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Compressible Gas Flow past a Plate with Upstream-Moving Surface

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Abstract—The numerical solution of the problem of compressible gas flow past a plate with upstream-moving surface is obtained. Unstable time-periodic and stable steady-state solutions of this problem are discussed. A new type of unsteady periodic flow in which the varying characteristics turn out to be integrally asymmetric at transonic velocities is obtained.

Keywords: unsteadiness, vortex, recirculation flows, moving surfaces, asymmetry

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In [1–9] the problem of incompressible fluid flow past a plate with upstream-moving surface was investigated. In [1–4] the solution was sought by integrating the time-independent boundary layer equations at zero pressure gradient. In this case it turned out that a solution of equations exists both for semi-infinite and finite plates if the ratio $\beta = -u_w/u_\infty$ of the plate surface velocity u_w to the free-stream velocity u_∞ is less than the critical value $\beta < \beta_1 = 0.3541$. Disappearance of the solution when $\beta > \beta_1$ can be explained. In the neighborhood of the plate surface the streamline jets go counter the oncoming stream because of the no-slip condition. As the distance from the leading plate edge decreases, the jets must be decelerated and then turn back. When $\beta < \beta_1$, the streamline jets are turned owing to the friction force. When $\beta > \beta_1$, the friction force is already insufficient to turn the streamline jets back. For this process it is necessary that another force appears. Such a force was introduced in [5–7]. It arises as a result of the pressure drop created by the recirculation flow zone. Inside such zones flow obeys Euler's equations. Consequently, the entire flow cannot already be described using the boundary layer equations only.

In [5, 6] the solutions were constructed for finite plate. In this case it was shown that there exists a second critical number β_2 such that for $\beta \le \beta_2$ the solution of the problem tends to a time-independent solution as time increases indefinitely, while for $\beta > \beta_2$ we obtain an infinitely growing self-similar solution. In [8, 9] it was established that when $\beta \ge 1$ the solution becomes unstable to asymmetric perturbations, a time-periodic solution being observed on the segment $1 \le \beta \le \beta_2$. The boundary of recirculation zones above and under the plate begins to depend on time. Vortices shedding to the wake behind the plate splash out periodically from the vortex zones either above the plate or beneath it. The lift force exerted on the plate develops. At the same time, the integral of the lift force over the time period is equal to zero, i.e., the flows of this kind may be called integrally symmetric. Time variation in the vorticity field over the second half of the period corresponds to the first half in the case of specular reflection about the plate surface and change in vorticity sign. In [7] the self-similar solution was constructed for the semi-infinite plate.

The present study is devoted to taking compressibility into account in the problem of flow past finite plate. The consideration includes the subsonic range of the free-stream transonic velocities. This coincides with the cruising speed regime of modern airplanes. At present, the possibility of using moving surfaces to control the flow is considered [10]. However, the theoretical investigations of such flows at transonic velocities are practically non-existent.

1. FORMULATION OF THE PROBLEM AND METHODS OF SOLVING

We will consider plane laminar flow of a viscous perfect gas past a stationary plate with moving surfaces. As mentioned above, the free-stream velocity is u_∞, the velocities of both plate surfaces are identical, directed counter the free stream, and are equal to u_w so that $\beta = -u_w/u_\infty$. We will assume that u_∞ and u_w take nonzero values starting from a certain instant of time. The plate is mounted at zero angle of attack, its length is equal to L. The dynamic viscosity coefficient can be determined from the Sutherland law:

$$\frac{\mu}{\mu_\infty} = \left(\frac{T}{T_\infty}\right)^{3/2} \frac{T_\infty + C}{T + C},$$

where T is the absolute temperature, $C = 110.4\,\mathrm{K}$, and the subscript ∞ corresponds to the values of quantities at infinite distance from the plate. The gas characteristics can be determined from the Navier–Stokes equations [11]

$$\rho\frac{d\mathbf{V}}{dt} = -\mathrm{grad}\left(p + \frac{2}{3}\mu\mathrm{div}\mathbf{V}\right) + 2\mathrm{Div}(\mu\dot{\mathrm{S}}),$$

the heat balance equation

$$\rho\frac{d}{dt}\left(h + \frac{V^2}{2}\right) = \frac{\partial p}{\partial t} + \mathrm{div}\left(2\mu\mathbf{V}\dot{S} - \frac{2}{3}\mu\mathbf{V}\mathrm{div}\dot{V} + \frac{\lambda}{c_p}\mathrm{grad}\,h\right),$$

and the continuity equation

$$\frac{\partial\rho}{\partial t} + \mathrm{div}(\rho\mathbf{V}) = 0.$$

Here, $\mathbf{V}$ is the velocity vector, ρ is the gas density, p is the pressure, h is the enthalpy, and $\dot{S}$ is the strain rate tensor.

Carrying out standard nondimensionalization by means of the quantities that depend on the gas parameters at infinity and the plate length, we obtain that, in addition to dependence on β and time t nondimensionalized by L/u_∞, the solution will depend on the Mach number at infinity M_∞, the Reynolds number $\mathrm{Re} = L\rho_\infty u_\infty/\mu_\infty$, the Prandtl $\mathrm{Pr} = c_p\mu/\lambda$, the specific heat ratio k, and the temperature factor η which is equal to the ratio of the plate temperature (identical along the plate) to the free-stream stagnation temperature. In the present study the effect of only the parameters M_∞ and β for fixed values of other parameters $\mathrm{Re} = 1000$, $\mathrm{Pr} = 0.72$, $k = 1.4$, and $\eta = 1$ is investigated.

The no-slip condition is imposed on the plate surface and the free-stream parameters are given at infinite distance from the plate.

The method of solving is based on numerical integration of the governing equations using the finite volume method. The solution is found on structured sub-grids. The boundary of the computational domain represented a rectangular located at the distance of $25L$ from the plate.

The program of numerical calculation was tested by comparing the calculation results for various grids and by means of comparison with the results obtained for incompressible fluid. For example, when $\beta = 1.6$ in the case of incompressible fluid the period t_0 at tending to the periodic solution is equal to 25.6, while the periodic solution with $t_0 = 25.7$ was obtained for compressible flow at $\mathrm{M}_\infty = 0.1$.

2. CALCULATION RESULTS

As in the case of incompressible fluid, in compressible gas recirculation zones are formed in the neighborhood of each of the plate sides (Fig. 1). However, the relative thickness of these zones, i.e., the ratio of the zone width to the length, decreases with increase in M_∞. In Fig. 2 we have compared the zero streamline (the streamline corresponding to the plate surface) obtained for $\beta = 1.6$ and at two free-stream velocities $\mathrm{M}_\infty = 0$ and 0.9.

Decrease in the relative zone thickness leads to reduction in the period of change in characteristics. For example, when $\beta = 1.6$ the period is equal to $t_0 = 25.7$ at $\mathrm{M}_\infty = 0.1$, while at $\mathrm{M}_\infty = 0.6$ the period is already equal to $t_0 = 2.2$ and at $\mathrm{M}_\infty = 0.9$ the flow becomes almost stable (Fig. 1). In the case of small Mach numbers M_∞ the unsteady process includes the entire gas inside the recirculation zone. The large period of flow variation is attributable to this fact. Approaching of the recirculation zones to the plate leads to the

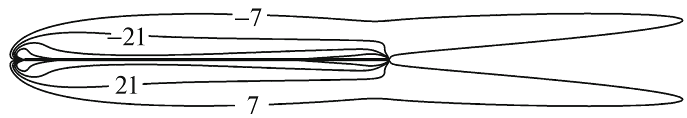

Fig. 1. Isolines of vorticity from –49 to 49 in step of 14 for $M_\infty = 0.9$ and $\beta = 1.6$.

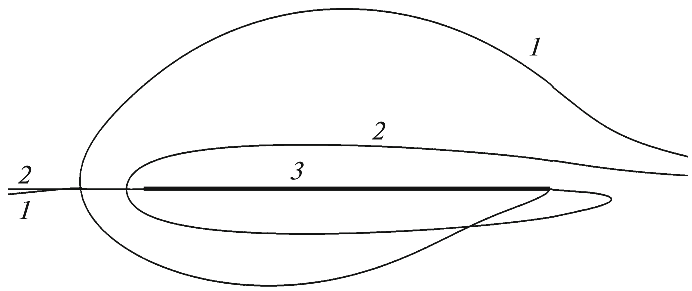

Fig. 2. Zero streamlines for $\beta = 1.6$: curves *1* and *2* correspond to $M_\infty = 0$ and 0.9, and straight line *3* corresponds to the plate.

fact that the instability domain is localized in the rear part of the plate when $M_\infty = 0.6$ (approximately from the mid-plate) and with further increase in the velocity the instability domain is initially localized in the neighborhood of the rear edge and then disappears.

By continuing to analyze the case $\beta = 1.6$, we note that local supersonic zones terminating by a shock appear in the stream starting from $M_\infty = 0.7$ (Figs. 3a and 3b). At $M_\infty = 0.9$ the vertical dimension of the supersonic zones becomes much greater than the horizontal dimension.

If increase in the number M_∞ when $\beta = 1.6$ leads to stabilization of flow, then increase in the parameter β at a given M_∞ again leads to instability. However, in these cases the vortices shedding from the vortex zones to the wake behind the plate begin to interact with the supersonic velocity field and shocks. In Fig. 4 we have shown an example of such an interaction that corresponds to the case $M_\infty = 0.9$ and $\beta = 4.2$. A new kind of unstable periodic flow that is characterized by integral asymmetry of flow develops at the numbers $M_\infty = 0.8$ and 0.9 and fairly large β. In Figs. 4 and 5 we have reproduced the characteristic forms of vortex zones. In the neighborhood of the leading edge we can see a flow deviation directed upward (or downward, this is a random process related to the form of disturbing factors) which does not change its direction during the entire period or the larger part of the period. In the second half period the vorticity field will not already correspond to the first half in the specular reflection of recirculation zones across the plate surface and the lift force will oscillates about a nonzero value. In Fig. 6 we have reproduced time variation of the lift force for $M_\infty = 0.8$ and $\beta = 3.2$. On the most part of the period the lift force is positive and large and on the smaller part of the period it is negative and small in the absolute value. For fairly large M_∞ or β the lift force ceases to change the direction with time and remains always either positive or negative.

In order to demonstrate that the choice of sign of the lift force is random, we carried out the following numerical experiment (Fig. 7). We considered flow with $M_\infty = 0.9$ and $\beta = 4$. An integrally asymmetric flow with negative lift force is established in a certain time. In this case the lift force is negative over the entire time range. At the instant of time $t = 568$ the velocity of motion of the lower surface increases to $\beta = 8$ and at the instant of time $t = 640$ the velocity of the lower surface again begins to corresponds to $\beta = 4$. Thereafter, the regime with positive lift force is established. In Fig. 7 lines I denote the mean value of the lift force coefficient before change in the velocity of motion of the plate surface and the same quantity with the other sign.

SUMMARY

The evolution of compressible gas flow past a plate with upstream-moving surface is investigated numerically. It is found that the length of zones in which unstable flow is observed and the period of vari-

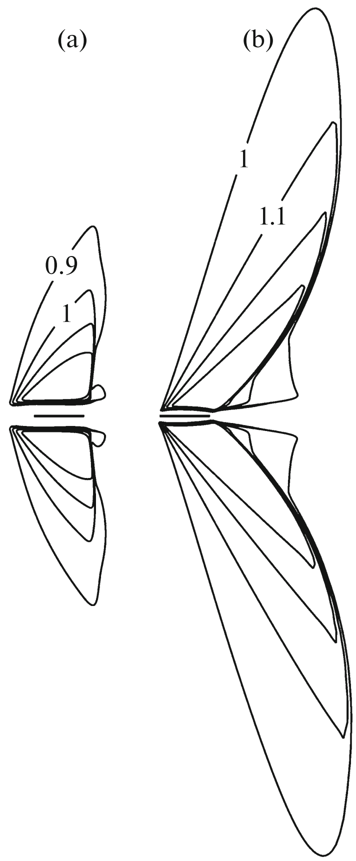

Fig. 3. Isolines of the Mach numbers at $\beta = 1.6$: $M_\infty = 0.8$ (a) and 0.9 (b).

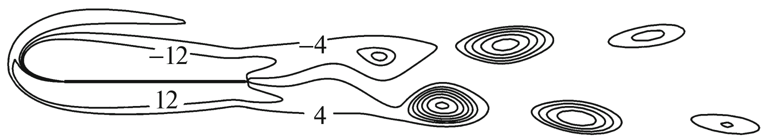

Fig. 4. Isolines of vorticity from –12 to 12 in step of 8 (in vortices the step is equal to 0.5) for $M_\infty = 0.9$ and $\beta = 4.2$.

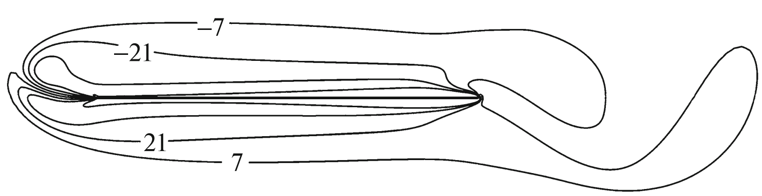

Fig. 5. The same as in Fig. 1 for $M_\infty = 0.8$ and $\beta = 3.2$.

ation in the flow characteristics with time decreases with increase in the free-stream Mach number. When $\beta = 1.6$ the further increase in the Mach number M_∞ leads to establishment of stable symmetric flow.

A new type of unsteady periodic flow with integral asymmetry of flow characteristics in time is established. Such type of flow is inherent to transonic velocities and fairly large β. In these regimes extended supersonic zones terminated by shocks develop in flow. In the case of periodic flow the onset and maintenance of integral asymmetry seem caused by vortices shedding from the vortex zones to the stream in the

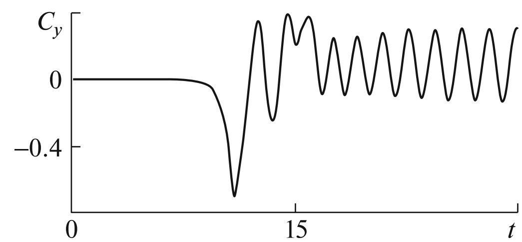

Fig. 6. Variation in the lift force coefficient for $M_\infty = 0.8$ and $\beta = 3.2$.

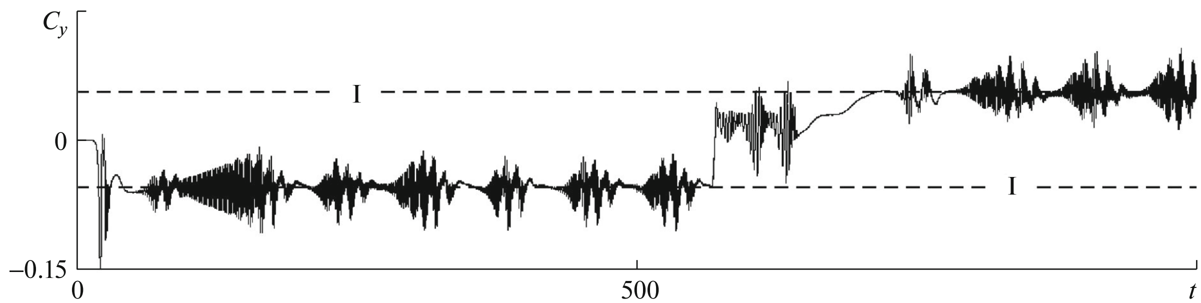

Fig. 7. Change in sign of the lift force coefficient after a strong perturbation of flow for $M_\infty = 0.9$ and $\beta = 4$: I correspond to the mean values of the lift force.

neighborhood of the plate. Coming into the wake zone behind the plate, these vortices interact with the supersonic zones and shocks.

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