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Interrelation between the Amplitude and Length of the 11-Year Sunspot Cycle

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Abstract—Alternative procedures for determining the length (duration) of the 11-year sunspot cycle are considered. For cycles 12–23 it is shown that the cycle and the cycle growth branch latitudinal lengths are much more closely related to the cycle amplitude than similar traditional lengths, if the time when the exponent, describing the average sunspot latitude drift, reaches the "reference" latitude value (the latitude phase reference time) is selected as a cycle starting time.

Two relationships are obtained. The first relationship makes it possible to rather accurately determine the cycle amplitude based on information about two cycle time intervals: the shift in the latitude phase reference point and the cycle latitudinal length. The second relationship relates the value of the interval between the above exponents of adjacent cycles to the cycle amplitudes.

The found relationships between the important amplitude, latitude, and time parameters of the 11-year solar magnetic cycle should be taken into account in the construction of adequate physical models.

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1. INTRODUCTION

The length of the 11-year solar (sunspot) activity cycle is the main cycle characteristic (together with amplitude) and is among the key parameters when dynamo models of the solar magnetic cycle are constructed (Karak and Choudhuri, 2011; Nandy et al., 2011).

Cycle length is determined as a time interval between the initial and final reference points of the cycle phase. For obvious reasons, times of successive cyclic minimums are usually selected as phase reference points for the 11-year sunspot cycle (Waldmeier, 1961; Wilson, 1987; Solanki et al., 2002; Du et al., 2006; Richards et al., 2009). The selection of such minimums evidently depends on the activity index used and the initial data processing algorithm.

Since the cycles evolve in time (the Schwabe–Wolf law) and space (the Spörer law), the sunspots of a new cycle usually appear at middle heliolatitudes, whereas the sunspot activity of an old cycle still continues at low heliolatitudes. Thus, adjacent cycles overlap in time, and such an overlapping can reach two years. Therefore, a minimum characterizes only the time of minimal total activity of such cycles. However, it is also difficult to distinguish the time of minimum (e.g., during multimonth periods without sunspots) in the absence of such overlapping.

To overcome these difficulties, some researchers (Hathaway et al., 1994; Roshchina and Sarychev, 2011) selected the so-called "starting" time, which is obtained from the description of the curve of this cycle using the parametric functional dependence where this starting time is among the dependence parameters, as a reference point for the 11-year cycle phase.

We (Ivanov and Miletsky, 2014) proposed another approach, according to which the cycle latitudinal phase reference point (LPRP) is selected as a cycle phase reference point, for which the exponent describing the sunspot average latitudes drift reaches a certain "reference" latitude value (26.6° in magnitude) in a cycle. Note that the exponent on average reaches this value of latitude at times of minimums (for cycles 12–23). However, in each specific cycle, this time is shifted relative to a minimum toward advance or delay. At such an approach, the LPRP is determined by the time of the exponent approximating the average latitudes cyclic drift.

2. DATA AND THEIR PROCESSING

We used the data on sunspots presented in the Greenwich catalog and its continuation NOAA/USAF (http://solarscience.msfc.nasa.gov/greenwch.shtml) for 1874–2013. We formed a series of rotation averages of sunspot latitudes and the sunspot group number index (G). The times and values of cyclic minimums (TGmin, Gmin) and maximums (TGmax, Gmax) are calculated based on the average values for a rotation, smoothed by a 13-point window with sinusoidal weights for the entire Sun and independently for its hemispheres.

Table 1. Traditional (TTmin) and latitudinal (TTlat) lengths of the 11-year sunspot activity cycles for the entire solar disk and separately for the hemispheres (cycles 12–23)

Cycle	Northern Hemisphere		Southern Hemisphere		Entire disk	
	TTmin	TTlat	TTmin	TTlat	TTmin	TTlat
12	10.37	11.58	11.50	11.47	11.12	11.85
13	12.4	12.92	11.21	11.47	11.28	12.02
14	10.61	12.04	12.17	12.18	12.10	12.68
15	11.35	11.26	9.86	10.07	9.86	10.65
16	10.00	10.60	9.93	10.23	10.38	10.74
17	10.54	10.29	10.83	10.03	10.45	10.15
18	9.78	10.00	10.15	10.59	10.08	10.31
19	10.38	9.52	10.60	10.23	10.31	9.69
20	11.65	11.55	10.39	10.15	11.57	11.41
21	9.78	9.92	11.20	9.50	10.16	9.90
22	10.83	9.75	9.93	9.52	10.15	9.36
23	11.05	11.04	12.62	11.89	12.47	12.03

Let us assume that TTmin = TGmin(n + 1) – TGmin(n) is a traditional cycle length in the nth cycle. We proposed the TTlat(n) = Tmin(n + 1) – Tlst(n) value as the cycle latitudinal length. Here Tlst(n) is LPRP. The TTmst(n) = Tlst(n) value indicates the LPRP value and shift sign relative to a minimum. The TTmin and TTlat values for cycles 12–23 are presented in Table 1.

For example, for the Northern (N) Hemisphere of cycle 19 presented in Fig. 1, we have: TTmin(19N) =

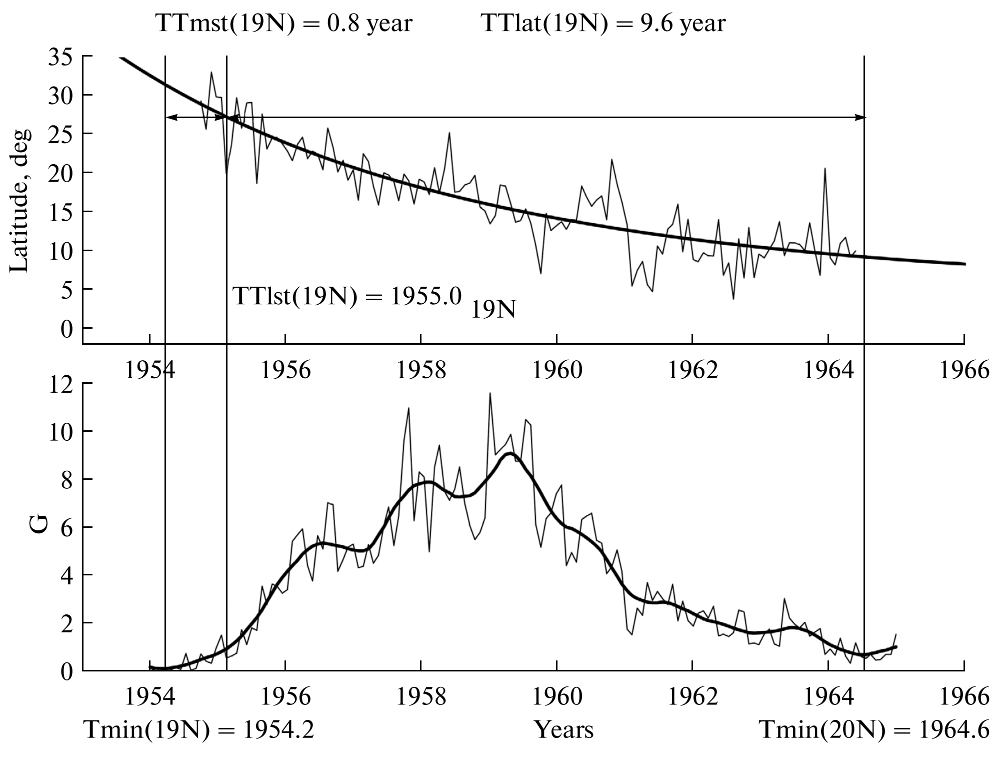

Fig. 1. Time variations (average for rotation and smoothed values) in average latitudes (top panel) and sunspot group number index G (bottom panel) for the Northern (N) Hemisphere in cycle 19. Time intervals are for the Northern (N) Hemisphere in cycle 19. TTmin(19N) and TTmin(20N) are times of minimums. Traditional cycle length is TTmin = TTmin(20N) – TTmin(19N). Latitudinal phase reference point is Tlst(19N). Cycle latitudinal length is TTlat(19N) = Tmin(20N) – Tlst(19N).

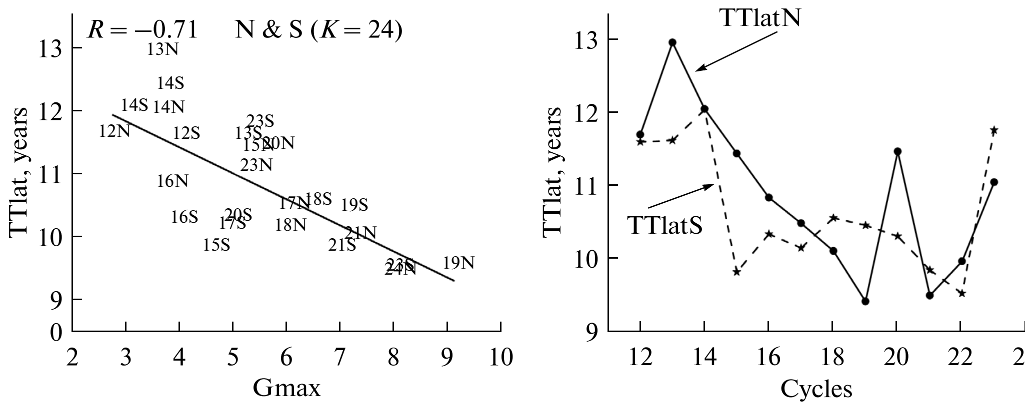

Fig. 2. Left panel: the dependence of the cycle latitudinal length (TTlat) on the cycle amplitude (Gmax) with regard to hemispheres N and S (cycle numbers are shown by numerals with letters). Right panel: the dependence of the cycle latitudinal lengths TTlatN (solid line and circles) and TTlatS (dotted line and asterisks) for hemispheres N and S, respectively, on the cycle number.

Tmin(20N) – Tmin(19N) = 1964.6 – 1954.2 = 10.4 yr, TTmst(19N) = Tlst(19N) – Tmin(19N) = 1955.0 – 1954.2 = 0.8 yr, TTlat(19N) = Tmin(20N) = Tlst(19N) = 1964.6 – 1955.0 = 9.6 yr.

3. RESULTS

As is known, the amplitude and length of the 11-year cycle vary from cycle to cycle. The interrelation between these parameters is a problem of prime importance. Studies (Dicke, 1978; Hoyng, 1993) indicate that the correlation between the cycle amplitude and traditional length (TTmin) is negative and insignificant (from –0.27 to –0.35). Thus, a weak tendency toward cycle shortening with increasing cycle amplitude is detected.

We now determine the degree of interrelation between the amplitude (Gmax) and the traditional (TTmin) and latitudinal (TTlat) lengths for cycles 12–23, which are presented in Table 1.

The resultant correlation coefficients (R) between the cycle amplitude (Gmax) and the traditional (Gmax, TTmin) and latitudinal (Gmax, TTlat) lengths are R(Gmax, TTmin) = –0.55 (the confidence level is CL = 93.5%) and R(Gmax, TTlat) = –0.83 (CL = 99.9%), respectively. When the hemispheres are considered separately ($k = 24$), the correlation coefficients are R(Gmax, TTmin) = –0.32 (CL = 85.1%) and R(Gmax, TTlat) = –0.71 (CL = 99.98%) (Fig. 2, left-hand panel). The corresponding regression equations have the following form: for the entire disk, TTlat = $A0 - A1$*Gmax, where $A0 = (13.9 \pm 0.7)$, $A1 = (0.29 \pm 0.06)$, ($k = 12$, R(Gmax, TTlat) = –0.83). When the hemispheres are considered separately, TTlat = $A0 - A1$*Gmax, where $A0 = (13.0 \pm 0.5)$, $A1 = (0.40 \pm 0.09)$, (N & S, $k = 24$, R(Gmax, TTlat) = –0.71.

In all cases the cycle length decreases with increasing cycle amplitude. The value of the interaction (expressed by negative correlation) is pronouncedly larger for the latitudinal length (TTlat), when LPRP is taken as a cycle phase reference point.

Thus, the most significant "inverse" interrelation was revealed between the cycle amplitude and latitudinal length. In addition, the activity secular trend is clearly defined in (very similar) dependences of cycle lengths TTlatN and TTlatS (solar hemispheres N and S) on the cycle number (Fig. 2, right-hand panel).

We now elucidate the character and value of the interrelation between the cycle amplitude (Gmax) and other time intervals of the 11-year cycle. Thus, the inverse correlation between the cycle growth phase length and amplitude at a maximum (the Waldmeier rule) is known (Waldmeier, 1935). The Wolf number is used to calculate the corresponding values. In this case the correlation is –0.65 for cycles 12–23 corresponding to the Greenwich catalog epoch. Let us assume that TTmimx(n) = TGmax(n) – TGmin(n) is the traditional length of the cycle growth branch in the nth cycle, and TTltmx(n) = TGmax(n) – Tlst(n) is the latitudinal length of the cycle growth branch (by analogy with the cycle latitudinal length introduced above). The values of these parameters for cycles 12–23 are presented in Table 2.

For cycles 12–23 for the entire disk ($k = 12$), the correlations between the cycle amplitude (Gmax) and the traditional (Gmax, TTmimx) and latitudinal (Gmax, TTltmax) lengths of the cycle growth branch are R(Gmax, TTmimx) = –0.63 (CL = 97.1%) and R(Gmax, TTltmax) = –0.86 (CL = 99.96%), respectively. When the hemispheres are taken into account ($k = 24$), the corresponding correlation coefficients are R(Gmax, TTmimx) = –0.25 (CL = 75.5%) and R(Gmax, TTlat) = –0.56 (CL = 99.6%).

In all cases the cycle growth branch length (as well as the total length considered above) is inversely proportional to the amplitude of this cycle. In this case the interrelation is pronouncedly stronger when LPRP,

Table 2. Traditional (TTmimx) and latitudinal (TTltmx) growth branches of the 11-year sunspot activity cycles for the entire solar disk and separately for the hemispheres (cycles 12–23)

Cycle	Northern Hemisphere		Southern Hemisphere		Entire disk	
	TTmimx	TTltmx	TTmimx	TTltmx	TTmimx	TTltmx
12	2.68	3.89	5.46	5.43	5.08	5.81
13	4.93	5.45	3.44	3.70	3.51	4.25
14	4.34	5.77	7.02	7.03	4.86	5.44
15	5.08	4.99	4.18	4.39	4.18	4.97
16	5.97	6.57	3.96	4.26	5.07	5.43
17	3.51	3.26	5.08	4.28	3.65	3.35
18	5.45	5.67	3.36	3.80	3.44	3.67
19	5.15	4.29	3.51	3.14	3.74	3.12
20	2.69	2.59	5.31	5.07	5.75	5.59
21	3.28	3.42	3.88	2.18	3.36	3.10
22	3.66	2.58	4.93	4.52	3.43	2.64
23	4.85	4.84	5.97	5.24	5.60	5.16

rather than the cycle amplitude minimum, is taken as a cycle phase reference point, which pronouncedly increases the significance of the latitudinal version of the Waldmeier rule.

Since we established that the 11-year cycle amplitude (Gmax) depends on TTlat (see above) and TTmst (see (Ivanov and Miletsky, 2014)), we could obtain the linear regression relationship simultaneously relating amplitude Gmax to TTmst and TTlat (Gmax = *f*(TTmst, TTlat)) based on the data for cycles 12–23 for the entire solar disk ($K = 12$).

For the entire solar disk ($K = 12$), we have: Gmax = $A0 + A1$* TTmst − $A2$ *TTlat, where $A0 = (29.2 \pm 6.1)$, $A1 = (2.0 \pm 1.1)$, $A2 = (1.7 \pm 0.6)$, ($R = 0.88$, $k = 12$, SD = 1.6). With regard to the solar hemispheres N & S ($K = 24$), we have: Gmax = $A0 + A1$ *TTmst − $A2$* TTlat, where $A0 = (15.2 \pm 3.2)$, $A1 = (0.8 \pm 0.4)$, $A2 = (0.9 \pm 0.3)$, ($R = 0.76$, $k = 24$, rms is SD = 1.2).

The obtained relationship makes it possible to determine the 11-year cycle amplitude by dividing the time interval between two cyclic minimums (the classical cycle length) into two intervals, i.e., the LPRP shift relative to a minimum (TTmst) and the cycle latitudinal length (TTlat), using the curve of average latitudes. We should note that the Gmax representation accuracy will remain unchanged if we select, instead of TTmst, a shift of the exponent approximating the curve of average latitudes relative to the nearest (previous) minimum that is proportional to TTmst as the first interval at any other fixed latitude and the corresponding interval of this exponent up to the next minimum as the second interval (at the same latitude). These time intervals factually characterize the time position of the mean latitude curve relative to adjacent minimums. Thus, the considered cycle length division variant, when the first interval (TTmst) is specified equal to the LPRP shift relative to the nearest minimum and the second interval is specified equal to the cycle latitudinal length (TTlat), is an important but particular case, when a reference latitude value equal to 26.6° in magnitude is selected as a fixed latitude (see Fig. 1).

The problem of the time interval (we denote it TTcst (n, $n + 1$)) between the curves of average latitudes (more exactly, the exponents approximating these curves) in adjacent cycles numbered n and $n + 1$ (see Fig. 4) is also very interesting. This time interval can be represented as a sum of two intervals: the cycle n

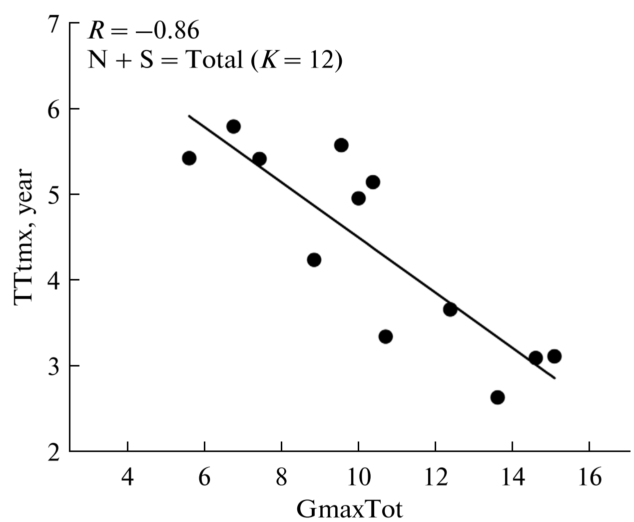

Fig. 3. Dependence of the latitudinal length of the cycle growth branch (TTltmx) on the cycle amplitude (GmaxTot) for the entire disk.

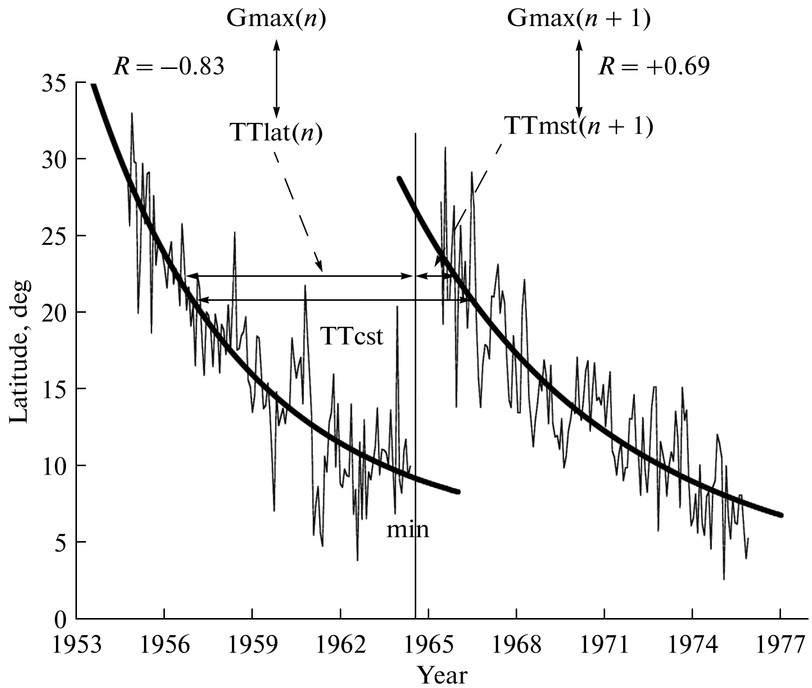

Fig. 4. A sample of determination of the latitudinal distance (TTcst) between adjacent cycles n and $n + 1$ and the latitudinal time intervals of the cycle: the latitudinal length for cycle n (TTlat(n)) and the LPRP shift relative to a minimum for cycle $n + 1$ (TTmst($n + 1$)). TTcst(n, $n + 1$) = TTlat(n) + TTmst($n + 1$).

latitudinal length TTlat(n) and the interval of the LPRP shift relative to a cycle $n + 1$ minimum TTmst($n + 1$).

We previously established that each interval-summand is related to the amplitude of the corresponding cycle. On this basis, we obtained the regression equation relating the TTcst(n, $n + 1$) value (based on the data for the entire solar disk) to the amplitudes of adjacent cycles Gmax(n) and Gmax($n + 1$): TTcst(n, $n + 1$) = $A0 - A1$*Gmax(n) + $A2$*Gmax($n + 1$), where $A0 = (11.9 \pm 0.7)$, $A1 = (0.19 \pm 0.06)$, $A2 = (0.08 \pm 0.06)$, ($R = 0.77$, $k = 11$, SD = 0.52).

4. CONCLUSIONS

For cycles 12–23, we studied the interrelation between the cycle amplitude (Gmax) and the traditional and latitudinal lengths, as well as the traditional and latitudinal lengths of the growth branches. We proposed selecting the latitudinal phase reference point (LPRP) as a cycle phase reference point. In all cases the cycle and cycle growth branch lengths decrease with increasing cycle amplitude.

The interrelation value (expressed by negative correlation) is pronouncedly larger for the cycle latitudinal length and the growth branch latitudinal length than for similar traditional lengths. Thus, these interrelations become stronger and the significance of the latitudinal version of the Waldmeier rule increases if LPRP is selected as a cycle phase reference point.

We obtained an equation that makes it possible to determine the 11-year cycle amplitude based on information about two cycle time intervals: the LPRP shift (TTmst) and the cycle latitudinal length (TTlat), which correlate with two successive cyclic minimums.

We established that the length of the time interval, which separates the exponents approximating curves of average latitudes of adjacent cycles, can be represented as an expression relating the interval to these cycle amplitudes.

The discovered interrelations between the 11-year cycle amplitude and the time intervals characterizing the curves of average latitudes time position relative to adjacent minimums show that important amplitude, latitude, and time parameters of the 11-year cycle correlate with one another and should be taken into account when physical models of the solar magnetic cycle are constructed.

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