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Unsteady Motion of a Viscous Conducting Fluid between Rotating Parallel Walls in a Transverse Magnetic Field

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Abstract—We study the motion of an incompressible viscous conducting fluid, which initially rotates as a solid at a constant angular velocity together with parallel bounding walls under the action of longitudinal vibrations of one of the walls beginning suddenly and a magnetic field suddenly applied to one of them. The walls make an arbitrary angle with the axis of rotation. The magnetic field is applied along the wall normal. In the general case, the solution is presented in the form of a series. The vectors of tangential stresses that act on the gap walls from the fluid are presented. Some particular cases of the wall motion are discussed. The results are used to study individual structures of the boundary layers at the walls. This study generalizes studies [1–3].

Keywords: incompressible viscous conducting fluid, magnetic field, analytical solution

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INTRODUCTION

In this study, we investigate an unsteady flow of an incompressible viscous conducting fluid in a rotating gap in an external uniform magnetic field. The unsteady flow is induced by nontorque vibrations of one of the gap walls. The formulation of the problem is schematically illustrated in Fig. 1. To the best of our knowledge, the problem is formulated in such a way for the first time. It is shown that, without rotation and magnetic field and with the fixed wall removed to infinity, the solution passes to the well-known solution of the problem on the unsteady motion of a fluid bounded by a moving plane wall [1]. In zero magnetic field at the fixed wall removed to infinity, the solution coincides with the results of [2] and, in zero magnetic field, the solution passes to the solution of [3]. In recent study [4], the flow of a conducting fluid between parallel walls was investigated, but the fluid was assumed to be ideal and the examined flow was stationary.

1. EXACT SOLUTIONS OF THE EQUATIONS OF MAGNETIC HYDRODYNAMICS

We solve the problem in the following formulation. A gap with width l formed by two infinite parallel walls Q_0 and Q_1 with insulating properties is filled with an incompressible viscous conducting fluid. The gap with the fluid rotates as a whole at the constant angular velocity $\vec{\omega}_0 = \text{const}$, such that the vector $\vec{\omega}_0$ and the walls make constant angle $\beta\left(0 < \beta \leq \frac{\pi}{2}\right)$. A particular case of zero magnetic field $\left(\beta = \frac{\pi}{2}\right)$ was discussed in [5].

We relate the Cartesian system of coordinates O_{xyz} with the basis vectors $\vec{e}_x, \vec{e}_y, \vec{e}_z$ to the plane Q_0 so that the plane O_{xz} coincided with the plane Q_0 and the y axis was directed along the normal to it inside the fluid. In this system of coordinates, the walls and liquid are at rest. At the instant of time $t > 0$, the wall Q_0 starts moving in the longitudinal direction at the velocity $\vec{u}(t)$. At the same instant of time, the external magnetic field $B_0 = \text{const}$ is applied along the wall normal. The problem is schematically illustrated in Fig. 1.

Below, we investigate the propagation of the perturbation in a homogeneous conducting medium under the action of a uniform magnetic field and longitudinal vibrations of the wall.

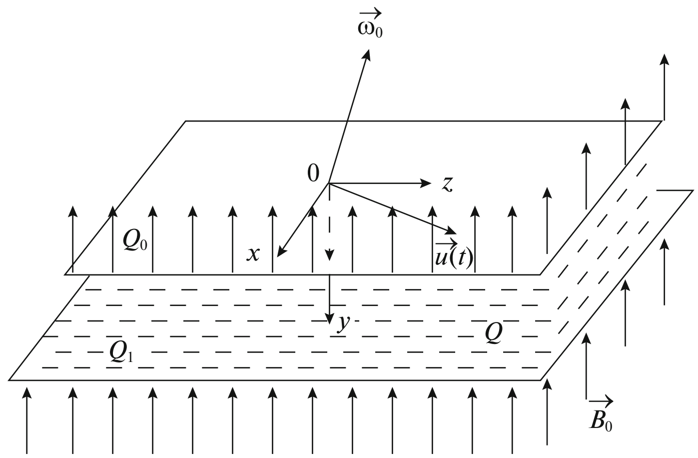

Fig. 1. Schematic of the problem.

The motion of a fluid in the system O_{xyz} rotating at angular velocity $\vec{\omega}_0$ in the magnetohydrodynamic approximation (an infinitely conducting fluid) is described by the equations of magnetic hydrodynamics and the boundary and initial conditions, which can be presented in the conventional notation as

$$\vec{\omega}_0 \times (\vec{\omega}_0 \times \vec{r}) + 2\vec{\omega}_0 \times \vec{v} + \frac{\partial \vec{v}}{\partial t} + (\vec{v}\nabla)\vec{v}$$

$$= -\frac{1}{\rho}\nabla \mathrm{P} + \nabla U + \nu \Delta \vec{v} + \frac{1}{\mu\rho} \mathrm{curl} \vec{B} \times \vec{B},$$

$$\frac{\partial \vec{B}}{\partial t} = \mathrm{curl}(\vec{v} \times \vec{B}), \quad \mathrm{div} \vec{v} = 0, \quad \mathrm{div} \vec{B} = 0 \quad \text{at} \quad \vec{r} \in Q,$$

$$\vec{v}(\vec{r}, t) = \vec{u}(t) \quad \text{at} \quad \vec{r} \in Q_0, \quad t > 0,$$

$$\vec{v}(\vec{r}, t) = 0 \quad \text{at} \quad \vec{r} \in Q_1, \quad t > 0,$$

$$\vec{B} = B_0 \vec{e}_y \quad \text{at} \quad \vec{r} \in Q_0, \quad t > 0; \quad \vec{B} = B_0 \vec{e}_y \quad \text{at} \quad \vec{r} \in Q_1, \quad t > 0, \tag{1.1}$$

$$\vec{v}(\vec{r}, 0) = 0 \quad \text{at} \quad t = 0; \quad \vec{B}(\vec{r}, 0) = 0 \quad \text{at} \quad t = 0,$$

where $\vec{r}$ is the radius vector relative to the pole O, $\vec{v}$ is the velocity of the fluid, P is the pressure, ρ is the density, ν is the kinetic viscosity, U is the potential of the external mass forces, B_0 is the magnetic induction, μ is the magnetic permeability, and Q is the fluid volume.

We will find the solution of system of Eqs. (1.1) in the form

$$\mathrm{P} = \rho (\vec{\omega}_0 \times \vec{r})^2 / 2 + \rho U + \rho q(y, t),$$

$$\vec{v} = v_x(y, t)\vec{e}_x + v_z(y, t)\vec{e}_z,$$

$\vec{B} = B_x(y, t)\vec{e}_x + B_0\vec{e}_y + B_z(y, t)\vec{e}_z$, where $q(y, t)$ is the unknown pressure function.

Then, system (1.1) falls into the two subsystems

$$\frac{\partial v_x}{\partial t} + 2\Omega v_z = \nu \frac{\partial^2 v_x}{\partial y^2} + \frac{1}{\mu\rho} B_0 \frac{\partial B_x}{\partial y},$$

$$\frac{\partial v_z}{\partial t} - 2\Omega v_x = \nu \frac{\partial^2 v_z}{\partial y^2} + \frac{1}{\mu\rho} B_0 \frac{\partial B_z}{\partial y}, \tag{1.2}$$

$$\frac{\partial B_x}{\partial t} = B_0 \frac{\partial v_x}{\partial y}; \quad \frac{\partial B_z}{\partial t} = B_0 \frac{\partial v_z}{\partial y}, \quad \Omega = \vec{\omega}_0 \cdot \vec{e}_y$$

with the boundary and initial conditions

$$\vec{v}(0,t) = \vec{u}(t) \quad \text{at} \quad y = 0, \quad t > 0; \quad \vec{v}(l,t) = 0, \quad t > 0,$$

$$\vec{B}(0,t) = B_0\vec{e}_y \quad \text{at} \quad y = 0, \quad t > 0; \quad \vec{B}(l,t) = B_0\vec{e}_y \quad \text{at} \quad y = l, \quad t > 0,$$

$$\vec{v}(y,0) = 0, \quad \vec{B}(y,0) = 0 \quad \text{at} \quad t = 0, \quad y > 0.$$

The equation for the pressure field is written in the form

$$\frac{\partial q}{\partial y} = 2\vec{v}\left(\vec{\omega}_0 \times \vec{e}_y\right) - \frac{1}{\mu\rho}\left(B_x\frac{\partial B_x}{\partial y} + B_z\frac{\partial B_z}{\partial y}\right); \quad 0 \le y \le l. \tag{1.3}$$

We introduce the complex structure

$$\hat{v} = v_x(y,t) + iv_z(y,t); \quad \hat{B} = B_x(y,t) + iB_z(y,t).$$

Then, system of Eqs. (1.2) takes the form

$$\begin{gathered}\frac{\partial \vec{v}}{\partial t} - i2\Omega\hat{v} = \nu\frac{\partial^2 \hat{v}}{\partial y^2} + \frac{B_0}{\mu\rho}\frac{\partial \hat{B}}{\partial y}, \\ \frac{\partial \hat{B}}{\partial t} = B_0\frac{\partial \hat{v}}{\partial y},\end{gathered} \tag{1.4}$$

and the boundary and initial conditions are

$$\hat{v}(0,t) = \hat{u}(t) \quad \text{at} \quad y = 0, \quad \hat{B}(0,t) = B_0 \quad \text{at} \quad y = 0,$$

$$\hat{v}(l,t) = 0 \quad \text{at} \quad y = l, \quad \hat{B}(l,t) = B_0 \quad \text{at} \quad y = l,$$

$$\hat{v}(y,0) = 0, \quad \hat{B}(y,0) = 0 \quad \text{at} \quad t = 0, \quad y > 0.$$

We exclude the magnetic induction from Eqs. (1.4) and obtain

$$\begin{gathered}\frac{\partial^2 \hat{v}}{\partial t^2} - \left(i2\Omega + \nu\frac{\partial^2}{\partial y^2}\right)\frac{\partial \hat{v}}{\partial t} - \frac{B_0^2}{\mu\rho}\frac{\partial^2 \hat{v}}{\partial y^2} = 0\ , \\ \hat{v}(0,t) = \hat{u}(t), \\ \hat{v}(l,t) = 0, \quad \hat{v}(y,0) = 0.\end{gathered} \tag{1.5}$$

We write the solution of Eq. (1.5) using the Duhamel integral

$$\hat{v}(y,t) = \frac{\partial}{\partial t}\int_0^t \hat{u}(t-\tau)\hat{v}_1(y,\tau)\,d\tau. \tag{1.6}$$

Here, $\hat{v}_1(y,t)$ is the solution of the boundary problem

$$\begin{gathered}\frac{\partial^2 \hat{v}_1}{\partial t^2} - \left(i2\Omega + \nu\frac{\partial^2}{\partial y^2}\right)\frac{\partial \hat{v}_1}{\partial t} - \frac{B_0^2}{\mu\rho}\frac{\partial^2 \hat{v}_1}{\partial y^2} = 0, \\ \hat{v}_1(0,t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0. \end{cases} \quad \hat{v}_1(l,t) = 0\end{gathered} \tag{1.7}$$

In the Laplacian images $\tilde{v}(y,p)$ [6], Eq. (1.7) takes the form

$$\begin{gathered}p^2\tilde{v}_1(y,p) - \left(i2\Omega + \nu\frac{\partial^2}{\partial y^2}\right)p\tilde{v}_1(y,p) - \frac{B_0^2}{\mu\rho}\frac{\partial^2}{\partial y^2}\tilde{v}_1(y,p) = 0, \\ \tilde{v}_1(0,p) = \frac{1}{p}, \quad \tilde{v}_1(l,p) = 0,\end{gathered} \tag{1.8}$$

and we have

$$\frac{\partial^2 \tilde{v}_1(y,p)}{\partial y^2} - \frac{p^2 - i2\Omega p}{\nu p + \dfrac{B_0^2}{\mu\rho}} \hat{v}_1(y,p) = 0,$$
$$\tilde{v}_1(0,p) = \frac{1}{p}; \quad \tilde{v}_1(l,p) = 0. \tag{1.9}$$

The solution of Eq. (1.9) takes the form

$$\tilde{v}_1(y,p) = c_1 e^{\lambda y} + c_2 e^{-\lambda y}, \quad \text{where} \quad \lambda^2 = \frac{p^2 - i2\Omega p}{\nu p + \dfrac{B_0^2}{\mu\rho}}.$$

Having determined the integration constants c_1 and c_2 from the boundary conditions, we obtain

$$\tilde{v}(y,p) = \frac{1}{p}\frac{\sinh(l-y)\lambda}{\sinh\lambda l}. \tag{1.10}$$

We introduce the function

$$\Psi = \frac{\sinh\lambda(l-y)}{\sinh\lambda l}$$

and decompose it into simple fractions [7]

$$\Psi = 1 - \frac{y}{l} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\frac{\lambda^2}{\lambda^2 + \left(\dfrac{\pi n}{l}\right)^2}\sin\pi n\left(1 - \frac{y}{l}\right).$$

We make the designation $\lambda_n = \pi n/l$. Then, we have

$$\Psi = 1 - \frac{y}{l} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\lambda_n(l-y)\frac{p^2 - i2\Omega p}{p^2 + p(\lambda_n^2\nu - i2\Omega) + \dfrac{\lambda_n^2 B_0^2}{\mu\rho}}. \tag{1.11}$$

Using the well-known operator calculus formulas [6], we obtain

$$L^{-1}\left(\frac{p+\alpha_n}{(p+\alpha_n)^2 + \omega_n^2} - \frac{\alpha_n + i2\Omega}{(p+\alpha_n)^2 + \omega_n^2}\right)$$
$$= e^{(-\alpha_n t)}\left(\cos\omega_n t - \frac{\alpha_n + i2\Omega}{\omega_n}\sin\omega_n t\right), \tag{1.12}$$

where L^{-1} is the reciprocal Laplacian and

$$\alpha_n = \frac{\lambda_n^2\nu - i2\Omega}{2}, \quad \omega_n^2 = \frac{\lambda_n^2 B_0^2}{\mu\rho} - \frac{(\lambda_n^2\nu - i2\Omega)^2}{4}.$$

Substituting (1.11) into (1.10), we obtain, with regard to (1.12), the solution of Eqs. (1.7) in the space of originals

$$\hat{v}_1(y,t) = 1 - \frac{y}{l} - \frac{2}{l}\sum_{n=1}^{\infty}\frac{\sin\lambda_n y}{\lambda n}e^{-\alpha_n t}\left(\cos\omega_n t - \frac{\alpha_n + i2\Omega}{\omega_n}\sin\omega_n t\right). \tag{1.13}$$

Thus, the solution of problem (1.5) is determined by formulas (1.6) and (1.13). Substituting (1.13) into (1.6), we obtain the desired field of velocities of the viscous conducting fluid.

$$\hat{v}(y,t) = \frac{\partial}{\partial t}\int_0^t \hat{u}(t-\tau)\hat{v}_1(y,\tau)\,d\tau. \tag{1.14}$$

The vectors of tangential stresses that act on the upper and lower gap walls from the fluid side are determined from the formulas

$$\hat{f}_0 = \rho\nu\frac{\partial}{\partial t}\int_0^t \hat{u}(\tau)\frac{\partial \hat{v}_1}{\partial y}(0, t-\tau)\,d\tau,$$

$$\left.\frac{\partial \hat{v}_1}{\partial y}\right|_{y=0} = -\frac{1}{l}\left(1 + 2\sum_{n=1}^{\infty} e^{-\alpha_n t}\left(\cos\omega_n t - \frac{\alpha_n + i2\Omega}{\omega_n}\sin\omega_n t\right)\right),$$

$$\hat{f}_l = \rho\nu\frac{\partial}{\partial t}\int_0^t \hat{u}(\tau)\frac{\partial \hat{v}_1}{\partial l}(l, t-\tau)\,d\tau,$$

$$\left.\frac{\partial \hat{v}_1}{\partial l}\right|_{y=e} = -\frac{1}{l}\left(1 + 2\sum_{n=1}^{\infty} (-1)^n e^{-\alpha_n t}\left(\cos\omega_n t - \frac{\alpha_n + i2\Omega}{\omega_n}\sin\omega_n t\right)\right).$$

The velocity field, magnetic induction vector, and vectors of the tangential stresses acting on the plates from the fluid can be used to take into account the force effects during the motion of a fluid in channels of different shapes, as well as in the filtration problems and modeling different physical phenomena in a moving fluid.

2. FIELD OF VELOCITIES OF THE FLOW INDUCED BY THE MOTION OF ONE OF THE WALLS

Let one of the planes Q_0 that forms the gap boundaries move in the longitudinal direction at the velocity $u(t) = u(0)e^{\lambda t}$, where $\lambda = -\alpha + i\omega$.

We investigate normal vibrations of a viscous conducting fluid inside a rotating gap, i.e., a class of movements in which all time factors depend on time with the factor $e^{\lambda t}$. Then, system of Eqs. (1.4) takes the form

$$\lambda\hat{v} - i2\Omega\hat{v} = \nu\frac{\partial^2\hat{v}}{\partial y^2} + \frac{B_0}{\mu\rho}\frac{\partial\hat{B}}{\partial y},\qquad \lambda\hat{B} = B_0\frac{\partial\hat{v}}{\partial y}, \tag{2.1}$$

The boundary and initial conditions are

$$\hat{v}(0,t) = \hat{u}(0) \quad \text{at} \quad y = 0, \quad \hat{B}(0,t) = B_0 \quad \text{at} \quad y = 0,$$

$$\hat{v}(l,t) = 0 \quad \text{at} \quad y = l, \quad \hat{B}(l,t) = B_0 \quad \text{at} \quad y = l,$$

$$\hat{v}(y,0) = 0, \quad \hat{B}(y,0) = 0 \quad \text{at} \quad t = 0, \quad y > 0.$$

Excluding the magnetic induction from Eq. (2.1), we obtain the ordinary differential equation for the function $\hat{v}(y)$

$$\frac{\partial^2\hat{v}}{\partial y^2} = \frac{\lambda - i2\Omega}{\nu + \dfrac{B_0^2}{\mu\rho\lambda}}\hat{v}, \quad 0 < y < l, \tag{2.2}$$

and the boundary conditions

$$\hat{v}(0) = \hat{u}(0), \quad \hat{u}(l) = 0. \tag{2.3}$$

The general solution of Eq. (2.2) has the form

$$\hat{v}(y) = \hat{C}_1 e^{qy} + \hat{C}_2 e^{-qy},$$

where $\hat{C}_1$ and $\hat{C}_2$ are the arbitrary complex constants and

$$q = \sqrt{\frac{\lambda - i2\Omega}{\nu + \frac{B_0^2}{\mu\rho\lambda}}}. \qquad (2.4)$$

Determining the integration constants from boundary conditions (2.3), we obtain the normal vibrations of a viscous conducting fluid in a dc magnetic field in the rotating gap

$$\tilde{v}(y,t) = e^{\lambda t}\hat{u}(0)\frac{\sinh(l-y)q}{\sinh ql}. \qquad (2.5)$$

Here, we can see that any of two roots of Eq. (2.4) can be taken as q. Using Eq. (2.5), we find the vectors of the tangential stresses that act on the upper and lower gap plates from the fluid side

$$\hat{f}_0 = -\rho\nu q e^{\lambda t}\hat{u}(0)\frac{\cosh lq}{\sinh ql}, \qquad (2.6)$$

$$\hat{f}_l = -\rho\nu q e^{\lambda t}\hat{u}(0)\frac{1}{\sinh ql}. \qquad (2.7)$$

It follows from Eqs. (2.5)–(2.7) that the field of the fluid velocities and the forces of friction strongly depend on the complex parameter q, which relates the parameters of the harmonic vibrations of the plates and rotation of the gap.

3. STRUCTURE OF THE BOUNDARY LAYERS

Let us consider Eq. (2.5) for the velocity field in more detail. We express the frequency q as

$$q = \frac{1}{\delta} + ik, \qquad (3.1)$$

where δ is the thickness of the boundary layer and k is the wavenumber.

We consider

$$\frac{\lambda - i2\Omega}{\nu + \frac{B_0^2}{\mu\rho\lambda}} = \frac{-\alpha + i(\omega - 2\Omega)}{\nu - \frac{\alpha B_0^2}{\mu\rho(\alpha^2+\omega^2)} - i\frac{\omega B_0^2}{\mu\rho(\alpha^2+\omega^2)}}.$$

We use the designations

$$m = \nu - \frac{\alpha B_0^2}{\mu\rho(\alpha^2+\omega^2)}; \quad n = -\frac{\omega B_0^2}{\mu\rho(\alpha^2+\omega^2)}$$

and obtain $m^2 + n^2 = \nu^2 + \frac{B_0^2(B_0^2 - 2\nu\alpha\mu\rho)}{\mu^2\rho^2(\alpha^2+\omega^2)}$.

We have

$$\frac{\lambda - i2\Omega}{\nu + \frac{B_0^2}{\mu\rho\lambda}} = \frac{[-\alpha + i(\omega - 2\Omega)](m - in)}{(m+in)(m-in)} = \frac{-\alpha m + n(\omega - 2\Omega) + i(\alpha n + m(\omega - 2\Omega))}{m^2+n^2}.$$

Let us make the designations

$$C = \frac{-\alpha m + n(\omega - 2\Omega)}{m^2+n^2}; \quad D = \frac{\alpha n + m(\omega - 2\Omega)}{m^2+n^2}.$$

We consider

$$q_{1,2} = \sqrt{C + iD}.$$

For the sake of convenience, we introduce the designations

$$\sqrt{C+iD} = q_{1,2} = \frac{1}{\delta_{1,2}} + ik_{1,2}.$$

Then, the quantities $\delta_{1,2}$ and $k_{1,2}$, which have the physical meaning of a boundary layer thickness and wavenumber, respectively, are determined by the formulas

$$\frac{1}{\delta_{1,2}^2} = \frac{\sqrt{C^2 + D^2} + C}{2}, \quad k_{1,2}^2 = \left(\frac{2}{\sqrt{C^2 + D^2} - C}\right)^{-1}. \tag{3.2}$$

We present velocity field (2.5) as a superposition of two traveling waves

$$\hat{v}(y,t) = \hat{A}e^{-i(ky+\omega t)} + \hat{B}e^{i(ky+\omega t)}, \tag{3.3}$$

$$\text{where} \quad \hat{A} = \frac{\exp[-\alpha t - y/\delta]\hat{u}(0)}{2\sinh ql}e^{ql}, \quad \hat{B} = \frac{\exp[-\alpha t + y/\delta]\hat{u}(0)}{2\sinh ql}e^{-ql}. \tag{3.4}$$

Then, the wavenumbers are

$$k = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{\alpha^2 + (\omega - 2\Omega)^2}{\nu^2 + \dfrac{B_0^2(B_0^2 - 2\nu\alpha\mu\rho)}{\mu^2\rho^2(\alpha^2 + \omega^2)}}} + \frac{\alpha\nu + \dfrac{B_0^2(\omega^2 - \alpha^2 - 2\omega\Omega)}{\mu^2\rho^2(\alpha^2 + \omega^2)}}{\nu^2 + \dfrac{B_0^2(B_0^2 - 2\nu\alpha\mu\rho)}{\mu^2\rho^2(\alpha^2 + \omega^2)}}\right)^{\frac{1}{2}}, \tag{3.5}$$

$$\delta = \sqrt{2}\left(\sqrt{\frac{\alpha^2 + (\omega - 2\Omega)^2}{\nu^2 + \dfrac{B_0^2(B_0^2 - 2\nu\alpha\mu\rho)}{\mu^2\rho^2(\alpha^2 + \omega^2)}}} - \frac{\alpha\nu + \dfrac{B_0^2(\omega^2 - \alpha^2 - 2\omega\Omega)}{\mu^2\rho^2(\alpha^2 + \omega^2)}}{\nu^2 + \dfrac{B_0^2(B_0^2 - 2\nu\alpha\mu\rho)}{\mu^2\rho^2(\alpha^2 + \omega^2)}}\right)^{-\frac{1}{2}}. \tag{3.6}$$

These waves propagate along the axis toward one another at the same phase velocity and depend on frequency. It means that the viscous conducting fluid is a dispersing medium.

$$v_\Phi = \frac{\omega}{k}. \tag{3.7}$$

The group velocities of these waves $v_{\text{gr}} = \dfrac{d\omega}{dk} = \dfrac{1}{dk/d\omega}$ also coincide. They depend on the damping coefficient, parameters of wall motion, velocity of rotation of the system, magnetic induction, and parameters of the fluid. The amplitudes of these waves depend on the gap depth, projection of the angular velocity onto the y axis, parameters of the wall motion, magnetic induction, and parameters of the fluid.

We assume the field induction to be $B_0^2 = 2\nu\alpha\mu\rho$. Let us consider the resonance case $\omega = 2\Omega$. Then,

$$k = \sqrt{\frac{\alpha}{\nu}}\frac{1}{\sqrt{1 + \dfrac{\alpha^2}{4\Omega^2}}}, \quad \delta = \sqrt{\frac{\nu}{\alpha}}\sqrt{1 + \frac{4\Omega^2}{\alpha^2}}. \tag{3.8}$$

In this resonance case, the wavenumber and boundary layer only depend on the fluid viscosity, damping coefficient, and projection of the angular velocity of the gap rotation onto the Oy axis. The wavenumber and boundary layer are independent of the magnetic permeability and conductivity of the fluid.

At $\alpha = 0$, the wavenumber is $k = 0$ and the motion of the fluid is reduced to the vibrations. In this case, the boundary layer fills the entire gap and is considered to be missing.

CONCLUSIONS

The problem of the unsteady flow of an incompressible viscous conducting fluid in the plane-parallel configuration was analyzed. The exact solutions of the three-dimensional nonstationary equations of magnetic hydrodynamics were found. No limitations on the nature of plate motion were imposed. The velocity field in the flow and the vectors of tangential stresses acting from the fluid on the gap walls were determined. For the case of normal vibrations of one of the walls, the resonance case was considered and the structure of the boundary layers adjacent to the walls was investigated. The mathematical procedure of integration of the system of differential equations of the investigated problem can be used to study more complex problems. The results can also be used to take into account the force effects during the motion of a fluid in channels of different shapes and solve the problems of filtration, as well as in modeling various physical phenomena in a moving fluid.

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