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Three-dimensional Turbulent Supersonic Flow over a Plate

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Abstract—The process of laminar-turbulent transition in a boundary layer is studied in terms of vortex evolution, from the growth of weak external disturbances to the formation of intense waves resulting in the development of a turbulent flow having internal scales of the problem. The self-sustained turbulence is obtained and the flow patterns on the plate surface and inside the turbulent boundary layer with fluid plumes propagating from the plate surface in the form of "bursting" are investigated. The basic flow parameters are calculated, such as the frequency and intensity of gas velocity fluctuations in turbulent spots. The validity of a local turbulence similarity law is confirmed. The study is performed on the basis of direct numerical simulation of a flow over a plate with Mach number M = 2 using unsteady Navier–Stokes equations without any closure model of turbulence.

Key words: supersonic flow, laminar and turbulent flow, Tollmien–Schlichting waves, turbulent spots

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The problem of laminar-turbulent transition in supersonic flows with the manifestation of nonlinear instability and the formation of Tollmien–Shclichting waves was considered in many publications presented theoretical, numerical, and experimental studies [1–11].

In this study, which continues [9, 10], we present the results related to the development of a turbulent flow over a plate in the absence of external disturbances (waves). Earlier in [10], it was obtained that when external disturbances in the form of harmonic pressure waves with the intensity of 1–2% p_∞ interact with the boundary layer, depending on the Mach number, layer thickness, wave length, and plate width, a resonant growth of the waves occurs up to the formation of strong, almost two-dimensional waves which disintegrate and form coherent structures. It means that the external disturbances, being transformed into strong waves and disintegrated, in some sense instantaneously shake the gas flow, after which the flow is turbulized [3]. When the intensity of the disintegrated waves (fluctuations) becomes small, for a fixed Reynolds number Re based on the plate length, a portion of "slow" gas is ejected from the viscous sublayer towards the outer edge of the boundary layer and the duplication of the boundary layer thickness (M = 2) occurs. Further, this flow pattern retains in the form of a structured vortex field which looks like an "established turbulence" [6]. The distributions of the flow parameters (pressure, temperature, and surface friction) along the plate surface agree with the existing experimental data.

This can be explained as follows: in turbulent flows, the major energy is carried by the large-scale eddies [3] which are modeled fairly accurately by the numerical method [12] and by many other methods, for instance [8]. Accordingly, if a stochastic flow with its natural frequency and formed turbulent spots is not developed, then the main average flow parameters in the calculation and in the experiment almost coincide.

Thus, as shown in [10], for fairly strong external disturbances the fluctuations with the natural frequency of the problem do not develop, because after the passage of the shock wave the disturbances travel over the boundary layer and, via the pressure fluctuations, impart their frequency and intensity to the flow, blocking the natural frequencies of the problem.

In the present study, we demonstrate that, with a decrease in the intensity of external waves by an order of magnitude, a stochastic motion with its own frequency and the formation of turbulent spots develops in the flow, and this motion retains after the external disturbances are "switched off." Thus, the energy dissipating in the turbulent region is taken from the free-stream energy.

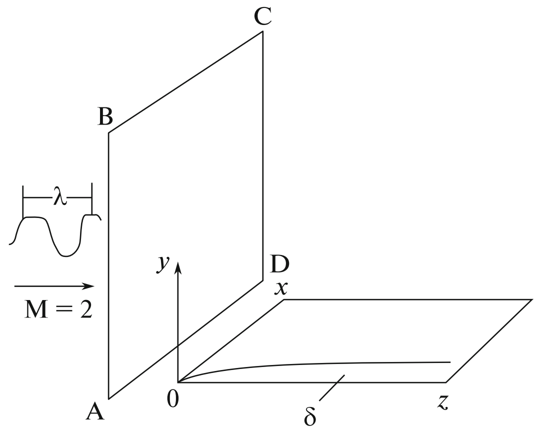

Fig. 1. Calculation domain for the flow over a plate.

1. FORMULATION OF THE PROBLEM

Figure 1 shows the scheme of a viscous supersonic flow over a rectangular plate (with z, x, and y being the longitudinal, transverse, and normal coordinate; and w, u, v being the velocity components along these coordinates).

In the frontal section ABCD of the rectangular calculation domain, the flow parameters are equal to those in the free stream. Initially, a quasi-steady flow over the plate with the boundary layer of thickness δ is established. Then, on the free-stream velocity w_∞ we impose a plane harmonic wave with a wavelength λ and a certain intensity. For a 3 D calculation, the Reynolds number Re based on the plate length is approximately equal to 10^6. A symmetric problem with respect to the y-axis is considered; accordingly, the flow patterns are presented for one surface of the plate. The results are obtained by the direct numerical simulation in the framework of the Navier–Stokes equations without using any turbulence models. An explicit two-step finite-difference scheme [12] is used. The accuracy of the calculations: for a 3 D flow over a short plate, the boundary layer thickness in the undisturbed flow coincides (within the error of 10–15%) with the value calculated in the central longitudinal section of the plate using an approximate formula for $w_{0.99}$. The geometrical dimensions given in the figures are scaled to the plate length, the pressure p and the density R are scaled to their values in the free stream, and the velocity components are scaled to the free-stream sonic velocity. The maximal number of the calculation points was 1.7×10^7; the spatial steps were equal to 10^{-6}–10^{-5} m, and the time steps were 10^{-9} s.

We note that in the first part of the paper for 3 D flows we present the calculations with a symmetric distribution of external disturbances in the entire initial section ABCD (Fig. 1). After the turbulization of a flow part the external disturbances are "switched off."

2. CALCULATION RESULTS

We will calculate the air flow over a thermally insulated plate with the flow velocity w_∞ and other parameters: $p_\infty = 1$ atm, $T_\infty = 278$ K, $R_\infty = 1.25$ kg s^2/m^4, $\mu_\infty = 1.72 \times 10^{-5}$ kg s/m^2, the Prandtl number Pr = 0.72, and the free-stream Mach number M = 2.

2.1. Distribution of Flow Parameters over the Plate Surface

Figures 2a and 2b show the pressure patterns on the surface of a plate immersed in gas flow with M = 2 and an imposed initially plane harmonic wave of frequency 780 kHz for two instants of time t_1 and $t_2 = t_1 + 2 \times 10^{-4}$ s. The frequency is chosen from the condition that for the wavelength of the order of the boundary layer thickness the maximal growth of the impinging disturbances occurs [10].

In Fig. 2a, the traces of the impinging low-intensity wave are clearly visible near the leading edge of the plate. However, this does not result in the development of a secondary flow in the form of longitudinal vortex structures along the side edges of the plate. Accordingly, for a weak intensity of the waves the flow is almost quasi-two-dimensional, and the transitional stage obeys almost the linear law. The pressure and entropy gradually grow, and the temperature is almost constant. For disturbances with fairly large initial

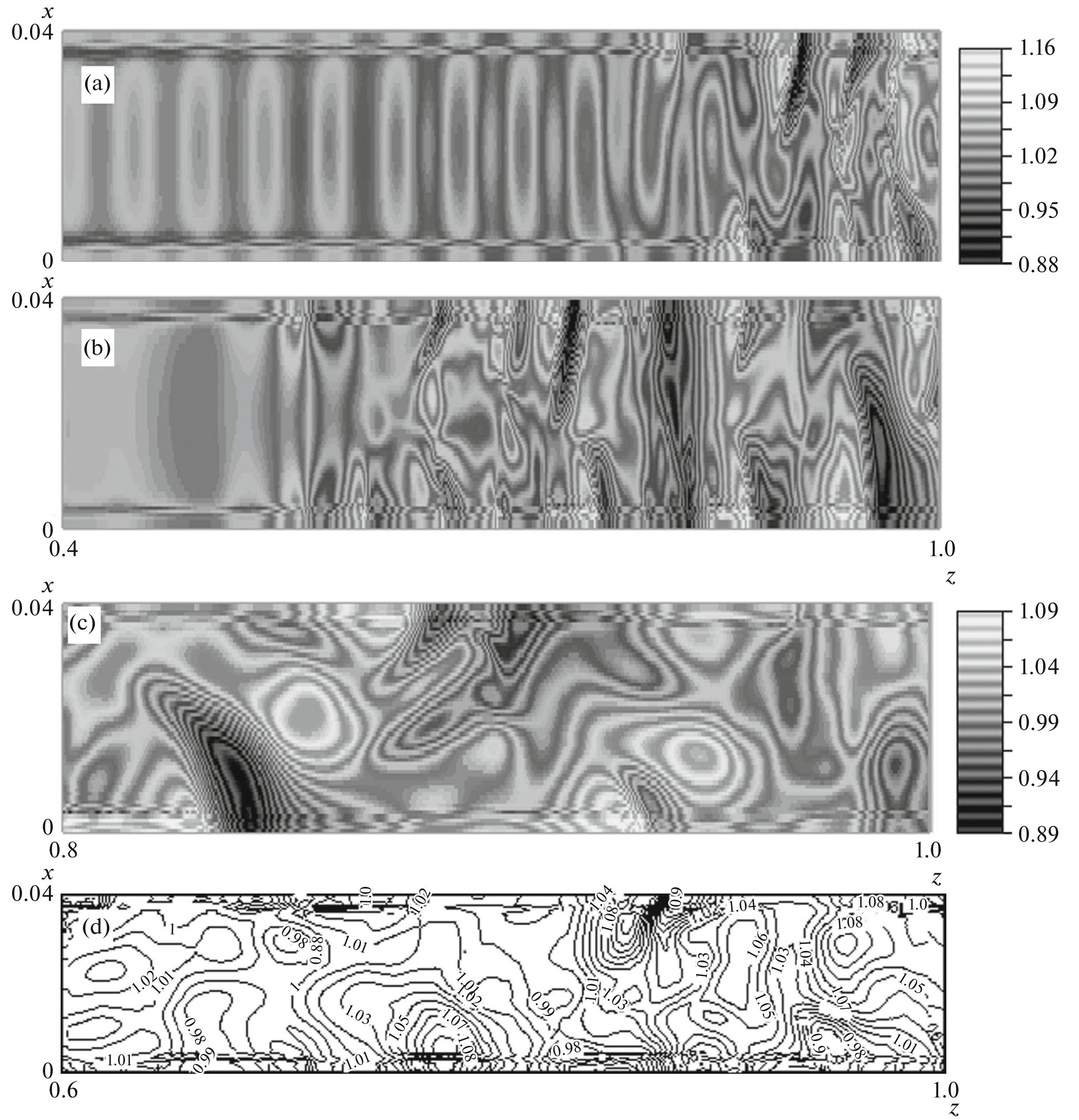

Fig. 2. Pressure distributions on the plate surface in a turbulent gas flow: (a) initial stage of flow turbulization, (b) developed stage of turbulent flow, (c, d) turbulent spots and the isobars on the plate.

amplitudes, there develops a 3D distribution of flow parameters over the plate, starting almost from its leading edge [10], and the flow parameters increase in the developing instability wave.

Near the trailing edge of the plate ($Re \approx 10^6$), the flow is turbulized. Here, the external waves are broken down by the formed intense fluctuations which have the natural frequency. Three-dimensional turbulent spots with different dimensions appear. The development of the turbulent region at instant t_2 is shown in Fig. 2b, where a large zone is clear with a chaotic motion of spots with diameters ranging from 0.1 to 1.5 mm. To this instant, the dimensions of the turbulent region become maximal, and at further instants its leading edge recedes and oscillates with the frequency of 10 kHz in the range $0.45 < z < 0.85$ ($0.45 \times 10^6 < Re < 0.85 \times 10^6$). The rate of the front propagation opposite the free stream from the position at t_1 to that at the current t is equal to 25 m/s. When the turbulization front recedes, the regions of a weakly disturbed gas in the form of crosswise waves remain on the plate surface. The length of these waves is equal

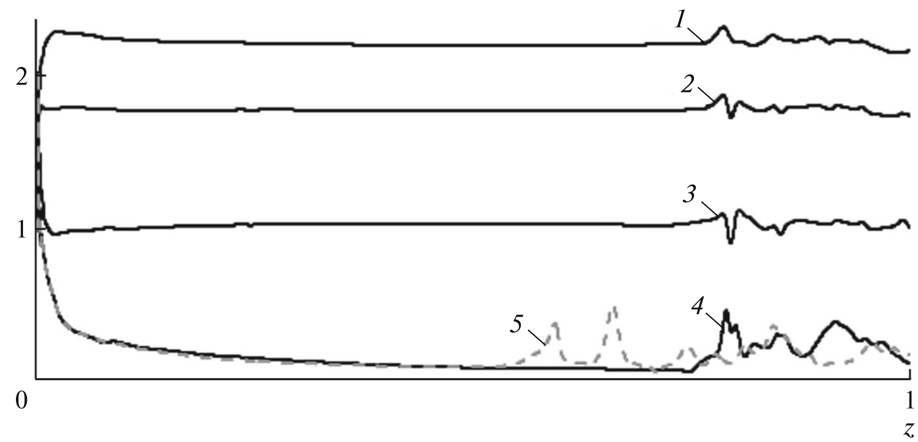

Fig. 3. Behavior of gasdynamic parameters: (*1–3*) s, T, p, (*4*) surface friction coefficient c_f on the plate in the laminar and turbulent regions, (*5*) c_f at the time instant t_2.

to half-width of the plate. This behavior of the gas is explained by the fact that, in the interaction of the turbulent region with side vortices, high-vorticity zones develop, which retain even after the turbulent region recedes. When the free-stream gas travels over these zones, due to the gas retardation crosswise waves are formed.

It should be noted that at instant t_2 the external disturbances ahead of the plate are “switched off”, and at this instant there are no traces of the external waves both near the leading edge of the plate and up to the turbulent region. It means that in this case a self-sustained turbulence is developed, which is characterized by the balance between the energy dissipated in the boundary layer and the energy entering from the free stream. A more detailed flow pattern in the turbulent region near the trailing edge of the plate is given in Fig. 2c and the isobars are shown in Fig. 2d.

In the course of flow turbulization, the gasdynamic parameters vary along the plate surface. Figure 3 shows the instant values of flow parameters on the plate surface at t_1 in the central longitudinal section. Curves *1–4* correspond to the distributions of entropy s, temperature T, pressure p, and surface friction coefficient c_f, respectively.

Clearly, when the flow turbulization occurs, the values of all parameters increase, which agrees to the existing experimental [1, 4] and numerical data [8]. Curve *5* shows the distribution of c_f at instant t_2. At this instant of time, the behavior of other flow parameters is similar.

The flow parameters distribution on the plate surface is directly correlated with the fluctuating gas flow over the plate.

2.2. Gas Flow in the Boundary Layer over the Plate

Figure 4a shows the instantaneous density distribution in the central longitudinal section of the calculation domain. The laminar boundary layer is shown (instant t_2), which transforms into the turbulent boundary layer consisting of various pulsating regions. Some of these regions tend to go out from the boundary layer into the supersonic free stream, which results in the formation of inclined waves outside the boundary layer. The inclination angle of these waves is determined by the intensity (velocity) of the gas entering the outer edge of the boundary layer.

For very weak disturbances propagating from the boundary layer, the inclined waves should be the Mach waves with 30° inclination angle. In the calculations, the inclination angle of the waves was varied in the range 45°–70°. For a more accurate calculation of the wave inclination angle, in Fig. 4b we present the isosurfaces of the fluctuations of the velocity component v normal to the plate surface in the central longitudinal section. The value of the dimensionless velocity component v varies between 0.3 and 0.6. The maximal value of v is attained at the outer edge of the boundary layer at the apexes of the bursts, i.e., the quasi-periodic ejections of viscous fluid out from the wall flow region (Figs. 4c, 4d). Taking into account the velocity of gas ejection from the boundary layer and using the formula for the Mach wave inclination angle, it is possible to estimate approximately the inclination angle of the waves as arcsin 0.8 ≈ 54°. Since the inclined waves formed are not of zero intensity, the value of this angle should be greater. As mentioned

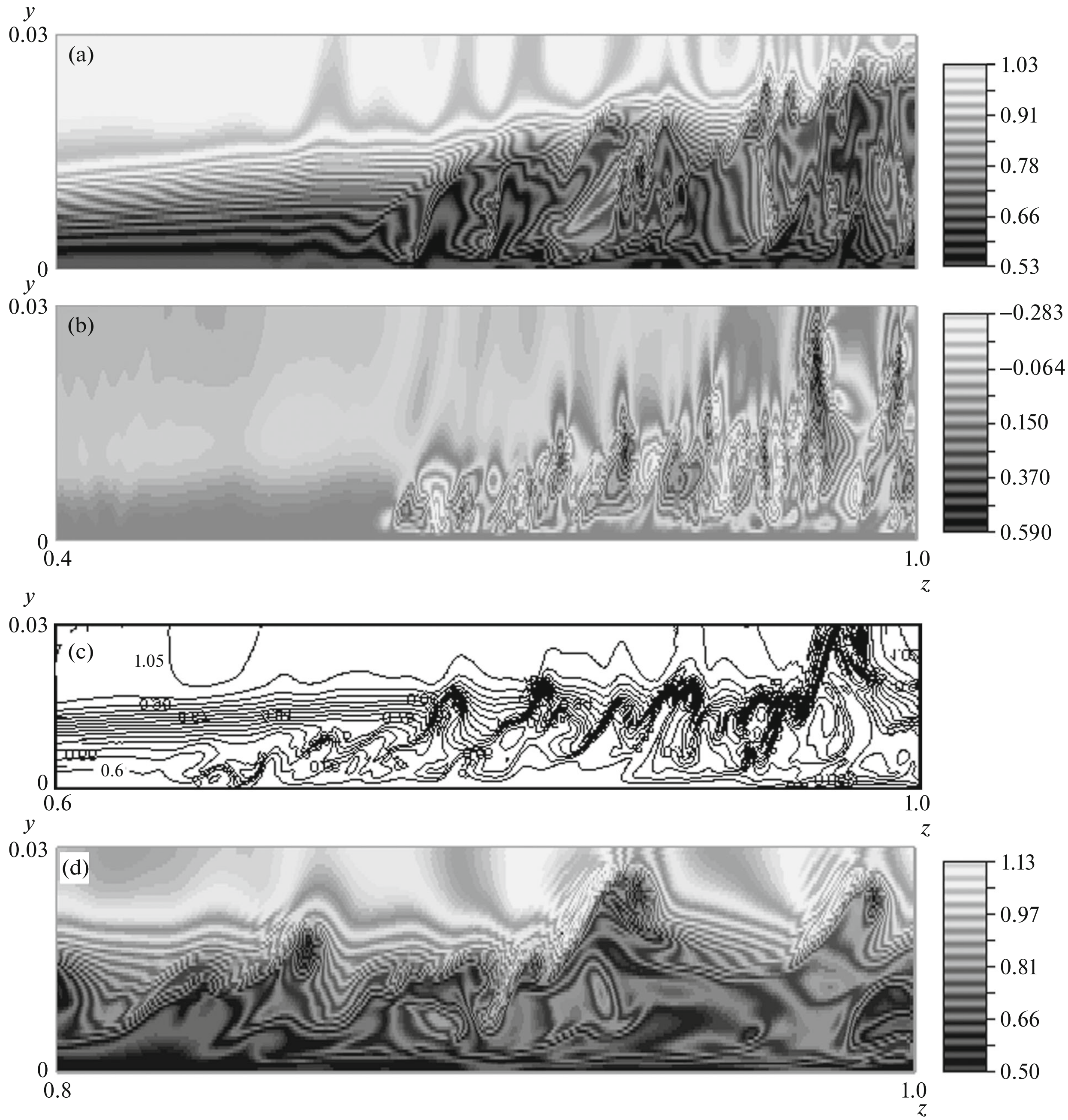

Fig. 4. Flow patterns in the boundary layer (instantaneous parameters in the central longitudinal section): (a) density field, (b) normal velocity component, (c) quasi-periodic ejections of viscous fluid from the wall region (bursts), (d) radiation of entropy waves in the interaction of a burst with the outer edge of the boundary layer.

above, the inclination angle of the waves developed in the flow turbulization was ranged in the limits 45°–70°. The minimal inclination angle corresponds to the wave formed from the moving leading front of the turbulent region. The introduction of imposed inclined waves in the experimental [4–6] and numerical studies [7, 8, 11] prompts the flow turbulization, although in real flows these waves develop independently, as the reaction of the outer flow on the turbulent fluctuations inside the boundary layer.

The dynamics of the turbulent flow development are illustrated in Fig. 5 for two instants of time, where the pressure distribution on the plate surface (a, d), the density distribution in the central longitudinal section of the calculation domain (b, e), and the pressure distribution in the central longitudinal section (c, f) are presented. From the figure, it is clear how far the turbulence front recedes during the time of about 10^{-5} s.

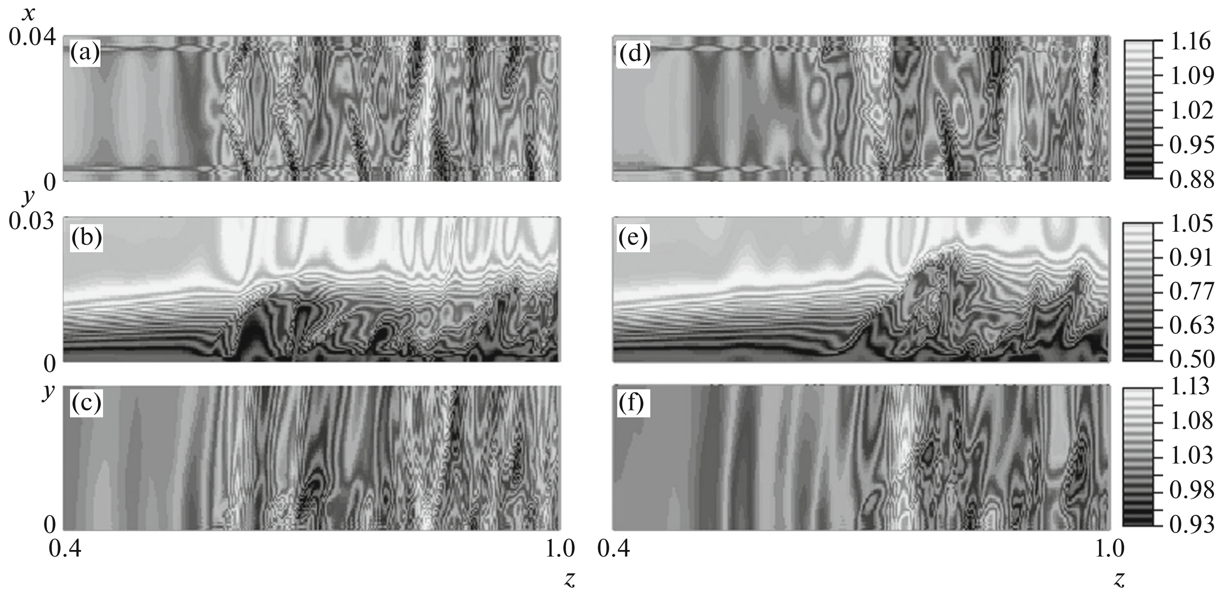

Fig. 5. Flow patterns over the plate for two instants of time: (a, d) pressure fields on the plate surface; (b, e) and (c, f) pressure and density distributions in the central longitudinal section of the calculation domain.

When the flow turbulization occurs, the velocity profile in the boundary layer changes. In Fig. 6, for two instants of time (a) and (b) (step 10^{-6} s) we plot the instantaneous profiles of the longitudinal velocity in the crosswise sections of the boundary layer. Curves *1–4* in this figure correspond to the velocity profiles in the sections where the flow is laminar ($z = 0.2$), transitional (0.4), and turbulent (0.8, 0.95).

The analysis indicates that due to the nonlinear amplification of disturbances the laminar flow transforms into the transitional one, and then into the turbulent flow characterized by a more flattened velocity profile. The horizontal lines in the figures, corresponding to the thicknesses of the laminar and turbulent boundary layer, show that the turbulent boundary layer thickness is two-fold greater. This agrees with the experimental data [2] and numerical simulations [10].

In Fig. 6, it is clear that, both for laminar and turbulent flow regimes, in the viscous sublayer the velocity is a linear function of the normal coordinate. With increase in the distance from the plate, this behavior is observed for the turbulent flow regimes (curves *3*, *4*) up to the Mach numbers M = 0.8–1, i.e., in the subsonic region. Depending on the Reynolds number Re (curves *3*, *4*), the "fullness" of the velocity profile for different instants of time can be smaller than for laminar flow. This means that the thickness of the viscous sublayer in the turbulent flow varies with time due to the ejection of gas portions from the plate surface in the form of bursts.

The density (curve *1*), pressure (*2*), and temperature (*3*) distributions in the section $z = 0.95$ are shown in Fig. 6c. An analysis indicates that entropy waves propagate from the turbulent boundary layer into the outer flow. These waves are clearly manifested in Fig. 4d at the instant of the burst interaction with the outer edge of the boundary layer. These waves are correlated with the frequency of fluid ejections, and hence they are not regular. Thus, in Figs. 5a and 5b it is clear that the behavior of curve *4* outside the boundary layer differs for different instants of time.

2.3. Distribution of the Flow Parameters on the Outer Edge of the Boundary Layer

During bursting, a gas ejection out off the plate surface occurs. In Fig. 6c, it is clear that a gas portion is ejected toward the outer edge of the boundary layer. This portion of gas has the values of parameters corresponding to those on the edge of the viscous sublayer, i.e., the gas is less dense and hotter than in the outer flow. A comparison of the distributions of the vorticity component normal to the plate surface and the density in the sections $y_1 = 1.5\delta$ (Fig. 6a) and $y_2 = 2\delta$ (Fig. 6b) shows that, at a height y_1 in the turbulent flow, the patterns of vorticity and density are very complex. At this height, there are regions in which the density is smaller (hotter gas) than on the plate surface. Above the turbulent region, in the section y_2 the burst apexes consisting of small-scale vortices are clearly marked.

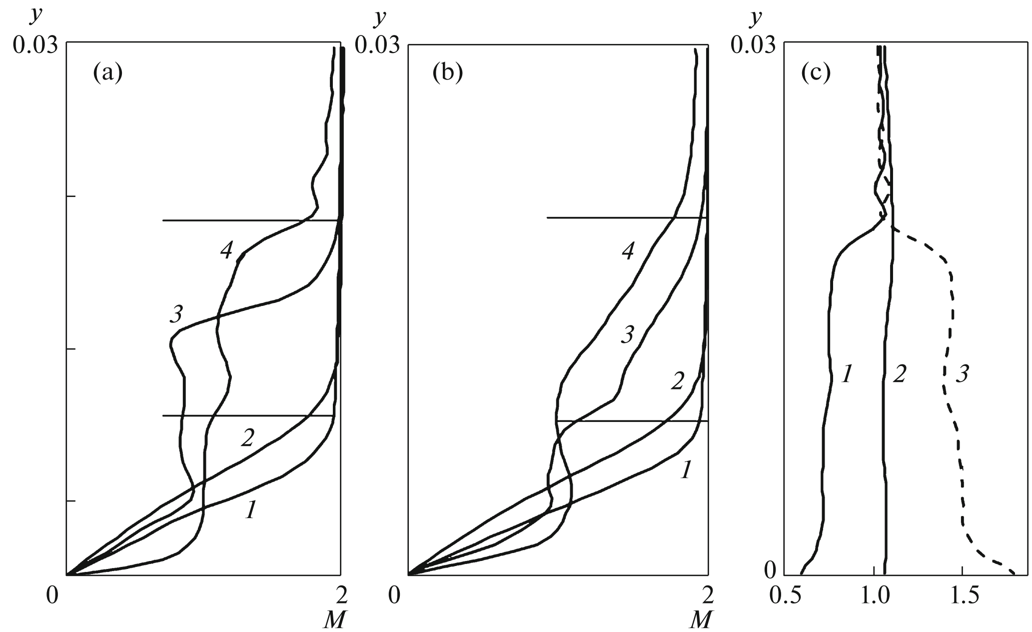

Fig. 6. Distributions of flow parameters across the boundary layer: (a) and (b) longitudinal velocity profiles for two instants of time, (*1*–*4*) velocity profiles in the sections where the flow is laminar ($z = 0.2$), transitional (0.4), and turbulent (0.8, 0.95); horizontal lines show the thickness of the laminar and turbulent boundary layer; (c) *1*–*3*—density, pressure, and temperature.

2.4. Fluctuations and the Discussion of the Results

A direct numerical simulation gives a huge body of information, with the most important data corresponding to the average flow parameters which can be compared with the data obtained by other methods. In Fig. 7a, we present the values of the pressure fluctuations on the plate surface in the turbulent flow region for three instants of time with a time step of 1.5×10^{-5} s. These fluctuations are generated in the absence of external disturbances. The time behavior of the fluctuations at three points ($z = 0.2, 0.8, 0.95$) is shown in Fig. 7b. The figure illustrates the reaction of the turbulent flow on the external disturbances entering from outside. Thus, at $z = 0.2$, a weak disturbance in the form of a harmonic wave (small oval) is clear, which, passing through the points $z = 0.8$ and 0.95, results in the increase in the fluctuation frequency from 300 to 3000 kHz.

The calculation of the average values of the pressure on the plate surface at the points $z = 0.2$, 0.7, and 0.95 gives the following results: 1.015, 1.052, and 1.017. Clearly, at the point with coordinate $z = 0.7$, where the leading front of the turbulent region is located, the pressure is greater than in the ambient gas. This forces the gas to move opposite the main free-stream flow.

The dialed calculation data for the flow over the plate make it possible to verify the validity of the main turbulence laws. Thus, an estimate of the self-similarity parameter of local turbulence $v_n/\Delta w \sim (n/L)1/3$ [3] (where n is the disturbance scale equal to approximately 0.002, v_n is the order of magnitude of the velocity fluctuations equal to approximately 0.6, L is the boundary layers thickness of the order of 0.02, and Δw is the order of variation of the leverage velocity equal to approximately 2) gives the range 0.3–0.4. Taking into account that for the estimates we took average values of parameters, we may assume that the self-similarity law is valid without additional requirements of homogeneity and isotropy of the flow. This was also obtained in [6].

The analysis of the calculation data for the flow over the plate for M = 2 indicates that the initial stage of the transition to turbulence strongly depends on the intensity of external waves. When the external harmonic disturbances are fairly intense, the internal fluctuations are blocked and the crosswise dimensions of the longitudinal vortices increase in the course of their diffusion with increase in Re. As a result of the interaction of two lateral longitudinal vortices, turbulization of the gas flow occurs. In the numerical modeling, this process depends substantially on the plate width. The dependence of the maximal growth of the disturbances on the plate width was mentioned in [10].

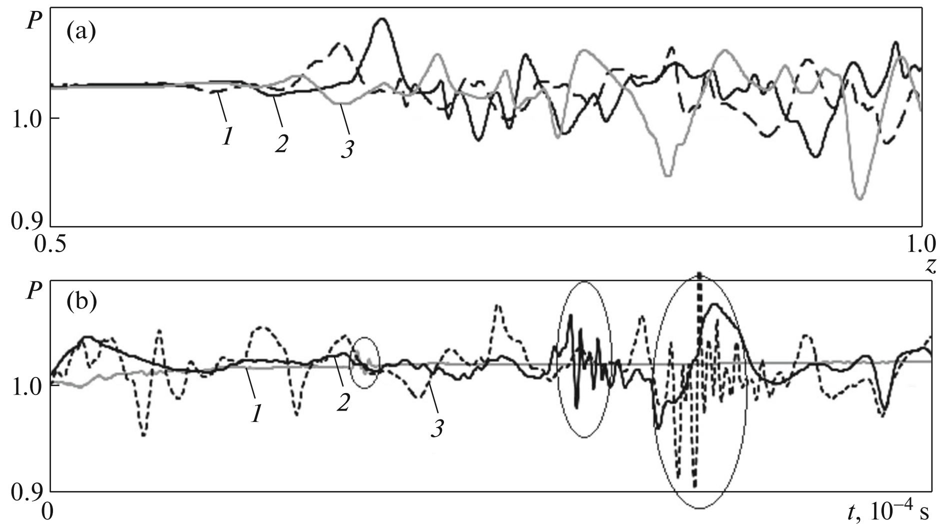

Fig. 7. Pressure fluctuations in the turbulent flow: (a) instantaneous pressure values on the plate for three instants of time (time step is 1.5×10^{-5} s); (b) time behavior of the fluctuations at three points ($z = 0.2, 0.8, 0.95$).

When the external disturbances imposed on the flow are weak, a linear-instability stage is observed, which then passes into a nonlinear stage resulting in the flow turbulization.

SUMMARY

Using the unsteady Navier–Stokes equations, a numerical solution of the problem of turbulization of gas flow over a thermally insulated flat plate with Mach number M = 2 is found up to the scales at which the fluctuations with natural frequencies of the problem develop. It is shown that the turbulent flow obtained can be realized without imposing external disturbances. The flow patterns on the plate surface and in the turbulent boundary layer are found. The correlation between the formed inclined waves and gas ejections from the plate surface (bursts) is explained. The validity of a self-similarity law for local turbulence is confirmed.

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