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Stratified Flow Structure near the Horizontal Wedge

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Abstract—In this work we studied the structure and dynamics of a two-dimensional flow of a continuously stratified fluid near a horizontal wedge using numerical methods. A mathematical model and a method of numerical implementation were developed. This allows for the simultaneous study of all the elements of multiscale stratified flows without additional hypotheses and connections. The numerical solution is implemented in the OpenFOAM open source package. The calculations were performed in the parallel mode with the use of computing resources of the UniHUB web-laboratory. The laws governing the flow formation are analyzed and the physical mechanisms that are responsible for the vortex formation in areas with high density gradients near the edges of the streamlined wedge are determined.

Keywords: stratification, numerical modeling, diffusion induced flows

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INTRODUCTION

Substances dissolved in liquids and particles suspended in gases under the action of gravity or global rotation of the Earth are unevenly distributed and form a stable stratification. In accordance with the type of state equation of a continuous medium, the density field is determined by temperature, impurity concentration, and pressure profiles. The non-equilibrium medium with the molecular flow of the stratifying components is at rest only if the density gradients are parallel to the direction of the gravity force. Interruption of the molecular flow at impermeable boundaries of arbitrary shape produces diffusion induced flows. Theoretical studies of stratified flows began in the early 1940s [1] and began to be modeled in the laboratory [2] a little later.

Interest in the study of gradient flows persists due to their prevalence in the environment. Under normal conditions, disturbances are concentrated in thin layers on impermeable surfaces and reach storm values, forming large temperature gradients in the atmosphere on steep slopes of the surface [3] and near glaciers [4]. Special attention has recently been paid to studying the effects of diffusion-induced currents on the dynamics and structure of processes in water bodies, lakes, seas [5], and oceans with allowance for the effect of Coriolis force [6]. Special attention was paid in some studies [7] to the analysis of the state equation of the marine environment and the presence of an additional stratifying component with its own diffusion coefficient ("differential diffusion").

At the end of the last century, along with analytical studies of stationary flows [1, 2, 5, 8], non-stationary models of current formation induced by diffusion on various obstacles (inclined and horizontal plate, cylinder, and sphere) began to develop. Laboratory experiments were also performed using highly sensitive shadow-imaging devices that showed the presence of beams of dissipative nonstationary gravitational waves at the obstacle poles, except for the previously observed large vortices [9].

An integral force that is absent on symmetrical obstacles (sphere, cylinder or horizontal plate, etc.), but takes a final value on an inclined plate and other bodies asymmetric with respect to the direction of the gravity force is formed along the slope currents. The resulting pressure gradients are large enough and can cause self-movement of free bodies of neutral buoyancy ("diffusion fish" [10]), which plays an important role in the dynamics of the marine environment. Stationary flows on a flat fixed wedge were calculated [11–13]. The study of the formation mechanisms of forces leading to the self-movement of bodies of different shapes is of practical interest.

With the onset of body movement, the diffusion-induced multi-scale elements of the currents do not disappear, but, on the contrary, turn into much more complex and subtle structures, such as quasi-stationary high-gradient layers separating different types of disturbances, vortex trail, and internal waves [9, 14]. The flow structure essentially depends on the problem parameters, such as the body shape, the stratification size, and the velocity of the external flow.

Vortices and waves in inhomogeneous media exist simultaneously and actively interact with each other along with the fine structure of the flow, which affects the processes of substance transfer, the separation of flow components, and the increase in the local impurity concentration. Neglecting small changes in density and imposing conditions of constant density, i.e., the approximation of a homogeneous incompressible fluid, lead to a degeneration of the system of basic equations due to the merging of some elements of the fine structure of different nature [15].

A method for constructing exact solutions of the basic system of equations for the mechanics of inhomogeneous liquids in a linear approximation was proposed in [14]. In the case of a viscous exponentially stratified fluid, the fields of two-dimensional coupled internal waves caused by the uniform motion of the plate along the underlying plane were calculated. The exact solution of the problem satisfying the physically justified boundary conditions was obtained in quadratures and visualized numerically. The resulting flow patterns clearly demonstrate two groups of internal waves, whose structure essentially depends on the surface inclination angle to the horizon and non-wave features near the obstacle edges.

At the same time, theoretical studies of diffusion-induced flows were performed for objects of infinite or semi-infinite geometry, that is, edge effects were not taken into account. Since real bodies have finite dimensions, studies of the flow structure near the extreme points of the body are of the greatest interest. This leads to the construction of numerical models of diffusion-induced flows and regularities of internal waves around bodies of finite size.

1. SYSTEM OF DEFINING EQUATIONS

In a two-dimensional approximation, we consider the evolution of the structure of an initially linearly stratified fluid, whose density $\rho = \rho_0(1 - z/\Lambda)$ is determined by the salinity distribution $S_0(z)$ and is characterized by the buoyancy scale $\Lambda = |\partial \ln \rho/\partial z|^{-1}$, frequency $N = \sqrt{g/\Lambda}$, and period $T_b = 2\pi/N$.

Mathematical modeling of the assigned problem is based on a system of equations for a multicomponent inhomogeneous incompressible fluid in the Boussinesq approximation [15] with allowance for the effects of buoyancy and diffusion of a stratifying impurity. In the study of slow compared with the sound speed fluid flow characterized by high thermal conductivity, the calculations can take into account only density changes associated with the concentration of the stratifying component, neglecting temperature changes. Thus, the defining system of equations is written as follows:

$$\begin{gathered}\rho = \rho_0(1 - z/\Lambda + s), \quad \nabla \cdot \mathbf{v} = 0, \\ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho_0}\nabla P + \nu\Delta\mathbf{v} - s\mathbf{g}, \\ \frac{\partial s}{\partial t} + \mathbf{v} \cdot \nabla s = \kappa_s \Delta s + \frac{v_z}{\Lambda}.\end{gathered} \tag{1.1}$$

Here, $S = S_0(z) + s$ is the total salinity including the degree of salt compression, s is the salinity perturbation, $\mathbf{v}$ is the velocity vector, P is the pressure, except the hydrostatic one, ν is the kinematic viscosity, κ_s is the salt diffusion coefficient, ∇ and Δ is the Hamilton and Laplace operators, and $\mathbf{g}$ is the gravity acceleration. The term with the vertical flow velocity component v_z in the last equation in (1.1) is the convective transfer of salinity that disturbs the initial stratification.

The sticking condition for the velocity components and non-leakage for salinity, as well as an unperturbed free flow at infinity, are given on the wedge surface:

$$s, \mathbf{v}|_{t=0} = 0, \quad s, \mathbf{v}|_{x,z\to\infty} = 0, \quad \mathbf{v}|_{\Sigma} = 0, \quad \left|\frac{\partial S}{\partial \mathbf{n}}\right\|_{\Sigma} = -\frac{1}{\Lambda}\frac{\partial z}{\partial \mathbf{n}} + \left|\frac{\partial s}{\partial \mathbf{n}}\right\|_{\Sigma}, \tag{1.2}$$

where U is the external flow velocity, and $\mathbf{n}$ is the external normal to the wedge surface Σ. The initial condition for a forced flow of a stratified medium is a steady-state flow caused by the interruption of the diffusion flow of a stratifying impurity on an impermeable surface of a fixed body. A detailed study of the structure and dynamics of diffusion induced flows on immobile obstacles was performed experimentally and numerically [9, 12, 13].

The system of equations (1.1) with initial and boundary conditions (1.2) is characterized by a number of characteristic scales, including the time scale $t = T_b$ and $\delta_U^L = L/U$, velocity scale ($U_N^\nu = \sqrt{\nu N}$, $U_N^{\kappa_s} = \sqrt{\kappa_s N}$, U), and length scales (buoyancy scale, Λ, horizontal and vertical dimensions of the obstacle, L and h, length of surface gravitational waves $\lambda_s = 2\pi U^2/g$, as well as internal $\lambda = UT_b$ and viscous $\Lambda_\nu = \sqrt[3]{g\nu}/N = \sqrt[3]{\Lambda(\delta_N^\nu)^2}$ waves, and viscous $\delta_N^\nu = \sqrt{\nu/N}$ and diffusion $\delta_N^{\kappa_s} = \sqrt{\kappa_s/N}$ microscales). The variety of linear scales and a wide range of their values (four–six orders of magnitude) indicate the complexity of the internal structure of the studied processes, which arises due to the heterogeneity of the molecular flow of a stratifying impurity.

Large dynamic scales, which are the internal wave length λ and the viscous wave scale Λ_ν, characterize the structure of the attached internal wave fields [16, 17]. The fine structure of the flow is characterized by universal microscales δ_N^ν and $\delta_N^{\kappa_s}$ determined by dissipative coefficients and buoyancy frequency [18]. Another pair of parameters, such as the Prandtl $\delta_U^\nu = \nu/U$ and Peclet $\delta_U^{\kappa_s} = \kappa_s/U$ scales, is determined by dissipative coefficients and the velocity of the body movement [15].

Significant differences in the values of linear scales indicate the complexity of the internal structure of a stratified flow that includes a set of regularly perturbed components characterizing waves or vortices and singularly perturbed components describing the components of the fine structure of the flow [9]. All components of the complete solution, both regularly and singularly perturbed ones, exist simultaneously in inhomogeneous media, actively interacting with each other and manifesting themselves in a wide range of defining parameters. Large length scales prescribe the choice of size for areas of observation or calculation, which should contain all the studied flow structural components, such as advanced disturbances, a downstream track, internal waves, and eddies, while microscopes determine the grid resolution and time step. At low speeds of body movement, the Stokes scale is critical while the Prandtl scale becomes dominant at high speeds.

In the nonlinear formulation, problem (1.1) and (1.2) provides the simultaneous study of all flow elements within the same description in natural physical variables without additional constants and relations. Due to the multiscale of the phenomenon under study and the nonlinearity of the governing equations, one of the main tools for analyzing such evolutionary processes is numerical modeling.

2. NUMERICAL MODELING

The solution of problem (1.1) and (1.2) is constructed numerically in a complete nonlinear formulation using the finite-volume method in the open-source OpenFOAM (www.openfoam.com) computing package. The package that was originally developed for the numerical calculation of three-dimensional problems of continuum mechanics can also effectively model two-dimensional problems. Technically, this is performed by selecting only one computational cell in the third dimension and setting special "empty" boundary conditions and on the front and back boundaries of the computational domain.

The standard numerical model of the open package OpenFOAM was supplemented with original software modules that take the effects of stratification and diffusion [19] into account. New variables (ρ, s), additional first and fourth equations in (1.1), and new physical parameters (N, Λ, κ_s, and $\mathbf{g}$) were added to the initial icoFoam model that implements the second and third equations in (1.1). The boundary condition of the salinity perturbation (the last condition (1.6)) was implemented using the extended utility funkySetBoundaryField that allows defining analytical expressions for physical variables. The utilities vorticity, forceCoeffs, funkySetFields, etc., were used to calculate additional physical variables.

For the interpolation of convective terms, a limited TVD scheme was used that provides minimal numerical diffusion and no solution oscillations. For discretization of the time derivative, we used an implicit asymmetric three-point scheme of the second order with differences back, which provides good time resolution of the physical process. On orthogonal sections of the grid, the normal velocity gradients required for calculating the diffusion terms by the Gauss theorem were on the cell surface from the velocity values in the centroids of the neighboring cells according to the second-order scheme. On non-orthogonal sections, an iterative error correction procedure caused by the grid non-orthogonality was used.

To solve the resulting system of linear algebraic equations, we used various iterative methods, such as the conjugate gradient method with the PCG pre-conditioner applied to symmetric matrices and the biconjugate gradient method with the PBiCG pre-conditioner for asymmetric matrices. As a pre-conditioner for symmetric and asymmetric matrices, we chose, respectively, the DIC and DILU procedures based on the simplified procedures of incomplete Cholesky factorization. To link the equations of

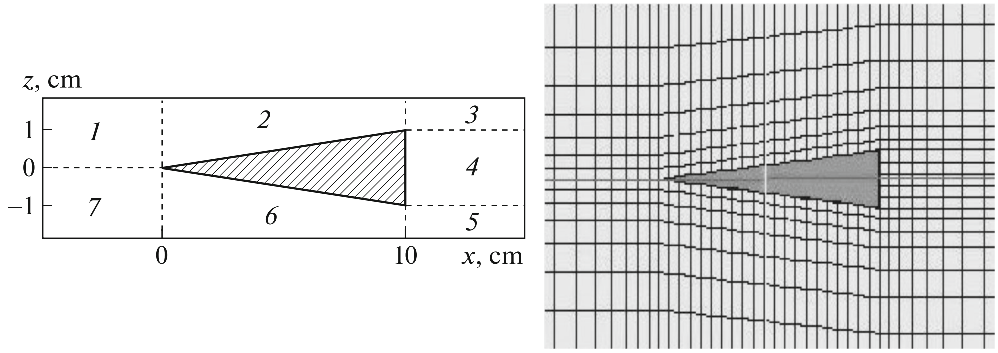

Fig. 1. Discretization of the computational domain: (left) division into blocks, and (right) grid pattern.

momentum and mass conservation, a stable well convergent PISO (Pressure-Implicit Split-Operator) algorithm that is most effective for solving non-stationary problems was used.

The computational domain was discretized using the SALOME open integrable platform that allows creating, editing, importing, and exporting CAD (Computer Aided Design) models and also building a grid for them using various algorithms and associating the physical parameters of the studied problem with the geometry. To build the computational grid, standard OpenFOAM utilities, such as blockMesh, topoSet, and refineMesh, were also used. The main OpenFOAM's polyMesh class that processes the mesh is created using the minimal amount of information needed to define the elements and parameters of the partition, such as vertices, edges, faces, cells, blocks, external boundaries, etc. By choosing a suitable type of computational grid, that is structured or unstructured, orthogonal or non-orthogonal, consistent with the area boundaries or not and each of which usually has its advantages and disadvantages, one can ensure a successful search for a solution to the problem under study. Therefore, the methods for constructing the grid for a specific problem were chosen individually based on the values of the characteristic length scales and the geometric complexity of the problem under consideration.

The computational domain is a rectangle divided into seven blocks. The streamlined body that is a horizontal wedge of length L and base height h (left side of Fig. 1) is located in the central part of the computational domain. The spatial discretization procedure of the problem was parameterized, which allows significantly reducing the duration of grid restructuration when changing the parameters of the problem. The simplicity of the wedge geometry allows the construction of a block-structured hexagonal computational grid with nodes that coincide at the boundaries between the blocks. Test calculations with various grid resolutions confirm the need for resolution of the smallest microscales of the problem since a relatively coarse mesh with a total cell number of $N_c \sim 10^6$ produces an unstable solution. Thus, numerical modeling of even two-dimensional problems of continuously stratified flows near impenetrable obstacles requires high-performance computations. The discretization algorithm of the computational domain includes the grid thickening towards the obstacle (right side of Fig. 1). The aspect ratio of the grid cell near the body is approximately equal to unity, which positively affects the convergence of the solution.

The need for a high spatial resolution of the assigned problem leads to a rather large number of computational nodes, which makes it irrational to perform calculations using a single-processor personal computer. The division of the computational domain for calculations in the parallel mode is performed by a simple geometric decomposition, in which the domain is divided into parts in certain directions with an equal number of computational cells in each block. Such an approach allows for the establishment of a high spatial resolution of the computational domain and investigating the problem in a wide range of basic parameters in quite reasonable time. Parallel calculations of the problem were performed in the web-laboratory UniHUB (www.unihub.ru).

The following values of the problem parameters were used in the calculations:

$$\rho_{00} = 1020 \text{ kg/m}^3, \quad g_z = 9.8 \text{ m/s}^2, \quad \nu = 10^{-6} \text{ m/s}^2, \quad \kappa_S = 1.41\times10^{-9} \text{ m/s}^2,$$
$$T_b = 6.28 \text{ s}, \quad L = 10 \text{ cm}, \quad \text{and} \quad h = 2 \text{ cm}.$$

The calculated time step, Δt, is determined by the Courant condition $\mathrm{Co} = |\mathbf{v}|\Delta t/\Delta r \le 1$, where Δr is the minimal mesh size, and $\mathbf{v}$ is the local flow velocity.

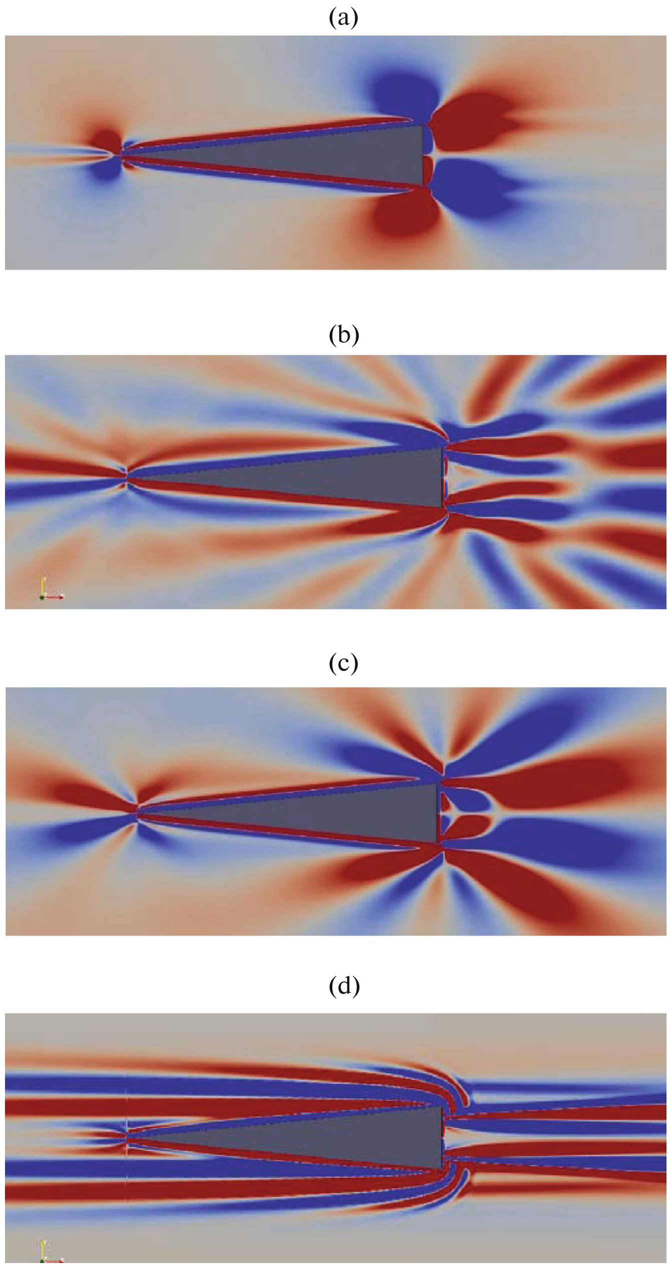

Fig. 2. Formation of the gradient field of the salinity perturbation when the wedge moves at a speed of $U = 10^{-4}$ m/s (a–d): t/T_b = 0.3, 1.1, 3.2, and 16.0 (positive values in red and negative values in blue).

3. CALCULATION RESULTS

The initial complex structure of the medium formed by diffusion-induced flows on a stationary wedge changes dramatically with the onset of forced motion even at low speed (Fig. 2). In a continuously stratified fluid, outrunning disturbances, rosettes of non-stationary internal waves and fields of the affiliated internal waves, as well as an extended trace behind the extreme points, are formed.

When the velocity of the external flow is comparable in order of magnitude with the characteristic velocity of the diffusion-induced flows $U_N^{\kappa_s} = \sqrt{\kappa_s N}$, the field structure retains the elements of the original field for a long time (Fig. 2a). In this case, the beams of alternating lanes remain attached to the sharp corners of the obstacles, which drift downstream at an increase in the velocity of the external flow. The length of the affiliated internal waves increases. Sources of internal waves are wedge corners that produce intense vertical displacement of the fluid. Therefore, the deflection of fluid layers from the initial position of neu-

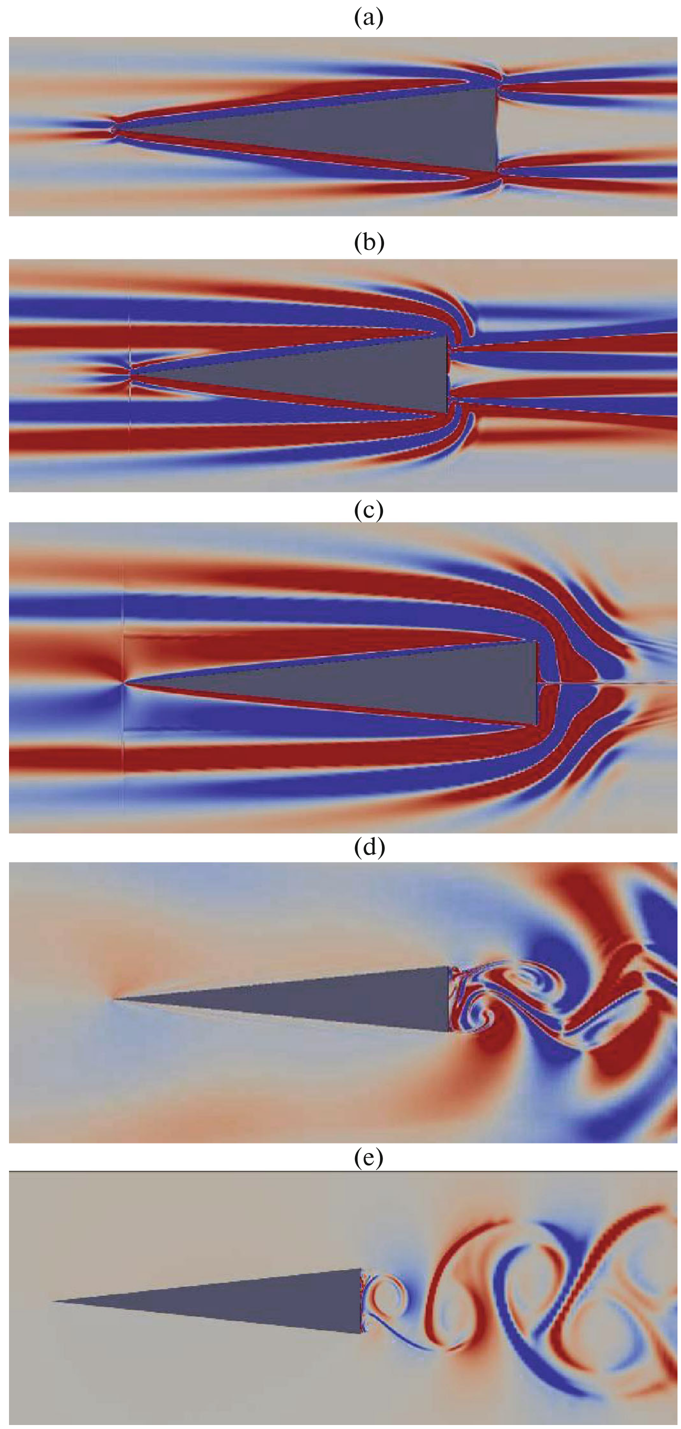

Fig. 3. Horizontal component of perturbations of the salinity gradient $\partial s/\partial x$ at an increase in the external flow velocity: (a–e): $U = 10^{-5}$, 10^{-4}, 10^{-3}, 10^{-2}, and 10^{-1} m/s (positive values in red and negative values in blue).

tral buoyancy produces their periodic oscillations. The irregularities of the crests and valleys of the internal wave forms reflect a complex pattern of interference between growing non-stationary and affiliated internal waves.

The number of observed non-stationary waves that do not penetrate behind the body increases with time. Sharp interfaces delineating the upper and lower boundaries of the density trace are clearly expressed far beyond the body. Non-stationary internal waves are well observed around the wedge corner points at $1 < t/T_b < 3$ in Figs. 2b and 2c.

The structure of the boundary flow near the wedge sides also strongly depends on the velocity of the external flow (Fig. 3). At an increase in the body speed, the introduced disturbances become more pronounced and overlap the flow pattern produced by weak slow initial diffusion-induced flows.

The observation of a sequence of calculated flow patterns shows that the flow around the wedge is non-stationary. Periodically formed on the leading edge, vortices move downstream along the body sides. Besides the vortices, high-gradient thin-structural layers and long internal waves are observed near the body. In this case, the internal wave pattern is stationary relative to the body.

As the speed of the external flow increases, the length of the affiliated internal wave increases proportionally according to the linear theory: $\lambda = UT_b$. The phase surface separating the wave perturbations of opposite signs is bent in the direction of the wedge movement. Well-defined periodic structures are formed beyond the extreme points of the wedge corners (Figs. 3a and 3b). Small-scale perturbations forming many fine structures in the flow near the body boundary (Fig. 3c) are produced near the anterior sharp tip of the wedge. At speeds of $U > 10^{-2}$ m/s, vortex disturbances form in the trace behind the wedge (Figs. 3d and 3e).

The calculated field structure of the salinity gradient when wrapped the wedge is in qualitative agreement with the shadow imaging of the gradient fields of the refractive index that was performed in the laboratory tank by the "color shadow method" with a horizontal slit and grating for bodies of other geometric shapes [9].

CONCLUSIONS

Calculations of stratified flows near the horizontal wedge of neutral buoyancy showed the high performance of the proposed numerical model.

Even near a stationary body, the field of the diffusion-induced flow under study in a stratified fluid is characterized by a complex internal structure: dissipative gravity waves appear at the body edges. The sharp edges of the streamlined obstacles produce expanding horizontal high-gradient layers that are clearly observed in laboratory experiments using high-resolution shadow imaging.

A group of affiliated waves with opposite phases relative to the neutral buoyancy horizon is formed around the slowly moving body at the edges. The main flow components then become eddies that are formed at the sharp edges of the wedge and drift behind it downstream. At a further increase in the flow rate of the body wrapping, the flow pattern becomes more complex and non-stationary.

Wrapping a wedge with a stratified fluid is a complex, multiscale, and non-stationary physical process that requires an additional detailed experimental and theoretical study with allowance for the effects of diffusion, thermal conductivity, and compressibility of the medium.

REFERENCES

1. Prandtl, L., *Essentials of Fluid Mechanics,* London: Blackie & Son, 1952.
2. Phillips, O.M., On flows induced by diffusion in a stably stratified fluid, *Deep-Sea Res.,* 1970, vol. 17, pp. 435–443.
3. Shapiro, A. and Fedorovich, E., A boundary-layer scaling for turbulent katabatic flow, *Bound. Layer Meteor.,* 2014, vol. 153, no. 1, pp. 1–17.
4. Oerlemans, J. and Grisogono, B., Glacier winds and parameterisation of the related surface heat fluxes, *Tellus A,* 2002, vol. 54, pp. 440–452.
5. Zyryanov, V.N., Hydrodynamic basis of formation of ladge-scale water circulation in the Caspian sea: 2. Numerical calculations, *Water Res.,* 2016, vol. 43, no. 2, pp. 292–305.
6. Garrett, C., MacCready, P., and Rhines, P.B., Boundary mixing and arrested Ekman layers: rotating stratified flow near a sloping boundary, *Annu. Rev. Fluid Mech.,* 1993, vol. 25, pp. 291–323.
7. Gargett, A.E., Differential diffusion: an oceanographic primer, *Progr. Oceanogr.,* 2003, vol. 56, no. 3, pp. 559–570.
8. Linden, P.F. and Weber, J.E., The formation of layers in a doublediffusive system with a sloping boundary, *J. Fluid Mech.,* 1977, vol. 81, pp. 757–773.
9. Chashechkin, Yu.D. and Mitkin, V.V., A visual study on flow pattern around the strip moving uniformly in a continuously stratified fluid, *J. Visualiz.,* 2004, vol. 7, no. 2, pp. 127–134.

10. Mercier, M.J., Ardekani, F.M., Allshouse, M.R., Doyle, B., and Peacock, T., Self-propulsion of immersed object via natural convection, *Phys. Rev. Lett.,* 2014, vol. 112, no. 20, p. 204501.
11. Page, M.A., Propelled by diffusion, *Nature Phys.,* 2010, vol. 6, pp. 486–487.
12. Dimitrieva, N.F. and Chashechkin, Yu.D., The structure of induced diffusion flows on a wedge with curved edges, *Phys. Oceanogr.,* 2016, vol. 3, pp. 70–78.
13. Dimitrieva, N.F. and Zagumennyi, Ia.V., Diffusion-driven flows on a wedge-shaped obstacle, *Phys. Scr.,* 2016, vol. 91, no. 8, p. 084002.
14. Bardakov, R.N. and Chashechkin, Yu.D., A stratified flow fine structure near a horizontally moving strip, *JSME Int. J.,* 2006, vol. 49, no. 3, pp. 601–604.
15. Chashechkin, Yu.D., Differential fluid mechanics—harmonization of analytical, numerical and laboratory models of flows, in *Mathematical Modeling and Optimization of Complex Structures, Computational Methods in Applied Sciences Series,* New York: Springer-Verlag, 2016, vol. 40, pp. 61–91.
16. Baydulov, V.G., Matyushin, P.V., and Chashechkin, Yu.D., Evolution of the diffusion-induced flow over a sphere submerged in a continuously stratified fluid, *Fluid Dyn.,* 2007, vol. 42, no. 2, pp. 255–267.
17. Baidulov, V.G. and Chashechkin, Yu.D., Invariant properties of systems of equations of the mechanics of inhomogeneous fluids, *J. Appl. Math. Mech.,* 2011, vol. 75, no. 4, pp. 390–397.
18. Landau, L.D. and Lifshitz, E.M., *Fluid Mechanics,* vol. 6: *Course of Theoretical Physics,* Oxford: Pergamon Press, 1986.
19. Chashechkin, Yu.D., Zagumennyi, Ya.V., and Dimitrieva, N.F., Dynamics of formation and fine structure of flow pattern around obstacles in laboratory and computational experiment, in *Communications in Computer and Information Science,* Springer Int. Publ., 2016, vol. 687, pp. 41–56.

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