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Modeling and Analysis of Ionospheric Parameters by a Combination of Wavelet Transform and Autoregression Models

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Abstract—We propose a method of the modeling and analysis of ionospheric parameters by combining wavelet transform with autoregression models (integrated moving average). The method makes it possible to reveal regularities in ionospheric parameters and to make forecasts on variations. Also, this method can be used to fill the gaps in ionospheric parameters, with consideration of their diurnal and seasonal variations.
The method was tested on *foF*2 data and data on the total electron content for the regions of Kamchatka and Magadan. The models constructed for the natural variation in ionospheric parameters allowed us to analyze its dynamical mode and build a forecast with a step of up to five hours. Based on estimates for model errors, we revealed anomalies arising during periods of increased solar activity and strong earthquakes in Kamchatka.

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1. INTRODUCTION

An important problem of ionospheric data analysis is to control the state of the ionosphere, to reveal and interpret the anomalies arising during ionospheric disturbances (Afraimovich and Perevalova, 2006; Liperovskaya et al., 2006; Deminov, 2008; Mandrikova et al., 2013a). Ionospheric anomalies may be caused by increased solar activity; in seismically active areas, they can also be observed in periods of increased seismic activity (Afraimovich and Perevalova, 2006; Liperovskaya et al., 2006; Mandrikova et al., 2013a). The structure of the ionosphere is variable and nonuniform, and its study is based on the analysis of variations in recorded parameters of the medium. The uncertainty of knowledge about the structure of recorded parameters and the absence of a formal model for their description make the given problem very difficult. Despite the rapid development of technologies for monitoring near-Earth space and methods for data analysis, the potential for controlling and forecasting the state of the ionosphere has still been very limited (Afraimovich and Perevalova, 2006).

The complex structure of variations in ionospheric parameters makes the traditional smoothing-based methods inefficient for modeling and analysis; these methods fail to study the fine features of data (Mandrikova and Polozov, 2012; Mandrikova et al., 2013a), which normally contain key information about the processes under consideration. A popular method of modeling and data analysis is the autoregressive integrated moving average (ARIMA) (Box and Jenkins, 1974; Nikiforov, 1983; Marple, 1990). ARIMA models make it possible to study the stable characteristics of the data structure and build their prediction. The practice has confirmed the power and flexibility of this mechanism in solving many applied problems (Marple, 1990; Geppener and Mandrikova, 2003). However, ARIMA methods have limitations, both in the possibility of their use for individual data structures and the regularities revealed in this case (Geppener and Mandrikova, 2003). The rapidly developing methods of wavelet transform (Donoho and Johnstone, 1998; Mallat, 2005), which couple the theory of data approximation and filtering techniques, make it possible to largely cope with this problem (Geppener and Mandrikova, 2003; Mandrikova and Polozov, 2012a; Mandrikova et al., 2012; Mandrikova et al., 2013a). Because of the great diversity of basis wavelet-functions with compact supports, this mechanism enables detailed study of the internal structure of complex data. Fast algorithms of wavelet transform allow its implementation in real time, which is very important for solving problems of operational data analysis.

The method of modeling and analysis of ionospheric parameters proposed in this paper combines the wavelet transform with ARIMA models. This method is based on the construct of multiscale analysis (fast wavelet transform (Mallat, 2005)), which represents the original time series as multiscale components. The resulting components have a simpler structure compared with original data and are approximated by ARIMA methods. This approach was first proposed for solving the problem of revealing anomalies in subsoil radon data and proved to be efficient

(Geppener and Mandrikova, 2003). The identification of *component models* involves noise suppression and the detection of stable characteristics of the data structure. This paper proposes to combine the resulting multiscale *component models* into a *common parametric construct* describing the time course of data, allowing one to build a forecast. Based on the estimation of residual errors of the resulting forecast, we develop an algorithm of anomaly detection.

The constructed multicomponent models of time series of the critical frequency of the ionosphere *foF*2 (data recorded by the Institute of Space Physics Research and Radiowave Propagation, Far East Division, Russian Academy of Sciences) and the total electron content (TEC) obtained from data of double-frequency ground-based GPS receivers (Afraimovich and Perevalova, 2006) over Kamchatka and Magadan confirmed the efficiency of the proposed method and made it possible to analyze the regular diurnal and seasonal variations in the parameters. On the basis of estimated deviations from the background level, we revealed ionospheric anomalies from several dozen minutes to several hours, resulting in periods of ionospheric disturbances. The analysis of anomalies indicated that they occur in periods of increased solar activity and during strong earthquakes in Kamchatka.

2. DESCRIPTION OF THE METHOD

2.1. Construction of the Model

Since the time series $f(t)$ has a structure that can vary at random times, the most effective way to describe it is to use approximation methods based on the expansion of functions with respect to the basis of the Lebesgue space $L^2(R)$:

$$f(t) = \sum_l c_l \xi_l(t),$$

where $c_l = \langle f, \xi_l \rangle$ is the expansion coefficients and ξ_l is the basis of the space $L^2(R)$.

To construct models that adapt to the structure of time series, we use nonlinear approximating schemes. In this case, f is approximated by M vectors that depend on its structure:

$$f_K = \sum_{k \in I} \langle f, \xi_k \rangle \xi_k,$$

where I is the set of indices that depends on the structure of f.

Since the analyzed features are diverse in form and have a local character and different scales, the most appropriate for their representation is the wavelet-space (Mallat, 2005):

$$f_K = \sum_{(j,k) \in I^j} d_{j,k} \Psi_{j,k},$$

where $d_{j,k} = \langle f, \Psi_{j,k} \rangle$ are the expansion coefficients, $\Psi_{j,k}(t) = 2^{j/2}\Psi(2^j t - k)$ is the orthonormal wavelet-basis of the space $L^2(R)$, and I^j is the set of indices that depends on the structure of f. The expansion coefficients $d_{j,k}$ are regarded as the result of mapping of f into the space of scale j.

Without a loss of generality, the basis of the space of recorded discrete data can be taken to be the closed space $V_j = \mathrm{clos}_{L^2(R)}(2^j \phi(2^j t - k)) : k \in Z)$ of scale $j = 0$ generated by the scaling-function $\phi \in L^2(R)$ (Mallat, 2005). Then, based on multiscale expansions (of fast wavelet-transform) up to level m, we can represent the data in the form proposed in (Mandrikova and Polozova, 2012a; Mandrikova et al., 2013a):

$$f_0(t) = \sum_{j=-1}^{-m} (g[2^j t] + e[2^j t]) + f[2^{-m} t], \quad (1)$$

where $g[2^j t] = \sum_k d_{j,k} \Psi_{j,k}(t)$ are detailing components, the expansion coefficients $d_{j,k} = \langle f, \Psi_{j,k} \rangle : (j,k) \in I^j$ describe units of different scales, $g[2^j t] \in W_j$, W_j is a space of scale j born by the wavelet-basis $\Psi_{j,k}(t) = 2^{j/2}\Psi(2^j t - k)$; $e[2^j t] = \sum_k e_{j,k} \Psi_{j,k}$ are noise components with the expansion coefficients $e_{j,k} = \langle f, \Psi_{j,k} \rangle : (j,k) \notin I^j$; $f[2^{-m} t] \in V_{-m}$ is the approximating component $f[2^{-m} t] = \sum_k c_{-m,k} \phi_{-m,k}(t)$, where the expansion coefficients $c_{-m,k} = \langle f, \phi_{-m,k} \rangle$ describe the trend of the series.

On the basis of mapping (1), we represent the time series as the sum of different-scale components $f[2^{-m} t]$, $g[2^j t]$, and $e[2^j t]$, where $j = \overline{-1, -m}$. To suppress the noise components $e[2^j t]$ and identify the components $f[2^{-m} t]$ and $g[2^j t]$ describing the *stable characteristics of the data structure*, we use the following operations.

(1) We restore each of the resulting components $f[2^{-m} t]$, $g[2^j t]$, and $e[2^j t]$, $j = \overline{-1, -m}$ to the original scale $j = 0$ and obtain restored components of the form $f_0^\mu(t) = \sum_k c_{0,k}^\mu \phi_{0,k}(t)$, $g_0^\mu(t) = \sum_k d_{0,k}^\mu \Psi_{0,k}(t)$ and $e_0^\mu(t) = \sum_k e_{0,k}^\mu \Psi_{0,k}(t)$, where μ is the number of components.

(2) Using the traditional approach (Box and Jenkins, 1974), we construct ARIMA models to approximate each of the resulting restored components $f_0^\mu(t) = \sum_k c_{0,k}^\mu \phi_{0,k}(t)$, $g_0^\mu(t) = \sum_k d_{0,k}^\mu \Psi_{0,k}(t)$, and $e_0^\mu(t) = \sum_k e_{0,k}^\mu \Psi_{0,k}(t)$.

(3) We conduct diagnostic tests for the resulting component models (Box and Jenkins, 1974). If these tests confirm that the model is adequate to data, we assume that the model of the component is ready for use and that this component describes the *stable characteristics* of the data structure in line with the theory of ARIMA methods.

(4) Using relationship (1), we combine the models of identified components into a common *parametric construct* (the remaining components of the series of relationship (1) are assumed to be noisy). Let us obtain a *parametric multicomponent model* describing the time course of data:

$$f_0(t) = \sum_{\mu=\overline{1,M}} \sum_{k=1,N_j^\mu} s_{j,k}^\mu(t) b_{j,k}^\mu(t), \qquad (2)$$

where $s_{j,k}^\mu(t) = \sum_{l=1}^{p_j^\mu} \gamma_{j,l}^\mu \omega_{j,k-l}^\mu(t) - \sum_{n=1}^{h_j^\mu} \theta_{j,n}^\mu a_{j,k-n}^\mu(t)$ is the estimated value of the μth component, $\gamma_{j,l}^\mu$ are the autoregressive parameters of the μth component, $\omega_{j,k}^\mu(t) = \nabla^{\nu^\mu} \beta_{j,k}^\mu(t)$; $\beta_{j,k}^1 = c_{j,k}^1$; $\beta_{j,k}^\mu = d_{j,k}^\mu$, $\mu = \overline{2,M}$; ν^μ is the order of the difference of the μth component, p_j^μ is the order of the autoregressive model of the μth component, h_j^μ is the order of the model, $\theta_{j,k}^\mu$ are the parameters of the moving average of the model of the μth component, $a_{j,k}^\mu$ are the residual errors of the model of the μth component, M is the number of modeled components describing the stable characteristics of the data structure, N_j^μ is the length of the μth component, $b_{j,k}^1 = \phi_{j,k}$ is the scaling-function, $b_{j,k}^\mu = \Psi_{j,k}$, $\mu = \overline{2,M}$ is the wavelet-basis of the μth component, and j is the scale.

Remark. Relationship (2) is true for any scale j. Therefore, the proposed model can be identified without restoring the components to the original scale $j = 0$ (see operation 1) (Mandrikova et al., 2013). Thus, based on a changed expansion level (see relationship (1)), different models of form (2) can be obtained for describing a time series. By minimizing the residual errors of the resulting models, one can chose the *best multicomponent time-series model*. The resulting estimate can also be improved by using different wavelet-functions.

2.2. Data Prediction and Anomaly Detection

The prediction of the value $s_{j,k+q}^\mu$, $q \geq 1$ (see relationship (2)) determines the forecast $s_{j,k}^\mu$ at time $t = k$ with a step q. The value of $s_{j,k+q}^\mu$ on the basis of model (2) is determined as

$$s_{j,k+q}^\mu(t) = \sum_{l=1}^{p_j^\mu} \gamma_{j,l}^\mu \omega_{j,k+q-l}^\mu(t) - \sum_{n=1}^{h_j^\mu} \theta_{j,n}^\mu a_{j,k+q-n}^\mu(t).$$

The *residual errors* of the μth component of the model of scale j are determined as the difference between predicted and actual data values at the time $t = k + q$:

$$a_{j,k+q}^\mu(t) = s_{j,k+q,\text{predicted}}^\mu(t) - s_{j,k+q,\text{actual}}^\mu(t).$$

The resulting model (2) describes the regular changes in approximated data. If there is an anomaly in data, their structure is changed, and the absolute values of residual errors increase. Therefore, the *procedure for separating anomalies* can be built on estimated residual errors of resulting component models obtained in the operation of forecasting.

The *detection of anomalies* in the component with number μ of scale j is done by checking the *condition*

$$D_{U_j} = \frac{1}{U_j} \sum_{k=1}^{U_j} \left(a_{j,k+q}^\mu(t)\right)^2 > T_{A_j}, \qquad (3)$$

where U_j is the length of the observation window on the scale j, and T_{A_j} is some preset threshold value indicating the presence of anomalies of scale j in the data.

3. MODELING RESULTS AND DATA ANALYSIS

This study used hourly data on the critical frequency *foF*2 and two-hour TEC data for regions of Kamchatka and Magadan. The *foF*2 data contain gaps, which significantly complicates the process of modeling and analysis. To reduce the error in the results obtained, time periods with the smallest number of gaps were chosen. Since the ionospheric process has a seasonal character, the data were arranged with respect to seasons and modeled separately. Below, we give a detailed description of the modeling stages of data on the critical frequency for winter and summer seasons.

In line with the selection criteria of basis wavelet-functions proposed in (Mandrikova and Polozov, 2012b), we specify Daubechies family basis functions for the expansions. Using a changed level of decomposition (see relationship (1)) and different wavelet-functions to describe the time course of *foF*2, different models of type (2) were obtained. The experimental results showed that the lowest approximation error in the *foF*2 data is provided by third-order Daubechies wavelets. The stage of identification and diagnostics of models of different components of the series (see operations 2 and 3 in section 2.1) showed that the original series of *foF*2 and their approximating components of the first and second expansion levels have a complex structure and cannot be approximated by an ARIMA model (there was a significant autocorrelation in the residual of models). By minimizing the residual errors in the resulting models, we obtain the *best model of the time course* of *foF*2, which includes three components

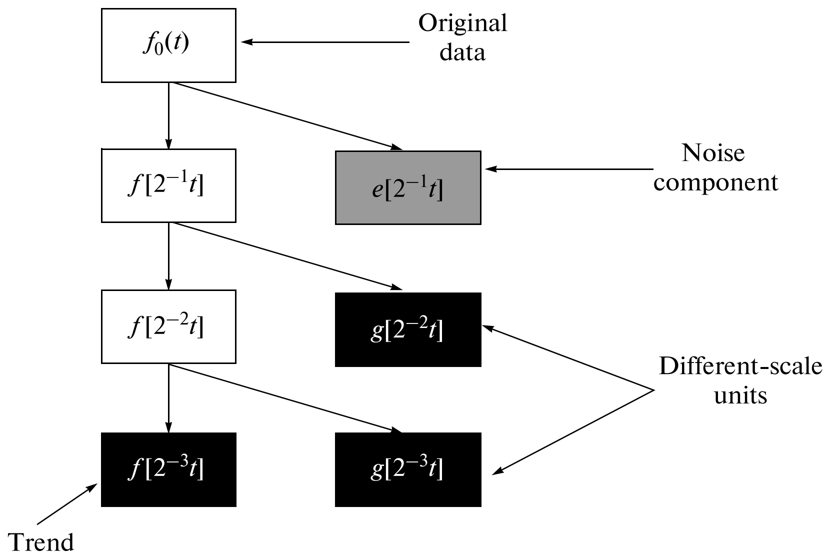

Fig. 1. Approximated components in the wavelet space (marked in black).

(for the winter and summer seasons) having the following form in the wavelet space (Fig. 1):

$$f_0(t) = f\left[2^{-3}t\right] + \sum_{j=-2}^{-3} g\left[2^j t\right] + e\left[2^{-1}t\right], \tag{4}$$

where the component $f\left[2^{-3}t\right] = \sum_k c_{-3,k}\phi_{-3,k}(t)$ describes the trend of the series, the components $g\left[2^j t\right] = \sum_k d_{j,k}\Psi_{j,k}(t)$, $j = -2, -3$, describe different-scale units, and $e\left[2^{-1}t\right]$ is the noise component.

For each of the restored components $f_0^1(t) = \sum_k c_{0,k}^1 \phi_{0,k}(t)$ (the scale of the component before restoring is $j = -3$; the time periods are 8 h or more), and $g_0^\mu(t) = \sum_k d_{0,k}^\mu \Psi_{0,k}(t)$, $\mu = 2,3$ (the scale of the components before restoring is $j = -2, -3$; the time periods are 2 to 4 h and 4 to 8 h, respectively) obtained from transform (4), we identified second-order integrated autoregressive models. The table shows evaluation parameters of the models of these components, obtained for data of different time periods. Due to the closeness between the values of model parameters and corresponding components, we specified a common model for the time course of *foF*2 for the winter and summer seasons:

$$f_0(t) = \sum_{\mu=\overline{1,3}} \sum_{k=\overline{1,N}} s_{0,k}^\mu(t) b_{0,k}^\mu(t), \tag{5}$$

where $s_{0,k}^1(t) = (1 + 0.27B)(1 - B)\omega_{0,k}^1(t) + a_{0,k}^1(t)$; $s_{0,k}^2(t) = (1 - 0.81B)(1 + 0.35B)\omega_{0,k}^2(t) + a_{0,k}^2(t)$; $s_{0,k}^3(t) = (1 - 0.35B)(1 + 0.65B)\omega_{0,k}^3(t) + a_{0,k}^3(t)$; $B^l\omega_{0,k}^\mu(t) = \omega_{0,k-l}^\mu(t)$; $a_{0,k}^\mu(t)$ are the residual errors of the model of the μth component; $\omega_{0,k}^\mu(t) = \nabla\beta_{0,k}^\mu(t)$; $\beta_{0,k}^1 = c_{0,k}^1$; $\beta_{0,k}^\mu = d_{0,k}^\mu$, $\mu = 2,3$; $a_{0,k}^\mu$ are the residual errors in the model of the μth component; N is the length of the series; $b_{0,k}^1 = \phi_{0,k}$ is the scaling-function; and $b_{0,k}^\mu = \Psi_{0,k}$, and $\mu = 2, 3$, is the wavelet-basis of the μth component.

Figure 2 shows the results of modeling and forecasting of *foF*2 data at Paratunka station using the resulting model (5) (for the period of analysis from Feb. 12, 2011 to Feb. 25, 2011). Analysis of Fig. 2 confirms the efficiency of the proposed method and shows that the resulting model can predict ionospheric parameters with a step of up to five hours. Certain times points are characterized by increased prediction errors (Figs. 2h–2j). To analyze the prediction errors, the model results were compared with geomagnetic data (for the *H*-component of the geomagnetic field), which were used to estimate the intensity of geomagnetic disturbances in the given time periods (the intensity of geomagnetic disturbances was estimated by the method proposed in (Mandrikova et al., 2013b)). The analysis results presented in Fig. 3 show that an increase i prediction errors indicating the occurrence of anomalies in the ionosphere was observed on the eve of the major earthquake in Kamchatka (Feb. 20, 2011; energy class of $E = 14.1$; the time of anomaly is shown in Fig. 3 by the dashed line). The periods of increased geomagnetic activity are also characterized by an increase in prediction errors. The statistical analysis of *foF*2 data for different years showed a substantial dependence of the intensity and frequency of anomalies in the ionosphere on the levels of solar (Fig. 4) and geomagnetic activities, which is consistent with the results of (Afraimovich and Perevalova, 2006). In a

Parameters of component models

Time period	$f_0^1(t)$		$g_0^3(t)$		$g_0^2(t)$	
	first parameter	second parameter	first parameter	second parameter	first parameter	second parameter
27.06.2005–10.07.2005 (Kamchatka)	1.013	–0.291	0.828	–0.339	0.443	–0.572
16.01.2006–04.02.2006 (Kamchatka)	1.012	–0.278	0.829	–0.344	0.371	–0.622
09.02.2011–27.02.2011 (Kamchatka)	1.009	–0.266	0.805	–0.355	0.437	–0.473
16.01.2006–04.02.2006 (Magadan)	1.009	–0.267	0.821	–0.348	0.366	–0.624
General model parameters (for summer and winter seasons)	1.01	–0.27	0.81	–0.35	0.35	–0.65

period of high solar activity, the background level of errors increases, while during increased geomagnetic activity, the deviation from the background level significantly increases, which points to the occurrence of anomalies.

The results of modeling of TEC data confirmed the relation of anomalies arising in the ionosphere with seismic events. Figure 5 shows as an example the results of a combined analysis of data on the critical frequency and TEC data. The anomalies in iono-

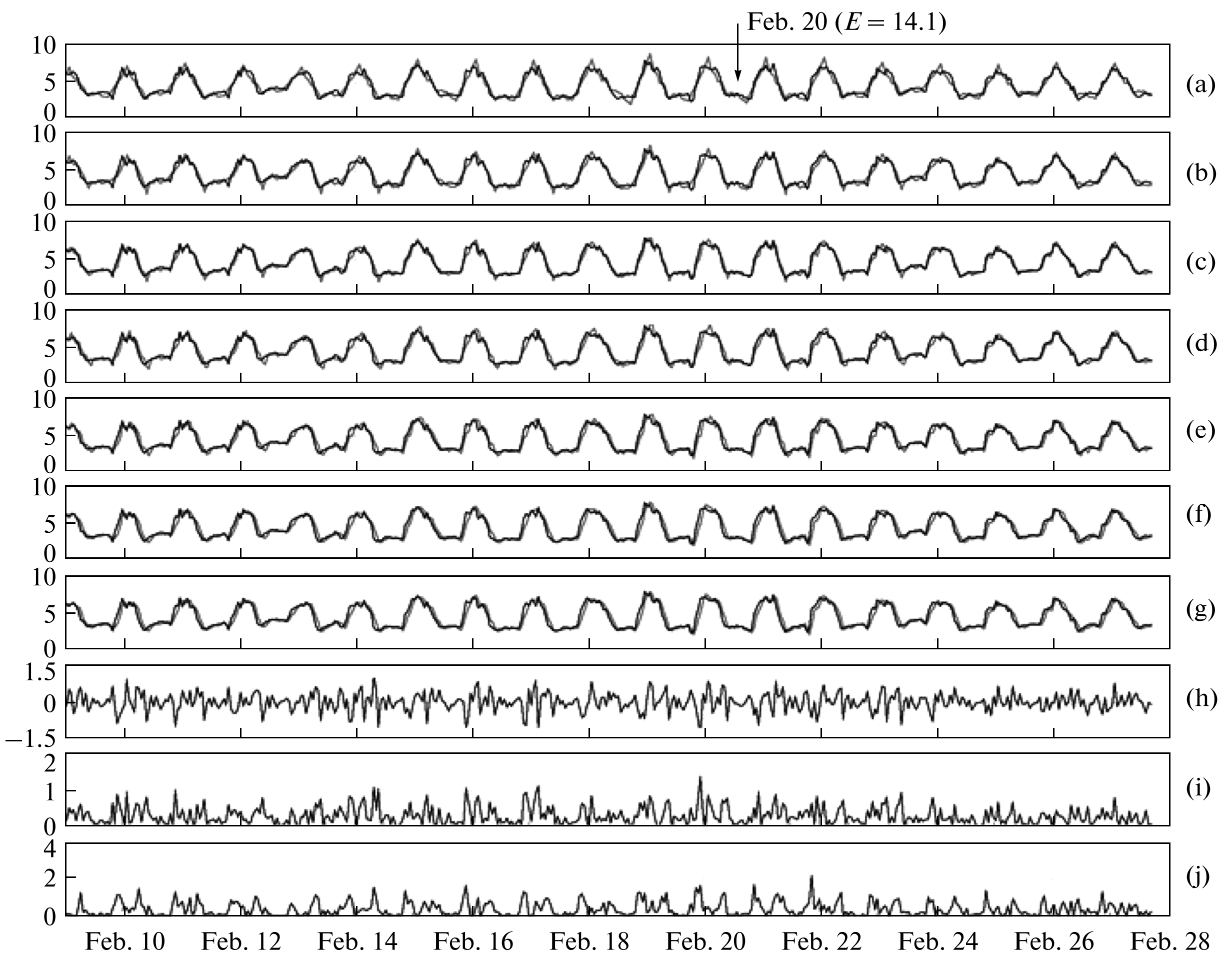

Fig. 2. Modeled and predicted data of *foF*2: (a) forecast of the component $f_0^1(t)$ with a step of $q = 1$; (b) forecast of the sum of components $f_0^1(t)$ and $g_0^3(t)$ with a step of $q = 1$; (c) forecast of the sum of components $f_0^1(t)$, $g_0^3(t)$, and $g_0^2(t)$ with a step of $q = 1$; (d), (e), (f), and (g) forecasts of the sum of components $f_0^1(t)$, $g_0^3(t)$, and $g_0^2(t)$ with steps of $q = \overline{2,5}$, respectively. The arrow indicates a seismic event.

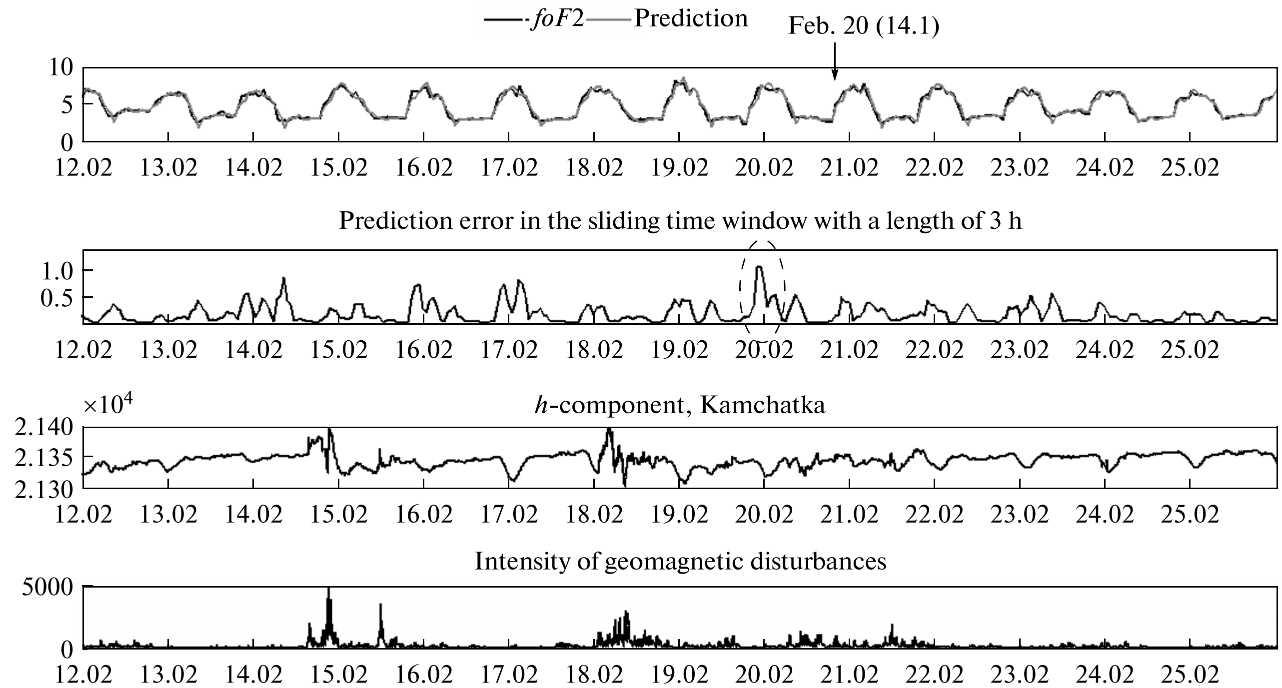

Fig. 3. Results of a detailed *foF*2 data analysis for the period from 12.02.2011 to 25.02.2011: the forecast was based on model (5) with a lead step of $q = 1$. The arrow indicates a seismic event.

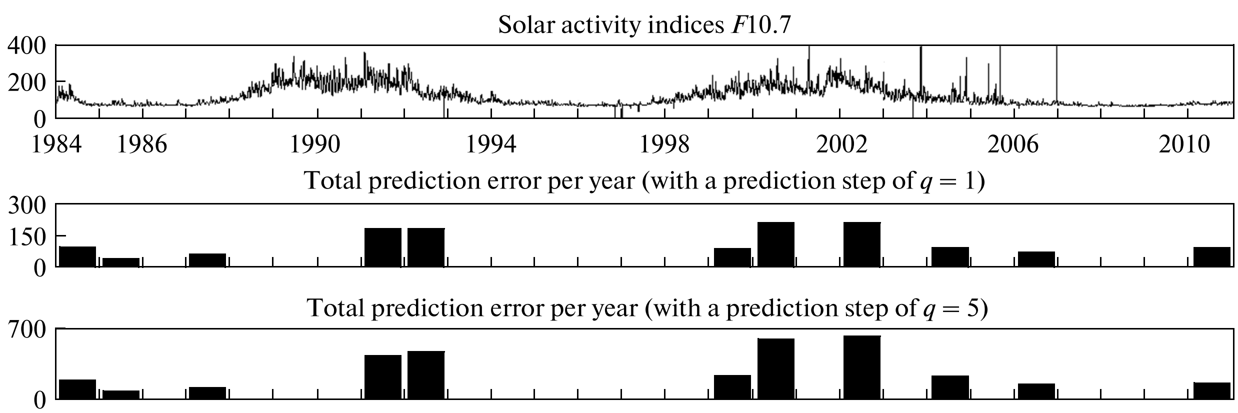

Fig. 4. Results of the estimation for errors of *foF*2 data forecast for different years.

spheric parameters were identified by checking condition (3) in a moving time window of 3 h for critical frequency data and 6 h for TEC data. On the eve of the series of seismic events (July 05, 2005, July 06, 2005, July 07, 2005, and July 10, 2005), there was a local increase in the prediction error in parameters of the critical frequency (Juny 30, 2005, the time of the anomaly was shown in Fig. 5 by the dashed line). At the time of the strongest earthquake (July 07, 2005; energy class of $E = 13$), the parameters of the critical frequency and TEC data are characterized by an arising large-scale anomaly of several days in length (shown in Fig. 5 by the dash-dotted line).

4. CONCLUSIONS

Using the proposed method of modeling and analysis of ionospheric parameters on the basis of a combination of wavelet-transform with ARIMA models, we approximate of the natural course of the critical frequency and TEC over Kamchatka and Magadan. Statistically, we have demonstrated that this method is efficient and can be used for studying regular changes in ionospheric parameters and predicting them with a step of up to five hours.

Based on estimated residual errors of the resulting models, we have revealed ionospheric anomalies with

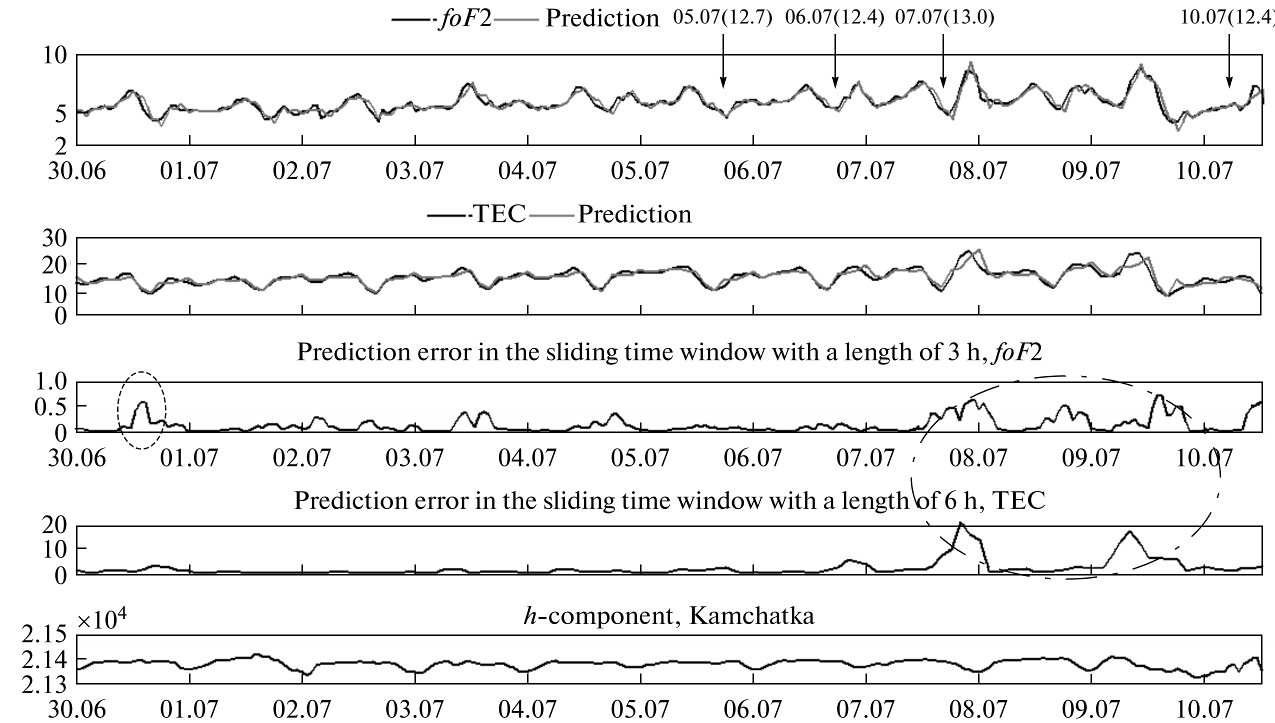

Fig. 5. Results of simulations of *foF*2 data and TEC data for the period from 30.06.2005 to 10.07.2005: the prediction was performed for the sum of components $f_0^1(t)$, $g_0^3(t)$, and $g_0^2(t)$ with a step of $q = 1$. The arrow indicates a seismic event.

a duration of several dozen minutes to several hours that arise in periods of ionospheric disturbances. The observed changes in ionospheric parameters have different scales and arise before and at the times of strong earthquakes in Kamchatka.

Analysis of the parameter variations in periods with different solar activity and comparison of model results with geomagnetic data showed that the intensity and frequency of anomalies in the ionosphere depends on the level of solar and magnetic activities. High magnetic activity is characterized by a significant increase in the deviation from the background level, which points to the occurrence of anomalies. High solar activity is characterized by an increase in the background level of errors and the frequency of anomalies.

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