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Effect of Viscous Friction Reduction by Blocking Dissipation

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Abstract—The steady-state Couette flow between two plane-parallel plates of finite thickness is considered. Fluids with the viscosity that decreases with increase in the temperature are considered. It is shown that the isothermality condition across the plates can be violated in the practically important case of small distances between the plates. This leads to the possibility of using dissipation to heat the fluid and, as a result, to significant reduction in friction without additional energy supply.

Keywords: Couette flow, energy dissipation, friction reduction

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For moving bodies friction drag reduction is one of the topical problems of aerodynamics and mechanics. For example, if it were possible to laminarize flow past a vehicle without expenditure of energy, this would lead to a considerable reduction in fuel consumption in the cruising regime of airplane flight. At present, there are reviews [1–7] devoted to the problem of fluid flow control in order to reduce friction. In [4, 5] the ways of fluid flow control are classified. One of such ways of control is passive flow control, i.e., when no energy is supplied to fluid. From the economic point of view, the active (energized) flow control may also be attractive if friction reduction results in lower total energy costs. In this connection, of interest is to investigate simple flows in which the physical mechanisms that make it possible to reduce friction are clearly demonstrated.

For internal flows including the channel and Couette flows the drag reduction problems are as important as the drag reduction problems in external aerodynamics. Friction reduction in channels is important to reduce the liquid and gas pipelining cost. Among the studies in this area, we can note [8–13].

For the Couette flow, that can be considered as a very simple model of a liquid lubricated plain bearing, the situation in which the viscosity of liquid is a function of temperature is of practical and scientific interest. In this case dissipative heating of fluid is able to change the flow parameters, in particular, the value of viscous friction. Investigations in this area are considered in monograph [14]. An interesting result can be obtained when the fluid viscosity coefficient depends on the temperature in accordance with the hyperbolic law [15]. In this case, viscous friction behaves nonmonotonically as a function of the relative velocity of plates. As the relative velocity of plates increases, friction initially also increases to a maximum and then, having reached a certain velocity, tends to zero.

In [14] the effect of dissipative heating on the torque acting on the cylindrical journal bearing at various angular velocities (the inner cylinder rotates and the outer cylinder is fixed) is theoretically analyzed and it is shown that the results of the numerical calculation and the experiment are in adequate agreement. It was established that the toque acting on the journal bearing decreases with the angular velocity within a certain range of the cylinder angular velocities. Thus, dissipative heating of fluid leads to friction reduction and decrease in the torque exerted on the cylinders.

In the present study, the way of viscous friction reduction without energy supply is investigated for given properties of fluid, relative velocity of plates, and surface temperature as applied to the Couette problem.

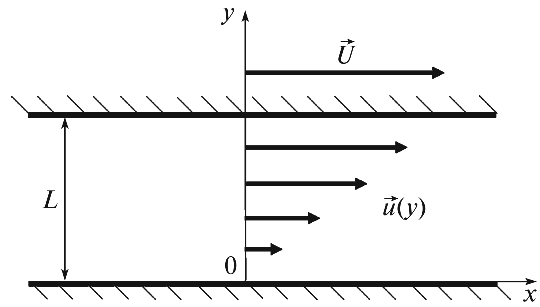

Fig. 1. Couette flow between two plane-parallel plates.

1. CLASSICAL COUETTE PROBLEM

Being within the framework of the Navier–Stokes equations, we will consider steady-state flow of a viscous heat-conducting fluid between two plane–parallel plates that move relatively each other. Without loss of generality, we will assume that the lower plate is fixed and the upper plate moves at a velocity U (Fig. 1).

The flow can be described by the following system of equations:

$$\begin{cases} \dfrac{d}{dy}\left(\mu\dfrac{du}{dy}\right) = 0 \\ \dfrac{d}{dy}\left(\lambda\dfrac{dT}{dy} + u\mu\dfrac{du}{dy}\right) = 0, \end{cases} \tag{1.1}$$

with the boundary conditions

$$u(0) = 0, \quad T(0) = T_1, \quad u(L) = U, \quad T(L) = T_2. \tag{1.2}$$

Here, x and y are the longitudinal and transverse coordinates, respectively, L is the distance between the plates, u and T are the fluid velocity and temperature, T_1 and T_2 are given temperatures of the lower and upper plates, respectively, μ is the dynamic viscosity coefficient, and λ is the thermal conductivity coefficient. We will assume that the viscosity and thermal conductivity coefficients depend only on the temperature: $\mu = \mu(T)$ and $\lambda = \lambda(T)$.

To describe the Couette flow, it is necessary to solve the boundary-value problem (1.1), (1.2). The system of equations (1.1) represents the momentum and energy conservation laws in the Navier-Stokes approximation, i.e., the following relations hold for the friction stress tensor P_{xy} and the component q_y of the heat flux vector:

$$P_{xy} = -\mu\frac{du}{dy}, \quad q_y = -\lambda\frac{dT}{dy}. \tag{1.3}$$

If we introduce the transformation of coordinates

$$\bar{s} = \frac{\int_0^y d\tilde{y}/\mu}{\int_0^L d\tilde{y}/\mu}, \tag{1.4}$$

then the momentum conservation law reduces to the second-order linear differential equation

$$\frac{d^2u}{d\bar{s}^2} = 0$$

and the velocity distribution must satisfy this equation with regard to the boundary conditions

$$u(\bar{s}) = U\bar{s}. \tag{1.5}$$

With regard to (1.5) and the boundary conditions for the temperature, the energy conservation law can be reduced to the equation

$$\phi(T_1,T)+\frac{U^2}{2}\overline{s}^2-\left(\frac{U^2}{2}+\phi(T_1,T_2)\right)\overline{s}=0, \tag{1.6}$$

where

$$\phi(T_1,T)=\int_{T_1}^{T}\frac{\lambda(t)}{\mu(t)}dt.$$

Equation (1.6) is the nonlinear algebraic equation for T. For given $\mu(T)$ and $\lambda(T)$ the temperature distribution of fluid between the plates can be found using Newton's method. Using (1.3) — (1.6), we can obtain the following simple expressions for the quantities P_{xy}, $q_{y_1}=q_y|_{y=0}$, and $q_{y_2}=q_y|_{y=L}$:

$$P_{xy}=-\frac{U}{L}\tilde{\mu},\quad q_{y_1}=-\left(\frac{U^2}{2}+\phi(T_1,T_2)\right)\tilde{\mu}/L,\quad q_{y_2}=\left(\frac{U^2}{2}-\phi(T_1,T_2)\right)\tilde{\mu}/L, \tag{1.7}$$

where $\tilde{\mu}=\int_0^1\mu(T(\overline{s}))d\overline{s}$.

Using expressions (1.7), we obtain

$$q_{y_2}-q_{y_1}=\frac{U^2}{L}\tilde{\mu}=-P_{xy}U.$$

We introduce the notation

$$\tau_{xy}=-P_{xy},\quad E_{y_2}=q_{y_2},\quad E_{y_1}=-q_{y_1},$$

hence

$$E_{y_2}+E_{y_1}=\tau_{xy}U. \tag{1.8}$$

Relation (1.8) represents the energy conservation law in the stationary case. Earlier, in [16] a similar relation was obtained for the Couette flow for a gas described by the Boltzmann equation. The quantity $\tau_{xy}U$ is the energy dissipated in the form of heat owing to viscosity in the fluid layer between the plates and $E_{y_2}+E_{y_1}$ is the heat released from the fluid volume by heat conduction. For channel flow of incompressible fluid of constant viscosity it was found that wall friction is proportional to dissipation [8].

In Figs. 2 and 3 we have reproduced the results of the numerical calculations of the classical Couette problem without heat conduction between the plates ($T_1=T_2$) at various plate temperatures. The distance between the plates $L=4\times10^{-5}$ m.

As the working fluids, we took engine oil and water. Below, we will give the approximate formulas for calculation of the dynamic viscosity and thermal conductivity coefficients as functions of the temperature T [17, 18].

For the M14G2TsS engine oil over the temperature range from 10 to 90°C the thermal conductivity and the viscosity are specified by the following functions

$$\lambda=0.1427e^{-0.0009971T}\,(\mathrm{W/m\,^\circ C}),$$

$$\mu=2.29042e^{(6.818-0.000718T)}T^{-2.60746}\ (\mathrm{Pa\ s}).$$

For water over the same the temperature range the thermal conductivity and the viscosity are equal to

$$\lambda=0.553(1+0.003T)\,(\mathrm{W/m\,^\circ C}),$$

$$\mu=\frac{0.001772346}{(0.984+0.000483T)(1+0.0337T+0.000221T^2)}(\mathrm{Pa\ s}).$$

From Fig. 2a it can be seen that for small values of U at $T_1=10$°C the dependence of the friction on the plate velocity is linear. This is caused by the fact that the dissipation effects are small and the viscosity can be assumed to be constant. Dissipative heating of fluid occurs at large values of U; thereby, the temperature increases, the viscosity decreases, and, as a consequence, τ_{xy} decreases. In the Couette problem with the viscosity dependent on the temperature the presence of the friction maximum was earlier revealed

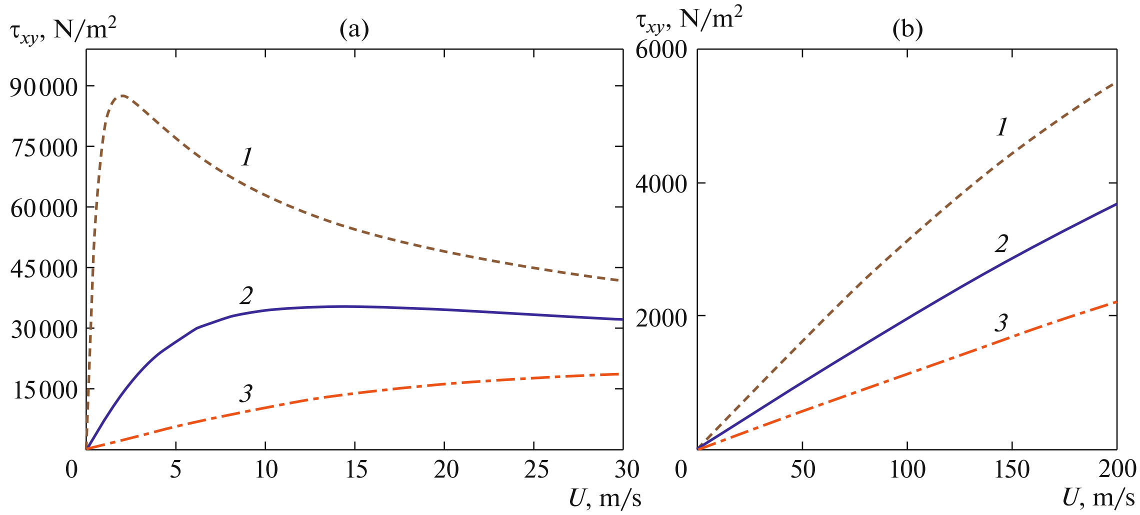

Fig. 2. Friction stress as a function of the plate velocity at various plate temperatures in the case of engine oil (a) and water (b): curves *1*–*3* correspond to T_1 = 10, 30, and 60°C, respectively.

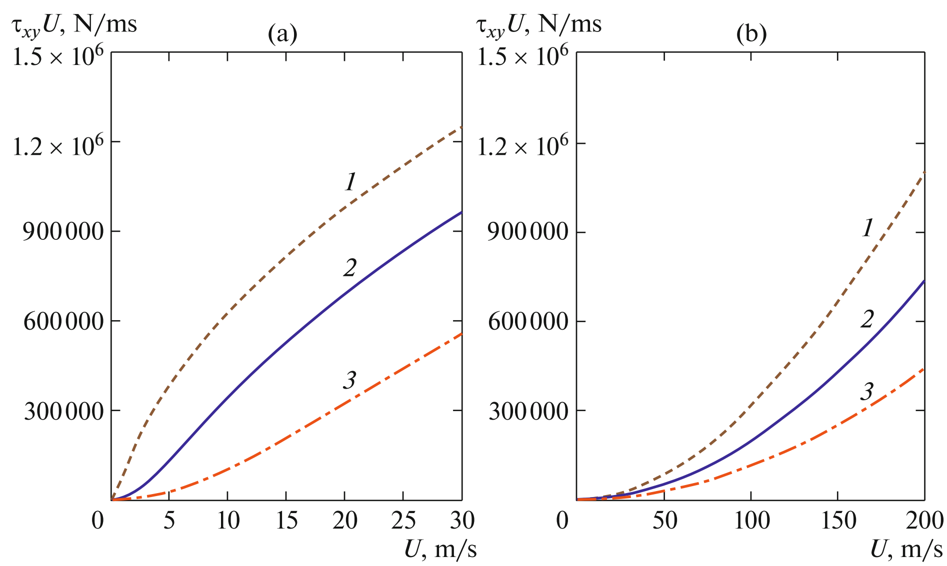

Fig. 3. Dissipated energy as a function of the plate velocity at various plate temperatures in the case of engine oil (a) and water (b): curves *1*–*3* correspond to T_1 = 10, 30, and 60°C, respectively.

in [15] from an analysis of the integral equation corresponding to the boundary-value problem. However, none particular calculations were made.

We note that, despite the decrease in friction with increase in the velocity (Fig. 2a), the dissipated energy increases with the velocity (Fig. 3a).

From Fig. 2 it can be seen that at the fixed velocity U increase in T_1 leads to decrease in τ_{xy}. For example, it is possible to increase T_1 using the energy supplied from the external sources but this is an additional expenditure of energy.

However, there exists an alternative energy source, namely, the dissipated energy. The idea of the approach proposed is to use the dissipated energy to reduce friction. In what follows, we will demonstrate how this approach can be implemented.

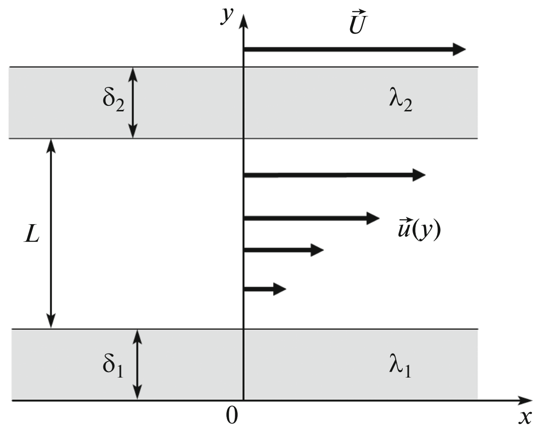

Fig. 4. Modified Couette flow.

2. MODIFIED COUETTE PROBLEM

We will consider the Couette flow with plates of finite thickness (δ_1 and δ_2) which have certain heat conductivity and given fixed temperatures T_{w_1} and T_{w_2} on the external boundaries of the plates (Fig. 4). The thermal conductivity coefficients λ_1 and λ_2 of the lower and upper plates are assumed to be constant.

To solve the modified Couette problem, it is necessary to solve the self-consistent problem in three following domains.

In the domain $0 \le y \le \delta_1$ the heat equation

$$\frac{d^2T}{dy^2} = 0$$

is solved for the lower plate with the boundary conditions

$$T(0) = T_{w_1}, \quad T(\delta_1) = T_1.$$

In the domain $\delta_1 \le y \le L + \delta_1$ the classical Couette problem described by the system of equations (1.1) with the boundary conditions

$$\begin{cases} \dfrac{d}{dy}\left(\mu \dfrac{du}{dy}\right) = 0 \\ \dfrac{d}{dy}\left(\lambda \dfrac{dT}{dy} + u\mu \dfrac{du}{dy}\right) = 0 \end{cases}.$$

$$u(\delta_1 = 0), \quad T(\delta_1) = T_1, \quad u(L + \delta_1) = U, \quad T(L + \delta_1) = T_2$$

is solved.

In the domain $L + \delta_1 \le y \le L + \delta_1 + \delta_2$ the heat equation

$$\frac{d^2T}{dy^2} = 0$$

is solved for the upper plate with the boundary conditions

$$T(L + \delta_1) = T_2, \quad T(L + \delta_1 + \delta_2) = T_{w_2},$$

where T_1 and T_2 are unknown temperatures on the interface between two phases (liquid—solid body) which can be found from the condition of continuity of the heat flux on the phase interface [19]

$$-\lambda \left.\frac{dT}{dy}\right|_{y=\delta_1} = -\lambda_1 \left.\frac{dT}{dy}\right|_{y=\delta_1}, \quad -\lambda \left.\frac{dT}{dy}\right|_{y=L+\delta_1} = -\lambda_2 \left.\frac{dT}{dy}\right|_{y=L+\delta_1}.$$

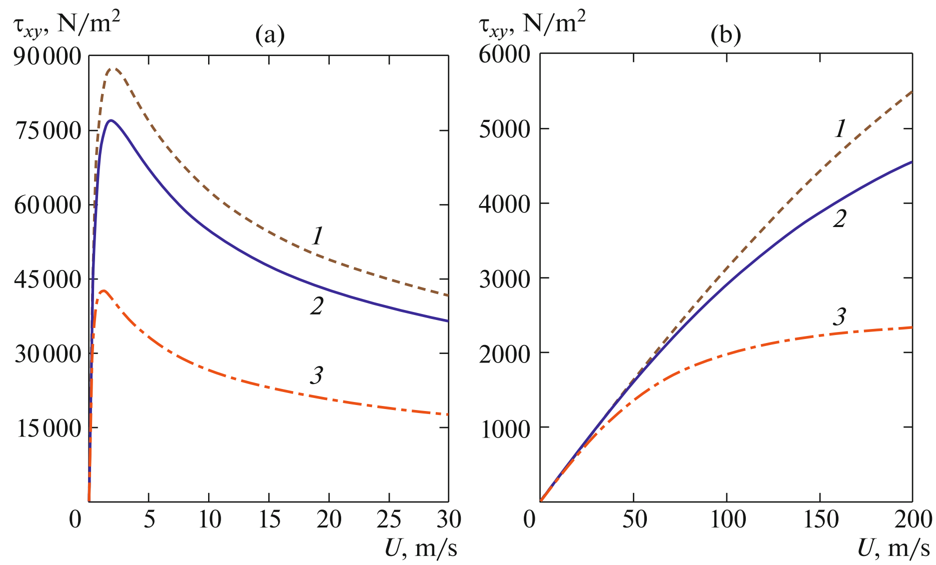

Fig. 5. Friction as a function of the plate velocity for various thermal conductivities λ_1 of the plates in the case of engine oil (a) and water (b) at the temperature of the external boundaries T_{w_1} = 10°C: curves *1–3* correspond to $\lambda_1 \to \infty$, λ_1 = 100 and 10 W/m °C, respectively.

For illustrative demonstration of the effect of viscous dissipation we will consider the Couette flow without heat conduction between the plates ($T_{w_1} = T_{w_2}$) in the symmetric case ($\lambda_1 = \lambda_2$ and $\delta_1 = \delta_2$).

From the condition of continuity of the heat flux on the interface between two phases at the constant thermal conductivity λ_1 it follows that

$$Q_{y_1} = -q_y|_{y=\delta_1} = \lambda_1 \frac{T_1 - T_{w_1}}{\delta_1},$$

hence

$$T_1 = T_{w_1}\left(1 + \frac{\delta_1 Q_{y_1}}{\lambda_1 T_{w_1}}\right). \tag{2.1}$$

We denote $f = rQ_{y_1}/T_{w_1}$ and $r = \delta_1/\lambda_1$, where f is the relative temperature drop across the plate thickness which characterizes the extent of plate nonisothermality and r is the thermal plate resistance per unit area. Then (2.1) takes the form:

$$T_1 = T_{w_1}(1 + f).$$

In [14] for calculating the Couette flow is was assumed that the plates are isothermal ($T_1 = T_{w_1}$) since for metal the thermal conductivity coefficient is considerably greater than that for fluid $\lambda_1 >> \lambda$ (this corresponds to $f << 1$). This case corresponds to the classical Couette problem considered earlier. In this case the entire dissipated energy is withdrawn outside and for given properties of fluid and given L and U friction can be change only by means of external heating of plates (by increasing T_{w_1}).

In our problem the parameter f can be varied from zero to values of the order of unity. As f increases, the plate becomes nonisothermal and fluid on the phase interface is heated. This leads to friction reduction. This heating of fluid occurs not due to the energy supplied from outside (variation in T_{w_1} but due to viscous dissipation).

As can be seen from definition of f, for fixed external plate temperatures T_{w_1} and given fluid the value of f can be increased due to:

– increase in r (to decrease the thermal conductivity of plates or to increase their thickness);

– increase in Q_{y_1} (to decrease the gap between the plates or to increase their relative velocity).

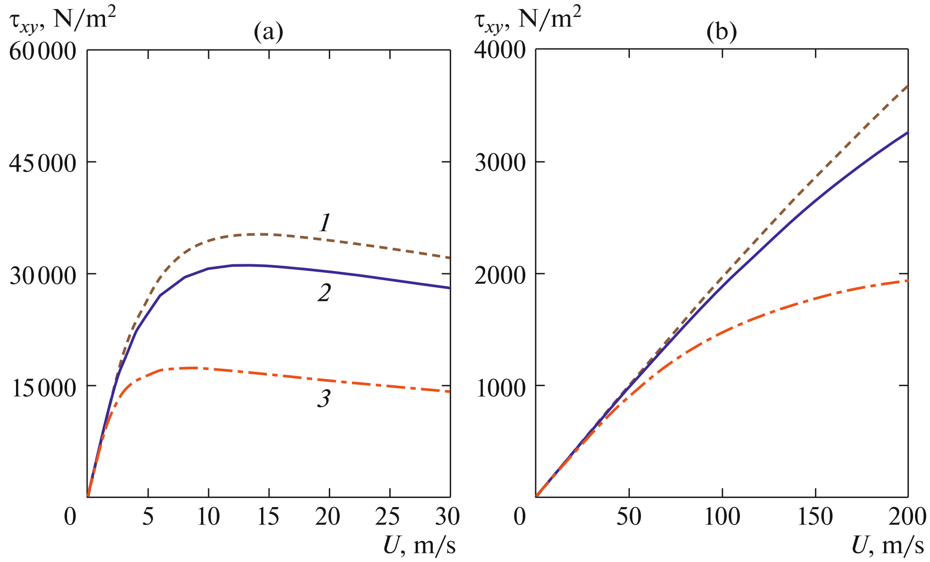

Fig. 6. Friction as a function of the plate velocity for various thermal conductivities λ_1 of the plates in the case of engine oil (a) and water (b) at the temperature of the external boundaries T_{w_1} = 30°C: curves *1*–*3* correspond to $\lambda_1 \to \infty$, λ_1 = 100 and 10 W/m °C, respectively.

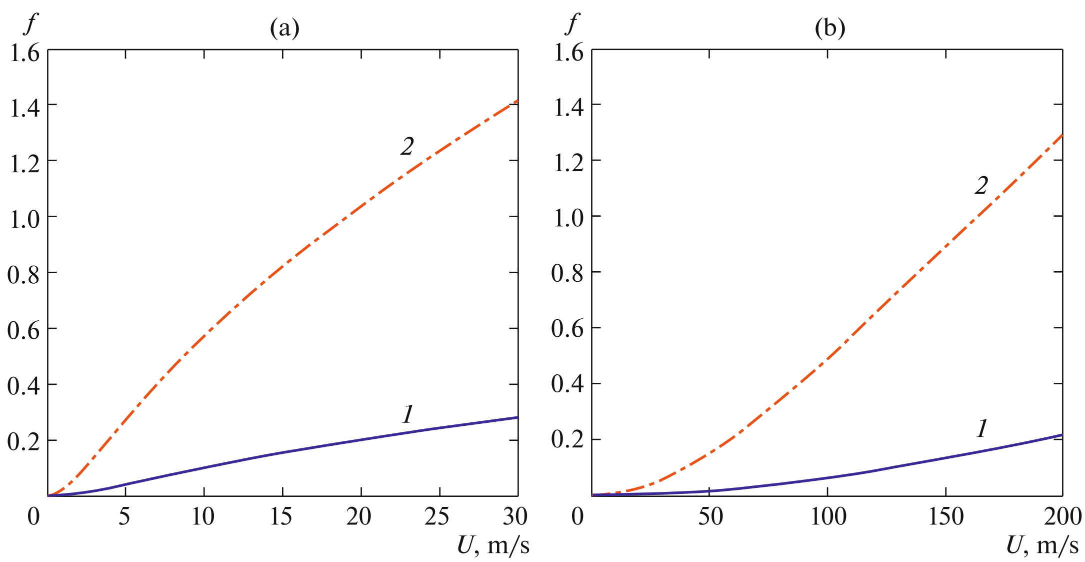

Fig. 7. Extent of nonisothermality *f* as a function of the plate velocity for various thermal conductivities λ_1 of the plates in the case of engine oil (a) and water (b) at the temperature of the external boundaries T_{w_1} = 30°C: curves *1* and *2* correspond to λ_1 = 100 and 10 W/m °C, respectively.

From the practical point of view, of interest is the case in which the increase in the thermal resistance r occurs without change in the problem geometry. In this case r can be increased due to replacement of the plate material by a material with a lower thermal conductivity.

Equation (2.1) is the nonlinear equation with respect to T_1 since Q_{y_1} depends on T_1. Determination of T_1 was implemented using Newton's method.

In Figs. 5–7 we have reproduced the results of calculations of the modified Couette problem without heat conduction between the plates ($T_{w_1} = T_{w_2}$) in the symmetric case ($\lambda_1 = \lambda_2$ and $\delta_1 = \delta_2$) at various temperatures on the external boundaries of the plates. The problem parameters are as follows: the plate thickness $\delta_1 = 2 \times 10^{-3}$ m and the distance between the plates $L = 4 \times 10^{-5}$ m.

Our calculations show that decrease in λ_1 leads to a significant decrease in τ_{xy}. This relates to the fact that decrease in the thermal conductivity of the plates leads to blocking the heat extraction and thereby leads to stronger fluid heating by increasing the extent of plate nonisothermality (see Fig. 7).

SUMMARY

The effect of viscous friction reduction without any additional expenditure of energy is demonstrated with reference to the modified Couette problem.

Decrease in the withdrawn dissipated energy turns out to be effective means for viscous friction reduction. The calculation results show that viscous friction can be significantly reduced by decreasing the thermal conductivity coefficient of the plates.

The energy necessary for friction reduction is taken due to viscous dissipation rather than from external sources.

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