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Dipole Electromagnetic Radiation by a Charged Drop Oscillating in a Uniform Electrostatic Field

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Abstract—The intensity of electromagnetic radiation generated by a charged drop oscillating in a uniform electrostatic field is studied within the framework of analytical calculations retaining the terms of the second order of smallness with respect to the ratio of the droplet oscillation amplitude to the droplet radius. It is found that the charge induced in the drop surface oscillations generates a dipole radiation detected in the first-order calculations and a self-charge detected with allowance for the second-order terms only. It is shown that the order of the magnitude of the total intensity of radiation generated by a cloud can be determined from small-droplet radiation. Among two radiation sources, namely, the radiation generated by small droplets oscillating at low modes and the radiation generated by hydrometeors oscillating at high modes, the first plays a dominant role.

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Dipole electromagnetic radiation (generated by charged cloud-droplets oscillating in an external electrostatic field) is of interest in connection with the problems of sounding thunderstorm clouds [1–3]. Earlier, this question has been considered in connection with the quadrupole electromagnetic radiation generated by charged cloud-droplets [4–6] without regard for the external electrostatic field. To be more exact, no conclusion on the quadrupole radiation detected in calculations linear with respect to the ratio of the droplet oscillation amplitude to the droplet radius can be made, namely, the calculation methods used in [4–6] made it possible to estimate only the total radiation generated by a droplet without its partition into quadrupole and dipole constituents [7]. The thing is that the asymptotic expansions are carried out in two small parameters, namely, these are δ (the ratio of the droplet radius to the length of radiated wave) and ε (the ratio of the droplet oscillation amplitude to the droplet radius). The dipole and quadrupole radiations are characteristic of expansions of the first and second orders of smallness in δ, respectively [8, p. 130]. The order of magnitude of this parameter is $\delta \sim 10^{-15}$ and it characterizes the electromagnetic field strengths at large distances from the system of charges generating this field. The magnitude of the second small parameter ε is much greater, namely, $\varepsilon \sim 0.1$; this parameter characterizes the mechanism of electromagnetic wave generation. If $\varepsilon = 0$, then there is no accelerated motion of surface charges and there is no radiation. However, excitation of oscillations of the first (translational) mode, which just can generate the dipole radiation, is forbidden by the condition of immobility of the mass center in the absence of the external electric field [9, p. 345] when the calculations of the first order in ε are carried out in the reference frame connected with charged-droplet mass center (just such calculations were carried out in [4–6]). As a result, in [4–6] the quadrupole radiation which is next in the intensity was recorded by virtue of the calculation methods. In order to reveal the dipole radiation of a charged droplet in the absence of the external electric field it is necessary to carry out calculations without neglecting the second-order terms in ε (there is no radiation in the first approximation).

The main object of clouds which is of interest in connection with the topic of our investigation is a charged drop oscillating in the intracloud electric field. Under the conditions existing inside a thunderstorm cloud the droplet dimensions and charges and the electric field strengths vary over fairly wide ranges (within several orders) [10, 11].

The problem of electromagnetic radiation of an oscillating charged ideally conducting droplet was first formulated and solved in [4] in which the dispersion relation was derived. From the dispersion relation

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there followed the presence of oscillation damping and in the ideal fluid model this can be related only to radiation of electromagnetic waves by the droplet. In [5] the viscosity of fluid and its finite conductivity were taken into account. In [6] the calculations carried out in [4, 5] were refined and attached to the conditions inside a thunderstorm cloud.

We note again that in [4–6] the calculations were carried out for a charged droplet in the absence of the external electric field in the first approximation in the parameter ε (ratio of the oscillation amplitude to the droplet radius). As follows from the general theory of radiation [8], only the electromagnetic radiation can be revealed in this order of smallness, whereas the dipole radiation of an oscillating charged drop can be detected only in taking the second-order terms in ε into account.

It should be noted that under the assumption that the distance R_0 is much greater than the dimensions of the system l the electric field of a system of charges can be expanded in an asymptotic series in terms of the ratio l/R_0 [8]. In the zeroth approximation, we obtain the Coulomb law for variation in the electric potential φ of this system as a function of the distance $\varphi \sim 1/r$, in the first and second approximations these are the dipole and quadrupole laws $\varphi \sim 1/r^2$ and $\varphi \sim 1/r^3$, respectively. When the charges move with an acceleration, the system will radiate electromagnetic waves. They differ by the relation with variation in some or other terms of the asymptotic expansion.

The combination $\mathbf{d} \equiv \Sigma q_i \cdot \mathbf{r}_j$, where q_i is one of the charges of the system and $\mathbf{r}_j$ is its location with respect to an arbitrary origin taken inside the system is called the dipole moment (vector). The combination $D_{ik} \equiv \Sigma q_i(3x_i x_k - r^2\delta_{ik})$ is the quadrupole moment (tensor). Here, δ_{ik} is the Kronecker delta [8].

As distinct from the papers published earlier, in the present study we carry out the investigation of the intensity of radiation of a spheroidal charged droplet in an external electrostatic field with reference to the type of radiation (dipole). It is shown that for the uncharged droplet in an electric field the radiation appears in the first-order calculations, while for the charged droplet this occurs only in the second-order approximation.

Only the most intense sources of dipole radiation are estimated, namely, oscillations of induced charges (calculated in the first order of smallness) and oscillations of the drop self-charge (calculated in the second order of smallness). The quadrupole and magnetic-dipole components of electromagnetic radiation are not considered.

In should be noted that in the current stage the investigation is carried out within the framework of extension of basic knowledge concerning the electromagnetic radiation sources. So far, this is not about the practical applicability. However, nevertheless, we can indicate electromagnetic sounding of clouds [1–3] as a possible application of the phenomena under investigation. This determines the topicality of the investigation.

We can note that at present the problem under consideration is investigated only slightly.

1. PHYSICAL FORMULATION OF THE PROBLEM

We will consider the problem of electromagnetic radiation of an oscillating charged (a charge Q) drop of an ideal incompressible ideally conducting fluid of a density ρ which has a surface tension coefficient σ. The drop is suspended without motion inside a thunderstorm cloud in the superposition of gravitational and uniform electrostatic fields, the latter of the strength $\mathbf{E}_0$. We will assume that the drop is located in a vacuum and its volume is determined by the volume of a sphere of radius R_0. The presence of the external electrostatic field which specifies a distinguished direction leads to appearance of induced charges on the drop surface and, as a result, to distortion of the equilibrium spherical shape of the drop. In the linear approximation (linear with respect to the ratio of the drop strain to the radius) the equilibrium shape of the drop can be considered to be spheroidal elongated along the electric field [12–14].

The self-charge and the charges induced by the electrostatic field, being distributed over the drop surface disturbed by capillary oscillations, will move with an acceleration (under oscillations) and radiate electromagnetic waves.

We will restrict our attention to consideration of axisymmetric oscillations of the drop surface. All the calculations of the problem will be carried out in dimensionless variables in which $R_0 = \rho = \sigma = 1$ in the spherical coordinates (r, θ, φ) with the origin at the center of mass of the spheroidal drop, the angle

θ will be reckoned from the axis of symmetry of the drop whose direction coincides with the direction of $\mathbf{E}_0$. The remaining quantities of the problem will be expressed in fractions of their characteristic values:

$$Q^* = R_0^{3/2}\sigma^{1/2}, \quad E_0^* = R_0^{-1/2}\sigma^{1/2}, \quad t^* = R_0^{3/2}\rho^{1/2}\sigma^{-1/2},$$
$$V^* = R_0^{-1/2}\rho^{-1/2}\sigma^{1/2}, \quad r^* = R_0, \quad P^* = R_0^{-1}\sigma.$$

Let $\nu(r,\theta,t)$ be the surface density of the charge induced by the external electrostatic field on the drop surface disturbed by oscillations. Integrating $\nu(r,\theta,t)$ over the surfaces of two different (with respect to the orientation of the electric field) halves of the drop (S_1 and S_2), we obtain the values of unlike induced charges in the form [12]:

$$q_+(r,\theta,t) = \int\limits_{S_1} \nu(r,\theta,t)dS_1, \quad q_-(r,\theta,t) = \int\limits_{S_2} \nu(r,\theta,t)dS_2.$$

These charges will be put into correspondence with equal point charges located on the axis of symmetry of the drop. The radius-vectors of the exact location of these point charges will be determined from the formulas

$$\mathbf{R}_{q_+}(t) = \frac{1}{q_+(r,\theta,t)}\int\limits_{S_1} r\cos\theta\nu(r,\theta,t)\cdot\mathbf{e}_z dS_1(r,\theta,t),$$
$$\mathbf{R}_{q_-}(t) = \frac{1}{q_-(r,\theta,t)}\int\limits_{S_2} r\cos\theta\nu(r,\theta,t)\cdot\mathbf{e}_z dS_2(r,\theta,t),$$
$$S_1 \equiv \left[r = R(\theta,t); \quad 0 \le \theta \le \frac{\pi}{2}; \quad 0 \le \varphi \le 2\pi\right],$$
$$S_2 \equiv \left[r = R(\theta,t); \quad \frac{\pi}{2} \le \theta \le \pi; \quad 0 \le \varphi \le 2\pi\right], \tag{1.1}$$

where $r = R(\theta,t)$ is the equation of the drop surface.

The drop self-charge Q will be put into correspondence with the equal point charge located at the center of mass of the drop.

The "centers" of all the charges will oscillate under oscillations of the drop surface. This will lead to radiation of electromagnetic waves. We will estimate the intensity of radiation of a charged drop in an external electric field as the sum of radiations of a system of point charges moving with an acceleration, namely, the self- and induced charges. For a charge q which moves with an acceleration the intensity of dipole radiation can be determined from the expression [8, p. 227]:

$$I = \frac{2}{3c^3}\left(\frac{\partial^2\mathbf{d}}{\partial t^2}\right)^2, \quad \mathbf{d} \equiv q\cdot\mathbf{R}_q, \tag{1.2}$$

where $\mathbf{R}_q$ is the radius-vector of the charge location and $\mathbf{d}$ is the dipole moment.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Let at $t = 0$ the equilibrium spheroidal shape of the charged drop $\kappa(\theta)$ undergo a virtual axisymmetric perturbation $\xi(\theta,t)$ of a given amplitude ε which is significantly less than the drop radius. In dimensionless variables the equation describing the drop surface in the reference frame with the origin at the center of mass of the drop takes the form:

$$R(\theta,t) = \kappa(\theta) + \xi(\theta,t), \quad |\xi| \ll 1. \tag{2.1}$$

The motion of fluid in the drop will be assumed to be potential and we adopt that the velocity field of fluid in the drop $\mathbf{V}(\mathbf{r},t) = \nabla\psi(\mathbf{r},t)$ is completely determined by the velocity potential $\psi(\mathbf{r},t)$. The electric field around the drop will be characterized by the potential $\Phi(\mathbf{r},t)$. In dimensionless variables the amplitude values of the velocity field of fluid flow have the same order of smallness as the drop surface oscillation amplitude $\psi(\mathbf{r},t) \sim \xi(\theta,t) \sim \varepsilon$.

The mathematical formulation of the problem of electromagnetic radiation by a charged drop oscillating in an external electrostatic field takes the form:

$$\Delta\psi(\mathbf{r},t)=0,\quad \Delta\Phi(\mathbf{r},t)=0, \tag{2.2}$$

$$r\to 0:\quad \psi(\mathbf{r},t)\to 0,\quad r\to\infty:\quad \Phi(\mathbf{r},t)\to -E_0 r\cos\theta, \tag{2.3}$$

$$r=\kappa(\theta)+\xi(\theta,t):\quad \frac{\partial\xi(\theta,t)}{\partial t}=\frac{\partial\psi(\mathbf{r},t)}{\partial r}-\frac{1}{r^2}\frac{\partial\psi(\mathbf{r},t)}{\partial\theta}\left(\frac{\partial\kappa(\theta)}{\partial\theta}+\frac{\partial\xi(\theta,t)}{\partial\theta}\right), \tag{2.4}$$

$$P(\mathbf{r},t)+P_q(\mathbf{r},t)=P_\sigma(\mathbf{r},t), \tag{2.5}$$

$$\Phi(\mathbf{r},t)=\Phi_s(t), \tag{2.6}$$

$$t=0:\quad \xi(\theta)=\varepsilon\sum_{j\in\Xi}h_jP_j(\mu),\quad \sum_{j\in\Xi}h_j=1,\quad \varepsilon\ll 1,\quad \frac{\partial\xi(\theta)}{\partial t}=0, \tag{2.7}$$

where h_j are the coefficients which determine the partial contribution of the jth vibrational mode to the total initial perturbation; Ξ is the set of numbers of the initially excited vibrational modes, $P_j(\mu)$ is the Legendre polynomial of the jth order, and $\mu\equiv\cos\theta$.

In these expressions $\Phi_s(t)$ is the value of the drop electric potential (2.6) which is constant along the drop surface.

The pressures entering into the dynamic boundary condition (2.5) (the hydrodynamic pressure, the pressure constant inside the drop in the equilibrium state, the pressure of the electric field, and the capillary pressure) are as follows:

$$P(\mathbf{r},t)=P_0-\frac{\partial\psi}{\partial t}-\frac{1}{2}\left(\nabla\psi\right)^2,\quad P_0,\quad P_q(\mathbf{r},t)=(\nabla\Phi)^2/8\pi,\quad P_\sigma(\mathbf{r},t)=\mathrm{div}\mathbf{n}(\mathbf{r},t),$$

where $\mathbf{n}(\mathbf{r},t)$ is the unit normal vector to the drop surface determined by the relation:

$$\mathbf{n}(\mathbf{r},t)=\left.\frac{\nabla(r-R(\theta,t))}{|\nabla(r-R(\theta,t))|}\right|_{r=R(\theta,t)}. \tag{2.8}$$

We will supplement the above system with the conditions of invariability of the total drop volume (consequence of incompressibility of fluid) and immobility of the center of mass of the drop, as well as with the condition of conservation of the total drop charge:

$$\int_V r^2dr\sin\theta d\theta d\varphi=\frac{4}{3}\pi,\quad \int_V \mathbf{r}\cdot r^2dr\sin\theta d\theta d\varphi=0, \tag{2.9}$$

$$V=[0\le r\le\kappa(\theta)+\xi(\theta,t),0\le\theta\le\pi,0\le\varphi\le 2\pi],$$

$$-\frac{1}{4\pi}\iint_S(\mathbf{n},\nabla\Phi)\,dS=Q, \tag{2.10}$$

$$S=[r=\kappa(\theta)+\xi(\theta,t),0\le\theta\le\pi,0\le\varphi\le 2\pi].$$

3. SOLUTION

We will expand the unknown quantities in series in terms of the small dimensionless oscillation amplitude ε [15]:

$$\xi(\theta,t)=\xi^{(1)}(\theta,t)+O(\varepsilon^2),\quad \psi_j(\mathbf{r},t)=\psi_j^{(1)}(\mathbf{r},t)+O(\varepsilon^2),$$

$$\Phi(\mathbf{r},t)=\Phi^{(0)}(\mathbf{r})+\Phi^{(1)}(\mathbf{r},t)+O(\varepsilon^2),$$

$$P(\mathbf{r},t)=P^{(0)}(\mathbf{r})+P^{(1)}(\mathbf{r},t)+O(\varepsilon^2),$$

$$P_\sigma(\mathbf{r},t)=P_\sigma^{(0)}(\mathbf{r})+P_\sigma^{(1)}(\mathbf{r},t)+O(\varepsilon^2),$$

$$P_q(\mathbf{r},t)=P_q^{(0)}(\mathbf{r})+P_q^{(1)}(\mathbf{r},t)+O(\varepsilon^2), \tag{3.1}$$

where $\Phi^{(0)}(r,\theta)$ is the electric potential in the neighborhood of the equilibrium charged spheroid in the external electrostatic field and $\Phi^{(1)}(r,\theta,t)$ is the electric potential in the neighborhood of the disturbed charged elongated spheroid in the external electric field. The superscripts in parentheses denote the order of smallness in ε.

Substituting the expansions (3.1) in (2.2)–(2.10), we will distinguish the zero-order problem for finding the equilibrium shape of the drop surface $\kappa(\theta)$ and the equilibrium electric potential $\Phi^{(0)}(r,\theta)$. We will also consider the first-order problem for finding the disturbed shape of the drop surface $R(\theta,t)$ and the electric potential $\Phi^{(1)}(r,\theta,t)$ which is an addition arising in the neighborhood of the disturbed charged spheroid.

The mathematical formulation of the zero-order problem in ε takes the form:

$$\Delta\Phi^{(0)}(\mathbf{r}) = 0,$$

$$r \to \infty: \quad \Phi^{(0)}(\mathbf{r}) \to -E_0 r\cos\theta,$$

$$r = R^*(\theta): \quad \Phi^{(0)}(\mathbf{r}) = \text{const}, \quad P^{(0)} + P_q^{(0)} = P_\sigma^{(0)},$$

$$\int_V r^2 dr \sin\theta d\theta d\varphi = \frac{4}{3}\pi, \quad \int_V \mathbf{r}\cdot r^2 dr\sin\theta d\theta d\varphi = 0,$$

$$V = [0 \le r \le \kappa(\theta), 0 \le \theta \le \pi, 0 \le \varphi \le 2\pi],$$

$$-\frac{1}{4\pi}\iint_S \left(\boldsymbol{\tau}_0(\mathbf{r}), \nabla\Phi^{(0)}(\mathbf{r})\right) dS = Q, \quad S = [r = \kappa(\theta), 0 \le \theta \le \pi, 0 \le \varphi \le 2\pi],$$

where $\boldsymbol{\tau}_0(\mathbf{r})$ is the unit normal vector to the equilibrium drop surface determined by the relation (2.8) on the surface $\kappa(\theta)$.

The zero-order problem in ε can readily be solved and the solution takes the form:

$$\Phi^{(0)}(r,\theta) = \frac{Q}{r}\left(1 + \frac{1}{3r^2}e^2 P_2(\mu)\right)$$
$$+ E_0\left(rP_1(\mu)\left(\frac{1}{r^3} - 1\right) + \frac{2}{5}\frac{1}{r^2}e^2\left(P_1(\mu) + \frac{3}{2}\frac{1}{r^2}P_3(\mu)\right)\right), \tag{3.2}$$

$$P^{(0)}(\mathbf{r}) = P_0, \quad P_q^{(0)}(\mathbf{r}) = \frac{\left(\nabla\Phi^{(0)}\right)^2}{8\pi}, \quad P_\sigma^{(0)}(\mathbf{r}) = 2 - \frac{1}{3}e^2(2 + L_\theta)P_2(\mu),$$

$$L_\theta \equiv \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right), \quad \kappa(\theta) \sim 1 + e^2 h(\theta) + O(e^4) \equiv 1 + \frac{1}{3}e^2 P_2(\mu) + O(e^4),$$

$$e^2 = \frac{9w}{1-W}, \quad w = \frac{E_0^2}{16\pi}, \quad W = \frac{Q^2}{16\pi}, \tag{3.3}$$

where $P_n(\mu)$ is the nth-order Legendre polynomial, n is an integer, $\mu \equiv \cos\theta$, and $h(\theta)$ is a function which describes the deviation of the equilibrium shape of the drop from the sphere [12, 16]. The solution (3.2) is the expansion of the exact solution given in [17].

We will give the mathematical formulation of the first-order problem in ε in the form:

$$\Delta\psi(\mathbf{r},t) = 0, \quad \Delta\Phi^{(1)}(\mathbf{r},t) = 0,$$

$$r \to 0: \quad \psi(\mathbf{r},t) \to 0, \quad r \to \infty: \quad \Phi^{(1)}(\mathbf{r},t) \to 0,$$

$$r = \kappa(\theta) + \xi(\theta,t): \quad \frac{\partial\xi(\theta,t)}{\partial t} = \frac{\partial\psi(\mathbf{r},t)}{\partial r} - \frac{1}{r^2}\frac{\partial\psi(\mathbf{r},t)}{\partial\theta}\left(\frac{\partial\kappa(\theta)}{\partial\theta} + \frac{\partial\xi(\theta,t)}{\partial\theta}\right),$$

$$P^{(1)}(\mathbf{r},t) + P_q^{(1)}(\mathbf{r},t) = P_\sigma^{(1)}(\mathbf{r},t), \quad \Phi(\mathbf{r},t) = \Phi_s(t).$$

The solutions of Eqs. (2.2) for the hydrodynamic and electric potentials $\psi(\mathbf{r},t)$ and $\Phi^{(1)}(\mathbf{r},t)$ which must satisfy the boundary conditions (2.3), as well as the perturbation of the equilibrium shape of the drop surface $\xi(\theta, t)$, can be written in the form of the series in the Legendre polynomials:

$$\psi(r,\theta,t) = \varepsilon \sum_{n=0}^{\infty} D_n(t) r^n P_n(\mu), \tag{3.4}$$

$$\Phi^{(1)}(r,\theta,t) = \varepsilon \sum_{n=0}^{\infty} F_n(t) r^{-(n+1)} P_n(\mu), \tag{3.5}$$

$$\xi^{(1)}(\theta,t) = \varepsilon \sum_{n=0}^{\infty} M_n(t) P_n(\mu). \tag{3.6}$$

In the calculations in the first order approximation in ε, for the pressure we obtain

$$P^{(1)}(\mathbf{r},t) = -\left(\frac{\partial \psi(\mathbf{r},t)}{\partial t} + e^2 \frac{\partial^2 \psi(\mathbf{r},t)}{\partial r \partial t} h(\theta)\right),$$

$$P_q^{(1)}(\mathbf{r},t) = -\frac{1}{4\pi}\left(2\left(4Q^2 h(\theta) + \frac{1}{3}(Q + 3E_0\cos\theta)^2\right)\xi(\theta,t) + (Q + 3E_0\cos\theta)\frac{\partial \Phi^{(1)}(\mathbf{r},t)}{\partial r}\right.$$
$$\left. + Q\left(-\frac{\partial h(\theta)}{\partial \theta}\frac{\partial \Phi^{(1)}}{\partial \theta} + h(\theta)\left(\frac{\partial \Phi^{(1)}}{\partial r} + \frac{\partial^2 \Phi^{(1)}}{\partial r^2}\right)\right)\right),$$

$$P_\sigma^{(1)}(\mathbf{r},t) = -(2 + L_\theta)\xi(\theta,t) + 2e^2\left(\xi(\theta,t) L_\theta h(\theta) + h(\theta)(2 + L_\theta)\xi(\theta,t)\right).$$

Substituting (3.4)–(3.6) in the boundary conditions (2.4), (2.6), and (2.8), we can find the following relations which connect the coefficients $D_n(t)$ and $F_n(t)$ in (3.4) and (3.5) with the coefficients $M_n(t)$ in the form:

$$D_n(t) = \frac{1}{n}\left(\frac{\partial M_n(t)}{\partial t}\left(1 - \frac{1}{3n}e^2((n(n-1)K_{2,n,n} - \alpha_{2,n,n})\right)\right.$$
$$- \frac{e^2}{3}\left(\frac{\partial M_{n-2}(t)}{\partial t}\frac{((n-2)(n-3)K_{2,n-2,n} - \alpha_{2,n-2,n})}{(n-2)}\right.$$
$$\left.\left. + \frac{\partial M_{n+2}(t)}{\partial t}\frac{((n+2)(n+1)K_{2,n+2,n} - \alpha_{2,n+2,n})}{(n+2)}\right)\right), \quad (n \geq 0), \tag{3.7}$$

$$F_n(t) = \Phi_s^{(1)}\delta_{n,0} + E_0\big(3(\mu_{n-1}^+ M_{n-1}(t) + \mu_{n+1}^- M_{n+1}(t))$$
$$+ e^2(M_{n-3}(t)l_1 + M_{n-1}(t)l_2 + M_{n+1}(t)l_3 + M_{n+3}(t)l_4)\big)$$
$$+ Q(M_n(t) + e^2(M_{n-2}(t)l_5 + M_n(t)l_6 + M_{n+2}(t)l_7)), \quad (n \geq 0), \tag{3.8}$$

$$M_0(t) = -\frac{2}{3}e^2 M_2(t), \quad M_1(t) = -\frac{3}{5}e^2 M_3(t),$$

$$\mu_n^+ = \frac{n+1}{2n+1}, \quad \mu_n^- = \frac{n}{2n+1},$$

$$K_{m,k,n} = [C_{k0,m0}^{n0}]^2, \quad \alpha_{m,k,n} = -\sqrt{m(m+1)k(k+1)}C_{m0,k0}^{n0}C_{m-1,k1}^{n0},$$

where $C_{mk,lp}^{nq}$ are the Clebsh-Gordan coefficients [16] which are nonzero only when the indices satisfy the inequalities $|m - l| \leq n \leq m + l$, where $m + l + n$ is an even number and $k + p = q$. The coefficients l_j that depend only on n are omitted lest the presentation be loaded with trivial expressions.

Substituting these expressions in the dynamic boundary condition (2.5), we can write the second-order inhomogeneous differential equation for finding the coefficients $M_n(t)$ (evolutionary equation):

$$\frac{\partial^2}{\partial t^2}M_n(t) + \omega_n^2 M_n(t)$$

$$= e^2 \left(\frac{\partial^2}{\partial t^2} M_{n-2}(t)\chi_1 + M_{n-2}(t)\chi_2 + \frac{\partial^2}{\partial t^2} M_{n+2}(t)\chi_3 + M_{n-2}(t)\chi_4 \right)$$

$$+ \frac{1}{4\pi}\Big(Q^2 e^2\big(M_{n-2}(t)\chi_5 + M_{n+2}(t)\chi_6\big) + QE_0\big(M_{n-1}(t)\chi_7 + M_{n+1}(t)\chi_8\big)\Big), \qquad (3.9)$$

where ω_n is the nth mode oscillation frequency which can be determined from the expression:

$$\omega_n^2 = n \left((n-1)(n+2) - e^2 \frac{(2n^5 + 23n^4 + 21n^3 - 17n^2 - 7n - 2)}{(2n-1)(2n+1)(2n+3)} \right.$$

$$\left. + 4W \left(-(n-1) + e^2 \frac{(4n^4 + 6n^3 - 2n^2 - 2n - 1)}{(2n-1)(2n+1)(2n+3)} \right) \right). \qquad (3.10)$$

The coefficients χ_n that depend only on the mode number are omitted in view of their cumbersomeness.

We will seek the solution of the second-order inhomogeneous differential equation obtained in the form of the sum of the general solution of the corresponding homogeneous equation and a particular solution of the inhomogeneous equation.

The solution of the homogeneous equation takes the form:

$$M_n(t)_{\text{hom}} = a_n \exp\big(i(\omega_n t + b_n)\big) + \text{c.c.} \quad (n \geq 0),$$

where a_n and b_n are real constants to be determined from the initial conditions and the abbreviation "c.c." denotes the terms which are complex conjugate to the written terms.

The particular solution of the inhomogeneous equation can be found using the method of successive approximations in e^2.

Satisfying the initial condition (2.7), we can determine the real constants a_n and b_n:

$$a = \frac{1}{2} h_j \Big(\delta_{j,n} + e^2\big(\delta_{j,n-2}\alpha_1(n) + \delta_{j,n}\alpha_2(n) + \delta_{j,n+2}\alpha_3(n)\big)$$

$$- \frac{3}{4\pi} QE_0 \big(\delta_{j,n-1}\alpha_4(n) + \delta_{j,n+1}\alpha_5(n)\big)\Big),$$

$$b_n = 0 \quad (j \in \Xi, \quad n = 2, 3, 4, \ldots), \qquad (3.11)$$

where $\delta_{j,n}$ is the Kronecker delta. The expressions for the coefficients $\alpha_1(n) - \alpha_5(n)$ are omitted in view of their cumbersomeness.

Thus, substituting the expressions (3.11) in the solution of the inhomogeneous evolutionary equation, we obtain that in the linear approximation in square of the eccentricity e^2 the shape of the surface of the oscillating droplet considered, correct to the terms of the first order of smallness in the dimensionless oscillation amplitude ε, can be described by the function

$$R(\theta, t) = 1 + e^2 h(\theta) + \varepsilon \sum_{j \in \Xi} M_j(t) P_j(\mu), \qquad (3.12)$$

in which the amplitude coefficients $M_j(t)$ take the form:

$$M_j(t) = \big(h_j + S_1(j)\big)\cos(\omega_j t) + S_2(j)\big(\cos(\omega_{j-2} t) - \cos(\omega_j t)\big)$$

$$+ S_3(j)\big(\cos(\omega_j t) - \cos(\omega_{j+2} t)\big) + S_4(j)\cos(\omega_{j-1} t) + S_5(j)\cos(\omega_{j+1} t), \qquad (3.13)$$

where $S_i(j)$ are coefficients that depend on the initial amplitudes h_j, $h_{j\pm1}$, and $h_{j\pm2}$, the physical parameters W, w, and e^2 defined in (3.3), and on the index j. The analytical form of the coefficients is omitted in view of their cumbersomeness. We note that $S_1(j)$, $S_4(j)$, and $S_5(j)$ are of the order of $\sim QE_0$ and $S_2(j)$ and $S_3(j)$ are $\sim e^2$. When there is no external electric field these coefficients vanish and the perturbation amplitudes $M_j(t)$ will be determined only by the term at the frequency ω_j: $M_j(t) = h_j \cos(\omega_j t)$.

Using (3.13) which represents the explicit form of the solution of the evolutionary equation with regard to the initial conditions, we can write the expression for the correction to the electric potential which appears in the neighborhood of the perturbed spheroid in the final form as follows:

$$\Phi^{(1)}(\mathbf{r},t)=\varepsilon\Bigg(\frac{1}{35r}e^2\big(6E_0M_3(t)+14QM_2(t)\big)+\sum_{j\in\Xi}\Big[E_0\Big(3\big(\mu^+_{j-1}M_{j-1}(t)+\mu^-_{j+1}M_{j+1}(t)\big)$$
$$+e^2\big(M_{j-3}(t)l_1(j)+M_{j-1}(t)l_2(j)+M_{j+1}(t)l_3(j)+M_{j+3}(t)l_4(j)\big)\Big)$$
$$+Q\Big(M_j(t)+e^2\big(M_{j-2}(t)l_5(j)+M_j(t)l_6(j)+M_{j+2}(t)l_7(j)\big)\Big)\Big]r^{(-j+1)}P_j(\mu)\Bigg). \qquad (3.14)$$

4. RADIATION OF ELECTROMAGNETIC WAVES BY INDUCED CHARGES

By virtue of symmetry of the induced drop charge distribution about the equatorial plane, we will consider half the drop. The like induced charges can be expressed in terms of the surface electric charge density $\nu\equiv\nu(r,\theta)\equiv-(\mathbf{n}(\mathbf{r},t),\nabla\Phi)/4\pi$ on the disturbed drop surface in the form [12]:

$$q_\pm=\int\limits_{S_{1,2}}\frac{\nu(\theta,t)}{(\mathbf{n}(\mathbf{r},t),\mathbf{e}_r)}r^2\sin\theta d\theta d\varphi=-\frac{1}{4\pi}\int\limits_{S_{1,2}}\frac{(\mathbf{n}(\mathbf{r},t),\nabla\Phi)}{(\mathbf{n}(\mathbf{r},t),\mathbf{e}_r)}r^2\bigg|_{r=R(\theta,t)}\sin\theta d\theta d\varphi. \qquad (4.1)$$

Substituting the expressions for the electric potential (see (3.1), (3.2), and (3.14)) and the normal vector (2.10) in (4.1), we obtain the following expression in the dimensional form for the absolute values of the charges induced on the disturbed drop surface:

$$q_\pm=\frac{3}{4}E_0R^2\Bigg(1+\frac{3}{5}w+2\varepsilon\sum_{j\in\Xi}h_j\Big(\big(N_0(j)+N_1(j)\big)\cos(\omega_jt)+N_2(j)\cos(\omega_{j+1}t)$$
$$+N_3(j)\cos(\omega_{j-1}t)+N_4(j)\cos(\omega_{j+2}t)+N_5(j)\cos(\omega_{j-2}t)\Big)\Bigg). \qquad (4.2)$$

For the absolute values of the radius-vectors $\mathbf{R}_{q\pm}$, which determine the position of the equivalent charges on the axis of symmetry, from (1.1) we obtain in the dimensional form:

$$R_{qz}=\frac{2}{3}R\Bigg(1+3w+\varepsilon\sum_{j\in\Xi}h_j\Big(\big(\eta_0(j)+\eta_1(j)\big)\cos(\omega_jt)$$
$$+\eta_2(j)\big(\cos(\omega_jt)-\cos(\omega_{j+2}t)\big)+\eta_3(j)\big(\cos(\omega_{j-2}t)-\cos(\omega_jt)\big)$$
$$+\eta_4(j)\cos(\omega_{j+1}t)+\eta_5(j)\cos(\omega_{j-1}t)+\eta_6(j)\cos(\omega_{j-2}t)+\eta_7(j)\cos(\omega_{j+2}t)\Big)\Bigg). \qquad (4.3)$$

The numerical coefficients $N_0(j)$ and $\eta_0(j)$ (depending only on the index j) and the coefficients $N_i(j)$ $(i=1{-}5)$ and $\eta_m(j)$ $(m=1{-}7)$ depending, besides j, on the physical parameters of the system, have cumbersome form and are omitted in the present study. We note that the coefficients $N_1(j)-N_3(j)$, $\eta_1(j)$, $\eta_4(j)$, and $\eta_5(j)$ are of the order $\sim QE_0\sim e$ and $N_4(j)$, $N_5(j)$, $\eta_2(j)$, $\eta_3(j)$, $\eta_6(j)$, and $\eta_7(j)$ are $\sim e^2$.

In Fig. 1a we have reproduced the displacement ΔR_{qz} of the centers of induced charges from the steady-state equilibrium position $R^{(eq)}_{qz}=2R(1+3w)/3$ as a function of dimensionless time t.

Substituting (4.2) and (4.3) in (1.2), we obtain an expression for the intensity of dipole radiation of the drop oscillating in an external electrostatic field. In order to estimate the maximum intensity of dipole radiation we replace the cosines by their maximum value. Thus, we can write the final expression obtained in the the first order approximation in ε for the intensity of dipole radiation of a charged drop in an external electrostatic field. The dipole radiation is generated by the charges moving with an acceleration and induced by the external electrostatic field. In the dimensional form this expression (I^*) takes the form:

$$I^* = \frac{E_0^2 R^6 \varepsilon^2}{3c^3} \sum_{j\in\Xi} h_j^2 \Bigg[\left(1+\frac{3}{5}w\right) \Big(\omega_j^2 \big(\eta_0(j)+\eta_1(j)\big) + (\omega_j^2 - \omega_{j+2}^2)\eta_2(j) + (\omega_{j-2}^2 - \omega_j^2)\eta_3(j) + \omega_{j+1}^2\eta_4(j) + \omega_{j-1}^2\eta_5(j) + \omega_{j-2}^2\eta_6(j) + \omega_{j+2}^2\eta_7(j) \Big) + 2(1+3w)\Big(\omega_j^2\big(N_0(j)+N_1(j)\big) + \omega_{j+1}^2 N_2(j) + \omega_{j-1}^2 N_3(j) + \omega_{j+2}^2 N_4(j) + \omega_{j-2}^2 N_5(j) \Big) \Bigg]^2.$$

In accordance with (3.3), in this expression the drop self-charge affects only the eccentricity of drop. As was noted in the introduction, in the first-order calculations the oscillations of the self-charge itself generate only the quadrupole radiation whose intensity is less than the radiation of the charges induced by the external field by 10^{-14} times [7].

If we take $Q = 0$, then the droplet becomes uncharged and in the expression for the intensity the coefficients $\eta_4(j) - \eta_7(j)$ vanish:

$$I_1 = \frac{E_0^2 R^6 \varepsilon^2}{3c^3} \sum_{j\in\Xi} h_j^2 \Bigg[\left(1+\frac{3}{5}w\right) \Big(\omega_j^2 \big(\eta_0(j)+\eta_1(j)\big) + (\omega_j^2 - \omega_{j+2}^2)\eta_2(j) + (\omega_{j-2}^2 - \omega_j^2)\eta_3(j) \Big) + 2(1+3w)\Big(\omega_j^2\big(N_0(j)+N_1(j)\big) + \omega_{j+2}^2 N_4(j) + \omega_{j-2}^2 N_5(j) \Big) \Bigg]^2. \quad (4.4)$$

We note that the expressions (4.2) and (4.3) are obtained in the linear approximation in the dimensionless drop oscillation amplitude. In accordance with [8], the expression for the radiation intensity (4.4) is written the quadratic approximation in ε.

5. RADIATION OF ELECTROMAGNETIC WAVES BY THE SELF-CHARGE OF A SPHEROIDAL DROP

We can obtain the displacement of the center of the self-charge of a spheroidal droplet along the z axis under axisymmetric oscillations by carrying out calculations with regard to only the terms of the second order in ε with the use of an expression similar to (1.1) when integration is carried out over the entire drop surface. As a result, we have in the dimensional form:

$$R_{Qz} = R\Bigg(-\frac{6}{35}e^2\varepsilon h_3\cos(\omega_3 t) + \varepsilon^2 \sum_{j\in\Xi} \Bigg[h_j h_{j-1} \Bigg\{ \left(\frac{j}{2(2j+1)} + \beta_1(j) \right) \times \big(\cos(\omega_j - \omega_{j-1})t + \cos(\omega_j + \omega_{j-1})t \big) + \beta_2(j)\big(\cos(\omega_{j+1} - \omega_{j-1})t + \cos(\omega_{j+1} + \omega_{j-1})t\big) + \beta_3(j)\big(1+\cos(2\omega_j t)\big) + \beta_4(j)\big(1+\cos(2\omega_{j-1} t)\big) + \beta_5(j)\big(\cos(\omega_{j+2} - \omega_{j-1})t + \cos(\omega_{j+2} + \omega_{j-1})t\big) + \beta_6(j)\big(\cos(\omega_{j-2} - \omega_{j-1})t + \cos(\omega_{j-2} + \omega_{j-1})t\big) + \beta_7(j)\big(\cos(\omega_j - \omega_{j+1})t + \cos(\omega_j + \omega_{j+1})t\big) + \beta_8(j)\big(\cos(\omega_j - \omega_{j-3})t + \cos(\omega_j + \omega_{j-3})t\big) + \beta_9(j)\big(\cos(\omega_j - \omega_{j-2})t + \cos(\omega_j + \omega_{j-2})t\big) + \beta_{10}(j)\big(\cos(\omega_{j+1} - \omega_{j-2})t + \cos(\omega_{j+1} + \omega_{j-2})t\big) \Bigg\} + h_j h_{j-3}\beta_{11}(j)\big(\cos(\omega_j - \omega_{j-3})t + \cos(\omega_j + \omega_{j-3})t\big) \Bigg] \Bigg). \quad (5.1)$$

The expressions for the coefficients $\beta_i(j)$ ($i = 1$–11) that depend on the index j and all the physical parameters of the problem are omitted by virtue of their extreme cumbersomeness (their analytical writing requires about four standard pages). They contain the terms $\sim QE_0$ and $\sim e^2$.

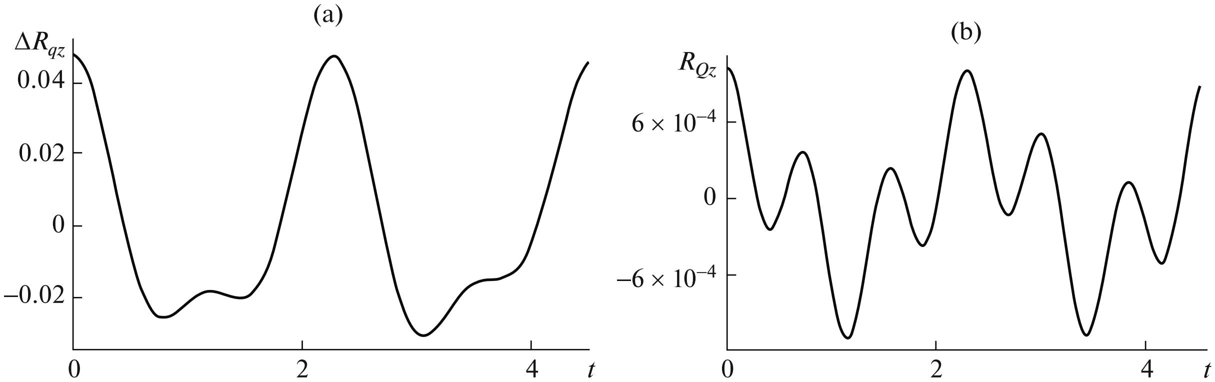

Fig. 1. Dimensionless displacements of the centers of the induced- and self-charge of the droplet (a and b, respectively) from the steady-state equilibrium position of the charge as functions of dimensionless time: the initial excitation of the equilibrium shape of the drop surface is $\varepsilon[P_2(\mu) + P_3(\mu)]/2$; $\varepsilon = 0.1$, $Q = 5.6 \times 10^{-7}$ electrostatic units ($\sim 3 \times 10^{-3}$ $Q_{\rm cr}$ and $R_0 = 3$ μm; $\sim 8 \times 10^{-5}$ $Q_{\rm cr}$ and $R_0 = 30$ μm); $E_0 = 50$ V/cm ($\sim 5 \times 10^{-5}$ $E_{0\rm cr}$ and $R_0 = 3$ μm; $\sim 2 \times 10^{-4}$ $E_{0\rm cr}$ and $R_0 = 30$ μm); $Q_{\rm cr}$ and $E_{0\rm cr}$ are the critical values of the charge and the field strength, respectively.

We note that the coefficients $\beta_i(j)$ appear as a result of spheroidal distortion of the equilibrium drop surface in the external electric field. The coefficients $\beta_1(j)$–$\beta_4(j)$ and $\beta_9(j)$ are of the order of QE_0 and $\beta_5(j)$–$\beta_8(j)$, $\beta_{10}(j)$, and $\beta_{11}(j)$ are $\sim e^2$. When there is no external electric field all the coefficients $\beta_i(j)$, as well as the eccentricity of drop e, vanish and the displacement of the center of the self-charge will be $\sim\varepsilon^2$ and contain only the frequencies $\omega_j - \omega_{j-1}$ and $\omega_j + \omega_{j-1}$.

In Fig. 1b we have plotted the graph of the function $R_{Qz}(t)$ calculated from (5.1) for the charged drop.

A comparison between Figs. 1a and 1b points to the appreciably greater oscillation amplitude of the induced charge "positions" (end of the vector R_{qz}) as compared with the oscillation amplitude of the self-charge "position" R_{Qz}. This relates to the fact that in the case of the polarization charges they are mainly concentrated at the vertices of spheroid and are almost completely involved in oscillations of the surface of the spheroidal drop along the OZ axis (the density of induced charges is fairly small in the neighborhood of spheroid's equator). At the same time, in the case of the charged drop the self-charge is distributed over the entire drop surface [17, p. 40] but only a part of the oscillations occurs along the OZ axis and contributes to formation of R_{Qz}.

In dimensional form the expression for the maximum intensity of the dipole electromagnetic radiation of the self-charge of the spheroidal charged drop found in the calculations carried out correct to the second order of smallness in ε takes the form:

$$I_3 = \frac{2}{3}\frac{Q^2}{c^3}R^2\left(\frac{6}{35}e^2\varepsilon h_3\omega_3^2 + \varepsilon^2\sum_{j\in\Xi}\left[h_j h_{j-1}\left\{\left(\frac{j}{2(2j+1)} + \beta_1(j)\right)\left((\omega_j - \omega_{j-1})^2 + (\omega_j + \omega_{j-1})^2\right)\right.\right.\right.$$
$$+ \beta_2(j)\left((\omega_{j+1} - \omega_{j-1})^2 + (\omega_{j+1} + \omega_{j-1})^2\right) + 4\beta_3(j)\omega_j^2 + 4\beta_4(j)\omega_{j-1}^2$$
$$+ \beta_5(j)\left((\omega_{j+2} - \omega_{j-1})^2 + (\omega_{j+2} + \omega_{j-1})^2\right) + \beta_6(j)\left((\omega_{j-2} - \omega_{j-1})^2 + (\omega_{j-2} + \omega_{j-1})^2\right)$$
$$+ \beta_7(j)\left((\omega_j - \omega_{j+1})^2 + (\omega_j + \omega_{j+1})^2\right) + \beta_8(j)\left((\omega_j - \omega_{j-3})^2 + (\omega_j + \omega_{j-3})^2\right)$$
$$\left. + \beta_9(j)\left((\omega_j - \omega_{j-2})^2 + (\omega_j + \omega_{j-2})^2\right) + \beta_{10}(j)\left((\omega_{j+1} - \omega_{j-2})^2 + (\omega_{j+1} + \omega_{j-2})^2\right)\right\}$$
$$\left.\left. + h_j h_{j-3}\beta_{11}(j)\left((\omega_j - \omega_{j-3})^2 + (\omega_j + \omega_{j-3})^2\right)\right]\right)^2. \qquad (5.2)$$

In this expression the drop sphericity is caused by the mutual influence of the fairly week external electrostatic field (which only specifies the preferential direction) and the drop self-charge which just ensures appearance of the nonzero finite eccentricity [18]. Therefore, in the case considered we can neglect the contribution to radiation introduced by the induced charges.

If we take $E_0 = 0$, then the equilibrium shape of the drop becomes spherical and the expression for the intensity I_3 can be significantly simplified since the coefficients $\beta_i(j)$ vanish. We will denote this intensity by I_2:

$$I_2 = \frac{2}{3}\frac{Q^2}{c^3}R^2\varepsilon^4\left[\sum_{j\in\Xi} h_j h_{j-1}\frac{j}{2(2j+1)}\left((\omega_j - \omega_{j-1})^2 + (\omega_j + \omega_{j-1})^2\right)\right]^2. \quad (5.3)$$

According to (5.3), the intensity of dipole radiation by a charged drop is nonzero in the absence of the external electric field when there are two neighboring modes in the spectrum of initially excited modes which determine the initial deformation of the equilibrium drop surface.

The most significant difference between the formulas (4.4) and (5.2) consists in the fact that (4.4) is obtained as a result of calculations of the first order of smallness in ε, while (5.2) with regard to only the second-order terms. Radiation of electromagnetic waves by the oscillating self-charge of the drop which can be seen in the first-order calculations is quadrupole and dipole radiation can be observed only in the second-order calculations. For the induced charge the pattern is different, namely, dipole radiation can be observed already in the first-order calculations.

The total intensity of dipole radiation I by a charged drop in the external electrostatic field can be simulated by the sum of the intensities of radiation generated by traveling self- and induced charges $I = I^* + I_3$. In this expression the first term is obtained in calculations of the first order of smallness in ε and the second term in calculations with regard to only the second-order terms.

The first possible source of electromagnetic radiation in clouds relates to finite-amplitude oscillations of small droplets over the dimension range from 3 to 30 μm which are the most abundant in clouds. The concentrations n of such droplets is $\sim 10^3$ cm^{-3} [10, 11]. The high-amplitude oscillations of the cloud-droplets can be initiated by various causes, namely, by coagulation, by fragmentation into smaller droplets as a result of collision processes or as a result of implementation of electrostatic instability, by the hydrodynamic and electric interaction between closely flying droplets, or by the aerodynamic interaction with the developed fine-scale turbulence characteristic of thunderstorm clouds. In accordance with the field observations [19, 20], the oscillation amplitudes of cloud-droplets can reach tens percents of the droplet radius.

In accordance with [4], the second possible source of electromagnetic radiation by a cloud relates to freely falling hydrometeors which coagulate with finer droplets and, therefore, are continuously oscillating and, consequently, radiating. However, in [4] drops of radius $R_0 = 1$ mm were proposed on the role of radiating hydrometeors. In accordance with the observation data [10, 11], the concentration of such drops in a cloud is fairly low $n \sim 1$ m^{-3}. Thus, the estimates of the intensity of electromagnetic radiation by a cloud based on the mechanism under consideration and carried out in [4] for the extreme numerical values of the charges and concentrations of the drops of $R_0 = 1$ mm are most likely significantly overestimated. Nevertheless, the mechanism itself proposed in [4] must be undoubtedly operating if finer drops with $R_0 = 100$ μm lie in its basis. In accordance with the observation data [10, 11], the concentration of such drops in a cloud is fairly high $\sim 10^3$ m^{-3} and their free-falling velocity is equal to ≈ 78 cm/s. At such a velocity of falling through a cloud of droplets of radii from 3 to 30 μm and the maximum drop concentration corresponding to the range from 3 to 7 μm, a hydrometeor will undergo approximately 22 collisions per second and, as a result, vibrational modes with $n \in \{2-30\}$ will be excited in the hydrometeor.

We will estimate the dipole electromagnetic radiation intensity I_2 for the first possible source related to fine charged droplets oscillations in the absence of the external electric field when two neighboring modes with the numbers $j = 2$ and $j = 3$ are excited at the initial instant. For the numerical estimates we take $\varepsilon = 1$, $h_2 = h_3 = 0.5$, $\sigma = 73$ dyn/cm, and $\rho = 1$ g/cm^3.

In Fig. 2 we have plotted the graph of the electromagnetic radiation intensity I_2 of a single charged droplet as a function of its radius and self-charge calculated from (5.3). We can see that the small heavily charged droplets have the highest electromagnetic radiation intensity. The intensity I_2 decreases rapidly with increase in the dimension and decrease in the charge of the droplets.

In Fig. 3 we have reproduced the results of calculations of the dipole radiation intensity I_1 of an uncharged drop in an external electric field as a function of the radius of equivalent droplet and the induced charge $q = 3E_0R_0^2/4$ calculated from (4.4). We can see that I_1 is almost independent of the

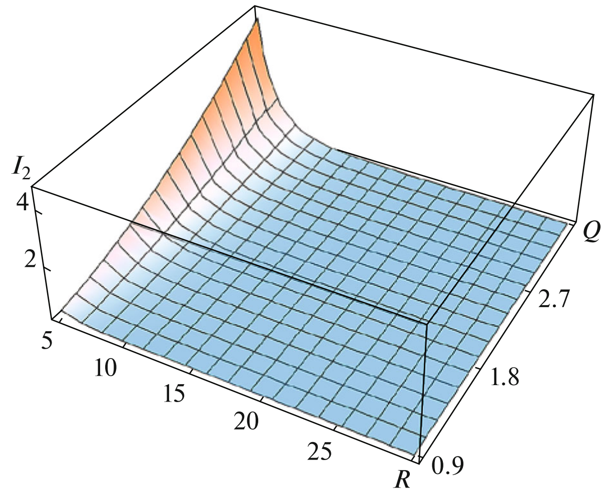

Fig. 2. Electromagnetic radiation intensity I_2 (10^{-29} erg/s) of a single charged drop as a function of the radius (in μm) and the drop charge (in 10^{-6} electrostatic units) in the absence of the external electric field: the initial excitation of the equilibrium shape of a droplet of the same form as in Fig. 1a: $\varepsilon = 0.1$, $\sigma = 73$ dyn/cm, and $\rho = 1$ g/cm^3.

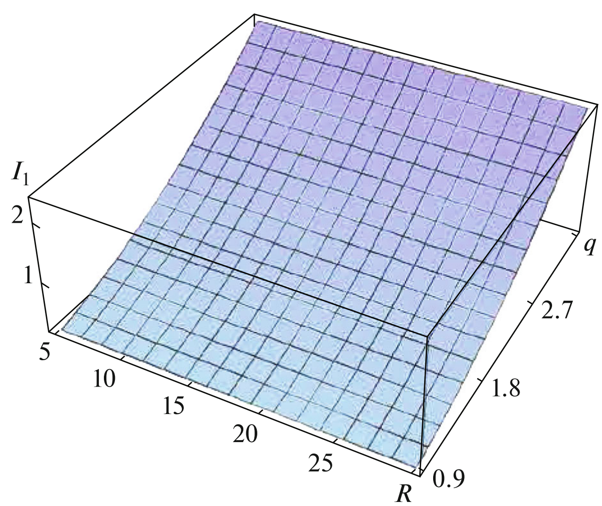

Fig. 3. The same as in Fig. 2 but for I_1; the electrostatic field strength $E_0 = 40-160$ V/cm.

droplet radius (this was reported in [21]) so that the intensity I_1 is determined by the electric field strength only.

In Fig. 4 we have reproduced the results of calculations of the dipole electromagnetic radiation intensity I_1 for the second source of radiation related to the hydrometeors freely falling through a cloud which coagulate with the smaller droplets when two neighboring modes with the numbers $j = 20$ and $j = 21$ are excited in the initial spectrum of the vibrational modes. The physical parameters are the same as in Fig. 3. From comparison of Figs. 3 and 4 we can see that the dependences of I_1 on R and q are qualitatively similar to those given in Fig. 3 and differ only quantitatively from the latter.

In Fig. 5 we have plotted the graph the radiation intensity I_2, related to radiation of the hydrometeors, as a function of the charge. The graph is qualitatively similar to that given in Fig. 2 for radiation of small droplets. However, in the case considered (for hydrometeors) the radiation intensity is higher by three orders of magnitude as compared with the small droplets. Primarily, this is connected with the fact that the hydrometeor self-charge is greater than the charge of a small drop by an order of magnitude or even higher.

In estimating the intensity of radiation by a cloud it is necessary to take into account that the number density of hydrometeors with $R_0 \approx 100$ μm is lower by six orders of magnitude than the number density

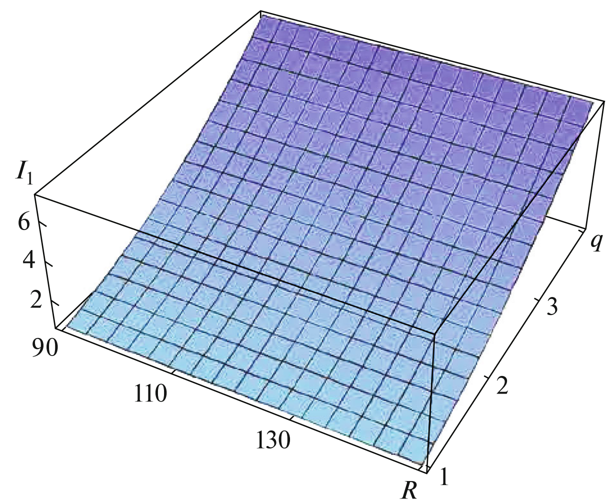

Fig. 4. Intensity I_1 (10^{-30} erg/s) for an uncharged drop oscillating in a weak electrostatic field as a function of the radius (in μm) and the induced charge q (in 10^{-5} electrostatic units): the initial excitation of the form $\varepsilon[P_{20}(\mu) + P_{21}(\mu)]/2$ for the same values of the quantities as those in Fig. 2; the electrostatic field strength $E_0 = 40-160$ V/cm.

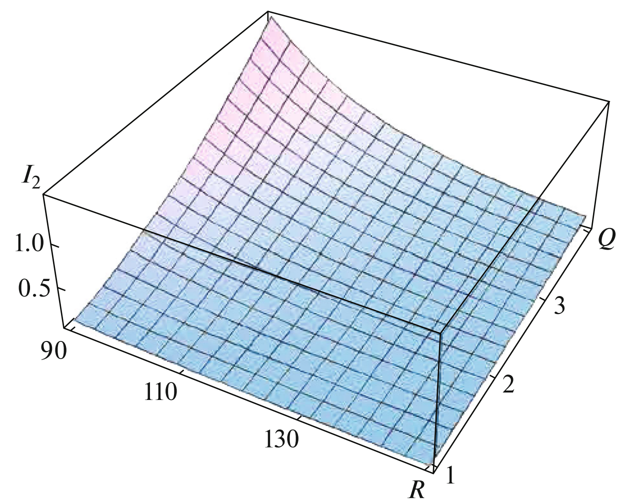

Fig. 5. The same as in Fig. 4 but for I_2 (10^{-26} erg/s) as a function of the radius (in μm) and the drop self-charge (in 10^{-5} electrostatic units).

of small droplets with $R_0 \approx 3$ μm. Thus, the order of magnitude of the intensity of electromagnetic radiation by a cloud is determined by small droplets.

The graphs of analogous dependences for the intensities I^* and I_3 constructed for the same parameters coincide qualitatively with the above-mentioned graphs for I_1 and I_2, respectively. In this case the radiation intensities become greater by several percents.

All the calculation was carried out for a drop of an ideal incompressible fluid in the potential flow model. Estimates show that taking the fluid viscosity into account is important only for the dimensionless viscosity of the order of unity. For water this relates to drops of radii less than 1 μm.

Summary. In model analytical calculations carried out with the accuracy of the second order of smallness with respect to the dimensionless oscillation amplitude it is found that there are asymptotic differences between radiation of electromagnetic waves by an oscillating charged droplet in the absence of the external electric field and by an uncharged droplet oscillating in an external electrostatic field. In the first case the less intense quadrupole radiation is revealed in the first-order calculations and the intense dipole radiation with regard to the second-order terms only, while in the second case other way round.

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