

# Energy and Power Spectra of the Potential Geomagnetic Field since 1840

S. V. Starchenko<sup>a, \*</sup> and S. V. Yakovleva<sup>a</sup>

<sup>a</sup>*Pushkov Institute of Terrestrial Magnetism, Ionosphere, and Radiowave Propagation,  
 Russian Academy of Science, Troitsk, Moscow, 142190 Russia*

\**e-mail: sstarchenko@mail.ru*

Received April 28, 2018; revised May 14, 2018; accepted September 27, 2018

**Abstract**—It is possible that the radially independent, spatial-spectral components of the energy and power of the potential part of the main geomagnetic field were determined and studied for the first time. Energy is obtained by integrating its known radial density from the core of the Earth to infinity, and power is a time derivative of energy. The total and spectral variations of energy and power from 1840 to 2020 are analyzed based on three generally accepted observational models of the geomagnetic field. The total energy ( $\sim 6 \times 10^{18}$  J) and power ( $\sim 10^8$  W) are determined by the sum of odd harmonics: dipole  $n = 1$ , octupole  $n = 3$ , etc. The dipole, the energy of which is close to the total energy symmetric with respect to the axis of rotation of the field, is predominant. The energy variations are  $\sim 10\%$  and are similar for all models with the exception of the “burst” of the international geomagnetic reference field (IGRF) model in 1945–1950. Comparative spectral analysis showed that the “burst” is concentrated at  $n = 9$  and 10, and the variations of the other harmonics are similar in all models. In this case,  $n = 3$  dominates over  $n = 2$ . From  $n = 3$  to 8, it decreases, and further  $n = 9$  dominates over 8 and 10. The mean powers close to zero for  $n > 1$  indicate an almost periodic behavior of the nondipole field, and significant power variations indicate a strong nonlinearity of the geodynamo. The results of the work are consistent with modern geodynamo-like models. The fact that such a significant IGRF “burst” that can have a non-linear geodynamic nature is a challenge. Alternatively, this may be some consequence of the imperfections of the IGRF model. Two other too-“quiet” models were subjected to excessive smoothing.

DOI: 10.1134/S0016793219010122

## 1. INTRODUCTION AND ENERGY HARMONICS OF THE POTENTIAL FIELD

Gauss showed that the main part of the observed geomagnetic field is the potential (see Gauss, 1952). Beginning in 1840, he developed a network of geomagnetic observatories that measured the full vector of a magnetic field, which made it possible to describe the field with sufficiently high accuracy. Numerous navigation measurements of its directions were also used. These directions were recorded in ship logs for more than five centuries, but their use to estimate the geomagnetic field requires a hypothesis about the magnitude of the field modulus. Different hypotheses lead to differences in models: compare, for example, those by Jackson et al. (2000) and Bondar et al. (2002). Therefore, we use field models no older than 1840 in this work.

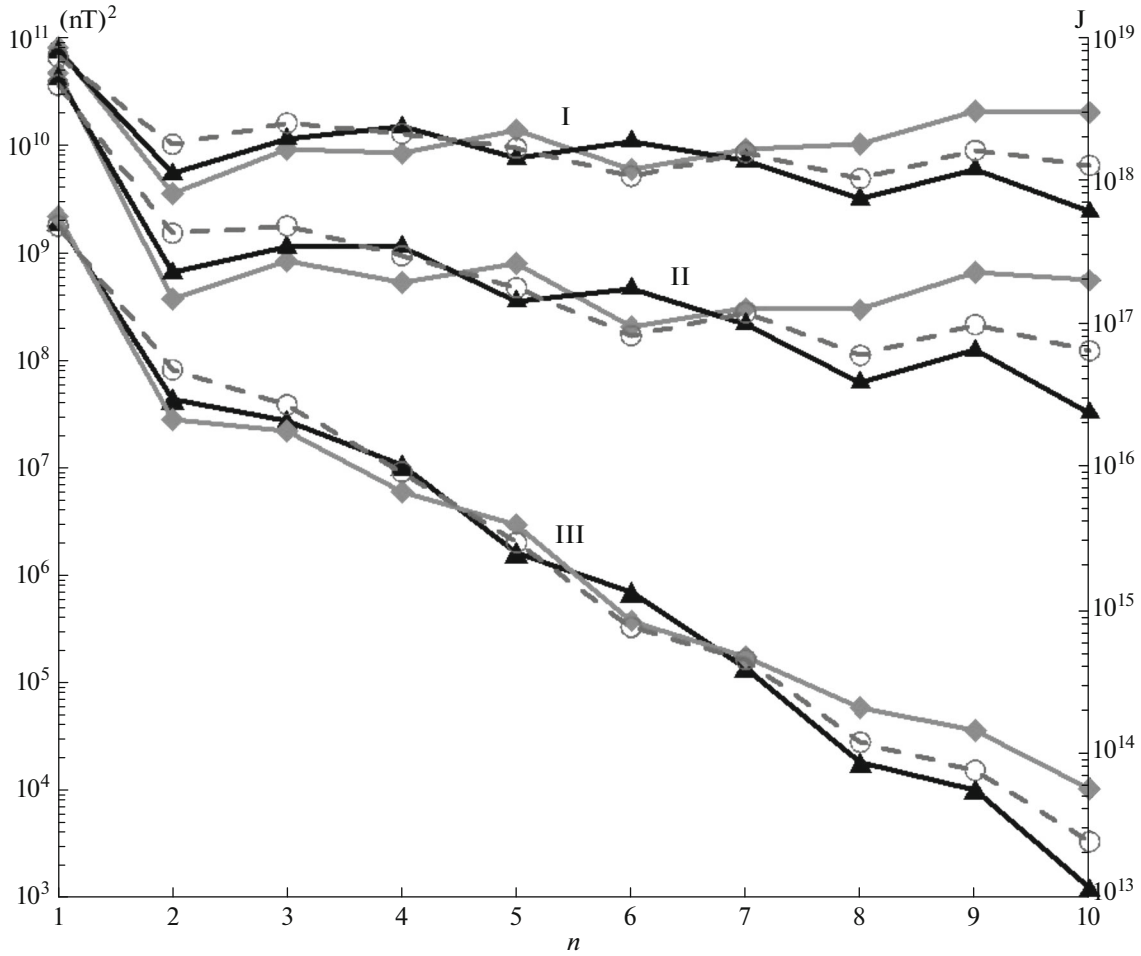
The study of the energy of the potential part of the main geomagnetic field was initiated by Mauersberger (1956). Based on this work, Lowes (1966, 1974) determined the normalized (per area of the sphere and expressed in  $T^2$ ) contribution of the multipole  $n$ -harmonic

$$R_n = (n+1) \left(\frac{a}{r}\right)^{2n+4} \sum_m^n [(g_n^m)^2 + (h_n^m)^2] \quad (1)$$

into radial energy density in terms of standard (Yanovsky, 1978) Gauss coefficients ( $g_n^m, h_n^m$ ), the Earth’s radius  $a$ , and spherical radius  $r$ . Thanks to the “heavy” hand of the discoverers, Eq. (1) is called the spatial power spectrum, although Russian authors (e.g., Zvereva 2015) use a physically more correct term—the energy spectrum. We correctly define the unnormalized (in J/m) contribution of the  $n$ -harmonic to the radial energy density of the potential field, which is considered to be

$$\frac{4\pi r^2}{2\mu_0} R_n = 2\pi a^2 \frac{n+1}{\mu_0} \left(\frac{a}{r}\right)^{2(n+1)} \sum_{m=0}^n [(g_n^m)^2 + (h_n^m)^2]. \quad (2)$$

Integrating (2) along the radius from the core-mantle boundary  $r = c$  to infinity, physically in an obvious way, we obtain the contribution of  $n$ -harmonics to all of the energy (in J):



**Fig. 1.** Three types of spectra based on COV\_OBS models (Gillet et al., 2015): the upper group (I) is traditional at the core, the lower (III) is traditional at the Earth’s surface, the middle group (II) is true as in (3) energy spectra (built on the right scale). The gray solid line with diamonds corresponds to the era of 1840, and the black line with triangle denotes 1930. The gray dashed line with empty circle shows 2020.

$$\begin{aligned}
 E_n &= 2\pi a^{2n+4} \frac{n+1}{\mu_0} \sum_{m=0}^n [(g_n^m)^2 + (h_n^m)^2] \int_c^{+\infty} \frac{dr}{r^{2n+2}} \\
 &= \frac{2\pi a^3}{\mu_0} \left(\frac{a}{c}\right)^{2n+1} \frac{n+1}{2n+1} \sum_{m=0}^n [(g_n^m)^2 + (h_n^m)^2].
 \end{aligned}
 \tag{3}$$

Expression (3) for the energy of the geomagnetic potential field is apparently proposed here for the first time, as is the obvious expression for the contribution of  $n$ -harmonics to power:

$$P_n = dE_n/dt. \tag{4}$$

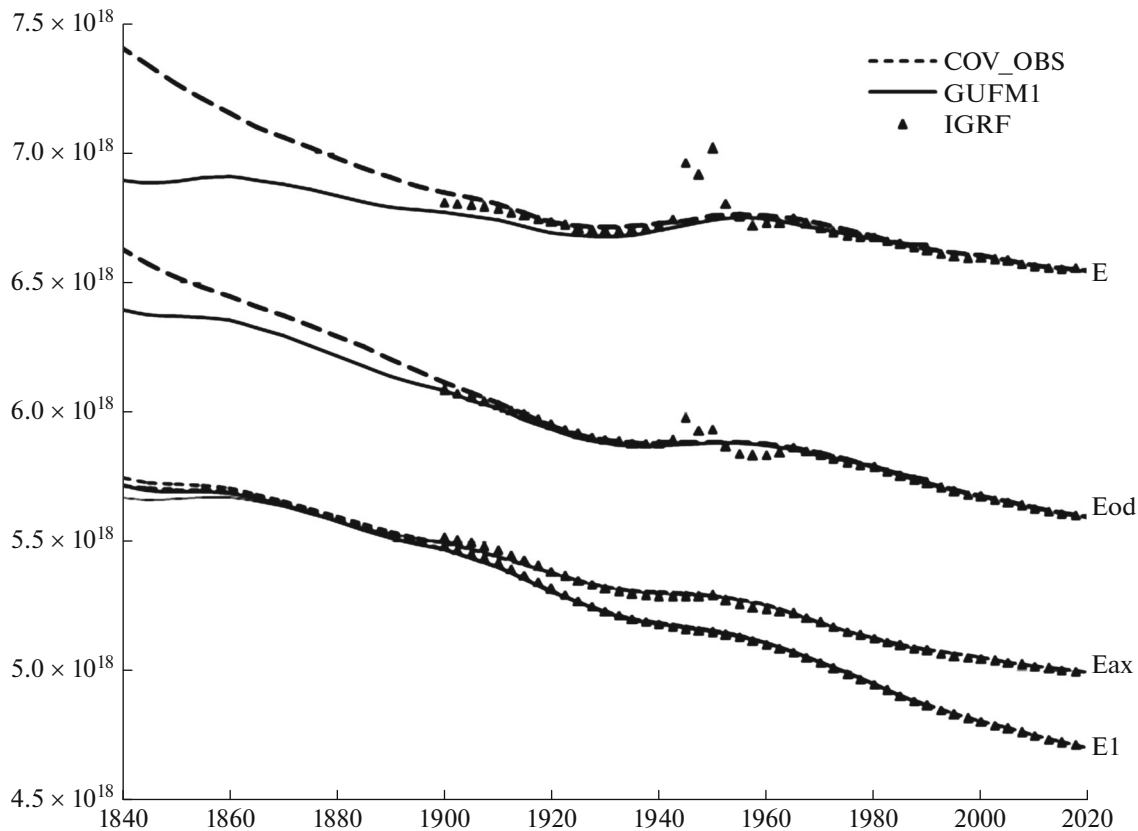
## 2. COMPARISON OF THE SUMS AND SPECTRA OF THREE MODELS

For comparison, we selected the three most generally accepted models of Gauss coefficients. The long-term 1590–1990 (Jackson et al., 2000) (GUFM1 below) and modern 1840–2020 (Gillet et al., 2015)

(COV\_OBS below) models have both been used since 1840. The main international model, International Geomagnetic Reference Field (IGRF), has been used since 1900 (see <http://www.ngdc.noaa.gov/IAGA/vmod/igrf.html>).

In Figure 1, the COV\_OBS model compares the aforementioned classical “power spectra” (1) on the surface of the core and the Earth with our radially independent energy spectrum (3) over the entire space except for the core of the Earth. We see the dominance of odd harmonics, which we detail later.

For all three considered models in Fig. 2, we compare the various total energies, which have variations of ~10%. The total energy  $E = E1 + E2 + \dots$  behaves differently for GUFM1 and COV\_OBS models later than 1900, and the IGRF model has a “burst” in 1945–1950. These discrepancies and the “burst” are concentrated in the sum  $E_{od} = E1 + E3 + \dots$  of odd components (3). The energies of the field  $E_{ax}$  ( $m = 0$ ) that are symmetric with respect to the axis of rotation,



**Fig. 2.** For the considered models (see legend): the total energy  $E$ , the sum of its odd  $n = 1, 3, \dots$  in (3)  $E_{od}$  components, the energy symmetric about the axis of rotation  $E_{ax}$  with  $m = 0$  from (3) and dipole energy  $E_1$  are compared (in J).

and those of the dipole energy  $E_1$  agree best with each other in the considered models. The relatively small difference between  $E - E_{od}$  is equal to the sum of the even components of the energy, which is close to  $E_{od} - E_1$ . This highlights the dominant dipole and energetically balances between the even and odd components of the nondipole field, which should be present in geodynamo-like models.

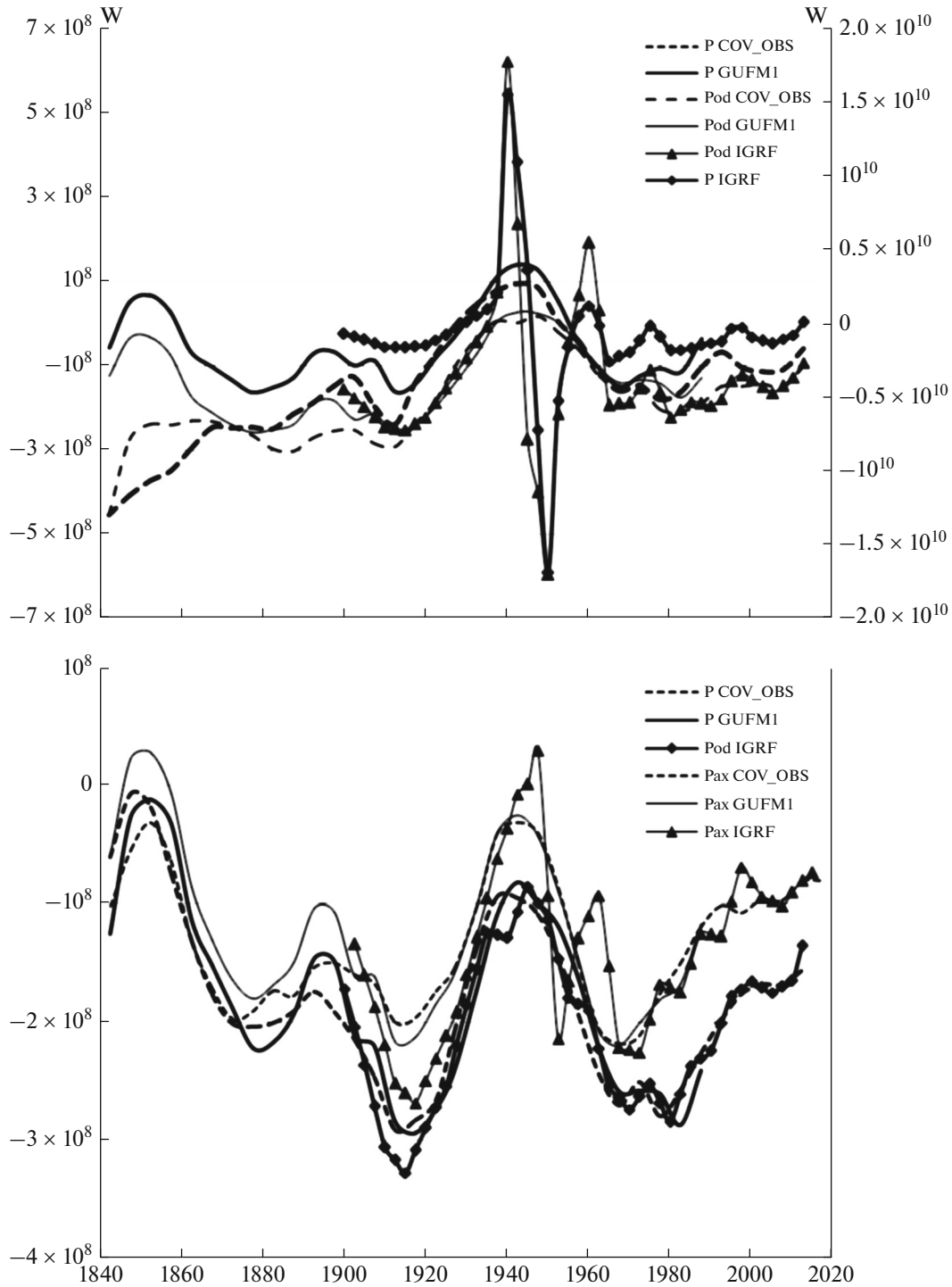
By analogy with Figure 2, Figure 3 shows the total power, which is significantly more variable than the energy and has an order of magnitude of several tenths of GW everywhere except for the IGRF burst of about 1 GW. This indicates a strong geodynamo nonlinearity, which should be reproduced in geodynamo-like models (Starchenko, 2014; Schaeffer et al., 2017 and references therein). The powers in Figure 3 are mostly negative, which is mainly due to the decrease in the dipole modulus in the modern era. Next, we take a closer look at the spectra of the nondipole field.

Let us compare the extreme and average spectral energies for all three considered models presented in Fig. 4. It is seen that the IGRF burst is concentrated in harmonics with  $n = 9$  and 10, and the variations of the other harmonics are similar in all models. At the same time,  $n = 3$  dominates over  $n = 2$ . With  $n = 3$  to 8, there is a decrease and then domination of  $n = 9$  over 8 and

10. This selection of several spectral scales is also present in theoretical estimates (Starchenko, 2014), and in the most advanced numerical geodynamo-like models (Schaeffer et al., 2017).

Even brighter is the presence of the IGRF burst in Fig. 5 with extreme and average spectral powers. The mean powers close to zero indicate an almost periodic behavior of the nondipole field in all harmonics with the exception of the tenth IGRF harmonics.

The most interesting evolution of “restless” harmonics with  $n = 9, 10$  from (3)–(4) is shown in Fig. 6 in comparison with their closest “quiet” harmonic,  $n = 8$ . From Figure 6 it is obvious that all of the “disturbance” was concentrated in the IGRF model beginning about 1940–1965. The two other models in question behave “quietly” everywhere. It is not clear whether such a significant IGRF burst is a real reflection of the highly nonlinear nature of the geodynamo or whether it is some kind of artificial imperfection of the IGRF model. Two other quiet models were subject to excessive smoothing. In any case, this feature deserves the closest attention of the world’s geomagnetic community.



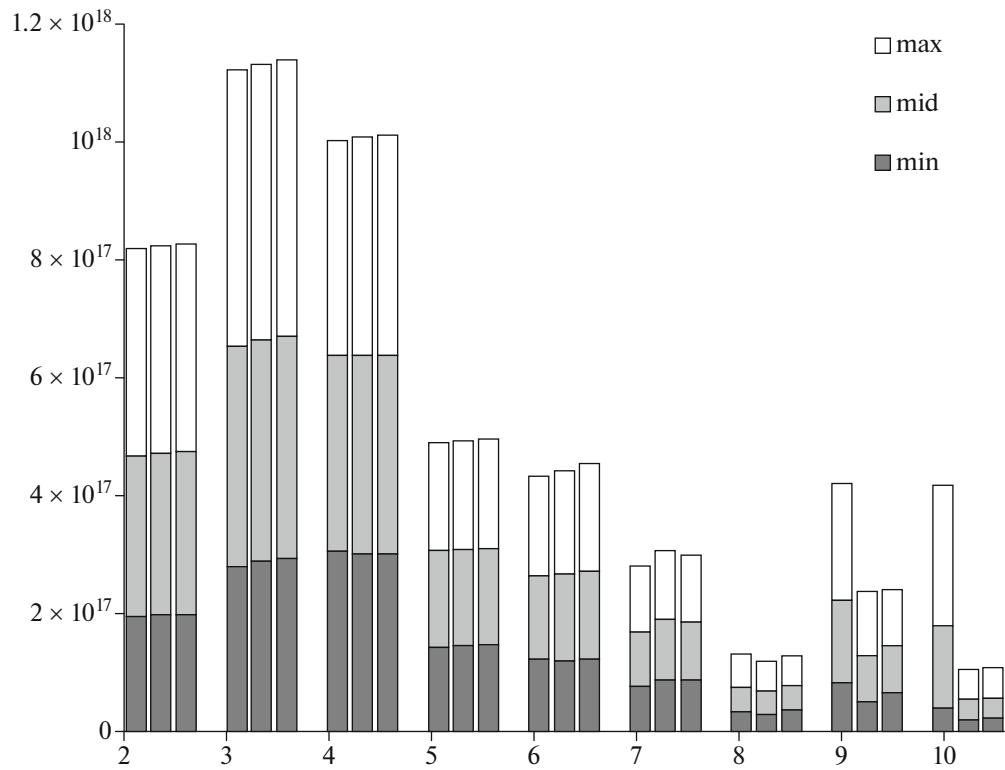
**Fig. 3.** Upper panel: comparison of the total power  $P$  and the sum of its odd  $n = 1, 3, \dots$  in (3–4) components Pod. For  $P$  of the IGRF model, the axis is on the right, and for the others, it is on the left. Lower panel: the power of the Pax field symmetrical with respect to the axis of rotation with  $m = 0$  from (3–4) and the dipole power  $P1$ .

### 3. CONCLUSIONS

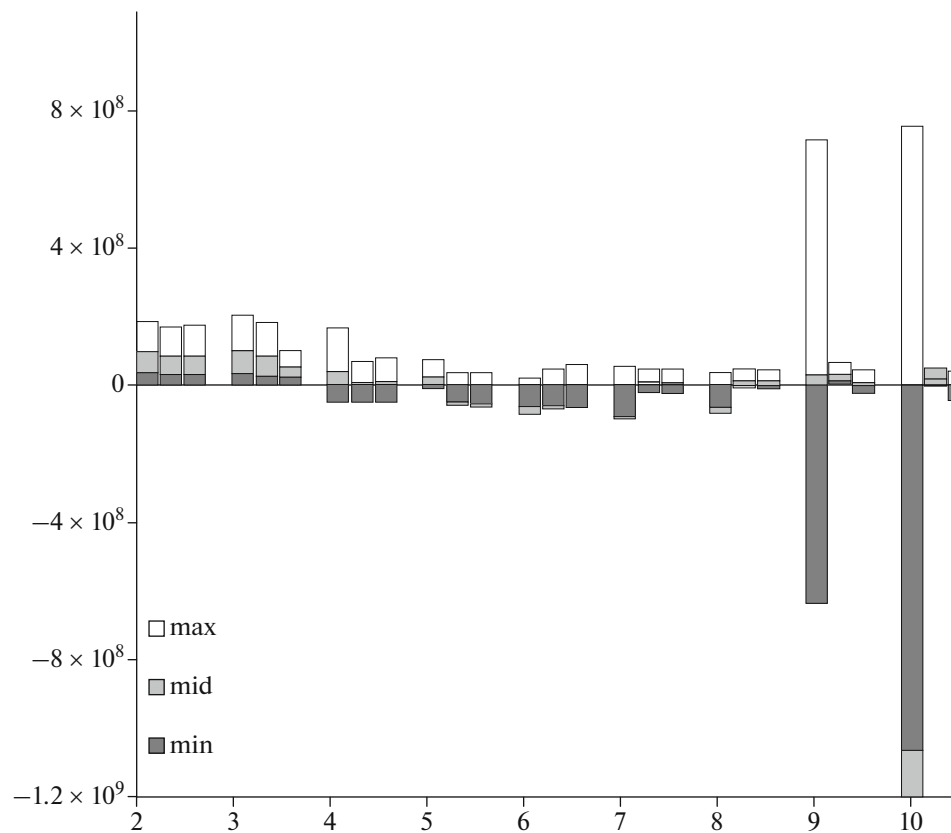
For the first time, multipole spectra of the energy and power of a potential part of the main geomagnetic field since 1840 were determined and studied. The energy was obtained by integrating its known radial

density from the Earth’s core to infinity, and power is the time derivative of energy.

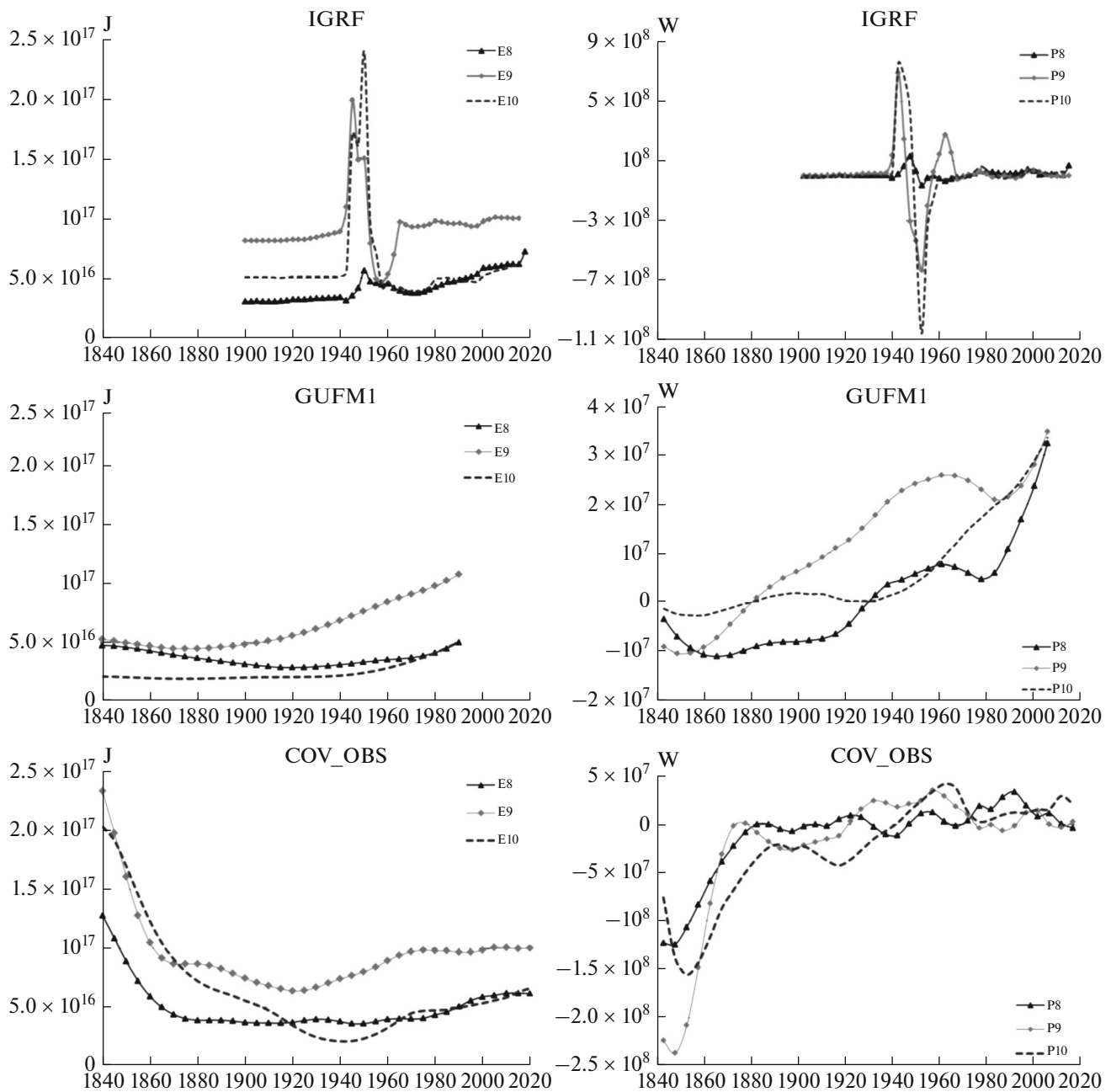
1. Based on three (Jackson et al., 2000; Gillet et al., 2015; and IGRF) generally accepted observational models of the geomagnetic field, the total and spectral



**Fig. 4.** Accumulative histograms in  $J$ . On the horizontal axis  $-n$ . The maximum values of the energy spectra (3) for nondipole  $n > 1$  are bright, the average values are gray, and the minimum values are black. Every first column is from the IGRF model, the second is from the GUFM1 model, and the third is from the COV\_OBS model for their mutually overlapping period of 1900–1990.



**Fig. 5.** As in Fig. 4, but for power spectra from (3–4) in  $W$ .



**Fig. 6.** Evolution of harmonics with  $n = 8, 9, 10$  in (3–4) for power (right) and energy (left). The upper pair is the IGRF model, the middle pair is the GUFM1 model, and the lower pair is the COV\_OBS model.

variations of energy and power from 1840 to 2020 were analyzed.

2. The total energy ( $\sim 6 \times 10^{18}$  J) and power ( $\sim 10^8$  W) were determined by the sum of odd multipoles: dipole  $n = 1$ , octupole  $n = 3$ , etc. The dipole, the energy of which is close to the total energy symmetric with respect to the axis of rotation of the field, dominates. The energy variations are  $\sim 10\%$  and are similar for all models with the exception of the burst of the IGRF model in 1945–1950.

3. Spectral analysis showed that the burst is concentrated at  $n = 9$  and  $10$ , and the variations of the other multipoles are similar in all models. In this case,  $n = 3$  dominates over  $n = 2$ . With  $n = 3$  to  $8$ , decreases, and further  $n = 9$  dominates over  $8$  and  $10$ .

4. The mean powers close to zero for  $n > 1$  indicate an almost periodic behavior of the nondipole field, and significant power variations indicate a strong non-linearity of the geodynamo. This result and all of the previous results are in good agreement with modern geodynamo models.

5. A new challenge for geomagnetic observations and the theory of hydromagnetic dynamo is the very significant IGRF burst noted in 2 and 3 above, which may be a reflection of the nonlinear nature of the geodynamo. Alternatively, this may be some artificial imperfection of the IGRF model. Two other quiet models were subject to excessive smoothing. In any case, this feature discovered by us deserves the close attention of the world's geomagnetic community.

#### ACKNOWLEDGMENTS

The authors are deeply grateful to the anonymous reviewer. His comments helped to improve significantly the presentation of this work, which was done mainly at the expense of the IZMIRAN budget. For geodynamo applications, partial support was provided by the Russian Foundation for Basic Research (project no. 16-05-00507-a).

#### REFERENCES

- Bondar, T.N., Golovkov, V.P., and Yakovleva, S.V., Spatio-temporal model of the secular variations of the geomagnetic field in the time interval from 1500 through 2000, *Geomagn. Aeron. (Engl. Transl.)*, 2002, vol. 42, no. 6, pp. 793–800.
- Gauss, K.F., *Izbrannye trudy po zemnomu magnetizmu* (Selected Works on Terrestrial Magnetism), Moscow: Izd. Akad. Nauk SSSR, 1952.
- Gillet, N., Barrois, O., and Finlay, C.C., Stochastic forecasting of the geomagnetic field from the COV-OBS.x1 geomagnetic field model, and candidate models for IGRF-12, *Earth, Planet and Space*, 2015, vol. 67, id 71. doi 10.1186/s40623-015-0225-z
- Jackson, A., Jonkers, A.R.T., and Walker, M.R., Four centuries of geomagnetic secular variation from historical records, *Phil. Trans. R. Soc. London*, 2000, vol. A358, pp. 957–990.
- Lowes, F.J., Mean-square values on sphere of spherical harmonic vector fields, *J. Geophys. Res.*, 1966, vol. 71, no. 8, p. 2179.
- Lowes, F.J., Spatial power spectrum of the main geomagnetic field, and extrapolation to the core, *Geophys. J. R. Astron. Soc.*, 1974, vol. 36, pp. 717–730.
- Mauersberger, P., Das Mittel der Energiedichte des geomagnetischen Hauptfeldes an der Erdoberfläche und seine säkulare Änderung, *Gerlands Beiträge Geophys.*, 1956, vol. 65, pp. 207–2015.
- Schaeffer, N., Jault, D., Nataf, H.-C., and Fournier, A., Turbulent geodynamo simulations: A leap towards Earth's core, *Geophys. J. Int.*, 2017, vol. 211, pp. 1–29.
- Starchenko, S.V., Analytic base of geodynamo-like scaling laws in the planets, geomagnetic periodicities and inversions, *Geomagn. Aeron. (Engl. Transl.)*, 2014, vol. 54, no. 6, pp. 694–701.
- Yanovskii, B.M., *Zemnoi magnetizm* (Terrestrial Magnetism), Leningrad: LGU, 1978.
- Zvereva, T.I., Dynamics of the main magnetic field of the Earth from 1900 until present days, in *Yubileinyi sbornik IZMIRAN-75 "Elektromagnitnye protsessy ot nedr Solntsa do nedr Zemli"* (Jubilee Volume IZMIRAN-75 "Electromagnetic Processes from the Sun's Interiors to the Earth's Interiors"), Moscow: IZMIRAN, 2015, pp. 36–45.

*Translated by E. Seifina*