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Variations in the Cyclic Characteristics of Solar Magnetic Activity on Long Time Scales

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Abstract—Prolonged variations in the duration of the Schwabe–Wolf (~11 years) and Suess (~200 years) cycles have been analyzed using different experimental data. It was shown that the duration of the Schwabe–Wolf cycle on a 2000-year time scale varied monotonically (on average, increasing) and cyclically (with a period of several hundred years); periods of 10.4, 11.0, and 11.4 years predominate on the occurrence frequency histogram. The Suess cycle duration was 200–290 years during the Holocene and tended to increase in the past. This was accompanied by cyclic variations with a period of 2300–2500 years corresponding to the Hallstatt cycle. Arguments for the assumption that the Suess cycle duration decreased by a factor of more than 1.5 over the past half billion years are presented. This may indicate that the solar rotation characteristics and convection zone parameters varied on long time scales during the Sun's evolution on the main sequence.

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1. INTRODUCTION

Solar activity manifests itself as cyclic variations in the solar magnetic field on different time scales. The following long-period cycles also exist in addition to the best-known 11-year (Schwabe–Wolf) cycle: 80–90 years (Gleissberg cycle), ~200 years (Suess cycle), and ~900 years or longer. The dynamo theory relates magnetic field cyclicity to two global field configurations (toroidal and poloidal) and to two effects of the conversion of one configuration into another: the so-called ω- and α-effects. Both of these effects depend on the global and local structure of the solar velocity field, which plays the main role in magnetic field self-excitation in a conducting plasma.

This paper concerns factual data on prolonged variations in cyclicity characteristics, which should to some extent be related to slow variations in the global solar velocity field.

First, we consider the data that form the basis for our study. We have direct data on solar activity only from the first half of the 17th century, and direct and comparatively regular data are available only since the 1920s. Therefore, to reconstruct cyclicity on longer time intervals, it is necessary use indirect data sources on the one hand and special approaches, which make it possible to obtain the general consistent pattern of solar activity variations, on the other hand.

An approach involving "time scales," which are distinguished in accordance with our possibilities for using observational material for reconstructions, was proposed in (Nagovitsyn et al., 2004, 2008). We distinguish the following successive scales: (a) 100–150 years, direct regular solar activity observations; (b) 400 years, irregular direct activity observations; (c) 1000–2000 years, the selection of indirect data on solar activity (auroras and sunspots observed by naked eye and radionuclides in natural archives); (d) 10000 years, indirect data (called "proxies" in English literature) on solar activity: data on the radiocarbon and beryllium-10 concentration in dated samples (tree rings, polar ice); (e) more than 10000 years, hypothetical data (thickness of varve and clay laminas), the origin of which is related to solar activity according to some researchers. The justifiability of such an approach follows a "logarithmic logic" on the one hand: it is necessary to use unfinished information in order to present longer intervals; on the other hand, each subsequent scale is based on the previous one as a reference scale, which is important for successive calibration of reconstructions.

In this work we consider a change in the cyclic characteristics of solar activity on long (~2000 years) and ultra-long (10000 years and longer) time scales. We will mainly be interested in variations in the current frequencies (periods) of the 11- and 200-year cycles. Note that it is always simpler to study variations in frequency characteristics than in amplitude characteristics as applied to the problems of reconstruction: frequency characteristics are less subjected to trends and random errors that distort information.

2. METHOD FOR STUDYING VARIATIONS IN CYCLE PERIODS

A wavelet approach is a method for studying the local frequency behavior of cyclic processes that makes it possible to simultaneously localize a function with respect to time and frequency, in contrast to a Fourier approach, when a signal is localized only with respect to frequency. The procedure of expansion in terms of wavelets—self-similar, adequately localized, soliton-like functions—was developed in many problems, starting from (Grosmann and Morlet, 1984). An orthonormal basis, which is used to expand a function, is constructed using linear dilations and translations of a basis wavelet, so that a wavelet transform is a certain "mathematical microscope" that scales a signal over time scales (Astaf'eva, 1996).

A real MHAT wavelet

$$\psi(t) = (1 - t^2)e^{-t^2/2}, \tag{1}$$

which is an element of the family formed by the *m*th derivative of Gaussian $\psi^{(m)} = (-1)^m \frac{\partial^m}{\partial t^m} e^{-t^2/2}$ at $m = 2$, is often used for problems that require good time localization of a signal. The Morlet wavelet

$$\psi(r) = \exp(imr)e^{-r^2/2} \tag{2}$$

at $m = 5$ or 6 is most frequently used to improve resolution in the frequency region.

An orthonormal basis as norm $\|p\| = \langle p, p\rangle^{1/2}$, $\langle p, q\rangle^{1/2} = \int_{-\infty}^{\infty} p(t)q^*(t)dt$ is formed from a basis wavelet as $\psi_{jk}(t) = 2^{j/2}\psi(2^j t - k)$.

A continuous wavelet transform of the studied function $f(t)$ is defined by the formula

$$[W_\psi f](a, b) = |a|^{-1/2} \int_{-\infty}^{\infty} f(t)\psi^*\left(\frac{t-b}{a}\right)dt, \tag{3}$$

where parameters *a* and *b* are related to the frequency and time scales, respectively. Thus, performing a wavelet transform (3), we find a correlation between $f(t)$ and an analyzing wavelet during its linear scale transformations (dilation) and its displacement along the implementation length (translation).

The calculation of the wavelet transform from sinusoid and the construction of the so-called "skeleton," which outlines variations in the position of maximums $[W_\psi f](a,b)$, make it possible to interpret expression (3) from $f(t)$ in terms of signal local frequencies and amplitudes.

In this work we will initially perform the following procedures using the MHAT wavelet in order to distinguish the cycles with a specific duration: (1) frequency filtering a time series (setting to zero all frequency components, except the required one, after a direct wavelet transform and then performing an inverse wavelet transform); and (2) analyzing current period variations using a skeleton of the Morlet transform of the sixth order.

3. VARIATIONS IN THE DURATION OF THE SCHWABE–WOLF CYCLE ON THE 2000-YEAR SCALE

It is known that the number of low-latitude auroras changes in parallel with the 11-year cycle of solar activity (Schove, 1983) and correlates well with the *aa* geomagnetic index (Pulkkinen et al., 2001). We use the historical data on these auroras as a "proxy" cycle duration and the extensive Silverman catalog (http://nssdcftp.gsfc.nasa.gov/miscellaneous/aurora/cat_ancient_auroral_obs_666bce_1951/) as a data source, having eliminated the events observed in high-latitude (higher than 50° N) countries from this catalog. It is necessary to perform the latter procedure in order to distinguish the most geoeffective events profiling the cyclicity behavior (see Fig. 1a).

As we mentioned in (Nagovitsyn, 2005), archived historical data are subjected to time scattering (loss of consideration). This is evident in Fig. 1a: variations decay as we go deeper into the past. We try to correlate this effect. Assume that $N(0, \tau)$ is the true number of auroras in year τ and that $N(t, \tau)$ is the number of auroras realized by us in time *t* after τ. In such a case, on the assumption that information is regularly scattered,

$$\frac{\partial N(t,\tau)}{\partial t} = -\gamma N(t,\tau). \tag{4}$$

From this we obtain

$$N(0,\tau) = N(t,\tau)\exp(\gamma t). \tag{5}$$

The information scattering coefficient (γ) can be found using the least squares method. It is clear that procedure (4)–(5) corrects information scattering only in a first approximation: this procedure cannot take into account individual episodes of local loss (such as, e.g., barbarous destruction of manuscripts, which took place in China in the 3th century BC under the Emperor Qin Shi Huang-di or in Alexandria in the 7th century AD under Caliph Umar). On the other hand, if we (as in this paper) are interested in frequency information, a monotonous continuous multiplicative correction does not change the frequency structure in the selected band. We also note that procedure (4)–(5) seems to be promising for future researchers dealing with reconstruction of the solar activity level.

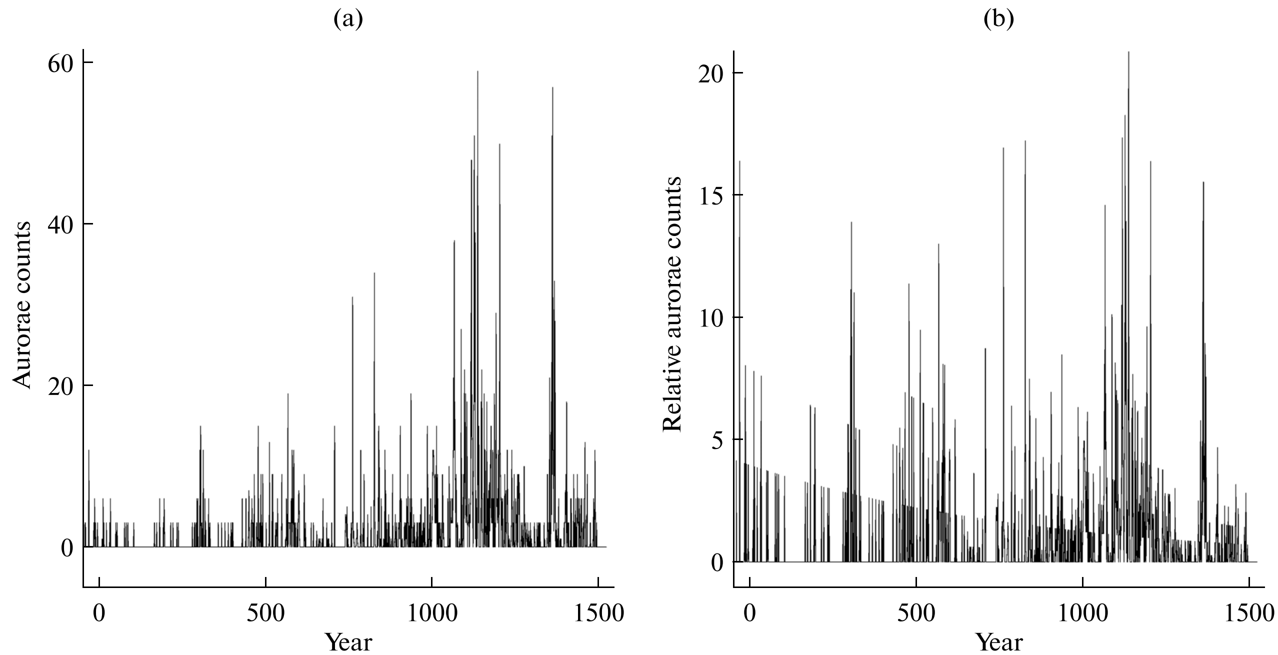

Fig. 1. The average annual number of auroras from the Silverman catalog: (a) observed and (b) corrected according to (4)–(5). Calendar years are plotted on the abscissa.

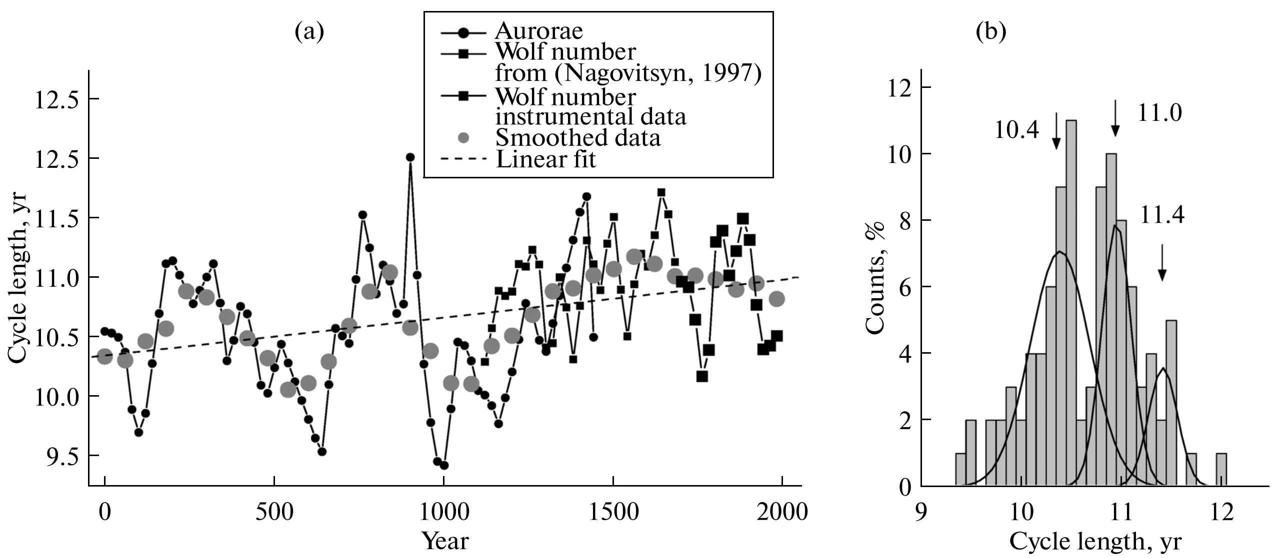

Fig. 2. (a) Variations in the duration of the 11-year solar activity cycle (T_{11}) during 0–2000 AD, obtained from different data sources (see the legend in the top part of the figure); (b) the histogram of the occurrence frequency of the T_{11} values during 0–2000 AD.

In addition to auroras, in this section we used the traditional Wolf number series and our "nonlinear version" of variations in this index since the 11th century (Nagovitsyn, 1997).

Figure 2 illustrates the obtained pattern of variations in the duration of the 11-year cycle (T_{11}) on the 200-year scale. It is evident that this pattern is not constant. A significant (at a level of 3.7σ) trend and prolonged (during several centuries) variations are observed over 2000 years. If we construct the occurrence frequency histogram for individual T_{11} values, it becomes clear that this histogram is bimodal and has

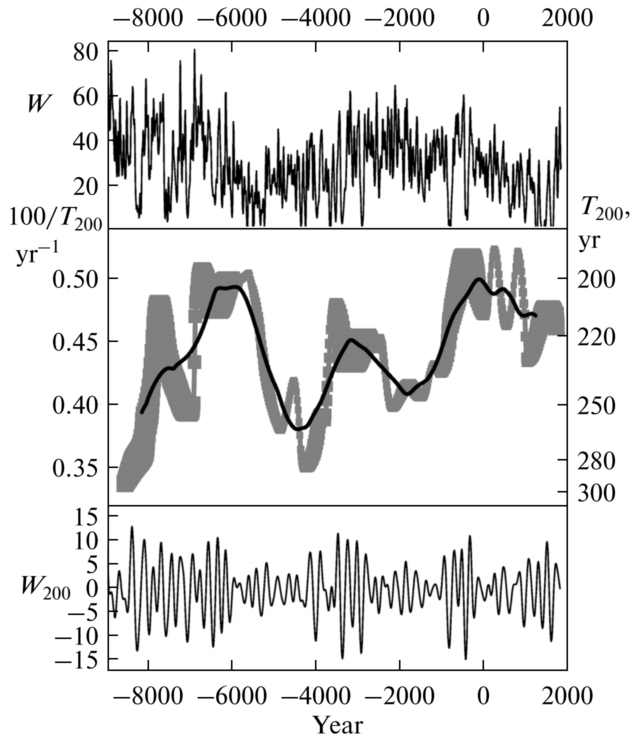

Fig. 3. Reconstructed solar activity variations during the Holocene based on the measurements of the radiocarbon fractional concentration in tree rings (top); the Suess component filtered with the MHAT wavelet (bottom); the component skeleton obtained using the Morlet wavelet (gray) and smoothed over 1000 years (black, middle). The dimensions of skeleton squares are proportional to the local cycle amplitudes. Calendar years are plotted on the abscissa.

maximums near 10.4, 11.0, and 11.4 years (see Fig. 3), which is in agreement with the results achieved in (Guseva and Nagovitsyn, 2012).

4. VARIATIONS IN THE SUESS CYCLE DURATION ON THE 10000-YEAR SCALE

For intervals longer than 2000 years, we have no admissible "proxies" for studying the 11-year cyclicity. However, we have information for studying longer cycles, and we are interested in the 200-year Suess cycle.

As initial data we used here the known decadal reconstruction of prolonged variations in solar activity from (Solanki et al., 2004), which is based on the fractional concentration of radiocarbon in tree rings.

Figure 3 illustrates the achieved results. It is clear that the Suess cycle duration considerably changes, so that the cycle period varies within 200-290 year during the Holocene and tends to increase toward the past. In this case we observe cyclic 2300–2500-year variations corresponding to the duration of the so-called Hallstatt cycle (Vasiliev and Dergachev, 2002).

5. ELATINA CYCLES AND HYPOTHETICAL VARIATIONS IN SOLAR ACTIVITY PARAMETERS DURING SEVERAL HUNDREDS OF MILLION YEARS

Thirty years ago it was widely discussed in the literature whether the varve thickness in the late Precambrian (~700 Ma) Elatina formation, South Australia, according to the Williams data (Williams and Sonett, 1985; Sonett et al., 1988) depends on solar activity (or on other factors). The author of the published series finally accepted the hypothesis that lamina size depends on tidal variations. However, we assume that this problem has not yet been finally solved.

We considered the fine time structure of the varve thickness series $E(t)$ in comparison with the solar activity series $W(t)$, including the so-called "alternate" version (Bracewell, 1988) (Fig. 4). It is interesting that the time structures of the series, especially those of alternate ones, are very similar (the main and secular modulating cycles are present). In addition, we constructed Fourier spectra $E(t)$ and $W(t)$ (Fig. 5a).

When compared, the Fourier spectra turn out to be surprisingly similar accurate to a certain continuous transformation of positions of individual frequency components (Fig. 5b).

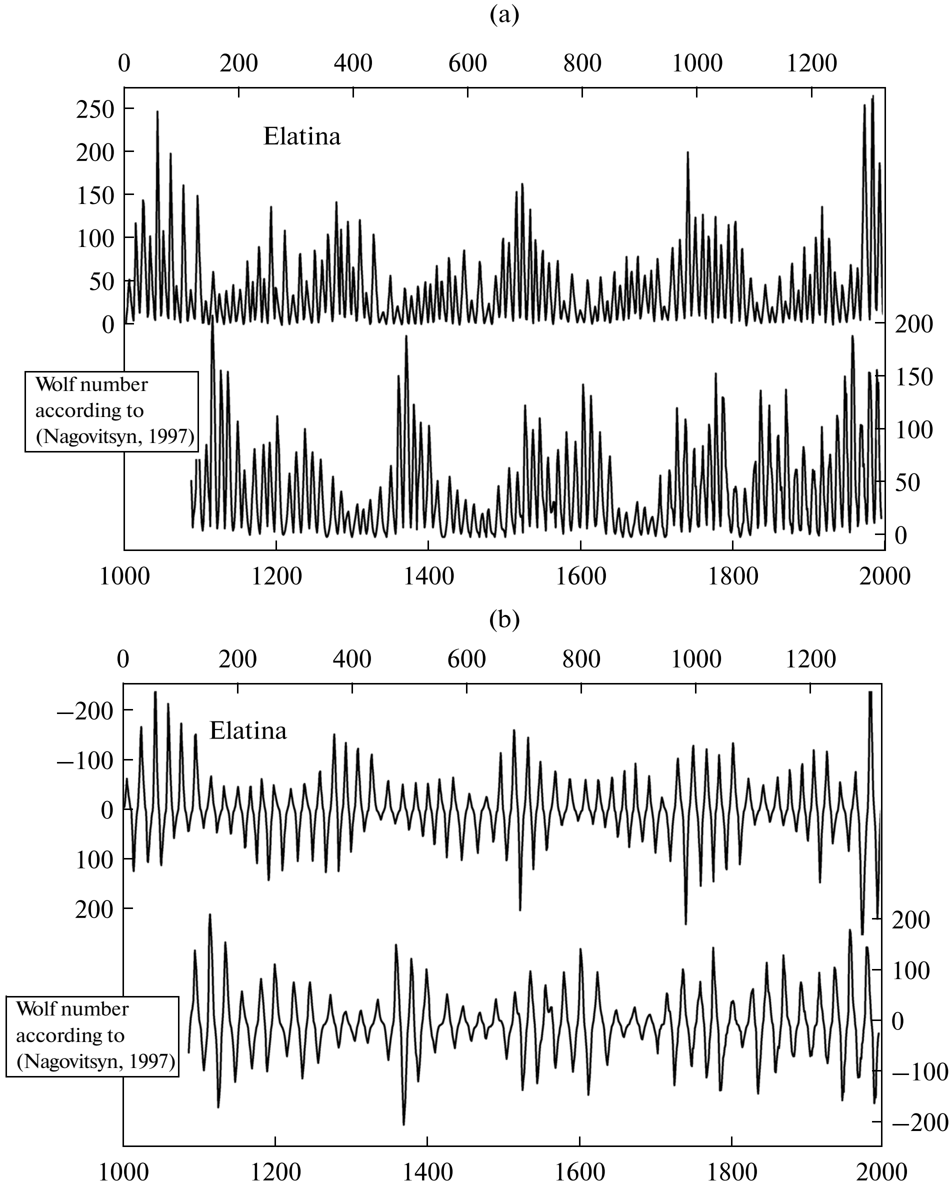

Fig. 4. Compared Elatina varve thickness series (upper panels) and solar activity variations according to (Nagovitsyn, 1997) (lower panels); (a) usual series, (b) alternate series. Calendar years for solar activity and the number of successive varve for the Elatina formation are plotted on the abscissa.

Summarizing, we note that the structure of time series $E(t)$ does not contradict the assumption that it is caused by the Sun. Bearing this in mind, we considered the variation in the Suess cycle on the ultra-long scale. Figure 6 presents the variation in the period of this cycle in the Holocene according to the data from the previous section, the linear extrapolation of this variation with confidence intervals at a 95% significance level, and the period value for the Elatina formation (late Precambrian, 680 Ma ago) independently obtained using a Fourier analysis. It is evident that the Suess cycle period has decreased by a factor of more than 1.5 for the past half billion years, if we accept the hypothesis that the Elatina cycles depend on solar activity.

This may indicate that the solar rotation characteristics and convection zone parameters varied on long time scales during the Sun's evolution on the main sequence.

6. CONCLUSIONS

We showed that the duration of the Schwabe–Wolf cycle on average increased over the last 2000 years and fluctuated on time scales of several hundred years. In

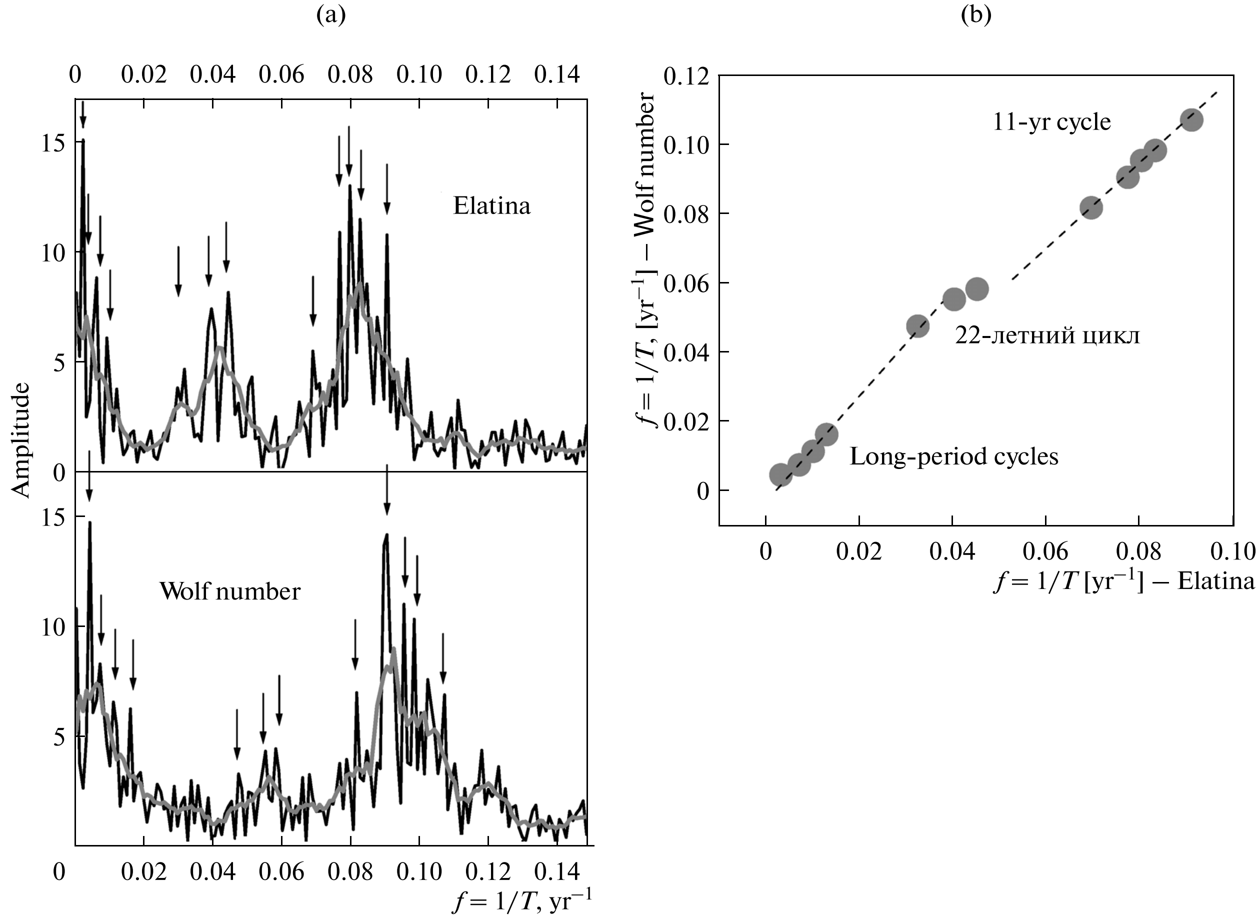

Fig. 5. (a) The Fourier spectra of the Elatina series and Wolf number according to (Nagovitsyn, 1997); (b) compared positions of the frequency components in these series.

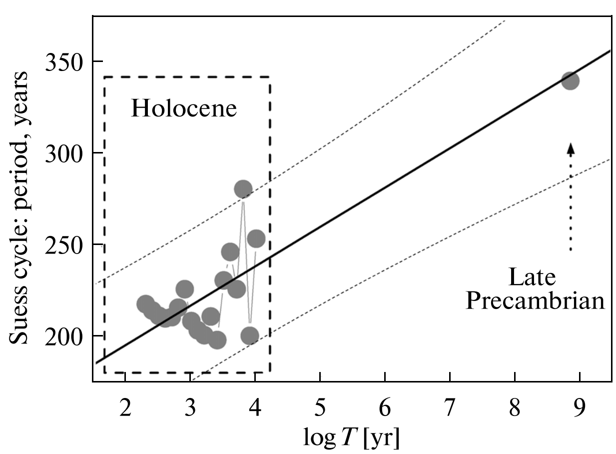

Fig. 6. Variations in the Suess cycle period for the past 680 Myr. The linear extrapolation of the cycle duration according to the data in the Holocene (a solid line); the confidence intervals at a 95% significance level (a dashed line). Logarithm of years toward the past is plotted on the abscissa.

this case 10.4, 11.0, and 11.4 years predominate on the histogram of the occurrence frequency of the duration values.

The duration of the Suess cycle during the Holocene varied within 200–290 years and tended to increase toward the past, which was accompanied by cyclic variations with a period of 2300–2500 years corresponding to the Hallstatt cycle.

We presented arguments for the assumption that the Suess cycle duration decreased by a factor of more than 1.5 over the past half billion years. This may indicate that the solar rotation characteristics and convection zone parameters varied on long time scales during the Sun's evolution on the main sequence.

Summarizing, we can state that the solar dynamo operates in varying modes both on the evolutionary time scales and on time scales of several thousand years to several tens of thousand years.

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