Model of Vertical Mixing Induced by Wind Waves

V. G. Polnikov*

Obukhov Institute of Atmospheric Physics, Russian Academy of Sciences, Moscow, 119017 Russia
*e-mail: polnikov@mail.ru

Abstract—Formulas for the vertical wind-wave-induced mixing coefficient are obtained. For this purpose, in the Navier−Stokes equations the flow velocity is decomposed into four components, namely, mean flow, wave orbital motion, wave-induced turbulent flow fluctuations, and background turbulent fluctuations. Such a decomposition makes it possible to distinguish the wave stress Re_w in the Reynolds equations as an addition to the background stress Re_b. For the background turbulent fluctuations the Prandtl approximation is used for closure of Re_w. This leads to an implicit expression for the wave-induced vertical mixing function B_v. The finite expression for B_v is determined using author's results for the turbulent viscosity in the wave zone found earlier within the framework of the three-layer conception for the air-water interface. The explicit expression for the function $B_v(a, u_*, z)$ is linear with respect to the wave amplitude $a(z)$ at the depth z and the friction velocity u_* in air. Since the wave amplitude decreases exponentially as a function of the depth, this result for $B_v(a)$ means the possibility of strengthening the wave impact on vertical mixing as compared with the well-known cubic dependence of $B_v(a)$.

From both scientific and applied points of view, it is of importance to describe adequately the mixing processes in the upper ocean layer. This is testified by the enormous number of studies in this direction, starting from studies by Kitaigorodskii and Lumley [1], their further development in [2−6], and ending with publications in the last years [7−9]. The scientific interest in the problem is determined by natural tendency of investigators to clarify the physics of mixing processes in the upper ocean layer. The practical significance of solving the problem is caused by the purposes of improvement of simulation of ocean circulation and of forecasting the weather and climate variability. In [7−9] particular geophysical applications of such solutions were described in detail; therefore, in what follows, we will consider only the physical aspect of this problem.

The traditional theoretical approach to the problem is based on using the multilevel schemes of closure of statistical moments for the fluctuating components of the velocity field. For the problems of upper ocean dynamics these schemes were systematized in the general form in [4]. In such an approach the full Navier−Stokes equations which contain the motions of various variability scales are considered, the chains of equations for moments of different orders are constructed, and the closures of the higher-order moments are made in terms of the lower-order moments on the basis of some or other hypotheses. Taking into account the great number of details of multiscale variability leads to necessity of introducing various turbulent variables (kinetic turbulence energy generation, diffusion, dissipation rates and etc.). This is characteristic of the entire turbulence theory [10]. In this direction, the obvious progress is reached and numerous models of the upper ocean layer dynamics are constructed (see, e.g., [6−14]). However, so general approach contains numerous physical assumptions, hypotheses, and simplifications. All these uncertainties lead to underestimation of the vertical mixing in the upper layer and of the depth of the mixed layer [2, 3, 7, 13].

These limitations of simulation can be removed by taking into account the effect of surface waves on the processes of vertical mixing. This concept was implemented in many studies (see, e.g., [1, 3, 5, 11−13] and the literature cited in these papers) in which the authors attempted to use various forms of involving the wave processes on the air−water interface into the traditional turbulence schemes. As a result, it was shown that the surface waves can significantly strengthen the mixing in the upper ocean layer [7−9]. How-

ever, so far the description of the mixing processes and their effective parametrization are far from to be done.

One of the successful results of solving the problem of mixing was obtained in [5] in which the turbulence induced by surface waves was investigated by means of direct closure of the Reynolds stresses. This approach is based on the idea of existence of the wave-induced turbulence in the upper water layer that has been justified and confirmed in numerous experimental [15−21] and theoretical [6, 22] studies. The analytical approach [5] seems to be the very simple, direct, and fairly effective solution of the problem under consideration.

In [5] the problem of wave mixing was considered by closing directly the Reynolds stresses appearing in the Navier−Stokes equations after averaging them over the scales of turbulent flow fluctuations [10]. Decomposing the flow velocity fluctuations into the background turbulent fluctuations u_i' and the wave-induced fluctuations $\tilde{u}_i'$, the authors of [5] investigated the question of closure of the stress $\langle \tilde{u}_i' u_j' \rangle$ (the angular brackets $\langle \ldots \rangle$ denote statistical averaging) in the very simple case of one-dimensional and homogeneous mean flow $\mathbf{U} = (U(z), 0, 0)$. For the stress of vertical mixing the following closure was used:

$$\langle u_1' \tilde{u}_3' \rangle = B_v \frac{dU}{dx_3}. \tag{0.1}$$

Here, the subscripts (1, 2, 3) denote the x, y, and z components of flow and the value of the coefficient B_v is treated as the turbulent diffusivity (viscosity) or the function of wave-induced vertical mixing.

Then in [5] it was taken parametrization of the quantity B_v represented in the form:

$$B_v = \langle \tilde{\lambda}_3' \tilde{u}_3' \rangle. \tag{0.2}$$

Here, the Prandtl mixing length λ_3' inherent in background turbulence (introduced implicitly in the closure (0.1)) is taken to be equal to the scale of the mixing length λ_3' which relates to wave-induced turbulence. Then, the wave-induced turbulent flow fluctuations $\tilde{u}_3'$ can be expressed in terms of the orbital velocities of wave motions $\tilde{u}_1$ by means of the same Prandtl approximation: $\tilde{u}_3' = \tilde{\lambda}_3'(d\tilde{u}_1/dz)$. As a result, the expression for the vertical mixing coefficient B_v gains the form that takes into account only the wave motions:

$$B_v = \langle (\tilde{\lambda}_3')^2 (d\tilde{u}_1/dx_3) \rangle. \tag{0.3}$$

The quantities $\tilde{\lambda}_3'$ and $\tilde{u}_1$ in (0.3) can be expressed with the use of formulas for the potential approximation for surface waves in terms of the wave number spectrum $S(\mathbf{k})$ and the wave-induced mixing functions B_v is defined in the form [5]:

$$B_v(S, z) = \alpha \int S(\mathbf{k}) \exp(2kz) d\mathbf{k} \frac{d(\int \omega^2(\mathbf{k}) S(\mathbf{k}) \exp(2kz) d\mathbf{k})^{1/2}}{dz}. \tag{0.4}$$

In (0.4) α is the dimensionless fitting coefficient, $\omega^2(\mathbf{k}) = g|\mathbf{k}|$ is the wave frequency which corresponds to the wave vector $\mathbf{k}$ for the gravity waves, g is the acceleration of gravity, and the exponential functions denote the dependence of the surface wave spectrum on the depth z reckoned from the middle interface between the media (the z axis is directed upward).

The main feature of Eq. (0.4) consists in cubic dependence of the function B_v on the wave amplitude $a_0 = \left(2 \int S(\mathbf{k}) d\mathbf{k}\right)^{1/2}$ on the surface. This leads a fairly strong damping of B_v with increase in the depth: $B_v(S, z) \propto (a_0)^3 \exp(3k_p z)$, where k_p is the wave number corresponding to the spectrum peak. Despite of the so strong dependence on the depth, in [5] it was noted that "adding B_v to the vertical diffusivity in a global ocean circulation model yields a temperature structure in the upper 100 m that is closer to the observed climatology than a model without the wave-induced mixing." Later, the fact of significant impact of the wave-induced mixing on the global circulation was confirmed in a large series of studies [7−9, 19, 23, 24].

In accordance with the above, the outlined approach seems to be a perspective semi-phenomenological solution of the problem of mixing induced by the presence of waves on the sea surface. However, taking

into account several vulnerable assumptions in the proposed version of the wave-induced mixing model (Eqs. (0.2) and (0.3)), this approach requires a certain modification.

The present study is directed to creation of a new version of the wind-wave-induced mixing model. Note that the new version of the model under consideration leads to the significantly stronger impact of waves on vertical mixing as compared with that yielded by Eq. (0.4).

1. CONSTRUCTION OF A MODIFIED MODEL

1.1. Theoretical Prerequisites

We will reproduce certain well-known analytical insights entering into solution of the problem under consideration. These insights clarify certain assumptions not discussed in [5] when postulating its governing equations. Note that the motions (flows) in the upper water layer located below the deepest troughs of wind waves on the fluid surface will be considered. It is also assumed that in the upper water layer there is a certain background fluid current which exists regardless of surface waves.

Following the well-known approach of turbulence theory [10, 11], we can write the governing Navier-Stokes equations in the tensor form:

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j^2}. \tag{1.1}$$

Here, as before, the quantities U_i ($i = 1, 2, 3$) represent the x–y–z-components of fluid flow in the upper layer below the wave troughs, ρ is the water density; P is the pressure, and ν is the kinematic viscosity of water; repeating indices mean summation over them. Following the approaches adopted in [5, 11], in (1.1) the effect of the Coriolis forces is neglected by reason of small scales of the motions considered.

In this problem, it is of first importance to distinguish different forms of motion before performing statistical averaging. For this purpose, we represent the expansion of the total fluid velocity in the presence of wind waves in the following form:

$$U_i = \langle U_i \rangle + u_i = (\bar{U}_i + \tilde{U}\delta_{i,1}) + \tilde{u}_i + \tilde{u}_i' + u_i'. \tag{1.2}$$

Here, $\langle U_i \rangle = (\bar{U}_i + \tilde{U}\delta_{i,1})$ is the general velocity (average over the ensemble of waves) in which $\bar{U}_i$ is the "background" average velocity and $\tilde{U}\delta_{i,1}$ is the average velocity of wind-wave drift induced by the wind and wave motions directed along the Ox axis (this is reflected by the Kroneker delta $\delta_{i,1}$), and the quantity u_i is the total addition to the mean flow. The addition u_i includes the following set of terms: the orbital wave motions $\tilde{u}_i$, the wave-induced turbulent components $\tilde{u}_i'$, and the background turbulent fluctuations u_i' which below will be called "background" turbulence. The Stokes drift induced by waves by virtue of their nonlinearity is included in the term $\tilde{U}\delta_{i,1}$. (The situation of waves without wind when only the Stokes drift occurs requires individual consideration.) Here, it should be noted that in decomposition (1.2) the following conditions are fulfilled: 1) the orbital velocity $\tilde{u}_i$ does not relates to turbulent velocity fluctuations; 2) by definition, the background turbulent motions u_i' are independent of the wave-induced turbulent motions $\tilde{u}_i'$; 3) in accordance with their nature, both types of fluctuations can be statistically correlated with one another. The last remark is of first importance for the further consideration.

The decomposition similar to (1.2) is assumed to be also fulfilled for the pressure P (without the term related to drift). However, in what follows, we will not consider the contribution of the terms related to the pressure in Eq. (1.1) assuming that the terms related to the pressure simply "correct" the fluctuations u_i' and $\tilde{u}_i'$ introduced above in this version of our constructions. Since no explicit expressions for the fluctuations u_i' and $\tilde{u}_i'$ are required in our model, this simplification of the system of equations seems to be fairly reasonable.

We adopt the following well-known statistical approximations [10, 11]:

1) All the wave, except for drift, and turbulent terms of flow in (1.2) are equal to zero in statistical averaging over the ensemble of waves

$$\langle \tilde{u}_i \rangle = 0; \quad \langle \tilde{u}_i' \rangle = 0; \quad \langle u_i' \rangle = 0. \tag{1.3a}$$

2) There are no correlations between the average, orbital, and turbulent motions

$$\langle \bar{U}_i \tilde{u}_i \rangle = 0, \quad \langle \bar{U}_i \tilde{u}_i' \rangle = 0, \quad \langle \bar{U}_i u_i' \rangle = 0. \tag{1.3b}$$

A similar statement is also valid for drift flow $\tilde{U}\delta_{i,1}$.

3) There are no correlations between the orbital wave motions and the turbulent fluctuations of both types

$$\langle \tilde{u}_i \tilde{u}'_j \rangle = 0, \quad \langle \tilde{u}_i u'_j \rangle = 0. \tag{1.3c}$$

4) The wave-induced turbulent fluctuations and the background turbulent terms correlate

$$\langle \tilde{u}'_i \tilde{u}'_j \rangle \neq 0, \quad \langle \tilde{u}'_i u'_j \rangle \neq 0, \quad \langle u'_i u'_j \rangle \neq 0. \tag{1.3d}$$

Moreover, the continuity condition holds for each component in (1.2).

Substituting (1.2) in (1.1), averaging over the ensemble of waves, and taking into account relations (1.3a) and (1.3b) and the continuity conditions for u, we obtain the equations

$$\frac{\partial \langle U_i \rangle}{\partial t} + \langle U_j \rangle \frac{\partial \langle U_i \rangle}{\partial x_j} + \frac{\partial \langle u_j u_i \rangle}{\partial x_j} = \left[pressure - terms \right] + \nu \frac{\partial^2 \langle U_i \rangle}{\partial x_j^2}. \tag{1.4}$$

Here, $\langle U_i \rangle = \bar{U}_i + \tilde{U}_i \delta_{i,1}$ and the third term on the left-hand side contains the Reynolds stress Re $= \langle u_j u_i \rangle$. In view of the adopted decomposition (1.2), the complete expression for the stress Re takes the form:

$$\langle u_j u_i \rangle = \langle \tilde{u}_j \tilde{u}_i \rangle + \langle \tilde{u}_j \tilde{u}'_i \rangle + \langle \tilde{u}_j u'_i \rangle + \langle \tilde{u}'_j \tilde{u}_i \rangle + \langle \tilde{u}'_j \tilde{u}'_i \rangle + \langle \tilde{u}'_j u'_i \rangle + \langle u'_j \tilde{u}_i \rangle + \langle u'_j \tilde{u}'_i \rangle + \langle u'_j u'_i \rangle. \tag{1.5}$$

From (1.5) with regard to relations (1.3c) and (1.3d) it follows that

$$\langle u_j u_i \rangle = \langle \tilde{u}_j \tilde{u}_i \rangle + \langle \tilde{u}'_j \tilde{u}'_i \rangle + \langle \tilde{u}'_j u'_i \rangle + \langle u'_j \tilde{u}'_i \rangle + \langle u'_j u'_i \rangle. \tag{1.6}$$

If there are no waves, then $\tilde{u}_i = 0$ and $\tilde{u}'_i = 0$ and the stress Re takes the standard form [10]:

$$\langle u_j u_i \rangle = \langle u'_j u'_i \rangle, \tag{1.7}$$

which corresponds to background turbulence existing without waves.

When there are no waves in the approximation of first-level closure [10], from (1.7) it follows that

$$\langle u_j u_i \rangle = K_{ji}(\partial \bar{U}_i / \partial x_j), \tag{1.8}$$

where the coefficient K_{ji} has the sense of the turbulent viscosity (diffusivity). In fact, at constant value of $K_{ji} = K_0$ the closure (1.8) leads the stress (1.7) to the standard form of the viscous term in Eq. (1.1):

$$\frac{\partial \langle u'_j u'_i \rangle}{\partial x_j} = \frac{\partial}{\partial x_j} K_{ji}(\partial \bar{U}_i / \partial x_j) = K_0 \frac{\partial^2 \bar{U}_i}{(\partial x_j)^2}.$$

Thus, carrying out closure of the "wave" terms in the stress Re of form (1.6), we can obtain the unknown wave-induced vertical mixing function.

1.2. Initial Detailed Elaboration

For the sake of simplicity, we will consider the case of the constant and homogeneous mean flow $\langle \mathbf{U} \rangle$ with regard to the drift $\tilde{U}$ and the vertical shear directed along the OX axis, i.e., $\langle \mathbf{U} \rangle = (U(z), 0, 0)$ and $\partial U / \partial x_i = 0$ for $i = 1, 2$.

Restricting our attention only to vertical flows and setting, to be specific, $j = 1$ and $i = 3$ in Eq. (1.6), we obtain the following form of the stress to be analyzed:

$$\langle u_1 u_3 \rangle = \langle \tilde{u}_1 \tilde{u}_3 \rangle + \langle \tilde{u}'_1 \tilde{u}'_3 \rangle + \langle \tilde{u}'_1 u'_3 \rangle + \langle u'_1 \tilde{u}'_3 \rangle + \langle u'_1 u'_3 \rangle, \tag{1.9}$$

for which it is necessary to find a final expression.

Firstly, we suppose that the orbital wave motion is potential. Then a monochromatic wave with the frequency ω, the wave number k, and the amplitude a_0 (at $z = 0$), which propagates in deep water in two-dimensional (x, z)-space, can be described in the linear approximation by the following formulas [25].

The elevation on the water surface has the form:

$$\eta(x, t) = a_0 \cos(kx - \omega t). \tag{1.10a}$$

The velocity potential is specified by the formula

$$\phi(x, z, t) = a_0 \frac{\omega}{k} \exp(kz) \sin(kx - \omega t). \tag{1.10b}$$

The orbital velocities can be given by the relations

$$\tilde{u}_1 = u_x = a_0 \omega \exp(kz) \cos(kx - \omega t) \quad \text{and} \quad \tilde{u}_3 = u_z = a_0 \omega \exp(kz) \sin(kx - \omega t), \tag{1.10c}$$

i.e., the amplitude of the wave motions damps with depth as $a(k, z) = a_0 \exp(kz)$.

Secondly, by reason of orthogonality of the oscillating functions, from (1.10c) it follows that

$$\langle \tilde{u}_1 \tilde{u}_3 \rangle = 0. \tag{1.11}$$

Using the Prandtl approximation for wave turbulent motions in the form:

$$\tilde{u}'_{1,3} = \lambda'_{1,3} \partial \tilde{u}_{1,3} / \partial x_3,$$

similarly to (1.11) we can obtain

$$\langle \tilde{u}'_1 \tilde{u}'_3 \rangle = 0. \tag{1.12}$$

Thirdly, eliminating the "background" turbulence in (1.9) (the last term) and using relations (1.11) and (1.12), we obtain the following wave-induced stress:

$$\langle u_1 u_3 \rangle_w = \langle \tilde{u}'_1 u'_3 \rangle + \langle u'_1 \tilde{u}'_3 \rangle. \tag{1.13}$$

The quantity $\langle u_1 u_3 \rangle_w$ is the wave stress which is additive to the background turbulent viscosity determined by the relation of form (1.8). In [5] an analog of Eq. (1.13) was first postulated and thereafter analyzed (see introduction). In what follows, we will present its final detailed elaboration.

1.3. Formulas of Closure

The new approach to closure of stress (1.11) includes the following steps.

1) Because of spatial homogeneity of wave-induced turbulence, we can assume that both terms in (1.13) have identical values. Then

$$\langle u_1 u_3 \rangle_w \cong 2 \langle \tilde{u}'_1 u'_3 \rangle. \tag{1.14}$$

2) We will now use the standard Prandtl approximation for the background turbulence fluctuations of the form:

$$u'_3 = \lambda'_3 \partial U / \partial x_3, \tag{1.15}$$

where λ'_3 is the unknown stochastic mixing length of background turbulence which, by definition, is not related to wave motions since it is inherent in turbulence which exists when there are no waves. As a result, the basic expression for the further analysis takes the form:

$$\langle u_1 u_3 \rangle_w \cong 2 \langle \tilde{u}'_1 \lambda'_3 \rangle \partial U / \partial x_3. \tag{1.16}$$

3) By analogy with relation (1.8), from (1.16) there follows the implicit expression for the wave-induced turbulent viscosity B_v of the form:

$$B_v \approx 2 \langle \tilde{u}'_1 \lambda'_3 \rangle. \tag{1.17}$$

It remains now to find closure of statistical averaging in (1.17).

4) We note that relation (1.17) for B_v differs from formula (0.2) proposed in [5]. Here, the mixing length λ'_3 inherent in background turbulence is used. The mixing length cannot be expressed in terms of any part of the wave motions as discussed above.

The function B_v has the following features:

— B_v is the statistical moment whose finite value can be postulated; this is usually made in turbulence theory [10].

— B_v is a linear function of the wave-induced turbulent fluctuation $\tilde{u}'_1$ which, in turn, in the Prandtl approximation, is also a linear function of the orbital wave velocity $(d\tilde{u}_1 / dx_3)$, i.e., of the wave amplitude
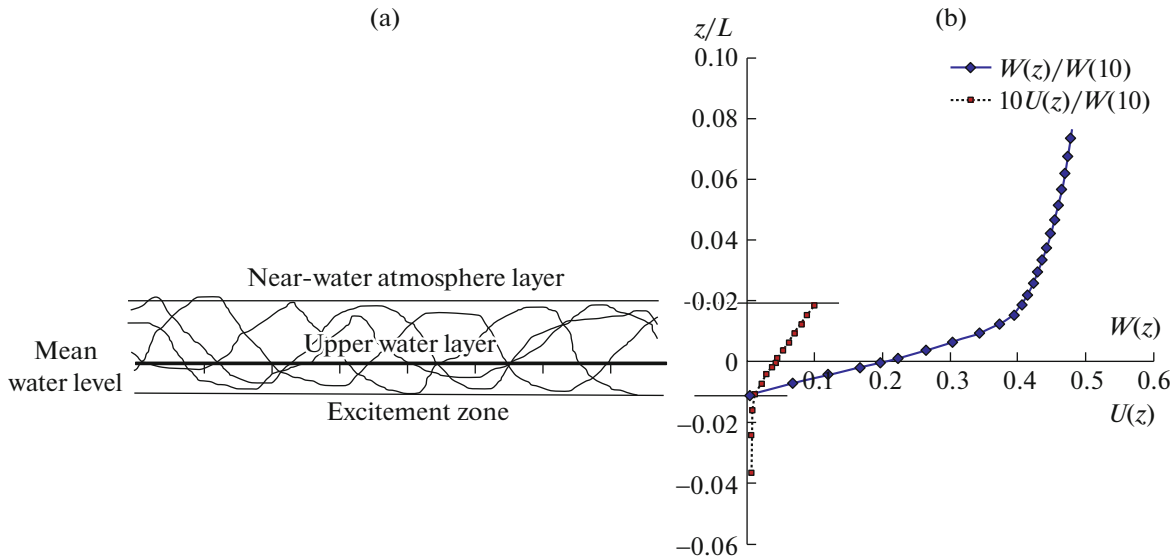
(a)

(b)

Fig. 1. Generalized pattern of the motions in the neighborhood of the interface between the media. (a) Diagram of location of the boundary atmosphere layer, the wave zone, and the upper water layer; curves describe ensemble of the surface elevations $\eta(t)$ at a fixed point in space. (b) Schematic view of the mean flow velocity profile $U(z)$ (on the basis of the measurement data in [28]) and the mean wind velocity profile $W(z)$ (on the basis of the calculations in [27]). The scale of height z is normalized by the wave length L corresponding to the peak frequency ω_p and the mean velocities by the wind velocity W_{10}.

(see formula (1.10c)). It can be expected that at any fixed depth z the quantity B_v must be a linear function of the local wave amplitude $a(z)$ that depends on the depth z.

5) In what follows, we do not suggest to use the Prandtl approximation for the wave-induced turbulent fluctuation $\tilde{u}_1'$ of the type (1.15) in order that thereafter not to seek a parametrization for the arising moment $\langle \tilde{\lambda}_3' \lambda_3' \rangle$, but to carry out closure for B_v by means of indirect determination of the statistical moment in (1.17) on the basis of recent theory of wind-induced drift flow constructed in author's study [26].

In accordance with this theory [26, 27] and empirical observations [28], the wavy air-water interface has the three-layer structure, namely, the near-water atmosphere layer, the wave zone in which air and water are alternatively present, and the upper (boundary) water layer (Fig. 1a). In [28] such a structure was obtained as a result of processing the empirical observations. The principal feature of the above three-layer structure of the wavy interface consists in the fact that in the wave zone the vertical profiles of both mean wind velocity $W(z)$ and mean flow $U(z)$ are linear functions of z. Outside the wave zone these profiles approach to logarithmic ones [27, 28].

The author has obtained theoretically [27], by processing the numerical experiments described in [29], the fact that the mean wind velocity profile $W(z)$ is linear, as shown in Fig. 1. In Fig. 1b we have reproduced schematically the profile $U(z)$ for the flow velocity in the wave zone. The exact results of measurements of $U(z)$ are given in [28]. The presence of the linear profile for the mean fluxes (both of wind and flows) means [10] that the wave zone plays a role of the viscous layer located between the near-water atmosphere and upper water layers. Therefore, from the point of view of statistical hydrodynamics [10] the following conditions of steady-state viscous shear flow must be satisfied in the wave zone: the momentum flux τ_w induced by wind and directed downward is constant and the turbulent viscosity coefficient K_{tw} which maintains the linear flow profile $U(z)$ is also constant.

In accordance with the classical turbulence theory [10], on the basis of the above consideration the following balance of the momentum flux must be fulfilled in the wave zone:

$$\tau_w/\rho_w = K_{tw} \partial U(z)/\partial z,$$

where on the left-hand side we have the vertical momentum flux in water τ_w divided by the water density ρ_w. Using this approach, in [26] the author established that in the wave zone the turbulent mixing (viscosity) coefficient K_{tw} has the form:

$$K_{\text{tw}} = c_{\text{tw}} u_* a\big|_{z=0}, \tag{1.18}$$

where $c_{\text{tw}} \sim 10^{-2}$ is a dimensionless coefficient and u_* is the friction velocity in the near-water atmosphere layer (see details in Appendix). Equation (1.18) means that in the wave zone the wave-induced turbulent viscosity K_{tw} is a linear function of the wave amplitude a.

6) In admitting unity of the mixing mechanism in the wave zone and the upper water layer, we can suggest that the unknown turbulent viscosity B_v which takes place in the upper water layer is the natural continuation of the turbulent viscosity K_{tw} existing in the wave zone. This hypothesis makes it possible to assume that parametrization of B_v is the analytical continuations of the well-known parametrization of the quantity K_{tw}. On this basis we can assume that the following relation will be satisfied at an arbitrary depth z:

$$B_v(z) \approx K_{\text{tw}}(z), \tag{1.19}$$

where $K_{\text{tw}}(z)$ is the analytical continuation of the function K_{tw} given by relation (1.18) at the undisturbed water level $z = 0$. Evidently, this continuation is implemented in terms of the well-known exponential dependence on the depth z for the amplitude of orbital motions for each of the spectral components of the wave spectrum, i.e., in accordance with (1.10c), $a(k, z) = a_0 \exp(kz)$, where a_0 is the wave amplitude at the level $z = 0$.

Thus, in the case of monochromatic surface wave the explicit expression for the function $B_v(a, u_*, z)$, dependent on the wave amplitude, the friction velocity u_*, and the depth z, takes the form:

$$B_v(a, u_*, z) \approx K_w(z) \approx c_v u_* a_0 \exp(kz), \tag{1.20}$$

where $c_v \sim 10^{-2}$ is a dimensionless fitting coefficient. Generalization of relation (1.20) to the case of the surface wave spectrum can be specified by the formula

$$B_v(z) = c_{Bv} u_* \left(\int S(\mathbf{k}) \exp(2kz) d\mathbf{k} \right)^{1/2}, \tag{1.21}$$

where $c_{Bv} \sim 10^{-2}$ is a fitting coefficient similar to the coefficient c_v in (1.20).

1.4. Concluding Remarks

Relations (1.20) and (1.21) complete derivation of the unknown function for the vertical mixing B_v induced by wind waves. As can be seen from these formulas, the main difference between (1.21) and the result (0.4) of [5] consists in the significantly stronger dependence of B_v on the local wave amplitude by virtue of the lower degree of damping exponent in Eq. (1.21). This result testifies to the possibility of implementation of the significantly higher intensity of wave-induced vertical mixing at the depth z than that following from the result (0.4). In practice, the result obtained can lead to the greater effect of waves on the processes in the upper sea layer and in large-scale circulation as compared with the estimates obtained earlier [5−9].

For physical understanding of the processes in the upper water layer it is of interest to verify the obtained dependences of the form (1.20) and (1.21). The dependences $B_v(a_0)$ and $B_v(u_*)$ can be verified using the numerical simulation of the wavy air-water interface by analogy with [6, 29, 30] and also in the laboratory experiments similar to [16, 19]. The successful verification of these dependences would make it possible to give preference to one or another version of the model that describes the processes of wave-induced mixing. In principle, methods of analytical verification of the dependences $B_v(a_0)$ and $B_v(u_*)$ are also possible. Some variants of them will be discussed below.

2. DISCUSSION OF THE RESULTS

We will verify the correspondence of the result (1.20) to certain well-known empirical dependences. Since the author do not know any empirical data on direct measurement of the turbulent viscosity coefficient B_v in the upper water layer, it is necessary to take suitable physical quantities which would make it possible to carry out verification of the dependences $B_v(a_0)$ and $B_v(u_*)$ on the basis of the available measurement data. The most convenient of such quantities is the dissipation rate of the turbulence kinetic energy ε whose value can be measured experimentally, see, for example, [11, 16, 17, 31].

For this purpose we can use the following relation between the wave-induced part of the dissipation rate of the turbulence kinetic energy ε_w and the viscosity B_v written in the form [25]:

$$\varepsilon_w \approx B_v \left(\partial \tilde{u}_1/\partial x_3\right)^2. \tag{2.1}$$

In relation (2.1) the vertical gradient of the velocity of orbital wave motions is used as an approximation suitable for qualitative estimates. Equation (2.1) makes it possible to carry out analytical verification of correspondence of relations (1.20) and (1.21) to the well-known empirical dependences of ε_w on the surface wave amplitude a_0 and the friction velocity u_* in the atmosphere [11, 16, 31].

In accordance with the measurements [16], in the case of the presence of the monochromatic wave on the water surface the following dependence takes place:

$$\varepsilon_w \propto a_0^3. \tag{2.2}$$

The correspondence between relations (1.20) and (2.2) becomes evident if we take into account that the wave velocity $\tilde{u}_1$ in (2.1) is linear with respect to the wave amplitude (see (1.10c)). Thus, the analytical verification of the dependence $\varepsilon_w(a_0)$ is quite successful.

In accordance with the measurements [11, 31], the well-known dependence of ε_w on u_* in the upper water layer takes the form:

$$\varepsilon_w \propto u_*^3. \tag{2.3}$$

Using relations (1.20), (1.21), and (1.10c), from formula (2.1) we can arrive at the relation

$$\varepsilon_w \propto u_* a(\omega_p k_p a)^2 \propto u_* \omega_p^6 a^3, \tag{2.4}$$

where ω_p is wave spectrum peak frequency and the subscript "0" of the wave amplitude is omitted for simplicity of the further notation. In order to obtain the dependence $\varepsilon(u_*)$ from formula (2.4), the well-known dependences for the dimensionless wave energy $\tilde{E}$ and the spectrum peak frequency $\tilde{\omega}_p$ on the dimensionless wave fetch $\tilde{X}$ can be used in the case of steady-statewaving. In accordance with the numerous experimental data, these dependences are as follows [32]:

$$\tilde{E} \equiv \frac{a^2 g}{u_*^4} \propto \tilde{X} \equiv \frac{Xg}{u_*^2} \quad \text{and} \quad \tilde{\omega}_p \equiv \frac{\omega_p u_*}{g} \propto \tilde{X}^{-1/3} \equiv \left(\frac{Xg}{u_*^2}\right)^{-1/3}, \tag{2.5}$$

where X is the wave fetch in units of length and g is the acceleration of gravity. From the first of the equations (2.5) it follows that the dependence of the (mean) wave amplitude on the friction velocity takes the form $a(u_*) \propto u_*$ and from the second we have $\omega_p(u_*) \propto (u_*)^{-1/3}$. In this case, relation (2.4) takes the form:

$$\varepsilon_w \propto u_* \omega_p^6 a^3 \propto u_*^2.$$

This corresponds to the empirical dependence (2.3) when taking into account the inevitable errors in estimates of the powers in Eqs. (2.2), (2.3), and (2.5).

Thus, the encouraging results of analytical verification of the correspondence of relations (1.20) and (1.21) to the well-known empirical dependences (2.2) and (2.3) raise the significance of their experimental verification. The latter can be implemented by estimating the empirical dependences $B_v(a_0)$ and $B_v(u_*)$ in the flume experiments similar to those described in [16, 19].

We also note that, in our opinion, the experimental verification of the dependences $B_v(a_0)$ and $B_v(u_*)$ can be made by two ways. The first (indirect) way is based on measurement of the dissipation rate of the turbulence kinetic energy ε_w as a function of the mean wave amplitude on the fluid surface a_0 and the friction velocity u_* with the successive use of the right-hand side of relation (2.4) for comparison with the theory, as presented above. The second (direct) way can be implemented by calculating the wave-induced vertical mixing B_v (as a function of a_0 and u_*) found from the rate of spatial spreading of a small droplet of colored ink in the water layer below the deepest wave troughs. For this purpose, we can use the well-known Einstein formula ($B_v \sim \langle \Delta z\rangle^2 /\Delta t$, where Δz is the ink-spot dimension and Δt is the ink-spot spreading time), bearing in mind the physical identity between the quantity B_v and the passive-particle diffusion

coefficient in water [10]. Naturally, the corresponding technique of carrying out the experiment is needed for the special detailed elaboration which can be developed in the case of necessity.

SUMMARY

The vertical turbulent mixing model is constructed. This model predicts the impact of wind waves on mixing in the upper fluid layer which is stronger as compared with the estimates [5]. The model is based on the following theoretical propositions.

In the Navier–Stokes equations the total flow velocity is decomposed into four components including the mean flow velocity, the wave orbital motions, the wave-induced turbulent flow fluctuations, and the background turbulent fluctuations. This makes it possible to separate the wave stress Re_w from the background stress Re_b and obtain the implicit expression for the wave-induced vertical mixing coefficient B_v.

The expression for B_v is specified more strictly using author's results [26] for the turbulent viscosity function K_{tw} in the wave zone located between the atmospheric boundary layer and the upper water layer. The unknown turbulent viscosity B_v in the upper water layer is set to be equal to analytical continuation of the turbulent viscosity function K_{tw} in the wave zone. Finally, B_v is specified as a linear function of the wave amplitude $a(z)$ at the depth z and the friction velocity u_* in air. This means the possibility of strengthening the wave impact on vertical mixing as compared with the well-known cubic dependence of $B_v(a)$ obtained earlier in [5].

The predicted analytic dependences $B_v(a_0)$ and $B_v(u_*)$ can be tested empirically by their determination from the data of measurements in the flume experiments of the type [16, 19].

APPENDIX

EXPLANATION OF FORMULA (1.18)

Formula (1.18) is constructed on the basis of three theoretical and experimental facts [26, 28]:

1) on the water surface the wind drift velocity is of the order of $0.5u_*$;

2) in the wave zone the drift velocity profile is linear with respect to z;

3) the thickness of the wave zone is of the order of $3a_0$, where a_0 is the mean wave height [27, 28] (see explanation to Fig. 1).

Under the assumption that in the wave zone the flow velocity varies from U_{d0} (on the boundary with the driving atmosphere layer) to the friction velocity in water $u_{*_w} \approx (ro)^{1/2}u_* \approx 0.03u_* \ll U_{d0}$ (on the boundary with the upper water layer), in the wave zone we can write the balance equation for the flux of momentum of the form:

$$\tau_w/\rho_w = K_{\mathrm{tw}}\frac{\partial U_d(z)}{\partial z} \approx K_{\mathrm{tw}}(U_{d0}/3a_0). \tag{A1}$$

Here,

$$\tau_w/\rho_w \approx ro\, u_*^2 = u_{*_w}^2 \tag{A2}$$

is the vertical flux of momentum in water divided by the water density; $ro = (\rho_a/\rho_w) \approx 10^{-3}$ is the ratio of the air and water densities; u_{*_w} is the friction velocity in water; and K_{tw} is the unknown turbulent mixing (or viscosity) coefficient in the wave zone. On the right-hand side of (A1) the flow velocity gradient is determined by the drift velocity gradient in the wave zone. In accordance with the above, the latter is of

the order $\partial U_d/\partial z \approx U_{d0}/3a$ since on the lower boundary of the wave zone the drift velocity is of the order of the friction velocity in water u_{*_w}, i.e., much less than $U_{d0} \approx 0.5u_*$.

Substituting expression (A2) and U_{d0} in Eq. (A1), we obtain the expression for the turbulent viscosity coefficient K_{tw} of the form (1.18).

REFERENCES

1. Kitaigorodskii, S.A. and Lumley, J.L., Wave-turbulence interaction in the upper ocean. Pt. I. The energy balance in the interacting fields of surface waves and wind-induced three-dimensional turbulence, *J. Phys. Oceanogr.*, 1983, vol. 13, pp. 1977−1987.

2. Kantha, L.H. and Clayson, C.A., An improved mixed layer model for geophysical applications, *J. Geophys. Res.*, 1994, vol. 99, pp. 25 235−25 266.

3. Ezer, T., On the seasonal mixed layer simulated by a basin-scale ocean model and the Mellor−Yamada turbulence scheme, *J. Geophys. Res.*, 2000, vol. 105, pp. 16843−16855.

4. Mellor, G.L. and Yamada, T., Development of a turbulence closure model for geophysical fluid problems, *Rev. Geophys. Space Phys.*, 1982, vol. 20, pp. 851−875.

5. Qiao, F., Yuan, Y., Yang, Y., Zheng, Q., Xia, C., and Ma, J., Wave-induced mixing in the upper ocean: Distribution and application to a global ocean circulation model, *Geophys. Res. Lett.*, 2004, vol. 31, p. L11303.
   https://doi.org/10.1029/2004GL019824

6. Babanin, A.V. and Chalikov, D., Numerical investigation of turbulence generation in non-breaking potential waves, *J. Geophys. Res.*, 2012, vol. 117, p. C00J17.
   https://doi.org/10.1029/2012JC007929

7. Qiao, F., Yuan, Y., Deng, J., Dai, D., and Song, Z., Wave−turbulence interaction-induced vertical mixing and its effects in ocean and climate models, *Phil. Trans. R. Soc.*, 2016, vol. A374, p. 20150201.
   https://doi.org/10.1098/rsta.2015.0201

8. Aijaz, S., Ghantous, M., Babanin, A.V., Ginis, L., Thomas, B., and Wake, G., Nonbreaking wave-induced mixing in upper ocean during tropical cyclones using coupled hurricane-ocean-wave modeling, *J. Geophys. Res. Oceans,* 2017, vol. 122, pp. 3939−3963.
   https://doi.org/10.1002/2016JC012219

9. Wals, K., Govekar, P., Babanin, A.V., Ghantous, M., Spence, P., and Scoccimarro, F., The effect on simulated ocean climate of a parameterization of unbroken wave-induced mixing incorporated into the k-epsilon mixing scheme, *J. Adv. Model. Earth Syst.*, 2017, vol. 9, pp. 735−758.
   https://doi.org/10.1002/2016MS000707

10. Monin, A.C. and Yaglom, A.Ya., *Statisticheskaya gidromekhanika* (Statistical Hydrodynamics), Vol. 2, Moscow: Nauka, 1967.

11. Anis, A. and Moum, J.N., Surface wave−turbulence interactions: Scaling ε(z) near the sea surface, *J. Phys. Oceanogr.*, 1995, vol. 25, pp. 2025−2045.

12. Ardhuin, F. and Jenkins, A.D., On the interaction of surface waves and upper ocean turbulence, *J. Phys. Oceanogr.*, 2006, vol. 36, pp. 551−557.
   https://doi.org/10.1175/JPO2862.1

13. Janssen, P.E.A.M., Ocean wave effects on the daily cycle in SST, *J. Geophys. Res.* 2012, vol. 117, p. C00J32.
   https://doi.org/10.1029/2012JC007943

14. Craig, P.D. and Banner, M.L., Modelling wave enhanced turbulence in the ocean surface layer, *J. Phys. Oceanogr.*, 1994, vol. 24, pp. 2547−2559.

15. Babanin, A.V., On a wave-induced turbulence and a wave-mixed upper ocean layer, *Geophys. Res. Lett.* 2006, vol. 33, p. L20605.
   https://doi.org/10.1029/2006GL027308

16. Babanin, A.V. and Haus, B.K., On the existence of water turbulence induced by non-breaking surface waves, *J. Phys. Oceanogr.*, 2009, vol. 39, pp. 2675−2679.
   https://doi.org/10.1175/2009JPO4202.1

17. Gemmrich, J.R. and Farmer, D.M., Near-surface turbulence in the presence of breaking waves, *J. Phys. Oceanogr.*, 2004, vol. 34, pp. 1067−1086.

18. Gemmrich, J.R., Strong turbulence in the wave crest region, *J. Phys. Oceanogr.*, 2010, vol. 40, pp. 583−595.
   https://doi.org/10.1175/2009JPO4179.1

19. Dai, D., Qiao, F., Sulisz, W., Han, L., and Babanin, A., An experiment on the nonbreaking surface-wave-induced vertical mixing, *J. Phys. Oceanogr.*, 2010, vol. 40, pp. 2180−2188.

20. Pleskachevsky, A., Dobrynin, M., Babanin, A.V., Günther, H., and Stanev, E., Turbulent mixing due to surface waves indicated by remote sensing of suspended particulate matter and its implementation into coupled modeling of waves, turbulence and circulation, *J. Phys. Oceanogr.*, 2011, vol. 41, pp. 708−724.

21. Sutherland, P. and Melville, W.K., Field measurements of surface and near-surface turbulence in the presence of breaking waves, *J. Phys. Oceanogr.*, 2015, vol. 45, pp. 943−965.
    https://doi.org/10.1175/JPO-D-14-0133.1

22. Benilov, A.Y., On the turbulence generated by the potential surface waves, *J. Geophys. Res.*, 2012, vol. 117, p. C00J30.
    https://doi.org/10.1029/2012JC007948

23. Qiao, F., Yuan, Y., Ezer, T., Xia, C., Yang, Y., Lü, X., and Song, Z., A three-dimensional surface wave-ocean circulation coupled model and its initial testing, *Ocean Dynamics*, 2010, vol. 60, pp. 1339−1355.

24. Huang, C.J., Qiao, F., Dai, D., Ma, H., and Guo, J., Field measurement of upper ocean turbulence dissipation associated with wave-turbulence interaction in the South China Sea, *J. Geophys. Res.*, 2012, vol. 17, p. C00J09.
    https://doi.org/10.1029/2011JC00780

25. Yuan, Y., Qiao, F., Yin, X., and Han, L., Analytical estimation of mixing coefficient induced by surface wave-generated turbulence based on the equilibrium solution of the second-order turbulence closure model, *Science China: Earth Sciences,* 2013, vol. 56. pp. 71−80.
    https://doi.org/10.1007/s11430-012-4517-x

26. Polnikov, V.G., A semi-phenomenological model for wind-induced drift currents, *Boundary-Layer Meteorol.*, 2019, vol. 172(3), pp. 417−433.
    https://doi.org/10.1007/s10546-019-00456-1

27. Polnikov, V.G., Features of air flow in the trough-crest zone of wind waves (2010).
    https://arxiv.org/abs/1006.3621.

28. Longo, S., Chiapponi, L., Clavero, M., Mäkel, T., and Liang, D., The study of the turbulence over the air-side and the water-induced boundary waves, *Coastal Engineering,* 2012, vol. 69, pp. 67−81.

29. Chalikov, D. and Rainchik, S., Coupled numerical modelling of wind and waves and theory of the wave boundary layer, *Boundary-Layer Meteorol.*, 2011, vol. 138, pp. 1−41.
    https://doi.org/10.1007/s10546-010-9543-7

30. Skote, M. and Henningson, D.S., Direct numerical simulation of a separated turbulent boundary layer, *J. Fluid Mech.*, 2002, vol. 471, no. 1, pp. 107−136.

31. Jones, N.L. and Monismith, S.G., The influence of whitecapping waves on the vertical structure of turbulence in a shallow estuarine embayment, *J. Phys. Oceanogr.*, 2008, vol. 38, pp. 1563−1580.
    https://doi.org/10.1175/2007JPO3766.1

32. Komen, G.I., Cavaleri, L., Donelan, M., Hasselmann, K., Hasselmann, S., and Janssen P.A.E.M., *Dynamics and Modelling of Ocean Waves,* Cambridge: Cambridge University Press, 1994.

Translated by E. A. Pushkar