far exceeds the probable errors of averages for one cycle of D. Figure 5 also shows that the cycle-averaged T changes almost exactly in antiphase with the change in D (the correlation coefficient between them is –0.87). However, from a formal statistical point of view, the latter conclusion cannot be

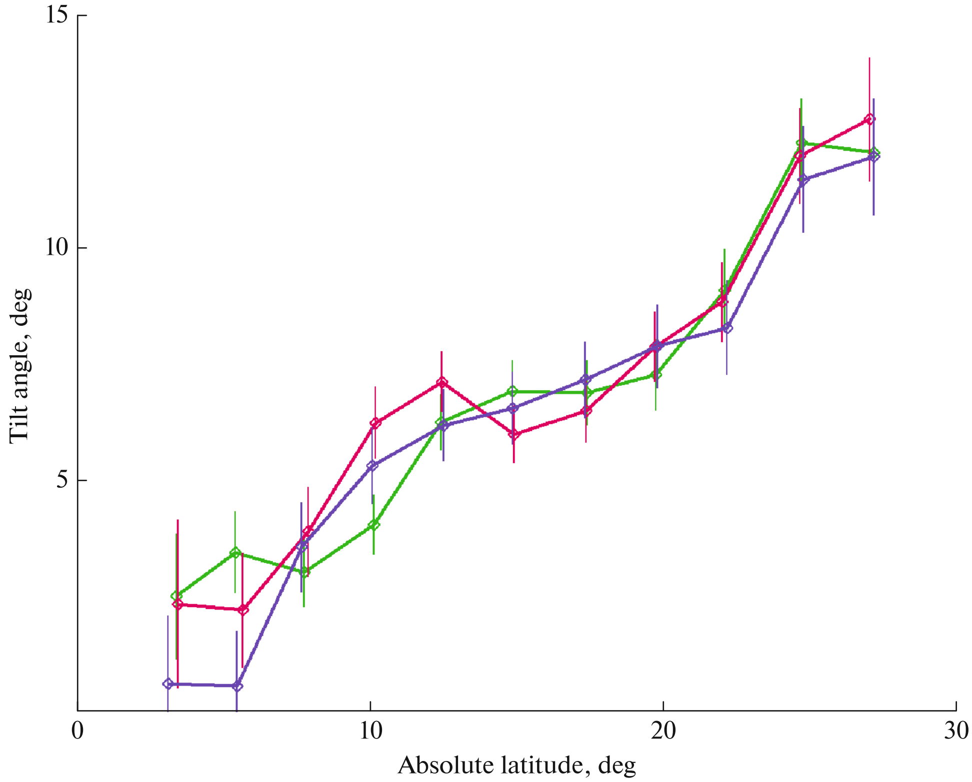

Fig. 3. Dependences of the mean tilt angles of sunspot groups on heliolatitude for groups with a maximum area of $20 < A < 100$ MH (blue line), $100 < A < 400$ MH (green line) and $A > 400$ MH (red line).

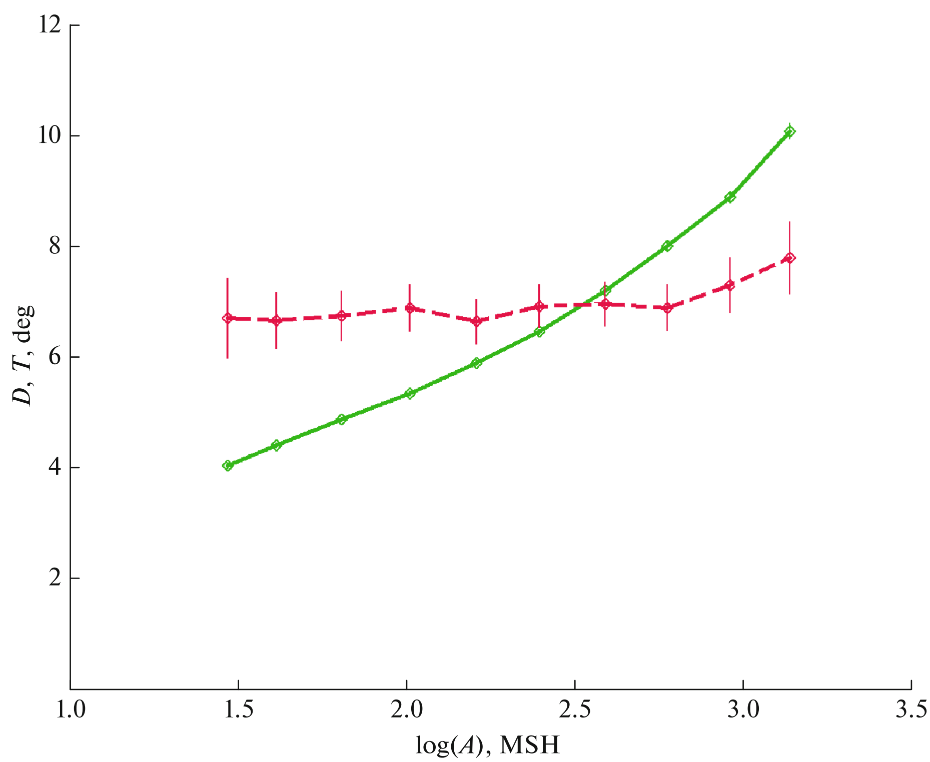

Fig. 4. The average pole separation values D (solid green line) and tilt angle T (red dashed line), depending on the maximum area of the spot group A. Vertical dashes show probable errors in the means.

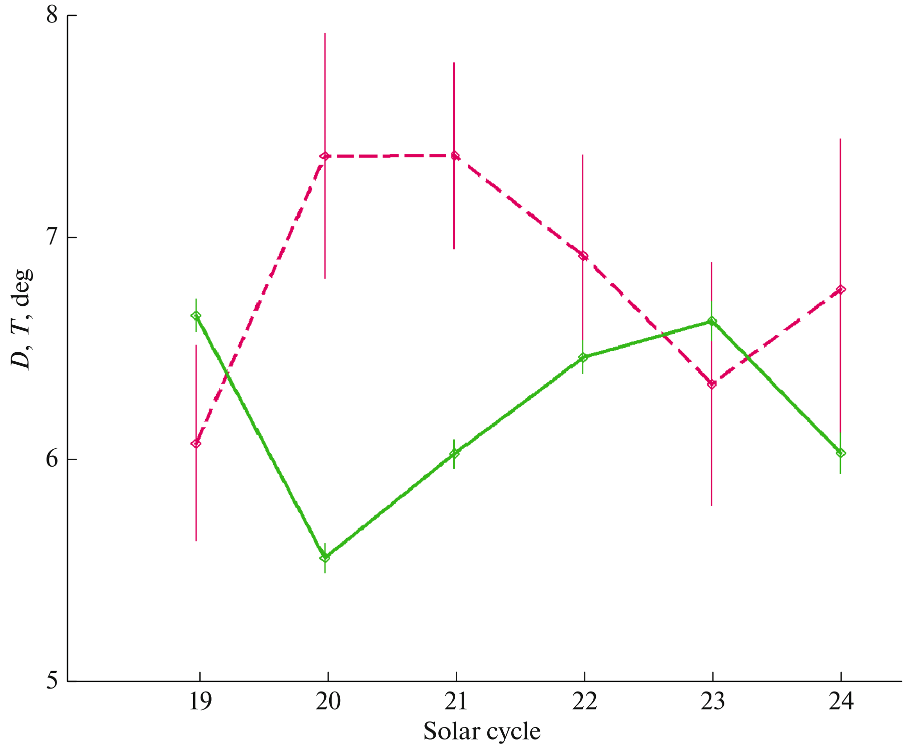

Fig. 5. Long-term variations in solar-cycle average pole separation values *D* (solid green line) and tilt angle *T* (red dashed line) of groups of spots. Vertical dashes show probable errors in the means.

considered well substantiated, since the probable errors in the estimates of the average values of the tilt angle are relatively large.

4. COMPARISON WITH THE RESULTS OF THE ANALYSIS OF OTHER DATA

In this section, we compare the results obtained from the UAO data with the results of the analysis of five other series of data on the tilt angles of sunspot groups described in the literature. The data series we selected for comparison were created on the basis of observations of sunspot groups in white light, although in one case information on the magnetic polarities of sunspots was also used. Two of these series were obtained from photoheliograms of the Mount Wilson Observatory for 1917–1985 (Howard, 1991) (hereinafter, this series is denoted as MW1) and the Kodaikanal Observatory for 1906–1987 (Sivaraman et al., 1999) (hereinafter KK). When creating these series, the same methods were used to determine the poles of sunspot groups and the angles of inclination of their axes. The same method was used to determine the tilt angles from the Debrecen Photoheliographic Data for 1974–2014. (Baranyi, 2015) (we will denote this data series as DPD). The fourth data series we consider on the tilt angles of sunspot groups covers the longest time interval: 1917–2016 (Tlatova et al., 2018). This series (hereinafter referred to as MW2) was obtained from Mount Wilson observations, but in a significantly different way than in (Howard, 1991): drawings of spots were used and their magnetic polarities were taken into account. The fifth series was created on the basis of the Pulkovo "Catalogues of solar activity" for 1948–1991 (Ivanov, 2012) and is henceforth referred to as CSA. A comparison of the results of the analysis of some of the data series listed above was previously carried out in (Ivanov, 2012; Barani, 2015; Wang et al., 2015).

Let us compare the average tilt angles obtained from different data (the average tilt angles according to MW2 data were estimated by us based on the table given in (Tlatova et al., 2018)). The comparison shows that the average UAO tilt angle of 6.8° is in close agreement with the MW2 data (7.1°) and somewhat worse agreement with CSA data (6.1°). At the same time, the average tilt angles according to the MW1, KK, and DPD data are much smaller (4.2°, 4.6° and 5.3° respectively). The relative closeness of the average tilt angles according to the UAO, CSA, and MW2 data, and their discrepancy with the MW1, KK, and DO data, is probably due to differences in the methods for determining the poles of sunspot groups.

As noted in Section 3, according to the UAO data, tilt angles in the northern hemisphere of the Sun are, on average, higher by 1.0° than in the southern hemisphere. Previously, a similar result was obtained when analyzing the KK series in (Sivaraman et al., 1999), where the difference in the tilt angles of the northern and southern hemispheres was 1.4°. The MW2 data also show that the average tilt of sunspot groups in the northern hemisphere is 0.8° higher than the south. Apparently, the north–south asymmetry of the tilt angles of about 1° is a real effect.

Let us now compare the dependences of the tilt angles of sunspot groups on the heliographic latitude. The slope of the linear function approximating the latitudinal dependence of tilt angles is a suitable parameter for quantitative comparison. According to the UAO data, 0.43 was obtained for this parameter, which can be considered to be in good agreement with the results of the analysis of the MW2 and CSA series (the corresponding values of the slope coefficient are 0.41 and 0.38). The data of the MW1 and KK series give significantly lower slope coefficients of 0.29, which is consistent with the lower average sunspot group tilt angles. A relatively low slope coefficient of 0.33 was also obtained for the DPD series.

When analyzing various data in the works cited above, certain deviations of the latitudinal dependence of the tilt angles from the linear function were found. In particular, a slowing of the growth of tilt angles, or even some decrease in them, was noted at latitudes above approximately 25°–30° (possibly, this property of tilt angles depends on the lifetime of sunspot groups (Nagovitsyn et al., 2021)). With regard to this feature, one can note the qualitative agreement between different series of data on the tilt angles of sunspot groups, including the UAO data. In quantitative terms, the similarity or difference in the course of the latitudinal dependences of tilt angles at latitudes above 25°–30° seems to be of little significance, since a rapid decrease in the number of sunspot groups is observed in this region, and the statistical errors in the estimates are quite large.

According to the UAO data, we found the presence of another interesting detail in the dependence of the tilt angle on the heliolatitude, an almost horizontal section ("plateau") near $F = 15°$. Since the maximum distribution of sunspot groups is located in this region of latitudes, the estimation errors are relatively small, and the "plateau" is quite distinct. The presence of this feature was found independently from samples of small, medium, and large sunspot groups, as well as from samples of sunspot groups in the northern and southern hemispheres of the Sun. The earlier existence of a "plateau" around $F = 15°$ was noted in (Baranyi, 2015), where the data series of three observatories, MW1, KK, and DPD, were studied, as obtained using the same method for determining the tilt angles. The agreement between the results of the analysis of four data series indicates the real existence of this feature in the latitudinal dependence of the tilt angles. However, this feature was not found in the analysis of the CSA series (Ivanov, 2012). The results of the MW2 data analysis presented in (Tlatova et al., 2018) do not allow one to unambiguously judge the presence or absence of a "plateau" near $F = 15°$, since the dependence of the tilt angle on the heliolatitude was calculated using large (10-degree) averaging intervals.

It can be concluded that the six data series are divided into two groups based on the relative closeness of the quantitative parameters (the average tilt angle of sunspot groups and the slope coefficient of the linear approximation of the latitudinal dependence of the tilt angles): the group with relatively large values of these parameters includes the MW2, CSA, and UAO series and the group with relatively low parameter values contains the MW1, KK, and DPD series. The data series of the second group were obtained using the same method for determining the centers and poles of sunspot groups; therefore, the relatively low values of the quantitative parameters characteristic of them are probably associated precisely with the features of the method. This is indirectly evidenced by one of the results of (Wang et al., 2015), where, according to the DPD data, it was found that the median tilt angles are significantly higher than the average values (the difference is about 1°). Since such a difference is not typical for UAO data, the agreement between UAO and DPD is better for the median values of the tilt angles than for the average values. In addition, the results of (Baranyi, 2015; Wang et al. 2015) show that filtering the MW1, KK, and DPD data by excluding sunspot groups with pole separation $D < 3°$ leads to an increase in the average tilt angles. In this case, the values of the coefficient of linear approximation of the latitudinal dependence of the tilt angles increase to 0.40–0.42, which agrees well with the MWO2, CSA, and UAO data. Exclusion of multispot groups with $D < 3°$ was justified in (Baranyi, 2015) by the fact that among these groups the majority are actually unipolar, that is, contain spots of only one polarity. We note that in the statistical sense the restriction of the sample of sunspot groups by the condition $D > 3°$ approximately corresponds to the condition $A > 20$ MH, which we used when creating the UAO series (see the $D(A)$ dependence plot in Fig. 4). Thus, a significant number of unipolar sunspot groups are implicitly excluded from the UAO data.

5. CONCLUSIONS

An analysis has been made of measurements of the tilt angles of the axes of 8732 sunspot groups observed at the Ussuriisk Astrophysical Observatory of the Institute of Applied Astronomy, Russian Academy of Sciences (UAO) in 1956–2021. As might be expected, the analysis of these data qualitatively confirmed Joy's law. In this case, the following results were obtained.

The mean over the entire sample value of the angle of inclination of the axes of sunspot groups is $6.8° \pm 0.2°$. In this case, the average tilt angle of sunspot groups in the northern hemisphere of the Sun is $1.0° \pm 0.3°$ more than in the southern one. No statistically significant relationship was found between the average tilt angles and the maximum areas of sunspot groups.

At heliographic latitudes below 30° the dependence of the average tilt angles T_m from the latitude F in the first approximation is approximated by a linear function $T_m = 0.16° + 0.43|F|$. At the same time, the $T_m(F)$ dependency shows significant deviations from the linear function, which look like two almost horizontal sections ("plateaus"). One of them is located at latitudes above 25° and the second occupies an interval of latitudes of 8°–10° centered around $F = 15°$. The $T_m(F)$ dependencies calculated separately for sunspot groups with small, medium, and large areas, do not show statistically significant differences, at least at latitudes below 30°.

The average distance between the poles of sunspot groups (pole separation) essentially depends on the maximum area of the group, but does not depend on the heliographic latitude. No correlation was found between pole separation and sunspot group tilt angles.

The results of the analysis of the UAO data are compared with the previously published results of the study of five other series of data on the tilt angles of sunspot groups. For the average tilt angles and the slope coefficients of their latitudinal dependences, good quantitative agreement was found with the results of the analysis of the MW2 series (Tlatova et al., 2018), and slightly worse agreement for the CSA series (Ivanov, 2012). Significant quantitative discrepancies between the UAO data and the MW1 (Howard, 1991), KK (Sivaraman et al., 1999), and DPD (Baranyi, 2015) data occur. The discrepancies are probably related to the peculiarities of methods for determining the poles of sunspot groups. A qualitative comparison of the dependences of tilt angles on heliolatitude showed that all six data series (including the UAO series) demonstrate a strong deviation from linearity at latitudes above 25°–30°, while the "plateau" centered around 15° latitude found in four series, that is, UAO, MW1, KK, and DPD.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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