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NUMERICAL METHODS

Boundary Value Problems with Integral Boundary Conditions for the Modeling of Magnetic Fields in Cylindrical Film Shells

V. T. Erofeenko*, G. F. Gromyko, and G. M. Zayats*****

Belarusian State University, Minsk, 220030 Belarus

Institute of Mathematics, National Academy of Sciences of Belarus, Minsk, 220072 Belarus

*e-mail: *bsu_erofeenko@tut.by, **grom@im.bas-net.by, ***zayats@im.bas-net.by*

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Abstract—We develop a mathematical model of the boundary value problem describing magnetic field shielding by a cylindrical thin-walled shell (screen) made of materials whose permeability depends nonlinearly on the magnetic field intensity. Integral boundary conditions on the shell surface are used. A numerical method is suggested for solving a nonlinear boundary value problem of magnetostatics with integral boundary conditions. The shielding efficiency coefficient characterizing the external magnetic field attenuation when passing into the interior of the cylindrical screen is studied numerically.

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1. INTRODUCTION

One main trend in the modern development of technology is the miniaturization of technical devices, whereby small volumes contain large numbers of high-precision elements and devices that serve as sources of electric and magnetic fields and are sensitive to external electromagnetic fields, which affect the accuracy of their operation. Hence it is increasingly topical to design screens and shells for shielding external constant and alternating magnetic and electric fields as well as for solving the problem of electromagnetic consistency of device elements. In particular, it is important to study the shielding properties of standard screens subjected to constant magnetic and electric fields from close sources. Experimental data concerning film screens can be found in [1–3]. The mathematical modeling of electromagnetic processes in shielding structures is presented in [4–9].

One important characteristic of film screens is the shielding efficiency coefficient, which characterizes the external magnetic field attenuation by the shielding material. If the permittivity of the material is constant, then the analytical computation of the efficiency for a screen with standard geometry can be carried out with the use of fairly simple formulas [6]. In actual screens, the distribution of permittivity across the thickness of the film material of the screen nonlinearly depends on the magnetic field intensity [5, 10]. This complicates finding magnetic characteristics of screens in analytical form. Some analytical-numerical and numerical studies of magnetic properties of film screens were carried out in [6–9]. Based on the approach developed in [8], the present paper suggests boundary value problems for the mathematical modeling of shielding of constant magnetic fields by film screens; these problems are boundary value problems for the nonlinear magnetostatic equation.

As a rule, a shielding boundary value problem is a three-domain problem with one domain inside the screen, one domain in the screen wall, and an infinite domain outside the screen. To implement the numerical solution of the problem, one transforms the infinite domain into a finite domain. To this end, various approaches are used. A technique excluding the domain inside the screen and the infinite domain outside the cylindrical screen was developed in [8, 9]. As a result, the original three-domain problem is transformed into a boundary value problem in the film layer with special boundary conditions on its face surfaces. For a constant external magnetic field, boundary conditions of the third kind are used. The extension of this approach to the modeling of arbitrary external field leads to integral boundary conditions on the screen surfaces. The method of integral boundary conditions in applied problems of electrodynamics was developed in [11, p. 207; 12–15]. Theoretical studies of initial–boundary value problems of mathematical physics with integral boundary conditions were carried out in [16–19].

2. STATEMENT OF THE SHIELDING PROBLEM

In the space $\mathbb{R}^3$ with cylindrical coordinate system $O\varrho\varphi z$, consider a cylindrical shell (screen) D ($R_1 < \varrho < R_2$, $0 \le \varphi < 2\pi$, $-\infty < z < \infty$) of thickness $\triangle = R_2 - R_1$ bounded by the cylindrical surfaces Γ_1 ($\varrho = R_1$) and Γ_2 ($\varrho = R_2$) (Fig. 1). The screen is made of the ferromagnetic material $Fe_{20}Ni_{80}$, which is characterized by high relative permittivity $\mu \approx 10^3 \div 10^4$. There is vacuum ($\mu = 1$) inside the screen in the domain D_1 ($0 \le \varrho < R_1$) and outside the screen in the domain D_2 ($\varrho > R_2$). There is a primary magnetic field $\mathbf{H}_0$ acting on the screen from the domain D_2. As a result of the interaction, the following fields occur: the field $\mathbf{H}_1$ in the domain D_1, the field $\mathbf{H}$ in the screen D, the reflected field $\tilde{\mathbf{H}}_2$ in the domain D_2, and the total field $\mathbf{H}_2 = \mathbf{H}_0 + \tilde{\mathbf{H}}_2$ in D_2.

We express the magnetic field via potential functions u_1, u, $\tilde{u}_2$, u_0, and $u_2 = u_0 + \tilde{u}_2$,

$$\mathbf{H}_j = -\operatorname{grad} u_j \quad (j = 1, 2), \qquad \mathbf{H}_0 = -\operatorname{grad} u_0, \qquad \mathbf{H} = -\operatorname{grad} u. \tag{1}$$

We determine the potentials by solving a boundary value problem of magnetostatics.

Boundary value problem 1. Given a potential u_0, determine potentials u_1, u, and $\tilde{u}_2$ satisfying the equations [11, p. 15]

$$\Delta u_1 = 0 \ \text{ in } \ D_1, \qquad \Delta \tilde{u}_2 = 0 \ \text{ in } \ D_2, \tag{2}$$

$$\operatorname{div}(\mu \operatorname{grad} u) = 0 \ \text{ in } \ D, \tag{3}$$

the transmission conditions

$$u|_{\varrho=R_1} = u_1|_{\varrho=R_1}, \qquad \mu \frac{\partial u}{\partial \varrho}\bigg|_{\varrho=R_1} = \frac{\partial u_1}{\partial \varrho}\bigg|_{\varrho=R_1}, \qquad 0 \le \varphi < 2\pi, \tag{4}$$

$$u|_{\varrho=R_2} = (u_0 + \tilde{u}_2)|_{\varrho=R_2}, \qquad \mu \frac{\partial u}{\partial \varrho}\bigg|_{\varrho=R_2} = \left(\frac{\partial u_0}{\partial \varrho} + \frac{\partial \tilde{u}_2}{\partial \varrho}\right)\bigg|_{\varrho=R_2}, \qquad 0 \le \varphi < 2\pi, \tag{5}$$

on the surfaces Γ_1 and Γ_2, and the condition $\lim_{\varrho\to\infty} \tilde{u}_2 = 0$ at infinity.

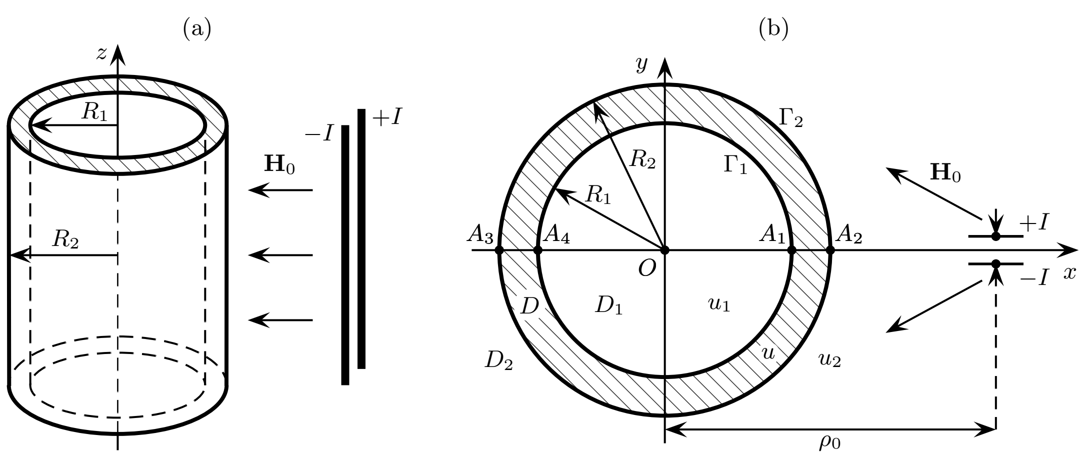

Fig. 1. Cylindrical screen: (a) a film screen subjected to the magnetic field generated by a twin line with a current; (b) a cross-section of the screen.

The relative permittivity in Eq. (3) nonlinearly depends on the absolute value $H = |\mathbf{H}| = |\operatorname{grad} u|$ of the magnetic field intensity [1].

For the external magnetic field $\mathbf{H}_0$ we take the field generated by a twin line with current I in the wires parallel to the screen (see Fig. 1) and placed at the points $(x = \varrho_0,\ y = \pm l/2)$, where l is the distance between the wires. A twin line with $l/R_2 \ll 1$ will be viewed as a point source $(x = \varrho_0,\ y = 0)$ lying in the plane $\varrho_0 > R_2$. In this case, the magnetic potential of the primary field in the coordinate system $O\varrho\varphi$ is given by the formula

$$u_0 = \frac{lI(\varrho\cos\varphi - \varrho_0)}{2\pi(\varrho^2 + \varrho_0^2 - 2\varrho\varrho_0\cos\varphi)}. \tag{6}$$

Let us take into account relations (1) and compute the magnetic field generated by the source. In the Cartesian coordinates, we obtain

$$\mathbf{H}_0 = \frac{lI}{2\pi}\frac{((x-\varrho_0)^2 - y^2)\mathbf{e}_x + 2y(x-\varrho_0)\mathbf{e}_y}{(y^2 + (x-\varrho_0)^2)^2}.$$

The intensity of the magnetic field of the source at the point $O(x = 0, y = 0)$ in the absence of the screen is given by the formula $H_0 = |\mathbf{H}_0(0)| = lI/(2\pi\varrho_0^2)$; consequently, $lI = 2\pi\varrho_0^2 H_0$. The primary potential for the constant external magnetic field $\mathbf{H}_0 = -H_0\mathbf{e}_x$ is

$$u_0 = H_0\varrho\cos\varphi. \tag{7}$$

We pose the problem of numerical analysis of the shielding efficiency coefficient

$$K_{\text{shield}} = \frac{|\mathbf{H}_0(0)|}{|\mathbf{H}_1(0)|} = \frac{H_0}{H_1},$$

which shows by what factor the exterior magnetic field of the source is attenuated when passing into the interior domain of the screen.

3. INTEGRAL BOUNDARY CONDITIONS

Problem (2)–(5) is a three-domain boundary value problem for the domains D_1, D, and D_2. Let us transform the original problem into a one-domain boundary value problem in the domain D by introducing integral boundary conditions posed on the screen surface and corresponding to the boundary conditions (4) and (5).

Consider problem (2)–(5) under the axial symmetry conditions $u_0(x,-y) = u_0(x,y)$, $u_j(x,-y) = u_j(x,y)$, $j = 1,2$. The potentials u_0, u_1, and $\tilde{u}_2$ in the domains D_1 and D_2 satisfy the Laplace equation (2), whose solutions are sequences of axisymmetric functions

$$\varrho^n\cos(n\varphi), \quad n = 0,1,2,\ldots; \qquad \varrho^{-n}\cos(n\varphi), \quad n = 1,2,\ldots$$

We represent the potential u_0 regular in the domain $0 \le \varrho < \varrho_0$ and the potential u_1 regular in the domain D_1 in the form of the series

$$u_0 = \sum_{n=0}^{\infty}\gamma_n\varrho^n\cos(n\varphi), \qquad 0 \le \varrho < \varrho_0, \tag{8}$$

$$u_1 = \sum_{n=0}^{\infty}\alpha_n\left(\frac{\varrho}{R_1}\right)^n\cos(n\varphi), \qquad 0 \le \varrho < R_1. \tag{9}$$

We represent the potential $\tilde{u}_2$ satisfying the condition at infinity in the form of the series

$$\tilde{u}_2 = \sum_{n=1}^{\infty}\beta_n\left(\frac{R_2}{\varrho}\right)^n\cos(n\varphi), \qquad \varrho > R_2. \tag{10}$$

Let us transform the boundary conditions (4) by substituting the series (9) into them,

$$u(\varrho,\varphi)|_{\varrho=R_1} = \sum_{n=0}^{\infty} \alpha_n \cos(n\varphi). \tag{11}$$

We integrate Eq. (11) from 0 to π and compute

$$\alpha_0 = \frac{1}{\pi}\int_0^{\pi} u(\varrho,\varphi)|_{\varrho=R_1}\, d\varphi. \tag{12}$$

The second boundary condition in (4) implies that

$$\mu\frac{\partial u}{\partial\varrho}\bigg|_{\varrho=R_1} = \frac{1}{R_1}\sum_{n=1}^{\infty} n\alpha_n \cos(n\varphi).$$

We use the orthogonality of the functions $\cos(n\varphi)$, $n\in\mathbb{N}$, and compute

$$\alpha_n = \frac{2R_1}{\pi n}\int_0^{\pi} \mu(\varrho,\varphi)\frac{\partial u(\varrho,\varphi)}{\partial\varrho}\bigg|_{\varrho=R_1}\cos(n\varphi)\,d\varphi, \qquad n\ge 1. \tag{13}$$

Let us substitute the integrals (12) and (13) into Eq. (11). After changing the order of summation and integration, we obtain the following integral boundary condition on the surface Γ_1 of the screen:

$$u(\varrho,\varphi)|_{\varrho=R_1} = \int_0^{\pi}\left(\frac{1}{\pi}u(\varrho,\psi) + R_1\mu(\varrho,\psi)\frac{\partial u(\varrho,\psi)}{\partial\varrho}K(\varphi,\psi)\right)\bigg|_{\varrho=R_1} d\psi, \tag{14}$$

where

$$K(\varphi,\psi) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\cos(n\varphi)\cos(n\psi).$$

According to [20, p. 576],

$$K(\varphi,\psi) = -\frac{1}{\pi}\ln\left[4\sin\left(\frac{|\psi-\varphi|}{2}\right)\sin\left(\frac{\psi+\varphi}{2}\right)\right]. \tag{15}$$

Let us transform the boundary conditions (5) by substituting the series (8) and (10) into them. Then

$$u|_{\varrho=R_2} = \sum_{n=0}^{\infty}\gamma_n R_2^n\cos(n\varphi) + \sum_{n=1}^{\infty}\beta_n\cos(n\varphi), \tag{16}$$

$$\mu\frac{\partial u}{\partial\varrho}\bigg|_{\varrho=R_2} = \frac{1}{R_2}\sum_{n=1}^{\infty} n(\gamma_n R_2^n - \beta_n)\cos(n\varphi). \tag{17}$$

We integrate Eq. (16) over $[0,\pi]$ and obtain

$$\gamma_0 = \frac{1}{\pi}\int_0^{\pi} u(\varrho,\varphi)|_{\varrho=R_2}\, d\varphi. \tag{18}$$

We multiply Eq. (17) by $\cos(n\varphi)$, integrate the resulting relation from 0 to π, and find, owing to the orthogonality of the functions $\cos(n\varphi)$, $n \in \mathbb{N}$, that

$$\beta_n = \gamma_n R_2^n - \frac{2R_2}{\pi n} \int_0^\pi \mu \frac{\partial u}{\partial \varrho}\bigg|_{\varrho=R_2} \cos(n\varphi)\, d\varphi. \tag{19}$$

Let us substitute the expressions (19) into Eq. (16). We take into account Eq. (17), sum the series (5) and (8), and obtain the following integral boundary condition on the surface Γ_2:

$$u(\varrho,\varphi)|_{\varrho=R_2} = 2u_0(\varrho,\varphi)|_{\varrho=R_2} - \int_0^\pi \left(\frac{1}{\pi} u(\varrho,\psi) + R_2 \mu(\varrho,\psi) \frac{\partial u(\varrho,\psi)}{\partial \varrho} K(\varphi,\psi)\right)\bigg|_{\varrho=R_2} d\psi. \tag{20}$$

4. MODELS OF SHIELDING BOUNDARY VALUE PROBLEMS WITH INTEGRAL BOUNDARY CONDITIONS

Let us state a one-domain shielding problem equivalent to the original problem (2)–(6) with the use of the boundary conditions (14) and (20) for the potential (6) of the primary magnetic field.

Boundary value problem 2. Given a potential u_0 (6), find a potential u satisfying the equation

$$\frac{\partial}{\partial \varrho}\left(\varrho \mu \frac{\partial u}{\partial \varrho}\right) + \frac{\partial}{\partial \varphi}\left(\frac{\mu}{\varrho} \frac{\partial u}{\partial \varphi}\right) = 0, \qquad R_1 < \varrho < R_2, \qquad 0 < \varphi < \pi, \tag{21}$$

the boundary conditions

$$\frac{\partial u}{\partial \varphi}\bigg|_{\varphi=0} = 0, \qquad \frac{\partial u}{\partial \varphi}\bigg|_{\varphi=\pi} = 0, \qquad R_1 \le \varrho \le R_2, \tag{22}$$

and conditions (14) and (20), where $2u_0(R_2,\varphi) = (2\varrho_0^2 H_0(R_2\cos\varphi - \varrho_0))/(R_2^2 + \varrho_0^2 - 2R_2\varrho_0\cos\varphi)$.

The variables in the problems have the following dimensions in SI units: $[\varrho, R_1, R_2, \Delta] = \mathrm{m}$, $[H, H_0] = \mathrm{A/m}$, $[u] = \mathrm{A}$, $[\mu] = 1$.

A model of a shielding boundary value problem in which a cylindrical shield is subjected to a constant magnetic field $\mathbf{H}_0$ with potential $u_0 = H_0 \varrho \cos\varphi$ ($H_0 = |\mathbf{H}_0|$) directed along the axis Ox is suggested in [8]. Here boundary conditions of the third kind are used. Let us state the corresponding boundary value problem.

Boundary value problem 3. Given a potential u_0 (7), find a potential u that satisfies Eq. (21), conditions (22), and the boundary conditions

$$\left(R_1 \mu(\varrho,\varphi) \frac{\partial u(\varrho,\varphi)}{\partial \varrho} - u(\varrho,\varphi)\right)\bigg|_{\varrho=R_1} = 0, \qquad 0 \le \varphi \le \pi, \tag{23}$$

$$\left(R_2 \mu(\varrho,\varphi) \frac{\partial u(\varrho,\varphi)}{\partial \varrho} + u(\varrho,\varphi)\right)\bigg|_{\varrho=R_2} = f_0(\varphi), \qquad 0 \le \varphi \le \pi.$$

For the primary potential $u_0 = H_0 \varrho \cos\varphi$, one has $f_0(\varphi) = 2H_0 R_2 \cos\varphi$.

For this potential u_0, boundary value problem 3 can be stated with the use of integral boundary conditions [21].

Boundary value problem 4. Given a potential u_0 (7), find a potential u that satisfies Eq. (21), conditions (22), and the boundary conditions

$$\left(R_1 \int_0^\pi \mu(\varrho,\psi) \frac{\partial u(\varrho,\psi)}{\partial \varrho} K(\varphi,\psi)\, d\psi - u(\varrho,\varphi)\right)\bigg|_{\varrho=R_1} = 0, \qquad 0 \le \varphi \le \pi, \tag{24}$$

$$\left(R_2 \int_0^\pi \mu(\varrho,\psi) \frac{\partial u(\varrho,\psi)}{\partial \varrho} K(\varphi,\psi)\, d\psi + u(\varrho,\varphi)\right)\bigg|_{\varrho=R_2} = f_0(\varphi), \qquad 0 \le \varphi \le \pi. \tag{25}$$

For the numerical implementation of boundary value problem 4, we make the change of variables

$$\mu = 10^3\bar{\mu}, \quad H_0 = 10^2\bar{H}_0, \quad H = 10^2\bar{H}, \quad \Delta = 10^{-3}h, \quad R_2 = 10^{-2}R,$$

where h is the screen thickness in millimeters, R is the outer radius of the cylindrical screen in centimeters, and $\bar{\mu}$ is the normalized relative permittivity.

We introduce new coordinates (x, y) by setting

$$\varphi = y, \qquad \varrho = (R_2 - R_1)x + R_1 = \Delta(x + \bar{\alpha}), \tag{26}$$

where $\bar{\alpha} = \alpha - 1$, $\alpha = R_2/\Delta = 10R/h$.

We write Eq. (21) and the boundary conditions (22) and (25) in the new variables (26) and state the following boundary value problem.

Boundary value problem 5. Given a potential u_0 (7), find a potential $\overline{u}(x, y)$ in the domain Ω $(0 \le x \le 1,\ 0 \le y \le \pi)$ such that

$$\frac{\partial}{\partial x}\left(A(x,\bar{\mu})\frac{\partial \overline{u}}{\partial x}\right) + \frac{\partial}{\partial y}\left(B(x,\bar{\mu})\frac{\partial \overline{u}}{\partial y}\right) = 0, \qquad 0 < x < 1, \qquad 0 < y < \pi, \tag{27}$$

$$\left(\int\limits_0^\pi A(x,\bar{\mu})\frac{\partial \overline{u}(x,\psi)}{\partial x}K(y,\psi)\,d\psi - 10^{-3}\overline{u}(x,y)\right)\Bigg|_{x=0} = 0, \qquad 0 \le y \le \pi, \tag{28}$$

$$\left(\int\limits_0^\pi A(x,\bar{\mu})\frac{\partial \overline{u}(x,\psi)}{\partial x}K(y,\psi)\,d\psi + 10^{-3}\overline{u}(x,y)\right)\Bigg|_{x=1} = 10^{-3}f_0(y), \qquad 0 \le y \le \pi, \tag{29}$$

$$\frac{\partial \overline{u}}{\partial y}\bigg|_{y=0} = 0, \qquad \frac{\partial \overline{u}}{\partial y}\bigg|_{y=\pi} = 0, \qquad 0 \le x \le 1, \tag{30}$$

where $\overline{u}(x, y) = u(\Delta(x + \overline{\alpha}), y)$ and

$$A(x,\bar{\mu}) = (x + \bar{\alpha})\bar{\mu}(\bar{H}), \qquad B(x,\bar{\mu}) = \frac{1}{x + \bar{\alpha}}\bar{\mu}(\bar{H}), \tag{31}$$

$$\bar{H} = \bar{H}\left(x, \frac{\partial \overline{u}}{\partial x}, \frac{\partial \overline{u}}{\partial y}\right) = \frac{10}{h}\sqrt{\left(\frac{\partial \overline{u}}{\partial x}\right)^2 + \frac{1}{(x+\bar{\alpha})^2}\left(\frac{\partial \overline{u}}{\partial y}\right)^2}. \tag{32}$$

Boundary value problem 5 is a problem with nonlocal integral conditions. Problems with nonlocal conditions are a rapidly developing trend in modern theory of differential equations. Problems with nonlocal integral conditions are an especially important subclass of such problems. The solvability of some classes of boundary value problems with integral conditions was studied in [16–18].

Ferromagnetics can have large magnetic flux density **B** even for relatively small magnetic field intensity **H**. The relationship between these variables is in general nonlinear owing to saturation and hysteresis. The dependence of permittivity on the magnetic field intensity is given by the formula [1, 9]

$$\bar{\mu}(\bar{H}) = \frac{B_m(h)\bar{H} + C_1(h)}{\bar{H}^2 + C_2(h)\bar{H} + C_1(h)}, \qquad 0 < h < 0.2, \tag{33}$$

where

$$C_1(h) = \frac{M(h)H_m^2(h)}{M(h) - 1}, \qquad C_2(h) = \frac{B_m(h)}{M(h)} - 2H_m(h),$$
$$M(h) = 40.386796h^2 + 9.94182h + 7.80194,$$
$$B_m(h) = 269.21679h^2 - 100.3814h + 12.346025,$$
$$H_m(h) = 13.461364h^2 - 5.0167164h + 0.6173063.$$

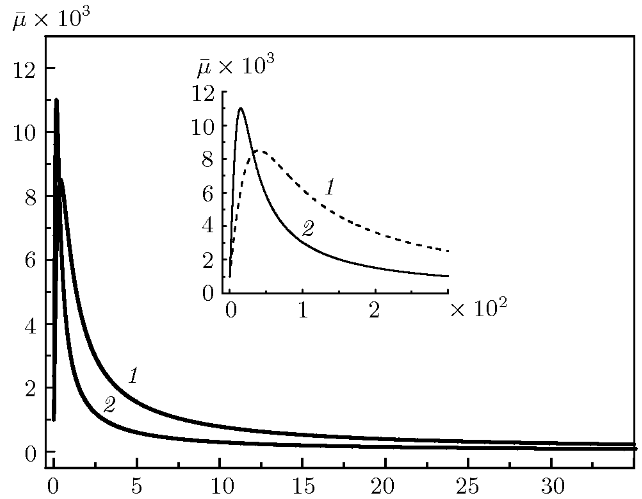

Fig. 2. Permittivity versus magnetic field intensity: *1*, $B_m = 8$, $H_m = 0.4$, and $M = 8.5$ ($h = 0.05\,\text{mm}$); *2*, $B_m = 3$, $H_m = 0.15$, and $M = 11$ ($h = 0.18\,\text{mm}$).

Figure 2 shows permittivity against magnetic field intensity computed by formula (33) for screens of various thickness, where $\bar{\mu}(\bar{H}) = (8\bar{H} + 0.18133)/(\bar{H}^2 + 0.14118\bar{H} + 0.18133)$ for $h = 0.05\,\text{mm}$ and $\bar{\mu}(\bar{H}) = (3\bar{H} + 0.02475) \times (\bar{H}^2 - 0.02727\bar{H} + 0.02475)^{-1}$ for $h = 0.18\,\text{mm}$.

5. NUMERICAL SOLUTION OF SHIELDING BOUNDARY VALUE PROBLEMS

In the papers [8, 9], a numerical method for solving shielding boundary value problems with boundary conditions of the third kind in the case of a constant magnetic field $\mathbf{H}_0$ with initial potential of the form $u_0 = H_0\varrho\cos\varphi$ was suggested and a series of numerical experiments were carried out to study the distributions of the potential, the magnetic field intensity, and the material permittivity in the film layer.

In what follows, we present a numerical method for solving a shielding boundary value problem with integral boundary conditions.

To solve the nonelliptic problem (27)–(32), we use the control volume method, in which differential equations are replaced by their finite-difference mesh counterparts [22, p. 156].

In the solution domain Ω, we construct a uniform grid $\bar{\omega} = \bar{\omega}_x \times \bar{\omega}_y$, where $\bar{\omega}_x = \{x_i = i\Delta x,\ \Delta x = 1/N,\ i = 0,\dots,N\}$, $\bar{\omega}_y = \{y_j = j\Delta y,\ \Delta y = \pi/M,\ j = 0,\dots,M\}$, Δx and Δy are the grid increments in the corresponding directions, and (x_i, y_j) are points of the grid $\bar{\omega}$. The values of the unknown function $\overline{u}(x,y)$ at the grid points will be denoted by $u_{i,j}$; i.e., $u_{i,j} = \overline{u}(x_i, y_j)$, $i = 0,\dots,N$, $j = 0,\dots,M$. The difference derivatives at the grid points will be denoted according to [22, p. 11] by $u_{x;i,j} = (u_{i+1,j} - u_{i,y})/\Delta x$, $u_{\bar{x};i,j} = (u_{i,j} - u_{i-1,j})/\Delta x$, $u_{y;i,j} = (u_{i,j+1} - u_{i,j})/\Delta y$, and $u_{\bar{y};i,j} = (u_{i,j} - u_{i,j-1})/\Delta y$.

By analogy with [9], we use the control volume method to construct an implicit finite-difference scheme approximating the differential equation (27) at the interior points of Ω. We integrate Eq. (27) over the control volume $[x_{i-0.5}, x_{i+0.5}] \times [y_{j-0.5}, y_{j+0.5}]$ surrounding the grid point (x_i, y_j) at which we seek the solution $u_{i,j}$, use some averaging of integrals, and arrive at the finite-difference scheme

$$(A_{i+0.5,j}u_{x;i,j})_{\bar{x};i,j} + (B_{i,j+0.5}u_{y;i,j})_{\bar{y};i,j} = 0, \qquad i = 1,\dots,N-1, \qquad j = 1,\dots,M-1, \tag{34}$$

where

$$A_{i+0.5,j} = (x_i + 0.5\Delta x + \bar{\alpha})\bar{\mu}_{i+0.5,j}, \qquad B_{i,j+0.5} = \frac{\bar{\mu}_{i,j+0.5}}{x_i + \bar{\alpha}},$$

$$\bar{\mu}_{i+0.5,j} = \frac{B_m\bar{H}_{i+0.5,j} + C_1}{\bar{H}^2_{i+0.5,j} + C_2\bar{H}_{i+0.5,j} + C_1}, \qquad \bar{\mu}_{i,j+0.5} = \frac{B_m\bar{H}_{i,j+0.5} + C_1}{\bar{H}^2_{i,j+0.5} + C_2\bar{H}_{i,j+0.5} + C_1},$$

$$\bar{H}_{i+0.5,j} = \frac{10}{h}\sqrt{(u_{x;i,j})^2 + \frac{1}{(x_i + 0.5\Delta x + \bar{\alpha})^2}(u_{cp,y}^{(i+0.5,j)})^2},$$

$$\bar{H}_{i,j+0.5} = \frac{10}{h}\sqrt{(u_{cp,x}^{(i,j+0.5)})^2 + \frac{1}{(x_i + \bar{\alpha})^2}(u_{y;i,j})^2},$$

$$u_{cp,y}^{(i+0.5,j)} = \frac{\tilde{u}_{i+0.5,j+0.5} - \tilde{u}_{i+0.5,j-0.5}}{\Delta y}, \qquad \tilde{u}_{i+0.5,j+0.5} = \frac{u_{i,j} + u_{i+1,j} + u_{i,j+1} + u_{i+1,j+1}}{4},$$

$$u_{cp,x}^{(i,j+0.5)} = \frac{\tilde{u}_{i+0.5,j+0.5} - \tilde{u}_{i-0.5,j+0.5}}{\Delta x}.$$

Assertion 1. *The finite-difference scheme* (34) *approximates Eq.* (27) *with second-order accuracy* $O((\Delta x)^2 + (\Delta y)^2)$.

Let us construct an approximation to the integral boundary conditions (28) and (29). Set

$$W(\tilde{x}, \psi) = A(x, \bar{\mu})\frac{\partial \overline{u}(x, \psi)}{\partial x}\bigg|_{x=\tilde{x}}, \quad \text{where} \quad \tilde{x} = \{0, 1\}.$$

Consider the integral $J^{(y)} = \int_0^\pi W(\tilde{x}, \psi)K(y, \psi)\,d\psi$. It follows from the form of the function $K(y, \psi)$ (15) that it has a singularity at $y = \psi$.

In view of the singularity of $K(y, \psi)$, for $y = 0$ we transform the integral

$$J^{(0)} = \int_0^\pi W(\tilde{x}, \psi)K(0, \psi)\,d\psi = \int_0^{\overline{y}} W(\tilde{x}, \psi)K(0, \psi)\,d\psi + \int_{\overline{y}}^\pi W(\tilde{x}, \psi)K(0, \psi)\,d\psi = J_0 + J_0^+, \quad (35)$$

where $\overline{y}$ is a small number.

Since $K(0, \psi) = -\pi^{-1}\ln(4\sin^2(\psi/2))$ and $\sin^2(\psi/2) \approx \psi^2/4$, because $\overline{y}$ is small, we obtain

$$J_0 = -\frac{1}{\pi}\int_0^{\overline{y}} W(\tilde{x}, \psi)\ln\left(4\sin^2\left(\frac{\psi}{2}\right)\right)d\psi \approx -\frac{1}{\pi}W(\tilde{x}, 0)\int_0^{\overline{y}}\ln(\psi^2)\,d\psi$$

$$= -\frac{2}{\pi}W(\tilde{x}, 0)\int_0^{\overline{y}}\ln\psi\,d\psi = -\frac{2}{\pi}\overline{y}(\ln\overline{y} - 1)W(\tilde{x}, 0).$$

The integral J_0^+ has no singularities.

At the points of the boundary $0 < y < \pi$, we have

$$J^{(y)} = \int_0^\pi W(\tilde{x}, \psi)K(y, \psi)\,d\psi = \int_0^{y-\overline{y}} W(\tilde{x}, \psi)K(y, \psi)\,d\psi$$

$$+ \int_{y-\overline{y}}^{y+\overline{y}} W(\tilde{x}, \psi)K(y, \psi)\,d\psi + \int_{y+\overline{y}}^\pi W(\tilde{x}, \psi)K(y, \psi)\,d\psi = J_y^- + J_y + J_y^+. \quad (36)$$

By analogy with J_0, since $\overline{y}$ is small, we obtain

$$J_y \approx -\frac{2}{\pi}\overline{y}(\ln 2\overline{y} - 1)W(\tilde{x}, y) + \int_{y-\overline{y}}^{y+\overline{y}} W(\tilde{x}, \psi)L(y, \psi)\,d\psi = -\frac{2}{\pi}\overline{y}(\ln 2\overline{y} - 1)W(\tilde{x}, y) + \widetilde{J}_y, \quad (37)$$

where $L(y, \psi) = -\pi^{-1}\ln\sin((y + \psi)/2)$ and the integral $\widetilde{J}_y$ does not have singularities.

For $y=\pi$, one has

$$J^{(\pi)}=\int\limits_0^{\pi}W(\tilde{x},\psi)K(\pi,\psi)\,d\psi=\int\limits_0^{\pi-\overline{y}}W(\tilde{x},\psi)K(\pi,\psi)\,d\psi+\int\limits_{\overline{y}}^{\pi}W(\tilde{x},\psi)K(\pi,\psi)\,d\psi=J_\pi^-+J_\pi. \quad (38)$$

It is easily seen that $J_\pi\approx-2\pi^{-1}\overline{y}(\ln\overline{y}-1)W(\tilde{x},\pi)$.

We replace the integrals in (35)–(38), which do not have singularities, with their approximate values given by the trapezoidal quadrature formula. For the partition step on the corresponding integration intervals we take the spatial grid increment Δy; then $\psi_j=y_j$, $j=0,\ldots,M$. Set $\overline{y}=\Delta y$. As a result, say, the integral (35) becomes

$$J^{(0)}\approx-\frac{2}{\pi}\overline{y}(\ln\overline{y}-1)W(\tilde{x},y_0)+\frac{\Delta y}{2}W(\tilde{x},y_1)K_{0,1}+\Delta y\sum_{k=2}^{M-1}W(\tilde{x},y_k)K_{0,k}+\frac{\Delta y}{2}W(\tilde{x},y_M)K_{0,M},$$

where

$$K_{0,k}=K(y_0,y_k)=-\frac{1}{\pi}\ln\left[4\sin\left(\frac{|y_k-y_0|}{2}\right)\sin\left(\frac{y_k+y_0}{2}\right)\right],\qquad k=1,\ldots,M.$$

We construct approximations to the flux $W(\tilde{x},\psi)$ at the boundary points $\tilde{x}=0$ and $\tilde{x}=1$ with regard to the main equation (27) and the corresponding boundary conditions (30). For example, on the boundary $\tilde{x}=0$ we have

$$W(0,y_k)=\begin{cases}A_{0.5,0}u_{x;0,0}+(\Delta x/\Delta y)B_{0,0.5}u_{y;0,0}, & k=0,\\ A_{0.5,k}u_{x;0,k}+0.5\Delta x(B_{0,k+0.5}u_{y;0,k})_{\bar{y};0,k}, & k=1,\ldots,M-1,\\ A_{0.5,M}u_{x;0,M}-(\Delta x/\Delta y)B_{0,M-0.5}u_{\bar{y};0,M}, & k=M.\end{cases} \quad (39)$$

We substitute the resulting approximations to the integrals (35)–(38) with regard to Eq. (39) into the boundary condition (28) and obtain a finite-difference approximation to the boundary condition (28) in the form

$$\begin{aligned}0.5\Delta y\left(A_{0.5,0}u_{x;0,0}+\left(\frac{\Delta x}{\Delta y}\right)B_{0,0.5}u_{y;0,0}\right)R_{j,0}+\Delta y\sum_{k=1}^{M-1}(A_{0.5,k}u_{x;0,k}+0.5\Delta x(B_{0,k+0.5}u_{y;0,k})_{\bar{y};0,k})R_{j,k}\\ +0.5\Delta y\left(A_{0.5,M}u_{x;0,M}-\left(\frac{\Delta x}{\Delta y}\right)B_{0,M-0.5}u_{\bar{y};0,M}\right)R_{j,M}-10^{-3}u_{0,j}=0,\quad j=0,\ldots,M. \quad (40)\end{aligned}$$

In a similar way, one can construct a finite-difference approximation to the boundary condition (29),

$$\begin{aligned}&0.5\Delta y(-A_{N-0.5,0}u_{\bar{x};N,0}+(\Delta x/\Delta y)B_{N,0.5}u_{y;N,0})R_{j,0}\\ &\quad+\Delta y\sum_{k=1}^{M-1}(-A_{N-0.5,k}u_{\bar{x},N,k}+0.5\Delta x(B_{N,k+0.5}u_{y;N,k})_{\bar{y};N,k})R_{j,k}+0.5\Delta y(-A_{N-0.5,M}u_{\bar{x};N,M}\\ &\quad-(\Delta x/\Delta y)B_{N,M-0.5}u_{\bar{y};N,M})R_{j,M}-10^{-3}u_{N,j}=-10^{-3}f_0j,\qquad j=0,\ldots,M, \quad (41)\end{aligned}$$

where

$$R_{0,0}=-\frac{4}{\pi}(\ln\Delta y-1),\quad R_{0,1}=\frac{1}{2}K_{0,1},\quad R_{0,k}=K_{0,k},\quad k=2,\ldots,M;$$

$$R_{1,0}=L_{1,0},\quad R_{1,1}=L_{1,1}-\frac{2}{\pi}(\ln 2\Delta y-1),\quad R_{1,2}=\frac{1}{2}(L_{1,2}+K_{1,2}),\quad R_{1,k}=K_{1,k},\quad k=3,\ldots,M;$$

$$R_{j,k}=K_{j,k},\quad k=0,\ldots,j-2,\quad R_{j,j-1}=\frac{1}{2}(K_{j,j-1}+L_{j,j-1}),\quad R_{j,j}=L_{j,j}-\frac{2}{\pi}(\ln 2\Delta y-1),$$

$$R_{j,j+1} = \frac{1}{2}(K_{j,j+1} + L_{j,j+1}), \quad R_{j,k} = K_{j,k}, \quad k = j+2, \ldots, M \quad \text{for} \quad j = 2, \ldots, M-2;$$
$$R_{M-1,k} = K_{M-1,k}, \quad k = 0, \ldots, M-3, \quad R_{M-1,M-2} = \frac{1}{2}(K_{M-1,M-2} + L_{M-1,M-2}),$$
$$R_{M-1,M-1} = L_{M-1,M-1} - \frac{2}{\pi}(\ln 2\Delta y - 1), \quad R_{M-1,M} = L_{M-1,M};$$
$$R_{M,k} = K_{M,k}, \quad k = 0, \ldots, M-2, \quad R_{M,M-1} = \frac{1}{2}K_{M,M-1}, \quad R_{M,M} = -\frac{4}{\pi}(\ln \Delta y - 1);$$
$$f_{0j} = f_0(y_j), \quad j = 0, \ldots, M,$$

and, in turn,

$$K_{j,k} = K(y_j, \psi_k) = K(y_j, y_k) = -\frac{1}{\pi} \ln \left[4 \sin\left(\frac{|y_k - y_j|}{2} \right) \sin\left(\frac{y_k + y_j}{2} \right) \right], \; j, k = 0, \ldots, M, \; j \neq k;$$
$$L_{j,k} = L(y_j, \psi_k) = L(y_j, y_k) = -\frac{1}{\pi} \ln \left[\sin\left(\frac{|y_j + y_k|}{2} \right) \right], \quad j, k = 1, \ldots, M-1.$$

We take into account the singularities of the integrand in the integrals (35)–(38) and obtain the following assertion.

Assertion 2. *The finite-difference schemes* (40) *and* (41) *approximate the respective boundary conditions* (28) *and* (29) *with accuracy* $O(((\Delta x)^2 + (\Delta y)^2)\ln(\Delta y))$.

For the boundary conditions (30), we construct the following finite-difference approximations:

$$B_{i,0.5} u_{y;i,0} + 0.5\Delta y (A_{i+0.5,0} u_{x;i,0})_{\bar{x};i,0} = 0, \qquad i = 1, \ldots, N-1, \tag{42}$$
$$B_{i,M-0.5} u_{\bar{y};i,M} - 0.5\Delta y (A_{i+0.5,M} u_{x;i,M})_{\bar{x};i,M} = 0, \qquad i = 1, \ldots, N-1. \tag{43}$$

Assertion 3. *The finite-difference schemes* (42) *and* (43) *approximate the respective boundary conditions in* (30) *with second-order accuracy* $O((\Delta x)^2 + (\Delta y)^2)$.

The finite-difference problem (34), (40)–(43) is a system of nonlinear equations for the unknowns $u_{i,j}$. To solve it, we use the matrix Thomas method [22, p. 557], which is implemented by the iterative process

$$\begin{aligned} \overset{s}{C}_0 \overset{s+1}{U}_0 - \overset{s}{B}_0 \overset{s+1}{U}_1 &= \overset{s}{F}_0, \qquad i = 0, \\ -\overset{s}{A}_i \overset{s+1}{U}_{i-1} + \overset{s}{C}_i \overset{s+1}{U}_i - \overset{s}{B}_i \overset{s+1}{U}_{i+1} &= \overset{s}{F}_i, \qquad i = 1, \ldots, N-1, \\ -\overset{s}{A}_N \overset{s+1}{U}_{N-1} + \overset{s}{C}_N \overset{s+1}{U}_N &= \overset{s}{F}_N, \qquad i = N, \end{aligned} \tag{44}$$

where $s = 0, 1, 2, \ldots$ is the iteration number.

Here $\overset{s+1}{U}_i = (\overset{s+1}{u}_{i,0}, \overset{s+1}{u}_{i,1}, \ldots, \overset{s+1}{u}_{i,M})^{\mathrm{T}}$, $i = 0, \ldots, N$, are the unknown vectors; $\overset{s}{F}_i$ are $(M+1)$-vectors, $F_i = (0, 0, \ldots, 0)^{\mathrm{T}}$, $i = 0, \ldots, N-1$, $F_N = (f_0^N, f_1^N, \ldots, f_M^N)^{\mathrm{T}}$, and $f_j^N = 10^{-3} f_0 j$, $j = 0, \ldots, M$; $A_i = \operatorname{diag}[a_{i,0}, a_{i,1}, \ldots, a_{i,M}]$, $i = 1, \ldots, N-1$, are the matrices with diagonal entries $a_{i,j} = A_{i-0.5,j}(\Delta x)^{-2}$, $j = 0, \ldots, M$, $A_N = (a_{j,k}^N)_{j,k=0}^M$ is the matrix with entries

$$a_{j,0}^N = A_{N-0.5,0} \frac{\Delta y}{2\Delta x} R_{j,0}, \qquad a_{j,k}^N = A_{N-0.5,k} \frac{\Delta y}{\Delta x} R_{j,k}, \qquad k = 1, \ldots, M-1,$$
$$a_{j,M}^N = A_{N-0.5,M} \frac{\Delta y}{2\Delta x} R_{j,M}, \qquad j = 0, \ldots, M;$$

$B_0 = (b^0_{j,k})^M_{j,k=0}$ is the matrix with entries

$$b^0_{j,0} = A_{0.5,0}\frac{\Delta y}{2\Delta x}R_{j,0}, \qquad b^0_{j,k} = A_{0.5,k}\frac{\Delta y}{\Delta x}R_{j,k}, \qquad k = 1,\dots,M-1,$$
$$b^0_{j,M} = A_{0.5,M}\frac{\Delta y}{2\Delta x}R_{j,M}, \qquad j = 0,\dots,M;$$

$B_i = \operatorname{diag}[b_{i,0}, b_{i,1},\dots,b_{i,M}]$, $i = 1,\dots,N-1$, are the matrices with entries $b_{i,j} = A_{i+0.5,j}(\Delta x)^{-2}$, $j=0,\dots,M$; C_i are the tridiagonal $(M+1)\times(M+1)$ matrices

$$C_i = \begin{pmatrix} p_{i,0} & s_{i,0} & 0 & 0 & \cdots & 0 \\ q_{i,1} & p_{i,1} & s_{i,1} & 0 & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \cdots & 0 & q_{i,M-1} & p_{i,M-1} & s_{i,M-1} \\ 0 & \cdots & \cdots & 0 & q_{i,M} & p_{i,M} \end{pmatrix}, \qquad i = 1,\dots,N-1,$$

with entries

$$p_{i,0} = \frac{2B_{i,0.5}}{(\Delta y)^2} + \frac{A_{i+0.5,0} + A_{i-0.5,0}}{(\Delta x)^2}, \qquad s_{i,0} = -\frac{2B_{i,0.5}}{(\Delta y)^2};$$
$$q_{i,j} = -\frac{B_{i,j-0.5}}{(\Delta y)^2}, \qquad p_{i,j} = \frac{A_{i+0.5,j} + A_{i-0.5,j}}{(\Delta x)^2} + \frac{B_{i,j+0.5} + B_{i,j-0.5}}{(\Delta y)^2},$$
$$s_{i,j} = -\frac{B_{i,j+0.5}}{(\Delta y)^2}, \qquad j = 1,\dots,M-1;$$
$$p_{i,M} = \frac{2B_{i,M-0.5}}{(\Delta y)^2} + \frac{A_{i+0.5,M} + A_{i-0.5,M}}{(\Delta x)^2}, \qquad q_{i,M} = -\frac{2B_{i,M-0.5}}{(\Delta y)^2};$$

$C_0 = (c^0_{j,k})^M_{j,k=0}$ and $C_N = (c^N_{j,k})^M_{j,k=0}$ are matrices such that the entries of C_0 have the form

$$c^0_{j,0} = \left(B_{0,0.5}\frac{\Delta x}{2\Delta y} + A_{0.5,0}\frac{\Delta y}{2\Delta x}\right)R_{j,0} - B_{0,0.5}\frac{\Delta x}{2\Delta y}R_{j,1} + 10^{-3}\delta_{j,0},$$
$$c^0_{j,k} = -B_{0,k-0.5}\frac{\Delta x}{2\Delta y}R_{j,k-1} + \left(A_{0.5,k}\frac{\Delta y}{\Delta x} + (B_{0,k+0.5} + B_{0,k-0.5})\frac{\Delta x}{2\Delta y}\right)R_{j,k}$$
$$- B_{0,k+0.5}\frac{\Delta x}{2\Delta y}R_{j,k+1} + 10^{-3}\delta_{j,k}, \quad k = 1,\dots,M-1,$$
$$c^0_{j,M} = \left(B_{0,M-0.5}\frac{\Delta x}{2\Delta y} + A_{0.5,M}\frac{\Delta y}{2\Delta x}\right)R_{j,M} - B_{0,M-0.5}\frac{\Delta x}{2\Delta y}R_{j,M-1} + 10^{-3}\delta_{j,M}, \quad j = 0,\dots,M,$$

and the entries of C_N have the form

$$c^N_{j,0} = \left(B_{N,0.5}\frac{\Delta x}{2\Delta y} + A_{N-0.5,0}\frac{\Delta y}{2\Delta x}\right)R_{j,0} - B_{N,0.5}\frac{\Delta x}{2\Delta y}R_{j,1} + 10^{-3}\delta_{j,0},$$
$$c^N_{j,k} = -B_{N,k-0.5}\frac{\Delta x}{2\Delta y}R_{j,k-1} + \left(A_{N-0.5,k}\frac{\Delta y}{\Delta x} + (B_{N,k+0.5} + B_{N,k-0.5})\frac{\Delta x}{2\Delta y}\right)R_{j,k}$$
$$- B_{N,k+0.5}\frac{\Delta x}{2\Delta y}R_{j,k+1} + 10^{-3}\delta_{j,k}, \quad k = 1,\dots,M-1,$$
$$c^N_{j,M} = \left(B_{N,M-0.5}\frac{\Delta x}{2\Delta y} + A_{N-0.5,M}\frac{\Delta y}{2\Delta x}\right)R_{j,M} - B_{N,M-0.5}\frac{\Delta x}{2\Delta y}R_{j,M-1} + 10^{-3}\delta_{j,M}, \quad j = 0,\dots,M.$$

Here $\delta_{j,k}$ is the Kronecker delta.

For the zero approximation $\overset{0}{U}_i = (\overset{0}{u}_{i,0}, \overset{0}{u}_{i,1}, \ldots, \overset{0}{u}_{i,M})^{\mathrm{T}}$, $i = 0, \ldots, N$, in the iterative process (44) we take the solution of problem (27)–(30) with $\bar{\mu} = \text{const}$ (for example, $\bar{\mu} = 6$). System (34), (40)–(43) is linear in this case and can be solved exactly by the matrix Thomas method.

We use this zero approximation to find the next iteration

$$\overset{1}{U}_i = (\overset{1}{u}_{i,0}, \overset{1}{u}_{i,1}, \ldots, \overset{1}{u}_{i,M})^{\mathrm{T}}, \qquad i = 0, \ldots, N.$$

The process is continued recursively. The current discrepancy is compared with the desired accuracy to decide whether to stop the iterative process.

As a result, we obtain a solution $u_{i,j} = \overline{u}(x_i, y_j)$, $i = 0, \ldots, N$, $j = 0, \ldots, M$.

6. COMPUTATION OF THE SHIELDING EFFICIENCY COEFFICIENT

For the numerical study of shielding properties of film screens, consider screens of radius $R_2 = 1.1 \times 10^{-2}$ m, 3.6×10^{-2} m, and 8.0×10^{-2} m and of thickness $\triangle = 5 \times 10^{-5}$ m and 1.8×10^{-4} m ($h = 0.05$ mm and 0.18 mm).

To compute the shielding efficiency coefficient, we take into account Eqs. (1) and the series (9) and compute the magnetic field in the domain D_1,

$$\mathbf{H}_1 = -\sum_{n=1}^{\infty} \frac{n\alpha_n}{R_1^n} \varrho^{n-1} (\cos(n\varphi)\mathbf{e}_\varrho - \sin(n\varphi)\mathbf{e}_\varphi).$$

At the screen center, one has $\mathbf{H}_1(0) = -(\alpha_1/R_1)\mathbf{e}_x$ and $H_1 = |\mathbf{H}_1(0)| = \alpha_1/R_1$.

We multiply Eq. (11) by $\cos\varphi$ and integrate; then we obtain $\int_0^\pi u(\varrho, \varphi)|_{\varrho=R_1} \cos\varphi \, d\varphi = \alpha_1 \pi/2$. As a result, the shielding coefficient is given by

$$K_{\text{shield}} = \frac{H_0}{H_1} = \frac{1}{2}\pi R_1 H_0 \left| \int_0^\pi u(\varrho, \varphi)|_{\varrho=R_1} \cos\varphi \, d\varphi \right|^{-1}.$$

The potential u was determined on the basis of a numerical solution of the boundary value problem (27)–(32).

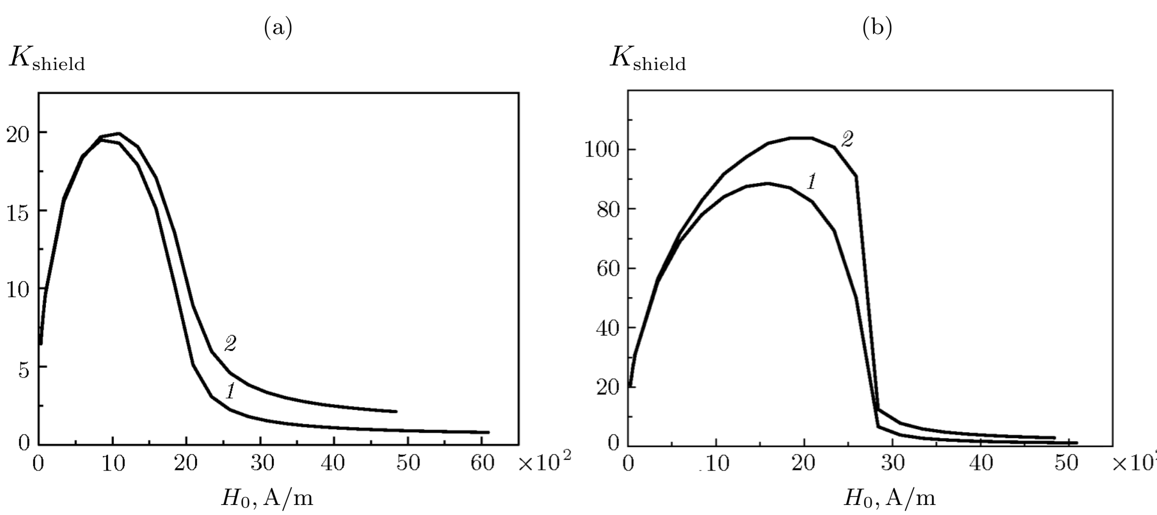

Fig. 3. Shielding efficiency coefficient versus the external magnetic field computed according to boundary value problem 4 (curves *1*) and boundary value problem 3 (curves *2*) for screens of radius $R_2 = 1.1 \times 10^{-2}$ m and of various thickness: (a) a screen of thickness $h = 0.05$ mm; (b) a screen of thickness $h = 0.18$ mm.

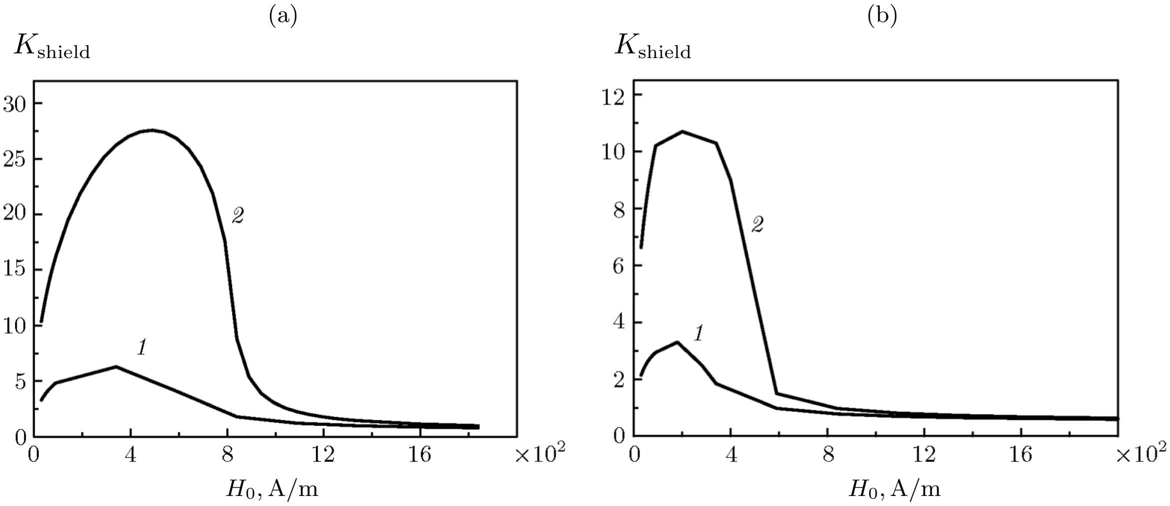

Fig. 4. Shielding efficiency coefficients for screens of various radii R_2 and thickness $h = 0.05$ (curves *1*) and $h = 0.18$ (curves *2*): (a) $R_2 = 3.6 \times 10^{-2}$ m; (b) $R_2 = 8.0 \times 10^{-2}$ m.

Figure 3 presents the shielding efficiency coefficient against the external magnetic field intensity H_0 as given by the solution of the boundary value problem with the integral boundary conditions (25) (curves *1*) and the problem with conditions (24) of the third kind on the screen surface (curves *2*) [8]. A comparison of curves *1* and *2* shows that, for practical studies of thin cylindrical screens, simple boundary conditions (i.e., conditions of the third kind) can be used.

Figure 4 presents the results of modeling of shielding efficiency coefficients for cylindrical screens of various radii and various thicknesses. It is shown that the larger the screen radius, the smaller the shielding efficiency is in accordance with the elementary formula given in [8].

7. CONCLUSION

A technique for modeling the penetration of the external magnetic field into an infinitely long cylindrical shell (screen) made of a material whose permittivity depends nonlinearly on the magnetic field intensity is developed. The original three-domain problem (the domain inside the shell, the infinite exterior domain, and the thin cylindrical film layer) is reduced to a boundary value problem for the nonlinear magnetostatic equation in the film layer with integral boundary conditions on the interior and exterior film surfaces. A number of models of boundary value problems are stated describing the action exerted on the screen by a constant magnetic field and a field generated by a conducting twin line parallel to the cylindrical screen. Integral boundary conditions of various types and boundary conditions of the third kind are used. For the comparative analysis of the models, a numerical method for solving boundary value problems with integral boundary conditions is developed. The dependence of the permittivity of the film on the magnetic field intensity is taken according to experimental data for permalloy $Fe_{20}Ni_{80}$. A numerical study of the shielding efficiency coefficient, which is the factor by which the external magnetic field is attenuated when passing through the film, is carried out. It is shown that, for thin films with a sufficiently small ratio of the film thickness Δ to the outer screen radius R_2 ($\Delta/R_2 < 10^{-2}$), modeling with integral conditions and modeling with boundary conditions of the third kind yield practically the same results for the case of a constant external magnetic field.

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