

Kink Oscillations of Coronal Loops with a Longitudinal Electric Current

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Abstract—Oscillations of coronal loops with electric currents from one base to another are studied. The energy of longitudinal currents can be a source of energy for loop flares (Alfvén and Carlqvist, 1967). A simplified model of a loop in the form of a composite cylindrical magnetic tube containing a shell with a potential azimuthal magnetic field is considered for the theoretical study of oscillations of loops with longitudinal currents. The dispersion features of the tube natural kink oscillations are considered based on numerical calculations.

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1. INTRODUCTION

The known idea of a loop flare is based on the assumption of the existence of electric currents flowing in coronal loops from one base to another (Alfvén and Carlqvist, 1967). The concept of coronal electric currents is generally accepted and used in the study of various dynamic processes in the corona (Török et al., 2014; Stepanov and Zaitsev, 2018). It is used to describe energy release processes and particle acceleration (Zaitsev and Stepanov, 2017). The magnetohydrodynamic (MHD) theory of coronal oscillations was developed in a rather detailed linear approximation (Zaitsev and Stepanov, 1975; Edwin and Roberts, 1983). The bending oscillations of coronal loops directly observed in the ultraviolet range are one of the main objects studied in coronal seismology (Aschwanden, 2005; Stepanov et al., 2012). New observations of coronal loops by the Solar Dynamics Observatory provide a wealth of detailed information on the oscillations (Abedini, 2018).

Oscillations of loops containing longitudinal electric currents are studied in this paper. The magnetic field of such loops is in the form of twisted magnetic flux ropes that influence the nature of oscillations (Bahari and Khalvandi, 2017). The transverse distribution of the current in loops is still controversial (Török et al., 2014). While the external magnetic field surrounding the loops is directed along their axis, the longitudinal currents, if there are any, are neutralized. These are the coronal loops to be considered here, although, according to some observation data, there are also loops with an electric current that flows in only one direction (Kontogiannis, 2017).

The adopted, simplified loop model looks like a straight magnetic tube containing two surface currents with longitudinal components flowing in opposite directions, such that their sum is zero, i.e., the currents are neutralized. We previously studied the radial oscillations of this magnetic tube (Mikhalyaev and Khongorova, 2012; Khongorova et al., 2012). Its kink oscillations are studied in this work. Note that the theory of kink oscillations in the solar atmosphere, which includes the study of damping mechanisms related, in particular, to the MHD wave emission into the environment (Tsap and Kopylova, 2001), encounters difficulties, even in the thin magnetic-tube approximation (Tsap et al., 2018). Therefore, in this paper, we limit ourselves to the case in which the eigenfrequencies of the waves are real values, i.e., we consider free, sustained oscillations.

2. CORONAL LOOP MODEL

We consider a magnetic tube consisting of two coaxial, nested parts (Fig. 1). The central part, called a filament, is a homogeneous magnetic tube of radius b ; it contains a homogeneous longitudinal field and plasma density distribution, and is characterized by a constant Alfvén velocity V_i . The field structure in the external environment is similar, and the Alfvén velocity is V_e . Below, the i and e indices denote values related to the filament and external environment, respectively. In the outer part of the tube with radius a , which we call the shell, the selected azimuthal field and potential are $(0, B_0/\alpha r, 0)$ (Mikhalyaev, 2005). Here, α is the parameter defining the spatial scale, and r is the radial coordinate in the cylindrical coor-

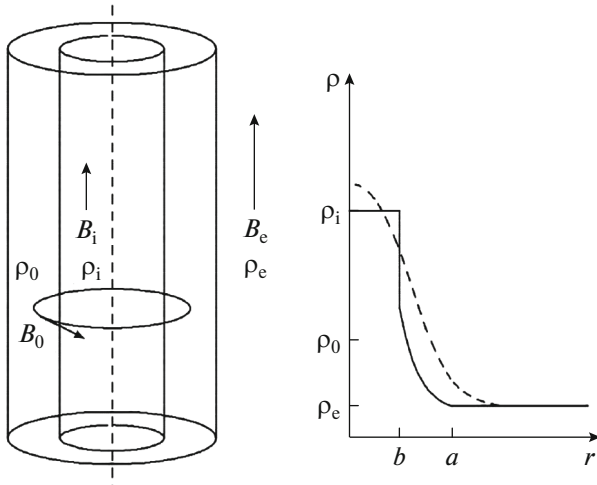


Fig. 1. General view of the composite magnetic tube (left) and radial plasma-density distribution (right).

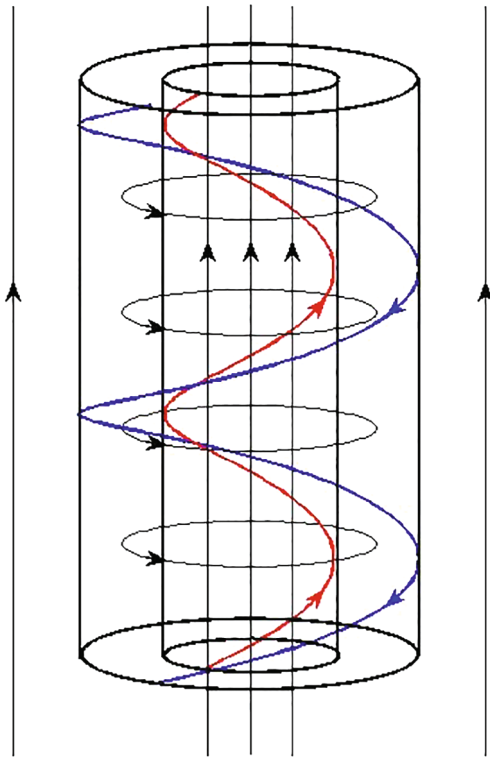


Fig. 2. Electric current flow on the surfaces that separate the filament, shell, and external environment. The electric currents lines are helically shaped.

dinate system, the Z axis of which coincides with the tube axis. We choose the plasma density in the shell, which decreases with the radius as $\rho_0/\alpha^2 r^2$. Here, the Alfvén velocity also takes a constant value denoted by V_0 (which is also used in Eq. (2)).

In coronal conditions, we disregard the gas pressure in comparison with the magnetic pressure. The

following ratios can be derived from the requirements of pressure balance at the cylindrical surface boundaries: $B_0 \alpha b = B_i = B_e \alpha a$. Discontinuities at the boundaries of the regions lead to the concentration of surface electric currents at the boundaries of these regions (Fig. 2): $j_\phi(b) = j_z(b) = c B_i / 4\pi$; $j_\phi(a) = j_z(a) = -c B_e / 4\pi$. The longitudinal components of the current have opposite directions, and the total current in the tube is zero. The considered model is used, because it provides explicit analytical solutions of the MHD equations, although magnetic tubes with diffuse current distribution would be more realistic models.

3. BENDING OSCILLATIONS OF THE CORONAL LOOP

We consider fast magnetoacoustic modes in the form of kink disturbances that correspond to an azimuthal number of $m = 1$. We are interested in the question of the existence of sustained free oscillations, which was studied earlier in detail for the case of a homogeneous magnetic tube (Zaitsev and Stepanov, 1975; Edwin and Roberts, 1983). The disturbances of the radial velocity can be written in the following form

$$v_r = V(r) \exp(ikz + i\phi - i\omega t). \tag{1}$$

The boundary conditions on the disturbed surfaces of the “filament–shell” and “shell–environment” interfaces for the velocity radial component and total pressure disturbance can be written as

$$V_i(b) = V_0(b), \quad V_e(a) = V_0(a), \tag{2}$$

$$P_i(b) = P_0(b) + \frac{B_0^2}{4\pi i \omega \alpha^2 b^3} V_0(b), \tag{3}$$

$$P_e(a) = P_0(a) + \frac{B_0^2}{4\pi i \omega \alpha^2 a^3} V_0(a).$$

The index “0” indicates the values related to the shell. In the filament and external environment, where the field is longitudinal and homogeneous, the dependence on the radial variable is expressed with Bessel functions. The corresponding expressions for the total pressure disturbance and the radial-velocity component are known from the theory of linear oscillations of a homogeneous magnetic tube:

$$P_i(r) = A_1 J_1(\lambda_i r),$$

$$V_i(r) = \frac{(-i\omega) \lambda_i A_1 J_1'(\lambda_i r)}{\varrho_i (\omega^2 - V_i^2 k^2)}, \quad \lambda_i = \sqrt{\frac{\omega^2}{V_i^2} - k^2}, \tag{4}$$

$$P_e(r) = A_4 K_1(\lambda_e r),$$

$$V_e(r) = \frac{(-i\omega) \lambda_e A_4 K_1'(\lambda_e r)}{\varrho_e (\omega^2 - V_e^2 k^2)}, \quad \lambda_e = \sqrt{-\frac{\omega^2}{V_e^2} + k^2}. \tag{5}$$

Here, the derivatives of the corresponding functions are marked with primes, and values A_1 and A_4 are arbitrary constants. In the external environment, the distribution decreases rapidly with the radius; thus, the solution describes a fast magnetoacoustic wave that is localized in the tube or, in other words, trapped by the tube (trapped modes).

In the shell, the dependence on the radial variable is determined by the equation of the following form, which we call radial:

$$\begin{aligned} & \xi^2(\xi^2 - 1) \frac{d^2 P_0}{d\xi^2} + \xi(3\xi^2 - 5) \frac{dP_0}{d\xi} \\ & + \left[(1 - \chi^2)\xi^4 + (\chi^2 - 2)\xi^2 - 3 \right] P_0 = 0, \quad (6) \\ & \xi = \frac{r\omega}{V_0}, \quad \chi = \frac{V_0 k}{\omega}. \end{aligned}$$

In addition to the zero singular point associated with the geometry of the problem, it has an additional end singular point $\xi_c = 1$, which has a resonance value. It corresponds to the value of the radial variable $r_c = V_0/\omega$. The resonance can be given the following physical meaning: exactly one wavelength with a phase velocity of V_0 and frequency of ω can be placed on the circumference of radius r_c .

We consider the solutions of the radial equation (6) in a unit circle:

$$|\xi| < 1, \quad (7)$$

thus excluding the resonance case that will be the subject of our special study. The determination of the equation for the zero singular point has solutions -1 and -3 , which differ by an integer. Linearly independent solutions of the radial equation have singularities in the form of poles, and the latter solution contains an attached vector in the form of logarithm. The solutions have the following expansions:

$$W_1(\xi) = \xi^{-1} \sum_{n \geq 0} a_n \xi^n, \quad W_2(\xi) = W_1 \ln \xi + \xi^{-3} \sum_{n \geq 0} b_n \xi^n. \quad (8)$$

The expansion coefficients are found from recurrent relations. The wave distribution in the cylindrical shell can be written in the form of their linear combination

$$P_0(r) = A_2 W_1(\lambda_0 r) + A_3 W_2(\lambda_0 r), \quad \lambda_0 = \omega/V_0, \quad (9)$$

$$(\xi^2 - 1) \frac{\rho_0 \omega^3}{(-i\omega)\alpha^2 V_0} V_{r0}(r) = \xi^2 \frac{d}{d\xi} \xi^2 P_0(r), \quad (10)$$

with the arbitrary coefficients A_2 and A_3 .

Substituting the found wave distributions into the boundary conditions, we obtain a homogeneous algebraic system for the arbitrary coefficients A_1, A_2, A_3 , and A_4 . The condition of system resolvability is met when its determinant is equal to zero. This leads to the dispersion equation, which binds the frequency ω and the longitudinal wave number k . The obtainment of the

dispersion equation is a standard procedure for this kind of problem, so we omit its full presentation. The dispersion equation for kink oscillations can be written as the following equality

$$D_1 D_4 - D_2 D_3 = 0, \quad (11)$$

where

$$D_1 = \frac{\lambda_i b J_1(\lambda_i b)}{J_1'(\lambda_i b)} (\lambda_0 b W_1'(\lambda_0 b) + 2W_1(\lambda_0 b)) - (\lambda_0^2 b^2 - 3)W_1(\lambda_0 b) + \lambda_0 b W_1'(\lambda_0 b), \quad (12)$$

$$D_2 = \frac{\lambda_i b J_1(\lambda_i b)}{J_1'(\lambda_i b)} (\lambda_0 b W_2'(\lambda_0 b) + 2W_2(\lambda_0 b)) - (\lambda_0^2 b^2 - 3)W_2(\lambda_0 b) + \lambda_0 b W_2'(\lambda_0 b), \quad (13)$$

$$D_3 = -\frac{\lambda_e a K_1(\lambda_e a)}{K_1'(\lambda_e a)} (\lambda_0 a W_1'(\lambda_0 a) + 2W_1(\lambda_0 a)) - (\lambda_0^2 b^2 - 3)W_1(\lambda_0 a) + \lambda_0 a W_1'(\lambda_0 a), \quad (14)$$

$$D_4 = -\frac{\lambda_e a K_1(\lambda_e a)}{K_1'(\lambda_e a)} (\lambda_0 a W_2'(\lambda_0 a) + 2W_2(\lambda_0 a)) - (\lambda_0^2 b^2 - 3)W_2(\lambda_0 a) + \lambda_0 a W_2'(\lambda_0 a). \quad (15)$$

4. SPECTRUM OF KINK OSCILLATIONS

The dispersion equation is solved with respect to ω for different values of k under the condition of the absence of resonance (7), which imposes the upper limit on the values of the wave number. On this basis, we limit ourselves to the consideration of long-wave oscillations ($ka \ll 1$). The equation is solved numerically with the finite sums in the expansions (8); the error related to the removal of the series remainder is controlled in the calculations. Figure 3 presents the results of the solution of the dispersion equation. For the typical coronal conditions, we choose $V_e > V_i$ and consider a number of values of the Alfvén velocity V_0 , the principal characteristic of the shell. The main conclusion is that the sustained free oscillations exist.

Dispersion curves are constructed for the case of a relatively thick shell ($a = 4b/3$) and a relatively thin shell ($a = 3.2b/3$). The results show that thin shell parameters have an insignificant influence on its oscillation spectrum. The phase velocity slowly decreases with an increase in the value of the wave number, which is typical of kink oscillations of a homogeneous tube in the fundamental mode. The numerical values of the oscillation periods for the typical coronal parameters are close to the periods of a homogeneous magnetic tube, which are used as a model in coronal seismology. This indicates that the

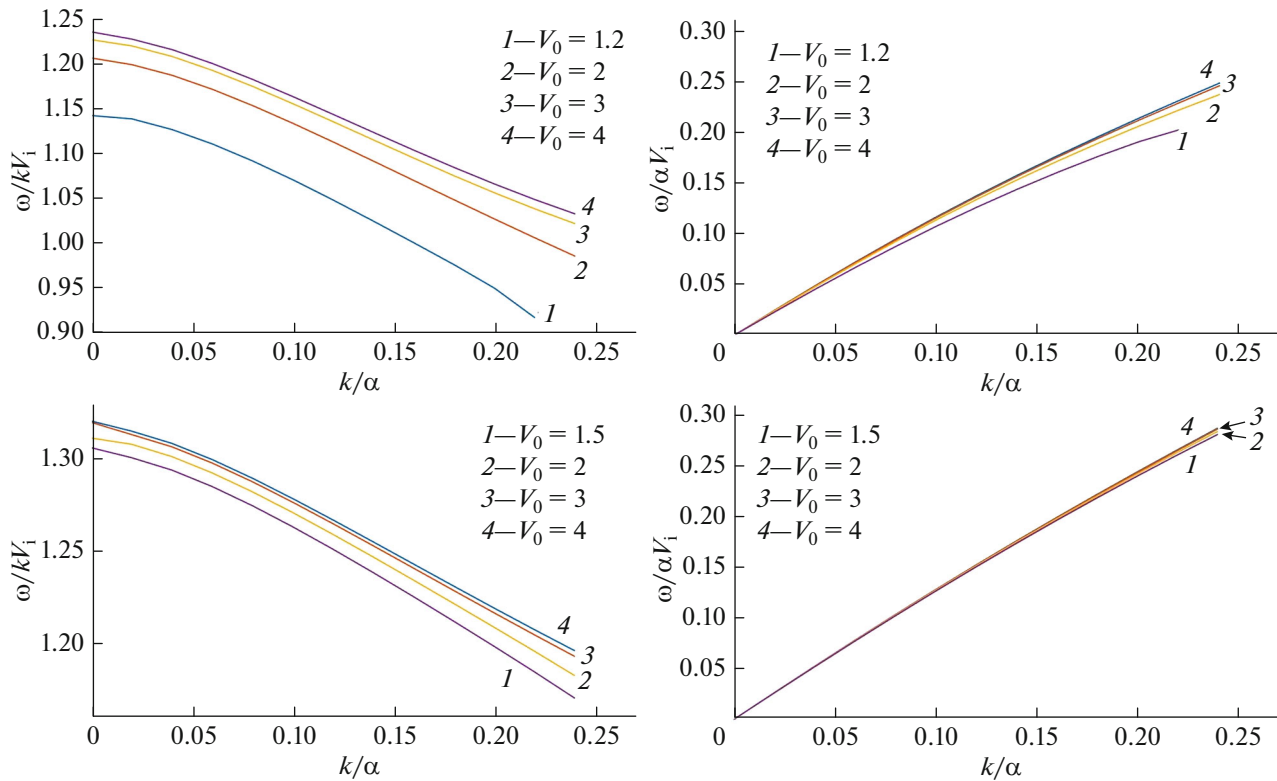


Fig. 3. Dependences of the kink oscillation phase velocity (left) and frequency (right) for the internal and external cylinder radius ratios: $a/b = 4/3$ (top panel) and $a/b = 3.2/3$ (bottom panel).

observed coronal loops may contain longitudinal electric currents.

The role of resonance in the behavior of kink oscillations must be studied for a more detailed analysis of oscillations over a wider range of the wave number values, and it is the subject of our further research.

5. CONCLUSIONS

The presence of electric currents in coronal loops is a universally recognized concept; therefore, the study of loop oscillations with these physical properties can be considered a relevant problem. The linear-oscillation model considered in the paper provides an algorithm to determine the spectrum of kink oscillations of a magnetic tube with an azimuthal component of the field. In the long-wave range, the oscillation-spectrum properties are similar to those of the homogeneous magnetic-tube oscillations.

The presence of resonance is a characteristic feature of the kink oscillations of the magnetic tube with an azimuthal component of the field. This is stipulated not by the usual radial variation of the Alfvén velocity, but by the existence of a surface, on which the time of wave propagation along a circumference of certain radius coincides with the Alfvén time. Here the value of the Alfvén velocity itself can be constant, indepen-

dent of the radius. The study of the resonance is the subject of our special study.

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CONFLICT OF INTEREST

The authors declare no conflict of interest.

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