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Efficient Approach to Description of Heat Transfer and Multicomponent Diffusion in Ionized Gases

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Abstract—The formulas for the heat fluxes of heavy components and electrons as well as the Stefan–Maxwell relations for the diffusion fluxes in a magnetic field are derived for a multicomponent two-temperature plasma with regard to the higher-order approximations in orthogonal expansions of the component distribution functions in Sonine polynomials. For the complex transport coefficients of heavy components and electrons exact formulas are obtained in the significantly simpler form as compared with the standard procedure of the Chapman–Enskog method with the minimum number of minimum-order matrix inversions.

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In [1–6] the expressions for the mass and heat fluxes in the completely ionized and three-component plasma in a magnetic field were obtained using the Chapman–Enskog method. For the multicomponent plasma moving in a magnetic field the transport equations were derived mainly using the 13th-moment Grad method [7–9]. In [10–19] the Chapman–Enskog method was used with these aims.

On the basis of systematic calculations of the transport coefficients [20–31] it was established that for ionized gases it is necessary to take into account at least the third approximation in terms of Sonine polynomials ($\xi \geq 3$). This leads to inversion of $N\xi$th-order matrixes, where N is the number of components. Additional complexities related to the necessity of solving high-order systems of linear algebraic equations for the complex transport coefficients arise in calculating the transport coefficients for multicomponent ionized gases in a magnetic field by means of the Chapman–Enskog method.

1. FEATURES OF CALCULATIONS OF THE TRANSPORT COEFFICIENTS FOR IONIZED GASES

In the literature there is a certain gap between the rigorous formulas of kinetic theory for diffusion and heat fluxes and the level of description of transport processes in particular problems of computational aerothermodynamics. In particular, the following classical expressions for the diffusion velocities $\mathbf{V}_i$ are generally not used in practical calculations of flows of multicomponent reacting gas mixtures and plasmas [32]

$$\mathbf{V}_i = \frac{n^2}{n_i\rho}\sum_j m_i D_{ij}\mathbf{d}_j^* - \frac{1}{n_i m_i} D_i^T \nabla \ln T,$$

$$\mathbf{d}_i^* = \nabla\left(\frac{n_i}{n}\right) + \left(\frac{n_i}{n} - \frac{n_i m_i}{\rho}\right)\nabla \ln p - \frac{n_i}{p}\mathbf{X}_i + \frac{n_i m_i}{p\rho}\sum_j n_j \mathbf{X}_j.$$

Here, n_i is the number of particles of the ith kind per unit volume, n is the total number of particles in unit volume, m_i is the mass of a particle of the ith kind, ρ is the mixture density, p is the pressure, T is the temperature, and $\mathbf{X}_i$ is the external force.

Calculations of the multicomponent diffusion and thermodiffusion coefficients D_{ij} and D_i^T using the formulas of kinetic theory for chemically nonequilibrium flows are fairly time-taking. For the

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multicomponent two-temperature plasma the formulas for the diffusion fluxes of heavy components (molecules, atoms, and ions) become significantly more complex and take the form [12, 13]:

$$\mathbf{J}_i = \frac{n^2}{\rho}\sum_j m_i m_j \left(D_{ij}^{\parallel}\mathbf{d}_j^{\parallel} + D_{ij}^{\perp}\mathbf{d}_j^{\perp} + D_{ij}^{\wedge}\mathbf{b}\times\mathbf{d}_j\right)$$
$$-\left(D_{ih}^{Th\parallel\nabla\parallel}\ln T_h + D_{ih}^{Th\perp\nabla\perp}\ln T_h + D_{ih}^{Th\wedge}\mathbf{b}\times\nabla\ln T_h\right)$$
$$-\left(D_{ie}^{Te\parallel\nabla\parallel}\ln T_e + D_{ie}^{Te\perp\nabla\perp}\ln T_e + D_{ie}^{Te\wedge}\mathbf{b}\times\nabla\ln T_e\right),$$
$$\mathbf{d}_j = \frac{1}{nkT_h}\nabla p_j - c_j\frac{p}{nkT_h}\nabla\ln p + \frac{1}{kT_h}\left(c_j\frac{q}{n} - \frac{n_j}{n}e_j\right)\mathbf{E}^{|}, \quad \mathbf{E}^{|} = \mathbf{E} + \mathbf{v}\times\mathbf{B}. \tag{1.1}$$

Here, T_h is the heavy particle temperature, T_e is the electron temperature, p_j is the partial pressure, c_j is the mass concentration, e_j is the electric charge of particles of the jth component, q is the space charge of the mixture, $\mathbf{E}$ is the electric field, $\mathbf{b}$ is the unit vector in the direction of the magnetic field $\mathbf{B}$, and $\mathbf{v}$ is the hydrodynamic velocity.

From (1.1) we can see that the complexities of description of multicomponent diffusion in the two-temperature plasma are aggravated by the circumstance that thermodiffusion constituents due to the electron temperature gradient appear in diffusion fluxes of heavy components. In this case it is necessary to calculate all the transport coefficients along the magnetic field (symbol $\|$) and in the perpendicular and transverse directions (symbols $\perp$ and $\wedge$, respectively).

From the point of view of reducing the computational burden and the level of complexity of the programm algorithms, the most rigorous and efficient approaches to description of multicomponent diffusion in particular problems of modern thermophysics are based on the use of the Stefan–Maxwell relations for the diffusion velocities of the components which have the following classic form for the one-temperature gas mixtures [32]:

$$\sum_j \frac{n_i n_j}{n^2 D_{ij}(1)}(\mathbf{V}_j - \mathbf{V}_i) = \mathbf{d}_i^* - \nabla\ln T\sum_j \frac{n_i n_j}{n^2 D_{ij}(1)}\left(\frac{D_j^T}{n_j m_j} - \frac{D_i^T}{n_i m_i}\right). \tag{1.2}$$

Here, $D_{ij}(1)$ are the binary diffusion coefficients in the first approximation which can be calculated from the formulas of kinetic theory [32]

$$D_{ij}(1) = \frac{3(m_i + m_j)}{16 n m_i m_j}\frac{kT}{\Omega_{ij}^{(1,1)}},$$
$$\Omega_{ij}^{(1,1)} = \sqrt{\frac{kT(m_i + m_j)}{2\pi m_i m_j}}\int_0^\infty e^{-\gamma_{ij}^2}\gamma_{ij}^5 Q_{ij}^{(1)}\,d\gamma_{ij},$$
$$Q_{ij}^{(1)} = 2\pi\int_0^\pi \sigma_{ij}(1-\cos\chi)\sin\chi\, d\chi.$$

In solving the particular problems of aerothermodynamics of multicomponent gas mixtures, the calculations of the binary coefficients $D_{ij}(1)$ are radically simpler than the calculations of the multicomponent diffusion coefficients D_{ij} and, as a rule, the contribution of the thermodiffusion constituent on the right-hand side of (1.2) is not taken into account at all.

In the present study, for the multicomponent two-temperature plasma formulas for the heat fluxes of heavy components and electrons as well as the Stefan–Maxwell relations for the diffusion fluxes in a magnetic field are derived with regard to the higher approximations in the orthogonal expansions of the distribution functions of the components in terms of the Sonine polynomials. For the transport coefficients of heavy components and electrons the formulas are obtained in the considerably simpler form as compared with the standard procedure of the Chapman–Enskog method. For the complex thermal conductivity coefficients the formulas generalize the formula [33] which corresponds to the second approximation and the exact formula in [34] to include the case of anisotropy of heat transfer in the multicomponent two-temperature plasma in the presence of the magnetic field.

2. INITIAL SYSTEM OF TRANSPORT EQUATIONS WITH REGARD TO THE HIGHER APPROXIMATIONS IN THE SONINE POLYNOMIALS

In what follows, we will assume that the particle collisions are elastic and the external electromagnetic field has no effect on them [11, 35]. In the N-component two-temperature plasma (the electron component has the number N) the initial system of equations, obtained in [12, 13] in the approximation $(m_e/m_h)^{1/2} \ll 1$, for the diffusion velocities $\mathbf{V}_i$ and the reduced partial heat fluxes $\mathbf{q}_i$ has the form:

$$\mathbf{d}_i = \frac{x_i m_i}{kT_h}\left(\sum_{j=1}^{N-1}(c_j\omega_j - \delta_{ij}\omega_j)\mathbf{b}\times\mathbf{V}_j - c_e\omega_e\mathbf{b}\times\mathbf{V}_e\right) - \sum_{j=1}^{N-1}\Lambda_{ij}^{00}\mathbf{V}_j - \Lambda_{ie}^{00}\mathbf{V}_e - \sum_{j=1}^{N-1}\sum_{p=1}^{M_h}\Lambda_{ij}^{0p}\boldsymbol{\xi}_{ip} - \sum_{p=1}^{M_e}\Lambda_{ie}^{0p}\boldsymbol{\xi}_{ep}, \tag{2.1}$$

$$\mathbf{d}_i = \frac{1}{nkT_h}\nabla p_i - c_i\frac{p}{nkT_h}\nabla\ln p + \frac{1}{kT_h}(c_i q - x_i e_i)\mathbf{E}^{|}, \quad i = 1,\ldots,N-1, \tag{2.2}$$

$$\mathbf{d}_e = \frac{m}{kT_h}c_e\omega_e\mathbf{b}\times\mathbf{V}_e - \Lambda_{ee}^{00}\mathbf{V}_e - \sum_{p=1}^{M_e}\Lambda_{ee}^{0p}\boldsymbol{\xi}_{ep}, \tag{2.3}$$

$$\mathbf{d}_e = \frac{1}{nkT_h}\nabla p_e + \frac{1}{kT_h}x_e e\mathbf{E}^{|}, \quad \sum_{i=1}^{N-1}\mathbf{d}_i + \mathbf{d}_e = 0, \tag{2.4}$$

$$x_i\nabla\ln T_h\delta_{r1} = \frac{3\sqrt{\pi}}{4}\frac{\Gamma(r+1)}{\Gamma(r+5/2)}\frac{m_i x_i\omega_i}{kT_h}\mathbf{b}\times\boldsymbol{\xi}_{ir} + \sum_{j=1}^{N-1}\Lambda_{ij}^{r0}\mathbf{V}_j + \sum_{j=1}^{N-1}\sum_{p=1}^{M_h}\Lambda_{ij}^{rp}\boldsymbol{\xi}_{jp}, \quad r = 1,\ldots,M_h, \tag{2.5}$$

$$x_e\nabla\ln T_e\delta_{r1} = -\frac{3\sqrt{\pi}}{4}\frac{\Gamma(r+1)}{\Gamma(r+5/2)}\frac{m_e x_e\omega_e}{kT_e}\mathbf{b}\times\boldsymbol{\xi}_{er} + \Lambda_{ee}^{r0}\mathbf{V}_e + \sum_{p=1}^{M_h}\Lambda_{ee}^{rp}\boldsymbol{\xi}_{ep}, \tag{2.6}$$

$$\boldsymbol{\xi}_{i1} = -\frac{1}{nkT_h}\mathbf{q}_i, \quad i = 1,\ldots,N-1; \quad \boldsymbol{\xi}_{e1} = -\frac{1}{nkT_e}\mathbf{q}_e, \tag{2.7}$$

$$x_i = \frac{n_i}{n}, \quad \omega_i = \frac{|e_i|}{m_i}|\mathbf{B}|, \quad i = 1,\ldots,N, \quad q = \sum_{i=1}^{N}x_i e_i, \quad \mathbf{b} = \frac{\mathbf{B}}{|\mathbf{B}|}.$$

The coefficients $\Lambda_{ij}^{rp} = \Lambda_{ji}^{pr}$ can be expressed in terms of the integral brackets Q_{ij}^{mp} [32]

$$\Lambda_{ij}^{mp} = \frac{3\pi}{8}\frac{m!p!}{\Gamma(m+5/2)\Gamma(p+5/2)}\frac{\sqrt{m_i m_j}}{nkT_h}\Omega_{ij}^{mp}.$$

The coefficients Λ_{ie}^{mp} andΛ_{ee}^{mp} take the form [12]:

$$\Lambda_{ie}^{mp} = \frac{3\pi}{8}\frac{m!p!}{\Gamma(m+5/2)\Gamma(p+5/2)}\frac{m_e}{nkT_e}\Omega_{ie}^{mp},$$

$$\Lambda_{ee}^{0p} = \Lambda_{ee}^{p0} = -\sum_{j=1}^{N-1}\Lambda_{je}^{0p}, \quad \Lambda_{ee}^{mp} = \Lambda_{ee}^{pm} = \sum_{i=1}^{N}\Lambda_{ie}^{mp} \quad (m,\ p > 0).$$

The vectors $\boldsymbol{\xi}_{ip}$ and $\boldsymbol{\xi}_{ep}$ $(p \geq 2)$ can be expressed in terms of the higher moments of the distribution functions [34]. The equations similar to (2.1)–(2.7) were also obtained in [9] from the solution of the linearized Boltzmann equation using the Grad method at $\mathrm{Kn} \ll 1$.

The form of the transport equations for the diffusion velocities $\mathbf{V}_i$ and the reduced partial heat fluxes $\mathbf{q}_i$ $(i = 1, \ldots, N)$ will depend on the way of resolution of the system (2.1)–(2.7) with respect to the fluxes. For example, if we resolve this system of linear equations at once with respect to all the fluxes on the right-hand side, then we obtain cumbersome expressions for the diffusion and heat fluxes as those in the standard procedure of the Chapman-Enskog method. The approach proposed here to solution of Eqs. (2.1)–(2.7) is the most efficient from the point of view of its application to the aerothermodynamic problems and reducing the calculation volume. This approach was used in [15, 34] for deriving the Stefan–Maxwell relations in the isothermal plasma, in [14] for the two-temperature plasma in the absence of the magnetic field, and in [17] in the general case.

3. HEAT FLUXES IN THE MULTICOMPONENT TWO-TEMPERATURE PLASMA

In complex form the system of equations (2.5) for the constituents of the flux vectors perpendicular to the magnetic field (denoted by the symbol $\perp$) takes the form:

$$\sum_{k=1}^{N-1}\sum_{p=1}^{M_h}\Lambda_{jk}^{rp*}\boldsymbol{\xi}_{kp}^{\perp} = x_j\nabla^{\perp}\ln T_h\delta_{r1} - \sum_{k=1}^{N-1}\Lambda_{jk}^{r0}\mathbf{V}_k^{\perp} \quad (j = 1, \ldots, N-1),$$

$$\Lambda_{jk}^{rp*} = \Lambda_{jk}^{rp} + i\frac{3\sqrt{\pi}}{4}\frac{\Gamma(r+1)}{\Gamma(r+5/2)}\frac{m_j x_j \omega_j}{kT_h}\delta_{jk}\delta_{rp}, \quad i^2 = -1, \tag{3.1}$$

where the vector product $\mathbf{b}\times\boldsymbol{\xi}_{kp}^{\perp}$ is formally represented in the complex form $i\boldsymbol{\xi}_{kp}^{\perp}$ $(i^2 = -1)$.

For the complex coefficients Λ_{jk}^{rp*} the properties of the coefficients Λ_{jk}^{rp} remain valid:

$$\Lambda_{jk}^{rp*} = \Lambda_{kj}^{pr*}, \quad \sum_{k=1}^{N-1}\Lambda_{jk}^{p0*} = \sum_{j=1}^{N-1}\Lambda_{jk}^{0r*} = 0.$$

The solution of system (3.1) with respect to the vectors $\boldsymbol{\xi}_{jp}$ takes the form:

$$\boldsymbol{\xi}_{jp}^{\perp} = \alpha_{jp}\nabla^{\perp}\ln T_h - \sum_{k=1}^{N-1}\beta_{jkp}\mathbf{V}_k^{\perp}, \quad j = 1, \ldots, N-1, \quad p = 1, \ldots, M_h, \tag{3.2}$$

$$\alpha_{jp} = -\frac{1}{\Lambda_h^*}\begin{vmatrix} 0 & 0 & 0 & \ldots & \delta_{js} & \ldots & 0 \\ x_r & \Lambda_{rs}^{11*} & \Lambda_{rs}^{12} & \ldots & \Lambda_{rs}^{1p} & \ldots & \Lambda_{rs}^{1M_h} \\ 0 & \Lambda_{rs}^{21} & \Lambda_{rs}^{22*} & \ldots & \Lambda_{rs}^{2p} & \ldots & \Lambda_{rs}^{2M_h} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \Lambda_{rs}^{M_h1} & \Lambda_{rs}^{M_h2} & \ldots & \Lambda_{rs}^{M_hp} & \ldots & \Lambda_{rs}^{M_hM_h*} \end{vmatrix},$$

$$\beta_{jkp} = -\frac{1}{\Lambda_h^*}\begin{vmatrix} 0 & 0 & 0 & \ldots & \delta_{js} & \ldots & 0 \\ \Lambda_{rk}^{10} & \Lambda_{rs}^{11*} & \Lambda_{rs}^{12} & \ldots & \Lambda_{rs}^{1p} & \ldots & \Lambda_{rs}^{1M_h} \\ \Lambda_{rk}^{20} & \Lambda_{rs}^{21} & \Lambda_{rs}^{22*} & \ldots & \Lambda_{rs}^{2p} & \ldots & \Lambda_{rs}^{2M_h} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \Lambda_{rk}^{M_h0} & \Lambda_{rs}^{M_h1} & \Lambda_{rs}^{M_h2} & \ldots & \Lambda_{rs}^{M_hp} & \ldots & \Lambda_{rs}^{M_hM_h*} \end{vmatrix},$$

$$\Lambda_h^* = \begin{vmatrix} \Lambda_{rs}^{11*} & \Lambda_{rs}^{12} & \ldots & \Lambda_{rs}^{1M_h} \\ \Lambda_{rs}^{21} & \Lambda_{rs}^{22*} & \ldots & \Lambda_{rs}^{2M_h} \\ \ldots & \ldots & \ldots & \ldots \\ \Lambda_{rs}^{M_h1} & \Lambda_{rs}^{M_h2} & \ldots & \Lambda_{rs}^{M_hM_h*} \end{vmatrix}.$$

Here, Λ_h^* is the determinant of the matrix whose elements are the matrices Λ_{rs}^{mp} and Λ_{rs}^{pp*} ($r,\ s = 1,\dots,N-1$). The matrices Λ_{rs}^{pp*} differ from Λ_{rs}^{pp} only by the diagonal (complex) elements. Symbol $\delta_{js} = 1$ if $j = s$ and $\delta_{js} = 0$ if $j \neq s$.

The constituent of the reduced heat flux of the jth component perpendicular to the magnetic field takes the form:

$$\mathbf{q}_j^{\perp} = -n_j k T_h \boldsymbol{\xi}_{j1}^{\perp} = -\lambda_j^* \nabla^{\perp} T_h + nkT_h \sum_{k=1}^{N-1} k_{T_h jk}^* \mathbf{V}_k^{\perp},$$
$$\lambda_j^* = nkx_j\alpha_{j1}, \quad k_{T_h jk}^* = x_j \beta_{jk1}. \tag{3.3}$$

The reduced heat flux of the heavy particles in the direction perpendicular to the magnetic field can be obtained by summing (3.3) over all heavy components

$$\mathbf{q}_h^{\perp} = \sum_{j=1}^{N-1} \mathbf{q}_j^{\perp} = -\lambda_h^* \nabla^{\perp} T_h + nkT_h \sum_{k=1}^{N-1} k_{T_h k}^* \mathbf{V}_k^{\perp}, \tag{3.4}$$

$$\lambda_h^* = -\frac{nk}{\Lambda_h^*} \begin{vmatrix} 0 & x_s & 0 & \dots & 0 \\ x_r & \Lambda_{rs}^{11*} & \Lambda_{rs}^{12} & \dots & \Lambda_{rs}^{1M_h} \\ 0 & \Lambda_{rs}^{21} & \Lambda_{rs}^{22*} & \dots & \Lambda_{rs}^{2M_h} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \Lambda_{rs}^{M_h 1} & \Lambda_{rs}^{M_h 2} & \dots & \Lambda_{rs}^{M_h M_h *} \end{vmatrix}, \tag{3.5}$$

$$k_{T_h k}^* = -\frac{1}{\Lambda_h^*} \begin{vmatrix} 0 & x_s & 0 & \dots & 0 \\ \Lambda_{rk}^{10} & \Lambda_{rs}^{11*} & \Lambda_{rs}^{12} & \dots & \Lambda_{rs}^{1M_h} \\ \Lambda_{rk}^{20} & \Lambda_{rs}^{21} & \Lambda_{rs}^{22*} & \dots & \Lambda_{rs}^{2M_h} \\ \dots & \dots & \dots & \dots & \dots \\ \Lambda_{rk}^{M_h 0} & \Lambda_{rs}^{M_h 1} & \Lambda_{rs}^{M_h 2} & \dots & \Lambda_{rs}^{M_h M_h *} \end{vmatrix}, \quad \sum_{k=1}^{N-1} k_{T_h k}^* = 0. \tag{3.6}$$

Here, λ_h^* is the complex thermal conductivity coefficient of the mixture of heavy components and $k_{T_h k}^*$ are the complex thermodiffusion ratios of heavy components. These complex transport coefficients can be represented in the form of the real and imaginary parts:

$$\lambda_h^* = \lambda_h^{\perp} + i\lambda_h^{\wedge}, \quad k_{T_h k}^* = k_{T_h k}^{\perp} + ik_{T_h k}^{\wedge}.$$

Similarly, we can derive the expression for the constituent of the heat flux of heavy components parallel to the magnetic field

$$\mathbf{q}_h^{\parallel} = -\lambda_h^{\parallel} \nabla^{\perp} T_h + nkT_h \sum_{k=1}^{N-1} k_{T_h k}^{\parallel} \mathbf{V}_k^{\parallel}. \tag{3.7}$$

The coefficients $\lambda_h^{\parallel}$ and $k_{T_h k}^{\parallel}$ can be calculated from the formulas (3.5) and (3.6) when $\mathbf{B} = 0$. Finally, for the reduced heat flux of heavy plasma components in the magnetic field we obtain

$$\mathbf{q}_h = \mathbf{q}_h^{\parallel} + \mathbf{q}_h^{\perp} = -\lambda_h^{\parallel} \nabla^{\parallel} T_h - \lambda_h^{\perp} \nabla^{\perp} T_h - \lambda_h^{\wedge} \mathbf{b} \times \nabla T_h$$
$$+ nkT_h \sum_{i=1}^{N-1} \left(k_{T_h i}^{\parallel} \mathbf{V}_i^{\parallel} + k_{T_h i}^{\perp} \mathbf{V}_i^{\perp} + k_{T_h i}^{\wedge} \mathbf{b} \times \mathbf{V}_i \right). \tag{3.8}$$

The heat flux transported by heavy components in the mixture of ionized gases takes the form:

$$\mathbf{J}_{qh} = \mathbf{q}_h + \frac{5}{2}kT_h \sum_{i=1}^{N-1} n_i \mathbf{V}_i = -\lambda_h^{\parallel}\nabla^{\parallel}T_h - \lambda_h^{\perp}\nabla^{\perp}T_h - \lambda_h^{\wedge}\mathbf{b}\times\nabla T_h$$
$$+ nkT_h \sum_{i=1}^{N-1}\left(\left(k_{T_h i}^{\parallel} + \frac{5}{2}x_i\right)\mathbf{V}_i^{\parallel} + \left(k_{T_h i}^{\perp} + \frac{5}{2}x_i\right)\mathbf{V}_i^{\perp} + k_{T_h i}^{\wedge}\mathbf{b}\times\mathbf{V}_i\right). \tag{3.9}$$

The formula for the electron heat flux can be similarly derived from Eqs. (2.6)

$$\mathbf{J}_{qe} = \mathbf{q}_e + \frac{5}{2}kT_e n_e \mathbf{V}_e = -\lambda_e^{\parallel}\nabla^{\parallel}T_e - \lambda_e^{\perp}\nabla^{\perp}T_e - \lambda_e^{\wedge}\mathbf{b}\times\nabla T_e$$
$$+ nkT_e\left(\left(k_{T_e}^{\parallel} + \frac{5}{2}x_e\right)\mathbf{V}_e^{\parallel} + \left(k_{T_e}^{\perp} + \frac{5}{2}x_e\right)\mathbf{V}_e^{\perp} + k_{T_e}^{\wedge}\mathbf{b}\times\mathbf{V}_e\right), \tag{3.10}$$

$$\lambda_e^{\perp} + i\lambda_e^{\wedge} = -\frac{nk}{\Lambda_e^*}\begin{vmatrix} 0 & x_e & 0 & \dots & 0 \\ x_e & \Lambda_{ee}^{11*} & \Lambda_{ee}^{12} & \dots & \Lambda_{ee}^{1M_e} \\ 0 & \Lambda_{ee}^{21} & \Lambda_{ee}^{22*} & \dots & \Lambda_{ee}^{2M_e} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \Lambda_{ee}^{M_e1} & \Lambda_{ee}^{M_e2} & \dots & \Lambda_{ee}^{M_eM_e*} \end{vmatrix}, \tag{3.11}$$

$$k_{T_e}^{\perp} + ik_{T_e}^{\wedge} = -\frac{1}{\Lambda_e^*}\begin{vmatrix} 0 & x_e & 0 & \dots & 0 \\ \Lambda_{ee}^{10} & \Lambda_{ee}^{11*} & \Lambda_{ee}^{12} & \dots & \Lambda_{ee}^{1M_e} \\ \Lambda_{ee}^{20} & \Lambda_{ee}^{21} & \Lambda_{ee}^{22*} & \dots & \Lambda_{ee}^{2M_e} \\ \dots & \dots & \dots & \dots & \dots \\ \Lambda_{ee}^{M_e0} & \Lambda_{ee}^{M_e1} & \Lambda_{ee}^{M_e2} & \dots & \Lambda_{ee}^{M_eM_e*} \end{vmatrix}, \tag{3.12}$$

$$\Lambda_e^* = \begin{vmatrix} \Lambda_{ee}^{11*} & \Lambda_{ee}^{12} & \dots & \Lambda_{ee}^{1M_e} \\ \Lambda_{ee}^{21} & \Lambda_{ee}^{22*} & \dots & \Lambda_{ee}^{2M_e} \\ \dots & \dots & \dots & \dots \\ \Lambda_{ee}^{M_e1} & \Lambda_{ee}^{M_e2} & \dots & \Lambda_{ee}^{M_eM_e*} \end{vmatrix}, \tag{3.13}$$

$$\Lambda_{ee}^{rp*} = \Lambda_{ee}^{rp} - i\frac{3\sqrt{\pi}}{4}\frac{\Gamma(r+1)}{\Gamma(r+5/2)}\frac{m_e x_e \omega_e}{kT_e}\delta_{rp}, \quad i^2 = -1.$$

Here, Λ_e^* is the determinant of the matrix of the elements Λ_{ee}^{rp} $(r \neq p)$ and Λ_{ee}^{pp*}.

It is significant that, as distinct from the standard Chapman–Enskog procedure [11], the thermal conductivity coefficients and the thermodiffusion ratios can be calculated directly from the formulas (3.5), (3.6), (3.11), and (3.12) without preliminary determination of the remaining transport coefficients and additional inversion of the matrixes. In the new formulation the heavy component transport coefficients are expressed at once in terms of the ratios of $(N-1)M_h$th-order complex determinants and the electron transport coefficients are expressed in terms of the ratios of M_eth-order complex determinants.

The formulas (3.5) and (3.11) for the thermal conductivity coefficients can be considered as a generalization of the Muckenfuss–Curtiss formula [33] to the case of the two-temperature plasma moving in the magnetic field in any approximation in the Sonine polynomials. For the one-temperature plasma the formulas for the heat flux in the mixture and the transport coefficients obtained in this section go over in the formulas [15, 16] correct to small terms of the order of $(m_e/m_h)^{1/2}$.

4. STEFAN–MAXWELL RELATIONS FOR THE DIFFUSION FLUXES IN THE MULTICOMPONENT TWO-TEMPERATURE PLASMA

In order to derive the Stefan–Maxwell relations for the diffusion velocities of heavy components in the magnetic field it is convenient to represent Eqs. (2.1) in the form:

$$\mathbf{d}_j^{\perp} = -\sum_{k=1}^{N-1} \Lambda_{jk}^{00*}\mathbf{V}_k^{\perp} - \Lambda_{je}^{00*}\mathbf{V}_e^{\perp} - \sum_{k=1}^{N-1}\sum_{p=1}^{M_h} \Lambda_{jk}^{0p}\boldsymbol{\xi}_{kp}^{\perp} - \sum_{p=1}^{M_e} \Lambda_{je}^{0p}\boldsymbol{\xi}_{ep}^{\perp},$$

$$\Lambda_{jk}^{00*} = \Lambda_{jk}^{00} - i\frac{x_j m_j}{kT_h}(c_k\omega_k - \delta_{jk}\omega_j), \quad \Lambda_{je}^{00*} = \Lambda_{je}^{00} + i\frac{x_j m_j}{kT_h}c_e\omega_e, \quad i^2 = -1. \tag{4.1}$$

After eliminating the vectors $\boldsymbol{\xi}_{ip}^{\perp}$ and $\boldsymbol{\xi}_{ep}^{\perp}$ in (4.1) using (3.2) and (2.6), we can express the vectors $\mathbf{d}_i^{\perp}$ in terms of the constituents of the diffusion velocities and the temperature gradients orthogonal to the magnetic field $\mathbf{B}$

$$\mathbf{d}_i^{\perp} = -\sum_{j=1}^{N-1} \left(\Lambda_{ij}^{00*} - \varphi_{ij}^{*}\right)\mathbf{V}_j^{\perp} - \left(\Lambda_{ie}^{00*} - \varphi_{ie}^{*}\right)\mathbf{V}_e^{\perp} - k_{T_h i}^{*}\nabla^{\perp}\ln T_h - k_{T_e i}^{*}\nabla^{\perp}\ln T_e,$$

$$\varphi_{ij}^{*} = \varphi_{ij}^{\perp} + i\varphi_{ij}^{\wedge} = -\frac{1}{\Lambda_h^{*}}\begin{vmatrix} 0 & \Lambda_{is}^{01} & \Lambda_{is}^{02} & \dots & \Lambda_{is}^{0p} & \dots & \Lambda_{is}^{0M_h} \\ \Lambda_{rj}^{10} & \Lambda_{rs}^{11*} & \Lambda_{rs}^{12} & \dots & \Lambda_{rs}^{1p} & \dots & \Lambda_{rs}^{1M_h} \\ \Lambda_{rj}^{20} & \Lambda_{rs}^{21} & \Lambda_{rs}^{22*} & \dots & \Lambda_{rs}^{2p} & \dots & \Lambda_{rs}^{2M_h} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \Lambda_{rj}^{M_h0} & \Lambda_{rs}^{M_h1} & \Lambda_{rs}^{M_h2} & \dots & \Lambda_{rs}^{M_hp} & \dots & \Lambda_{rs}^{M_hM_h*} \end{vmatrix},$$

$$\varphi_{ie}^{*} = \varphi_{ie}^{\perp} + i\varphi_{ie}^{\wedge} = -\frac{1}{\Lambda_e^{*}}\begin{vmatrix} 0 & \Lambda_{ie}^{01} & \Lambda_{ie}^{02} & \dots & \Lambda_{ie}^{0p} & \dots & \Lambda_{ie}^{0M_e} \\ \Lambda_{ee}^{10} & \Lambda_{ee}^{11*} & \Lambda_{ee}^{12} & \dots & \Lambda_{ee}^{1p} & \dots & \Lambda_{ee}^{1M_e} \\ \Lambda_{ee}^{20} & \Lambda_{ee}^{21} & \Lambda_{ee}^{22*} & \dots & \Lambda_{ee}^{2p} & \dots & \Lambda_{ee}^{2M_e} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \Lambda_{ee}^{M_e0} & \Lambda_{ee}^{M_e1} & \Lambda_{ee}^{M_e2} & \dots & \Lambda_{ee}^{M_ep} & \dots & \Lambda_{ee}^{M_eM_e*} \end{vmatrix},$$

$$k_{T_e i}^{\perp} + ik_{T_e i}^{\wedge} = -\frac{1}{\Lambda_e^{*}}\begin{vmatrix} 0 & x_e & 0 & \dots & 0 \\ \Lambda_{ie}^{10} & \Lambda_{ee}^{11*} & \Lambda_{ee}^{12} & \dots & \Lambda_{ee}^{1M_e} \\ \Lambda_{ie}^{20} & \Lambda_{ee}^{21} & \Lambda_{ee}^{22*} & \dots & \Lambda_{ee}^{2M_e} \\ \dots & \dots & \dots & \dots & \dots \\ \Lambda_{ie}^{M_e0} & \Lambda_{ee}^{M_e1} & \Lambda_{ee}^{M_e2} & \dots & \Lambda_{ee}^{M_eM_e*} \end{vmatrix}. \tag{4.2}$$

Finally, it is convenient to represent relations (4.2) in the form:

$$\mathbf{d}_i^{\perp} = \sum_{j=1}^{N-1}\left[c_i\omega_j(c_j - \delta_{ij})\frac{m}{kT_h} + \varphi_{ij}^{\wedge}\right]\mathbf{b}\times\mathbf{V}_j + \left(-c_ic_e\omega_e\frac{m}{kT_h} + \varphi_{ie}^{\wedge}\right)\mathbf{b}\times\mathbf{V}_e$$

$$+ \sum_{j=1}^{N-1}\frac{x_ix_j}{D_{ij}(1)f_{ij}^{\perp}}\left(\mathbf{V}_j^{\perp} - \mathbf{V}_i^{\perp}\right) + \frac{x_ix_e}{D_{ie}(1)f_{ie}^{\perp}}\mathbf{V}_e^{\perp}$$

$$- k_{T_h i}^{\perp}\nabla^{\perp}\ln T_h - k_{T_h i}^{\wedge}\mathbf{b}\times\nabla\ln T_h - k_{T_e i}^{\perp}\nabla^{\perp}\ln T_e - k_{T_e i}^{\wedge}\mathbf{b}\times\nabla\ln T_e,$$

$$f_{ij}^{\perp} = f_{ji}^{\perp} = \left(1 - \Delta_{ij}^{\perp}\right)^{-1}, \quad \Delta_{ij}^{\perp} = \Delta_{ji}^{\perp} = \varphi_{ij}^{\perp}/\Lambda_{ij}^{00}, \quad \varphi_{ij}^{\perp} = \varphi_{ji}^{\perp}, \quad \varphi_{ij}^{\wedge} = \varphi_{ji}^{\wedge},$$

$$f_{ie}^{\perp} = f_{ei}^{\perp} = \left(1 - \Delta_{ie}^{\perp}\right)^{-1}, \quad \Delta_{ie}^{\perp} = \varphi_{ie}^{\perp}/\Lambda_{ie}^{00},$$
$$D_{ij}(1) = D_{ji}(1) = \frac{3(m_i + m_j)}{16nm_i m_j}\frac{kT_h}{\Omega_{ij}^{(1,1)}}, \quad D_{ie}(1) = D_{ei}(1) = \frac{3}{16nm_e}\frac{kT_h}{\Omega_{ie}^{(1,1)}}. \tag{4.3}$$

For electron diffusion perpendicularly to the magnetic field we have

$$\mathbf{d}_e^{\perp} = \left(c_e\omega_e\frac{m}{kT_h} + \varphi_e^{\wedge}\right)\mathbf{b}\times\mathbf{V}_e - \sum_{j=1}^{N-1}\frac{x_e x_j}{D_{ej}(1)f_{ej}^{\perp}}\mathbf{V}_e^{\perp} - k_{T_e}^{\perp}\nabla^{\perp}\ln T_e - k_{T_e}^{\wedge}\mathbf{b}\times\nabla\ln T_e,$$
$$k_{T_e}^{\perp} = -\sum_{j=1}^{N-1}k_{Tej}^{\perp}, \quad k_{T_e}^{\wedge} = -\sum_{j=1}^{N-1}k_{Tej}^{\wedge}, \quad \varphi_e^{\wedge} = -\sum_{j=1}^{N-1}\varphi_{je}^{\wedge}. \tag{4.4}$$

For diffusion along the magnetic field we have

$$\mathbf{d}_i^{\|} = \sum_{j=1}^{N-1}\frac{x_i x_j}{D_{ij}(1)f_{ij}^{\|}}\left(\mathbf{V}_j^{\|} - \mathbf{V}_i^{\|}\right) + \frac{x_i x_e}{D_{ie}(1)f_{ie}^{\|}}\mathbf{V}_e^{\|} - k_{T_h i}^{\|}\nabla^{\|}\ln T_h - k_{T_e i}^{\|}\nabla^{\|}\ln T_e, \tag{4.5}$$
$$\mathbf{d}_e^{\|} = -\sum_{j=1}^{N-1}\frac{x_e x_j}{D_{ej}(1)f_{ej}^{\|}}\mathbf{V}_e^{\|} - k_{T_e}^{\|}\nabla^{\|}\ln T_e. \tag{4.6}$$

Equations (4.5) and (4.6) coincide in form with the Stefan–Maxwell relations in [36] when there is no magnetic field. We note that in [17, 37] the binary diffusion coefficients $D_{ie}(1)$ were defined in another way, namely, as follows:

$$D_{ie}(1) = \frac{3}{16nm_e}\frac{kT_e}{\Omega_{ie}^{(1,1)}}.$$

In connection with this definition, in the Stefan–Maxwell relations given in [17, 37] there is a factor T_e/T_h multiplying the diffusion velocities $\mathbf{V}_e$.

The transport coefficients with the superscript "$\|$" can be calculated from the same formulas as the coefficients with the superscript "$\perp$" if we set in them $\omega_i = 0$, i.e., replace Λ_{rs}^{mp*} by Λ_{rs}^{mp}.

In the first non-zero approximation for the binary diffusion coefficients ($f_{ij}^{\|}(1) = f_{ij}^{\perp}(1) = 1$, $\varphi_{ij}^{\perp}(1) = 0$), in the second approximation for the thermodiffusion ratios of heavy components, and in the third approximation for the thermodiffusion ratios of electrons the Stefan–Maxwell relations take the most convenient and simple form for implementation in computational algorithms for solving the magnetohydrodynamic and aerothermodynamic problems:

$$\mathbf{d}_i = \sum_{j=1}^{N-1}\left[c_i\omega_j(c_j - \delta_{ij})\frac{m}{kT_h}\right]\mathbf{b}\times\mathbf{V}_j - c_i c_e\omega_e\frac{m}{kT_h}\mathbf{b}\times\mathbf{V}_e$$
$$+ \sum_{j=1}^{N-1}\frac{x_i x_j}{D_{ij}(1)}(\mathbf{V}_j - \mathbf{V}_i) + \frac{x_i x_e}{D_{ie}(1)}\mathbf{V}_e - k_{T_h i}^{\|}(2)\nabla^{\|}\ln T_h - k_{T_h i}^{\perp}(2)\nabla^{\perp}\ln T_h$$
$$- k_{T_h i}^{\wedge}(2)\mathbf{b}\times\nabla\ln T_h - k_{T_e i}^{\|}(3)\nabla^{\|}\ln T_e - k_{T_e i}^{\perp}(3)\nabla^{\perp}\ln T_e - k_{T_e i}^{\wedge}(3)\mathbf{b}\times\nabla\ln T_e, \tag{4.7}$$
$$\mathbf{d}_e = c_e\omega_e\frac{m}{kT_e}\mathbf{b}\times\mathbf{V}_e - \sum_{j=1}^{N-1}\frac{x_e x_j}{D_{ej}(1)}\mathbf{V}_e - k_{T_e}^{\|}(3)\nabla^{\|}\ln T_e$$
$$- k_{T_e}^{\perp}(3)\nabla^{\perp}\ln T_e - k_{T_e}^{\wedge}(3)\mathbf{b}\times\nabla\ln T_e. \tag{4.8}$$

For the one-temperature plasma the relations (4.7) and (4.8) go over in the formulas of [15] correct to small terms of the order of $(m_e/m_h)^{1/2}$.

For practical calculations we recommend the following formulas for the electron transport coefficients in the third approximation:

$$\lambda_e^{\perp}(3) + i\lambda_e^{\wedge}(3) = -\frac{nkx_e^2\Lambda_{ee}^{22*}}{\Lambda_{ee}^{11*}\Lambda_{ee}^{22*} - (\Lambda_{ee}^{12})^2}, \tag{4.9}$$

$$k_{T_e}^{\perp}(3) + ik_{T_e}^{\wedge}(3) = x_e\frac{\Lambda_{ee}^{10}\Lambda_{ee}^{22*} - \Lambda_{ee}^{20}\Lambda_{ee}^{12}}{\Lambda_{ee}^{11*}\Lambda_{ee}^{22*} - (\Lambda_{ee}^{12})^2}, \tag{4.10}$$

$$k_{T_ei}^{\perp}(3) + ik_{T_ei}^{\wedge}(3) = x_e\frac{\Lambda_{ie}^{10}\Lambda_{ee}^{22*} - \Lambda_{ie}^{20}\Lambda_{ee}^{12}}{\Lambda_{ee}^{11*}\Lambda_{ee}^{22*} - (\Lambda_{ee}^{12})^2}. \tag{4.11}$$

Summary. For the multicomponent two-temperature plasma the formulas describing the heat fluxes of heavy components and electrons as well as the Stefan–Maxwell relations for the diffusion velocities in a magnetic field are derived with regard to arbitrary approximations in the orthogonal expansions of the component distribution function in the Sonine polynomials. For the transport coefficients of heavy components and electrons the formulas are obtained in the considerably simpler form as compared with the standard procedure of the Chapman–Enskog method. For the complex thermal conductivity coefficients the formulas generalize the formula which corresponds to the second approximation and the rigorous formula to the case of anisotropy of heat transfer in the multicomponent two-temperature plasma in the presence of the magnetic field.

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