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# Flows of Supersonic Underexpanded Jets on the Range of Moderate Reynolds Numbers

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**Abstract**—The results of an experimental investigation of the gasdynamic structure of supersonic underexpanded air jets flowing out of a sonic nozzle into a low-pressure medium are presented. This setting of the experiment makes it possible to achieve high values of the nozzle-to-ambient pressure ratio at moderate outflow Reynolds numbers characteristic of underexpanded jets issuing from micronozzles. The data on the supersonic core length, the laminar-turbulent transition location, and the jet flow characteristics are obtained. The results are compared with those obtained in microjets flowing out of sonic nozzles. Emphasis is placed on the earlier discovered effect of inverse transition of a turbulent jet into the laminar flow regime with increase in the Reynolds number.
*Key words:* supersonic underexpanded jets, modeling of microjets, experiments.

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Supersonic gas jets have been widely and for a long time been used in engineering processes, aviation, and astronautics. In recent years, the work on an investigation of the properties of supersonic microjets and their practical application has been intensely performed. In particular, supersonic microjets have been tested for suppressing the jet noise of jet engines [1–3] and in executive units of pneumonic devices [4]. They can be used for mixing gases and protecting surfaces from the action of chemically agressive or high-temperature media. The advantage of microjets is that they can be placed on a surface at a high density, thus producing a reliable curtain at small gas flow rates. In this case, the main characteristics of microjets are their penetrating capability and the intensity of mixing processes; here, the key role in the estimation of the efficiency of the jet action on a flow is played by the supersonic region length.

During the last 30 years some experimental studies [5–16] were devoted to an investigation of the gasdynamic structure of imperfectly expanded (off-design) axisymmetric and plane jets, the supersonic region length, and laminar-turbulent transition in microjets. In [10–15] the gasdynamic structure of axisymmetric jets flowing out into the atmosphere from sonic micronozzles, 10 to 340 $\mu$m in diameter, was studied; the off-design ratio varied from 1 to 4, which corresponded to the range of moderate jet-outflow Reynolds numbers from 200 to 10 000. In these studies, the effects of laminar-turbulent transition and laminar flow regime recovery in microjets were first described; the former process is accompanied by the shortening of the supersonic flow region, while the latter results in an increase in its length. In the same studies the preliminary experiments on the modeling of microjets by macrojets were described under the condition of the equality of the Reynolds numbers based on the nozzle diameter and the flow parameters in the critical section. The importance of investigations on the modeling of microjets by macrojets lies in the fact that they can answer the question whether there are fundamental differences in the gasdynamics and stability of microscopic and macroscopic jets and whether the Mach and Reynolds numbers and other gasdynamic similarity parameters are applicable in describing microscopic jet flows.

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In this study, we present the results of experimental investigations of the gasdynamic structure and stability of supersonic underexpanded jets of millimeter scale on the range of moderate Reynolds numbers, on which laminar-turbulent transition in microjets is observable. The results of the measurements are compared with the experimental data on an investigation of underexpanded microjet parameters.

## 1. EXPERIMENTAL EQUIPMENT AND MEASUREMENT TECHNIQUES

The experiments were performed on a low-pressure jet rig. The setup was a vacuum-tight chamber, 0.223 $m^3$ in volume, pumped down by a high-performance vacuum pump. A plenum chamber with a sonic nozzle, $D = 0.6$ mm in diameter, was attached to an orifice in the chamber wall. Air at a temperature of $297 \pm 1$ K was supplied to the plenum chamber through a low-flow-rate needle valve which fixed the jet stagnation pressure $P_0$. The chamber, into which the jet flowed out, was pumped down using a valve with controlled area of passage, which made it possible to settle a given pressure $P_c$ in the chamber. The stagnation pressure in the plenum chamber was monitored by the VO1227 reference vacuummeter (accuracy class of 0.15), while the pressure in the jet outflow chamber was measured by the differential TDM4-IV1 pressure gauge. The measurements were performed at the jet flow axis using the Pitot tube, 0.2 mm in the outer diameter and 0.1 mm in the inner diameter, which was mounted on a three-component coordinate device with the 0.15 mm accuracy of fixing the coordinates. The Pitot tube could be displaced relative to the nozzle from zero to 200 mm, the tube motion velocity being 0.3 mm/s. The Pitot pressure $P_0$ was measured by the differential TDM4-IV1 pressure gauge; the characteristic time of pressure stabilization in the tube was 0.1 s. The pressure was measured by the TDM4-IV1 gauges at an accuracy of 1%. The value of $P_0$ was recorded using a 12-digit analog-digital processor at a frequency of 10 Hz.

The length of the supersonic region in the jet normalized by the jet diameter $L_s/d$ was determined as a distance from the nozzle exit to the point at the jet axis $x/d$, where the flow velocity is equal to the local speed of sound. The flow velocity becomes equal to the local speed of sound (the Mach number is unity), as the Pitot pressure reaches the value

$$P_0 = P_c \left( \frac{\gamma + 1}{2} \right)^{\gamma/\gamma - 1},$$

where $\gamma$ is the adiabatic exponent. The quantity $L_s/d$ was determined at a fixed stagnation pressure $P_0$ or at a fixed value of the jet pressure ratio $n$

$$n = P_0 \left[ P_c \left( \frac{\gamma + 1}{2} \right)^{\gamma/\gamma - 1} \right]^{-1}.$$

The data array $L_s/d(n)$ makes it possible to construct the dependence of the supersonic core length on the ratio $n$.

Using the parameter $n$ is conventional in studies on supersonic jet flows. At a constant stagnation temperature its any value corresponds to a certain Reynolds number $\mathrm{Re}_d$ of the outflow from a micronozzle based on the nozzle diameter $d$ and the flow parameters at the sonic nozzle exit, $\mathrm{Re}_d = \rho^* u^* d/\mu^*$. The asterisked parameters correspond to the values in the critical section of the nozzle coinciding for sonic nozzles with the exit section. The quantities $u^*$ (it is equal to the speed of sound in the critical section) and $\mu^*$ are constant at a fixed stagnation temperature and independent of the stagnation pressure $P_0$. Only the density $\rho^*$ depends on the stagnation pressure; thus, it is a linear function of the jet pressure ratio $n$. Hence follows the linear relation between the jet-outflow Reynolds number $\mathrm{Re}_d$ and the parameter $n$.

## 2. RESULTS

In this study, we checked the supposition put forward in [13, 14] on the similarity of the gasdynamic structures and stability characteristics of the jets flowing out of micro and macronozzles provided that the Reynolds numbers based on the flow parameters in the critical section and the nozzle diameter are the same for the two flows. At a fixed macronozzle diameter $D$ and the pressure ratio $n$ this makes it possible to model the jet outflow from a micronozzle with any diameter by means of selecting the corresponding values of $P_0$ and $P_c$. The effective, or model, micronozzle diameter $d$ is given by the relation $d = DP_c/P_a$,

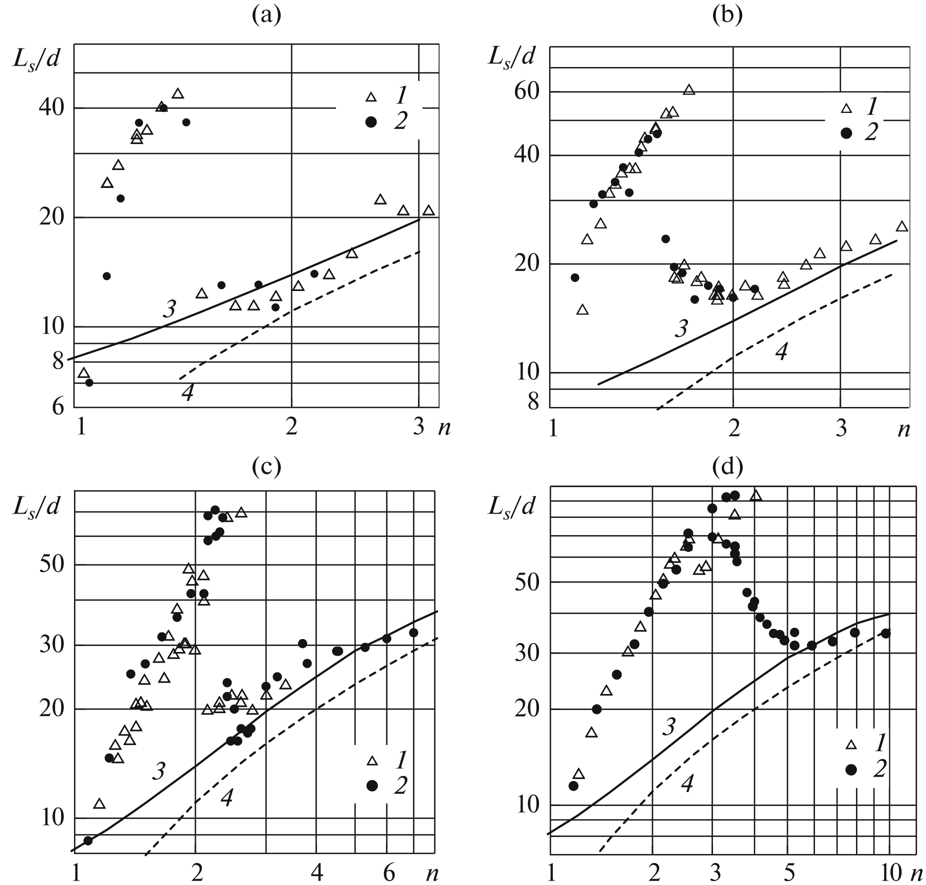


**Fig. 1.** Dependence of the normalized supersonic-region length on the jet pressure ratio for actual (*1*) and model (*2*) micronozzles; $d = 44.3$ (a), 34.8 (b), 21.4 (c), and 16.1 $\mu$m (d); (*3*), generalized data on hot turbulent macrojets [17] and (*4*), generalized data on cold turbulent macrojets [18].

while the pressures $P_0$ and $P_c$ are related by the equation for $n$ presented above. Here, $P_a$ is the pressure in the atmosphere, where the jets flowed out of actual micronozzles in the experiments. In what follows, it is the micronozzles used in the experiments with jets in [10–15] that will be referred to as the actual micronozzles. The effective nozzles of the micrometer scale calculated for $D = 0.6$ mm from the equality $\mathrm{Re}_D = \mathrm{Re}_d$ and the pressure $P_c$ determined from the relation for $d$ presented above will be called model micronozzles.

Earlier, in [13] the dependences $L_s/d(n)$ obtained in the experiments with actual and model micronozzles were compared; the shortcoming of that comparison was that the correspondence between the actual and model micronozzles was incomplete. In this study, we compare the dependences $L_s/d(n)$ obtained at exact equality of the diameters of the actual and model micronozzles with the diameters 10.4, 16.1, 21.4, 24.3, 26, 34.8, 36, 44.3, and 45.7 $\mu$m. In Fig. 1 the $L_c/d(n)$ dependences of the actual and model microjets are plotted for four nozzle diameters. The plots contain the generalized data obtained for turbulent hot (*3*) and cold (*4*) axisymmetric macrojets in [17, 18]. In this study, the longitudinal lengths $x$ and $L_c$ were normalized by the nozzle diameter $D = 0.6$ mm but, for the sake of convenience in comparing with actual microjets, everywhere on the plots the normalized diameter $d$ is noted.

As can be seen from the plots, the dependences $L_s/d(n)$ and the positions of the sharp decrease in the supersonic core length with respect to the coordinate $n$ agree fairly well. As shown in [14], in this region the integral value of the mass flow rate fluctuations rapidly increases and the frequency range of the fluctuations enlarges, which was interpreted as laminar-turbulent transition in the jet flow and, as a consequence, a decrease in the relative length of the supersonic region down to the level characteristic of turbulent macrojets. The results of the comparison show that the modeling of microjets in the Reynolds number $\mathrm{Re}_d$ is possible.

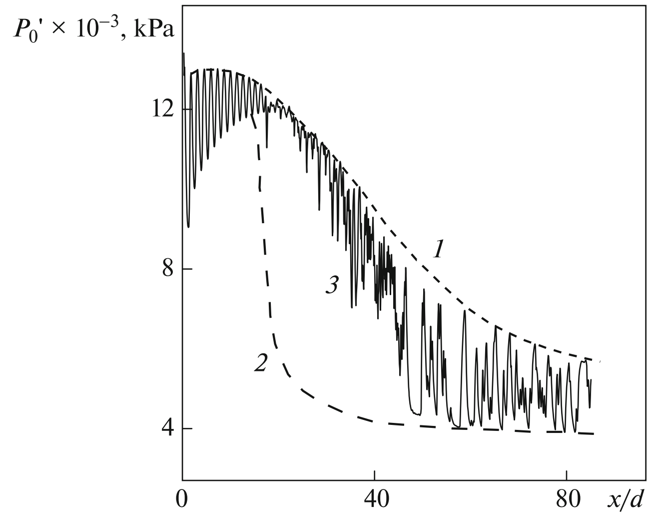


**Fig. 2.** Dependence of the Pitot pressure $P_0'$ in the jet on the normalized distance from the nozzle $x/d$ in the laminar (*1*), turbulent (*2*), and transitional (*3*) jet flow regimes; $d = 21.4$ $\mu$m.

In this study, the variation in the supersonic core length was investigated on a narrow $n$ range in the region of laminar-turbulent transition in the jet. In particular, a jet, whose Reynolds number $\mathrm{Re}_d$ corresponded the micronozzle with the diameter of 21.4 $\mu$m and the parameter $n$ ranging from 2.3 to 2.5 was taken. The lower boundary of the $n$ range corresponded to the laminar jet flow regime and the upper boundary corresponded to the turbulent regime. In Fig. 2 the longitudinal distributions of the Pitot pressure $P_0'$ along the normalized distance from the nozzle exit are plotted; they were obtained at continuous motion of the tube for the laminar (curve *1*, $n = 2.3$), turbulent (curve *2*, $n = 2.5$), and transitional (curve *3*, $n = 2.4$) jet flow regimes. At $x/d < 10$ visible in the figure are oscillations of $P_0'$ due to the passage of the shock-wave structure of the jet by the Pitot tube; they rapidly decay with the distance from the nozzle exit. Clearly that at $x/d > 10$ the pressure irregularly oscillates between the values corresponding to the laminar and turbulent jet flow regimes. As this passes, the time of the stay of $P_0'$ in one of the extreme positions can sometimes be considerable, amounting to several seconds.

The statistics of the time distribution of the pressure $P_0'$ between the values corresponding to the laminar and turbulent jet flow regimes depend on the distance from the nozzle. To obtain the statistical data the time dependences of the pressure $P_0'$ were measured in several jet sections at a constant value $n = 2.4$ and the expectation time greater than 40 s. By way of illustration, in Fig. 3 we have plotted the time dependences of the pressure $P_0'$ for four distances from the exit of the nozzle with the effective diameter 21.4 $\mu$m. On the plots the Pitot pressure levels in the laminar (*1*) and turbulent (*2*) jet flow regimes are also shown. Clearly that with the distance from the nozzle exit the frequency and amplitude of $P_0'$ variations increase and, at the same time, the mean value of $P_0'$ is displaced from the laminar to the turbulent level.

In Fig. 4 we have plotted the histograms of the probability **P** in percents of recording the pressure values $P_0'$ on the range between the laminar and turbulent flow regimes at six points on the jet axis. Unity on the abscissa axes of the histograms is associated with the pressure $P_0'$ in the laminar regime and zero with the turbulent jet flow regime (cf. Fig. 3). As can be seen in Fig. 4, with increase in the spacing between the nozzle and the Pitot tube the histogram width first increases and then again decreases, while the probability maximum is displaced from the values corresponding to the laminar jet flow regime to those corresponding to the turbulent regime.

This pressure $P_0'$ behavior must correspond to an increase or decrease of the supersonic core length in the jet. On the other hand, this must lead to a decrease in the pressure $P_0'$ with decrease in the supersonic region length and to its increase with increase in the supersonic core length. In turn, the variation in $P_0'$ must lead to large variations in the mass flow rate at the jet axis in the transitional region, considerably greater than the mass flow rate fluctuations in the developed turbulent jet. This effect can provide an

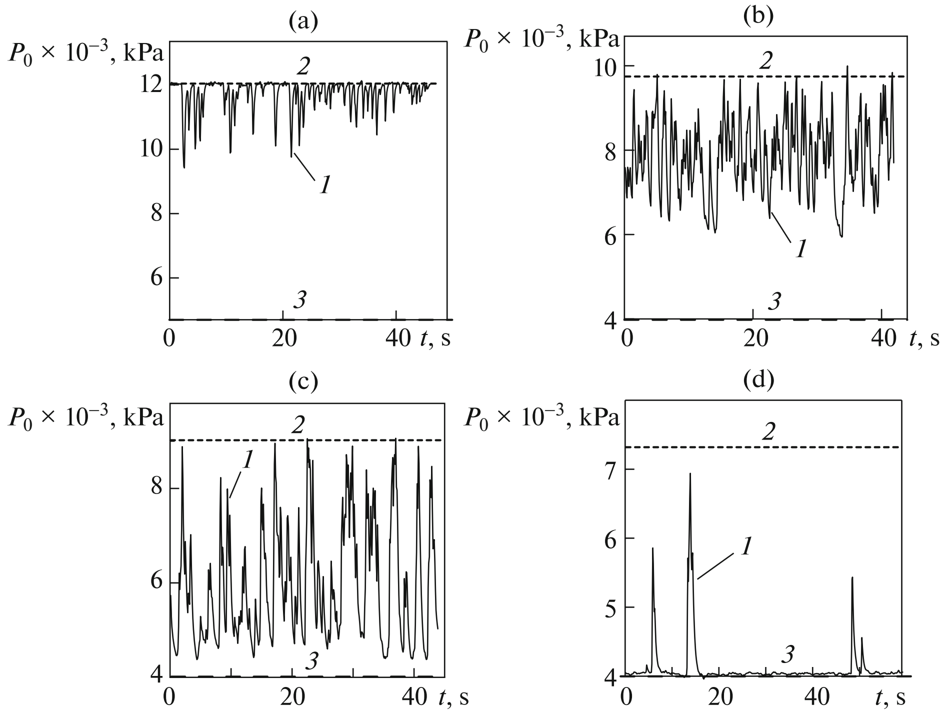


**Fig. 3.** Time dependence of the pressure $P_0'$ in the jet for four distances from the nozzle $x/d = 23.3$ (a), 34.7 (b), 44 (c), and 57.5 (d); *2* and *3*, pressure levels $P_0'$ in the laminar and turbulent jet flow regimes.

explanation for the occurrence of a considerable low-frequency signal of the hot-wire anemometer in the regions of sharp variation in the supersonic core length found in [14].

To verify this supposition we performed the measurements of the pressure $P_0'$ at certain points on the jet axis varying the pressure $P_0$ in the plenum chamber of the nozzle (thus varying the parameter $n$). Here, the jet rig made it possible to vary the pressure ratio $n$ on a considerably wider range than in the experiments on the actual microjet outflow into the atmosphere. The jet outflow of the nozzles with the effective diameters 10.4, 16.1, 21.4, 24.3, 26, 36, and 45.7 $\mu$m was studied. Some results of the measurements are shown in Figs. 5–7 in the form of the dependences of the normalized Pitot pressure $\alpha = 2P_0'/(\gamma + 1)P_c$ on the parameter $n$. This presentation makes it possible to illustrate the presence of subsonic ($\alpha < 1$) and supersonic ($\alpha > 1$) flows at the measurement point on the jet axis for any value of $n$. On certain plots light circles pertain to the data of the measurements made for actual microjets flowing out into the atmosphere at the same points on the jet axis.

For the purpose of comparing the quantity $\alpha(n)$ in the jet under consideration with the analogous quantity corresponding to the fully turbulent jet in the same figures we have also plotted the dependences $\alpha(n)$ for the turbulent microjet issuing from the nozzle of the same diameter (triangles). Here, we use the fact of the similarity of the dependences of $P_0'/P_0$ on $x/d$ for turbulent microjets experimentally established from an analysis of the measured data for actual and model micronozzles, 61.4, 65.3, 149, 215, and 341 $\mu$m in diameter. According to the earlier obtained data [10–12] the jets issuing from the micronozzles with these diameters are turbulent on the entire $n$ range studied. It turned out that the $\alpha(n)$ dependence of a turbulent microjet on the interval $x/d = 10–40$ can be approximated by the power-law function $\alpha = An^{1-B}$. The coefficients $A$ and $B$ are, in turn, approximated by the fourth-order polynomials in the parameter $x/d$. These approximations were used for calculating the pressure $P_0'$ in turbulent jets flowing out of the nozzles, less than 60 $\mu$m in diameter, for all values of $n$, where the outflow regime can actually be both laminar and turbulent.

From the plots presented in Figs. 5–7 it can be seen that the quantity $\alpha$ first diminishes and approaches the level of the on-design turbulent jet but then the normalized jet pressure again increases. At small values of $x/d$ this pressure does not reach the line of the separation between the supersonic and subsonic flows ($\alpha = 1$). With increase in $x/d$ the normalized pressure $\alpha$ intersects this line, which

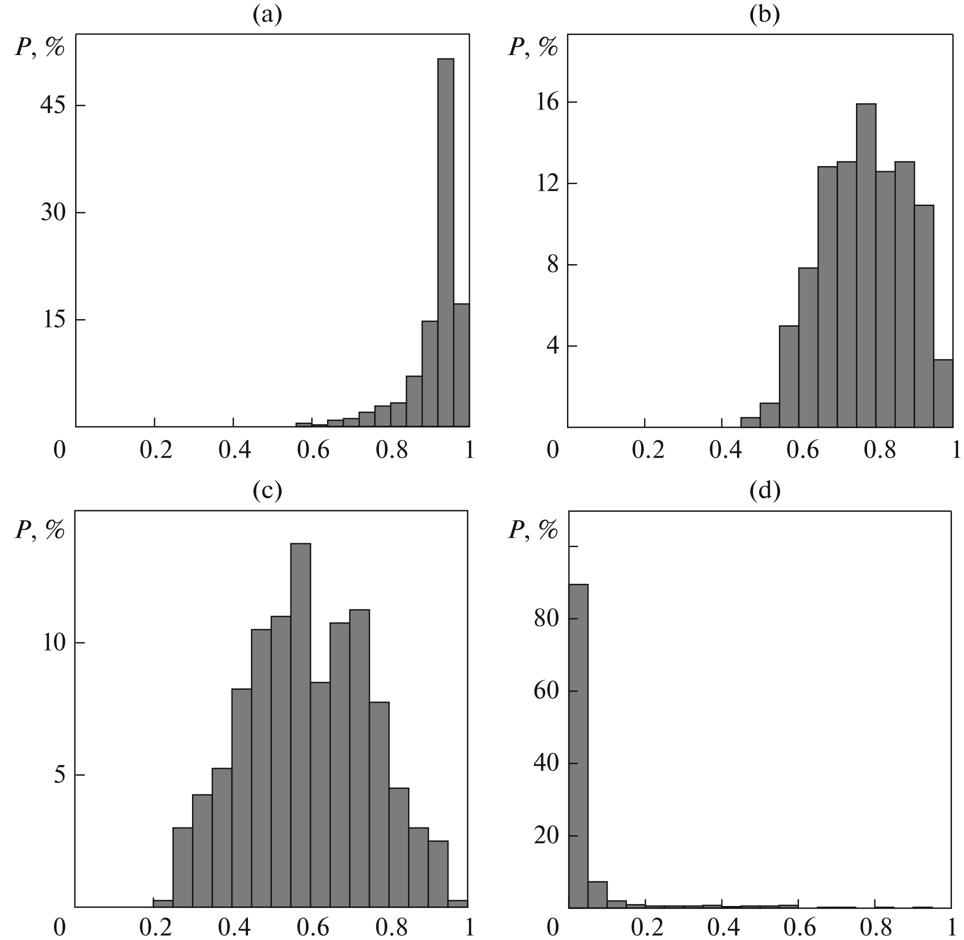


**Fig. 4.** Histograms of the probability of recording the pressure $P_0'$ in the transitional regime on the Pitot pressure interval between the laminar and turbulent jet flow regimes for different tube positions on the jet axis $x/d = 23.2$ (a), 34.7 (b), 44 (c), and 57.5 (d).

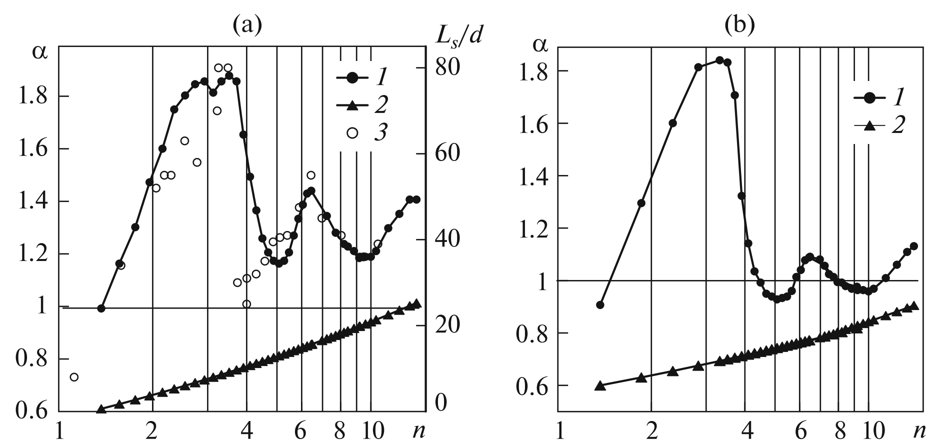


**Fig. 5.** Dependence of the quantity $\alpha$ on the jet pressure ratio $n$ for the model turbulent jet under study (*1*) and the fully turbulent jet (*2*) at two points on the axis; $d = 16.1$ $\mu$m; $x/d = 25$ (a) and 30 (b); (*3*) variations in the supersonic core length $L_s/d$.

means the jet flow transition to the subsonic regime. In Figs. 5 and 6 for the sake of comparison we have presented the dependences of the normalized supersonic core length $L_s/d(n)$ taken from [14]. In Fig. 7 the results of the measurements of $\alpha$ for the jets issuing from actual micronozzles are also presented. The synchronism of the $\alpha(n)$ variations at different distances $x/d$ from the nozzle exit indicates the

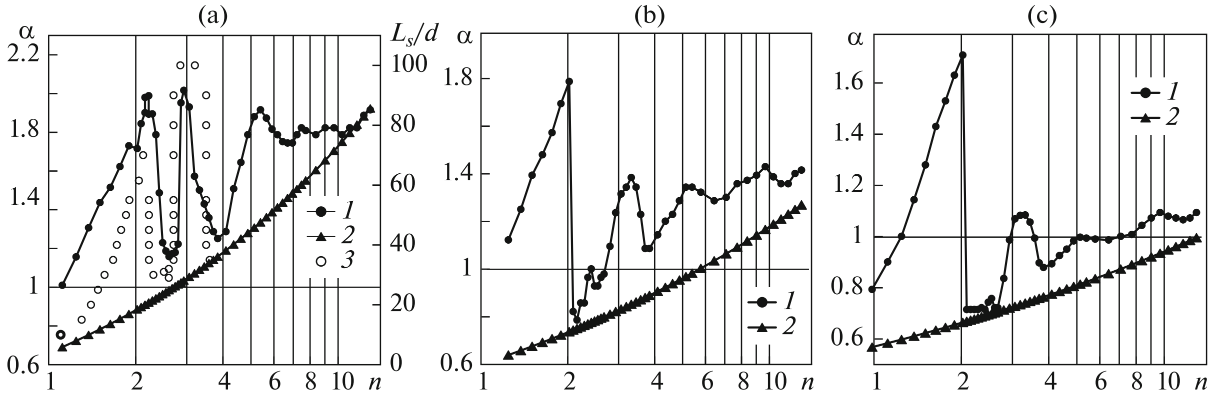


**Fig. 6.** Dependence of the quantity $\alpha$ on the jet pressure ratio $n$ for the model turbulent jet under study (*1*) and the fully turbulent jet (*2*) at three points on the axis; $d = 24.3$ $\mu$m; $x/d = 15$ (a), 20 (b), and 25 (c); (*3*) variations in the supersonic core length $L_s/d$.

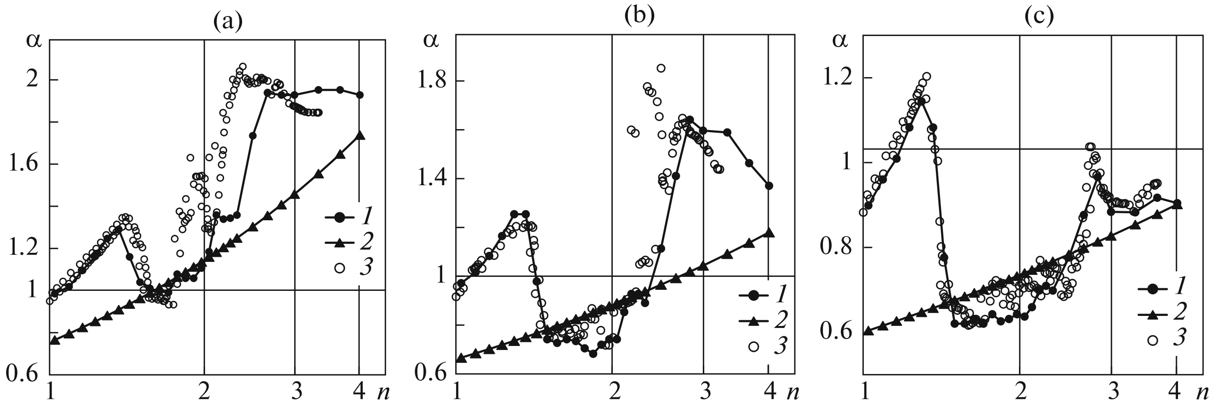


**Fig. 7.** Dependence of the quantity $\alpha$ on the jet pressure ratio $n$ for the model turbulent jet under study (*1*) and the fully turbulent jet (*2*) at three points on the axis; $d = 45.7$ $\mu$m; $x/d = 10$ (a), 15 (b), and 20 (c); (*3*) data of measurements of $\alpha$ in an actual microjet.

corresponding variations in the jet length and, in particular, the length of its supersonic region. The first decrease in $\alpha$ in Figs. 5–7 corresponds to laminar-turbulent transition in the jet which happens at the same values of $n$, as in the experiments with actual micronozzles [10–15]. After reduction in $\alpha$ its increase can be seen on the plots; it is due to an increase in the supersonic core length in the jets corresponding to the inverse transition discovered in microjets [13, 14]. The physical processes leading to the reverse of laminar-turbulent transition call for further investigation. These data and the comparisons of the model and actual microjets made above show that the stability characteristics of microjets can also be modeled with respect to the Reynolds number $\mathrm{Re}_d$.

*Summary*. The gasdynamic structure of supersonic, imperfectly expanded gas jets flowing out of a sonic nozzle, 0.6 mm in diameter, into a low-pressure region at the jet core ratios $n$ ranging from 1 to 20 is experimentally investigated. The measurements are performed for the Reynolds numbers $\mathrm{Re}_d$ based on the nozzle diameter and the flow parameters at its exit corresponding to the experiments with microjets flowing out into the atmosphere from micronozzles, 61 to 10.4 $\mu$m in diameter. It is shown that under the equality of the Reynolds numbers $\mathrm{Re}_d$ the jets issuing from actual and effective micronozzles have similar values of the relative supersonic core length and laminar-turbulent transition conditions. This confirms the possibility of modeling macrojets and microjets with respect to the Reynolds number.

The dynamics of the variation in the supersonic region jet is determined in the laminar-turbulent transition region and the statistics of the temporal variations of the supersonic core length as a function of the distance from the nozzle are presented.

The phenomenon of the laminar-turbulent transition reverse with increase in the jet pressure ratio (the Reynolds number $Re_d$) earlier established for microjets is confirmed in the case of the jets issuing from model micronozzles.

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