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PARTIAL DIFFERENTIAL EQUATIONS

Boundary Value Problems for a Nonstrictly Hyperbolic Equation of the Third Order

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Abstract—We study classical solutions of boundary value problems for a nonstrictly hyperbolic third-order equation. The equation is posed in a half-strip and a quadrant of the plane of two independent variables. The Cauchy conditions are posed on the lower boundary of the domain, and the Dirichlet conditions are posed on the lateral boundaries. By using the method of characteristics, we find the analytic form of the solution of considered problems. The uniqueness of the solutions is proved.

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1. INTRODUCTION

The study of boundary value problems for a nonstrictly hyperbolic third-order equation is motivated not only by the development of the theory of partial differential equations. They arise in the description of particular physical phenomena, for example, in the mathematical modeling of the propagation of linear acoustic waves in a dispersive medium [1, p. 87]. The properties of such equations and problems were studied in [2, 3].

A large part of publications on hyperbolic equations deal with the Cauchy problem. Generalized solutions of mixed problems of third-order hyperbolic equations were considered in [4–6], and existence and uniqueness theorems were proved for such solutions in appropriate function spaces. Note also the papers [7–9], where boundary value problems on the plane were studied by functional methods in the case of two independent variables.

The study or construction of classical solutions of problems is topical for the theory of partial differential equations and for numerical methods for boundary value problems. Note that the classical solutions of such problems are determined not only by the choice of the form of boundary conditions for partial differential equations but also by matching conditions for the functions occurring in the conditions and the equations. The paper [10] deals with the classical solution and considers the first mixed problem for the simplest third-order hyperbolic equation with distinct characteristics. To the best of the author's knowledge, the present paper is the first to construct the classical solutions of the mixed problem and the Cauchy problem with a nonsmooth boundary for a nonstrictly hyperbolic third-order equation with multiple characteristics in closed form.

2. STATEMENT OF THE PROBLEM

In the domain $Q = (0, +\infty) \times (0, l)$ of two independent variables, $(t, x) \in Q \subset \mathbb{R}^2$, consider the nonstrictly hyperbolic third-order equation

$$(\partial_t - a\partial_x + b)^3 u(t, x) = f(t, x), \qquad (t, x) \in \overline{Q} = [0, \infty) \times [0, l], \tag{1}$$

where a, b, and l are real numbers, $\overline{Q}$ is the closure of the domain Q, and ∂_t and ∂_x are the partial derivatives with respect to t and x, respectively. In the general case, $\partial_t^k \partial_x^p = \dfrac{\partial^{k+p}}{\partial_t^k \partial_x^p}$ are the partial derivatives with respect to t and x of order $k+p$, where k and p are nonnegative integers. To be definite, we assume that $a<0$. On the lower side of the domain Q, we supplement Eq. (1) with the Cauchy conditions

$$\partial_t^i u(0,x)=\varphi_i(x), \qquad i=0,1,2, \qquad x\in[0,l], \tag{2}$$

and, on the lateral part of the boundary ∂Q, we pose either the boundary conditions

$$\partial_x^i u(t,0)=\psi_i(t), \qquad i=0,1,2, \qquad t\in[0,+\infty), \tag{3}$$

or the conditions

$$\partial_x^i u(t,0)=\psi_i(t), \quad i=0,1, \quad t\in[0,+\infty), \quad u(t,l)=\mu(t), \quad t\in[-l/a,+\infty). \tag{4}$$

We study the boundary value problems (1)–(3) and (1), (2), (4) for the case in which Eq. (1) is homogeneous; i.e.,

$$(\partial_t-a\partial_x+b)^3u(t,x)=0, \qquad (t,x)\in\overline{Q}. \tag{5}$$

By [11], the general solution of Eq. (5) can be represented as a linear combination of three arbitrary functions,

$$u(t,x)=e^{-bt}f_1(x+at)+e^{-bt}f_2(x+at)t+e^{-bt}f_3(x+at)t^2 \tag{6}$$

with the corresponding domains $D(f_i)$, $i=1,2,3$. One can readily see that $D(f_i)=(-\infty,l]$, $i=1,2,3$, if $(t,x)\in\overline{Q}$. By $C^3(\overline{Q})$ we denote the set of three times continuously differentiable functions defined on $\overline{Q}$, and by $C^{i,j}(\overline{Q})$ we denote the set of functions that are defined on $\overline{Q}$ and are i times continuously differentiable with respect to the first argument and j times continuously differentiable with respect to the second argument.

We formulate the obtained result in the form of a lemma.

Lemma 1. *The general solution of Eq.* (5) *in the class* $C^3(\overline{Q})$ *can be represented in the form* (6), *where the* f_i *are arbitrary three times continuously differentiable functions on the half-open interval* $(-\infty,l]$.

Therefore, to find the solutions $u:\ \mathbb{R}^2\supset\overline{Q}\ni(t,x)\to u(t,x)\in\mathbb{R}$ of problems (2), (3), (5) and (2), (4), (5), one should choose functions f_i, $i=1,2,3$, such that the sum (6) satisfies conditions (2), (3) and (2), (4), respectively.

3. MATCHING CONDITIONS

First, consider problem (2), (3), (5).

For the values of functions and their derivatives in the case of one independent variable, we introduce the following notation. Let $g:\ \mathbb{R}\to g(z)$ be a function of the variable z. Then $d^kg(z)=\dfrac{d^k}{dz^k}g(z)$ is the kth derivative; $g(a)$ and $d^kg(a)$ are the values of the function g and its derivative d^kg of order k at a point a, etc.

Lemma 2. *Suppose that the given functions satisfy the smoothness conditions*

$$\varphi_i\in C^{5-i}([0,l]), \qquad i=0,1,2, \qquad \psi_j\in C^{5-j}([0,+\infty)), \qquad j=0,1,2;$$

then problem (2), (3), (5) *is uniquely solvable in the class* $C^3(\overline{Q})$ *if and only if*

$$\begin{gathered}\varphi_0(0)=\psi_0(0), \qquad d\varphi_0(0)=\psi_1(0), \qquad d^2\varphi_0(0)=\psi_2(0),\\ a^3d^3\varphi_0(0)=b^3\psi_0(0)+3b^2(d\psi_0(0)-a\psi_1(0))\\ +3b(d^2\psi_0(0)-2ad\psi_1(0)+a^2\psi_2(0))+(d^3\psi_0(0)-3ad^2\psi_1(0)+3a^2d\psi_2(0)),\\ \varphi_1(0)=d\psi_0(0), \qquad d\varphi_1(0)=d\psi_1(0), \qquad d^2\varphi_1(0)=d\psi_2(0),\end{gathered} \tag{7}$$

$$
\begin{aligned}
&a^4d^4\varphi_0(0) - 3a^3bd^3\varphi_0(0) + 3a^2b^2d^2\varphi_0(0) - ab^3d\varphi_0(0)\\
&\qquad = a^3d^3\varphi_1(0) + 3a^2d^2\psi_2(0) - a(3b^2d\psi_1(0) + 9bd^2\psi_1(0) + 5d^3\psi_1(0))\\
&\qquad\quad + 2(b^3d\psi_0(0) + 3b^2d^2\psi_0(0) + 3bd^3\psi_0(0) + d^4\psi_0(0)),
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
&\varphi_2(0) = d^2\psi_0(0), \qquad d\varphi_2(0) = d^2\psi_1(0),\\
&a^3d^3\varphi_1(0) = a^2(d^2\varphi_2(0) + 2d^2\psi_2(0)) + b^3d\psi_0(0) + 3b^2d^2\psi_0(0) + d^4\psi_0(0)\\
&\qquad + 3b(a^2d^2\varphi_1(0) - 2ad^2\psi_1(0) + d^3\psi_0(0)) - 3a(b^2d\psi_1(0) + d^3\psi_1(0)),\\
&a^4(3bd^4\varphi_0(0) + 2d^4\varphi_1(0)) + a^2(b^3d^2\varphi_0(0) + d^3\psi_2(0)) + a(b^3d\psi_1(0) - 3bd^3\psi_1(0) - 2d^4\psi_1(0))\\
&\qquad + b^3d^2\psi_0(0) + 3b^2d^3\psi_0(0) + 3bd^4\psi_0(0) + d^5\psi_0(0)\\
&\qquad = a^3(a^2d^5\varphi_0(0) + 3b^2d^3\varphi_0(0) + 3bd^3\varphi_1(0) + d^3\varphi_2(0)).
\end{aligned} \tag{9}
$$

Proof. By substituting the general solution (6) into the initial conditions (2), we obtain the system of equations

$$
\begin{aligned}
f_1(x) &= \varphi_0(x),\\
-b\varphi_0(x) + adf_1(x) + f_2(x) &= \varphi_1(x),\\
-b\varphi_1(x) - abdf_1(x) + a^2d^2f_1(x) + 2adf_2(x) - bf_2(x) + 2f_3(x) &= \varphi_2(x)
\end{aligned} \tag{10}
$$

for $x \in [0, l]$. By solving (10), we find the functions f_i, $i = 1, 2, 3$, on the interval $[0, l]$,

$$
\begin{aligned}
&f_1(y) = \varphi_0(y),\\
&f_2(y) = \varphi_1(y) + b\varphi_0(y) - ad\varphi_0(y),\\
&f_3(y) = \frac{1}{2}\varphi_2(y) + b\varphi_1(y) + \frac{a^2}{2}d^2\varphi_0(y) - ad\varphi_1(y) - abd\varphi_0(y) + \frac{b^2}{2}\varphi_0(y).
\end{aligned} \tag{11}
$$

Therefore, the functions f_i, $i = 1, 2, 3$, are uniquely determined by the initial conditions (2) on the closed interval $[0, l]$. Consequently, for each point $(t, x) \in Q$, $0 \le x + at \le l$, the solution of problem (2), (3), (5) is given by the formula

$$
\begin{aligned}
u(t, x) = e^{-bt}\Biggl(&\varphi_0(x + at) + [\varphi_1(x + at) + b\varphi_0(x + at) - ad\varphi_0(x + at)]t\\
&+ \biggl[\frac{1}{2}\varphi_2(x + at) + b\varphi_1(x + at) + \frac{a^2}{2}d^2\varphi_0(x + at) - ad\varphi_1(x + at)\\
&- abd\varphi_0(x + at) + \frac{b^2}{2}\varphi_0(x + at)\biggr]t^2\Biggr).
\end{aligned}
$$

Next, we find the functions f_i, $i = 1, 2, 3$, on the half-open interval $(-\infty, 0]$.

By substituting the general solution (6) into the boundary conditions (3), we obtain the system of equations

$$
\begin{aligned}
e^{-bt}f_1(at) + e^{-bt}f_2(at)t + e^{-bt}f_3(at)t^2 &= \psi_0(t), \qquad t \in [0, +\infty),\\
e^{-bt}df_1(at) + e^{-bt}df_2(at)t + e^{-bt}df_3(at)t^2 &= \psi_1(t), \qquad t \in [0, +\infty),\\
e^{-bt}d^2f_1(at) + e^{-bt}d^2f_2(at)t + e^{-bt}d^2f_3(at)t^2 &= \psi_2(t), \qquad t \in [0, +\infty),
\end{aligned}
$$

which, after the multiplication by e^{bt}, acquires the form

$$
\begin{aligned}
f_1(at) + f_2(at)t + f_3(at)t^2 &= e^{bt}\psi_0(t), \qquad t \in [0, +\infty),\\
df_1(at) + df_2(at)t + df_3(at)t^2 &= e^{bt}\psi_1(t), \qquad t \in [0, +\infty),\\
d^2f_1(at) + d^2f_2(at)t + d^2f_3(at)t^2 &= e^{bt}\psi_2(t), \qquad t \in [0, +\infty).
\end{aligned}
$$

By computing the derivative of the first equation, we obtain the relation

$$
a[df_1(at) + df_2(at)t + df_3(at)t^2] + f_2(at) + 2f_3(at)t = be^{bt}\psi_0(t) + e^{bt}d\psi_0(t).
$$

Using the second equation of the system, we rewrite the last relation in the form

$$f_2(at) + 2f_3(at)t = be^{bt}\psi_0(t) + e^{bt}d\psi_0(t) - ae^{bt}\psi_1(t). \tag{12}$$

Likewise, from the second equation in the system, we obtain the relation

$$df_2(at) + 2df_3(at)t = be^{bt}\psi_1(t) + e^{bt}d\psi_1(t) - ae^{bt}\psi_2(t). \tag{13}$$

Next, we compute the derivative of Eq. (12),

$$a[df_2(at) + 2df_3(at)t] + 2f_3(at) = b^2e^{bt}\psi_0(t) + 2be^{bt}d\psi_0(t) + e^{bt}d^2\psi_0(t) - abe^{bt}\psi_1(t) - ae^{bt}d\psi_1(t).$$

We use Eq. (13); to this end, we rewrite it in the form

$$2f_3(at) = b^2e^{bt}\psi_0(t) + 2be^{bt}d\psi_0(t) + e^{bt}d^2\psi_0(t) - 2abe^{bt}\psi_1(t) - 2ae^{bt}d\psi_1(t) + a^2e^{bt}\psi_2(t);$$

then we obtain a system of equations without derivatives for the functions f_i, $i = 1, 2, 3$,

$$\begin{aligned} f_1(at) + f_2(at)t + f_3(at)t^2 &= e^{bt}\psi_0(t), \\ f_2(at) + 2f_3(at)t &= be^{bt}\psi_0(t) + e^{bt}d\psi_0(t) - ae^{bt}\psi_1(t), \\ 2f_3(at) &= b^2e^{bt}\psi_0(t) + 2be^{bt}d\psi_0(t) + e^{bt}d^2\psi_0(t) \\ &\quad - 2abe^{bt}\psi_1(t) - 2ae^{bt}d\psi_1(t) + a^2e^{bt}\psi_2(t), \\ &\quad t \in [0, +\infty). \end{aligned} \tag{14}$$

By solving it, we find the functions

$$\begin{aligned} f_1(at) &= e^{bt}\left((1 - bt)(\psi_0(t) + at\psi_1(t) - td\psi_0(t)) + \frac{t^2}{2}(d^2\psi_0(t) - 2ad\psi_1(t) + a^2\psi_2(t) + b^2\psi_0(t))\right), \\ f_2(at) &= e^{bt}(b(1 - bt)\psi_0(t) + (1 - 2bt)(d\psi_0(t) - a\psi_1(t)) - td^2\psi_0(t) + 2atd\psi_1(t) - a^2t\psi_2(t)), \\ f_3(at) &= e^{bt}\left(\frac{1}{2}b^2\psi_0(t) + bd\psi_0(t) + \frac{1}{2}d^2\psi_0(t) - ab\psi_1(t) - ad\psi_1(t) + \frac{1}{2}a^2\psi_2(t)\right), \qquad t \in [0, +\infty), \end{aligned}$$

which can be reduced by the change of variables $y = at$ to the form

$$\begin{aligned} f_1(y) &= \frac{e^{by/a}}{2a^2}\Bigg[2(a - by)\left(a\psi_0\left(\frac{y}{a}\right) + ay\psi_1\left(\frac{y}{a}\right) - yd\psi_0\left(\frac{y}{a}\right)\right) \\ &\quad + y^2\left(b^2\psi_0\left(\frac{y}{a}\right) + d^2\psi_0\left(\frac{y}{a}\right) - 2ad\psi_1\left(\frac{y}{a}\right) + a^2\psi_2\left(\frac{y}{a}\right)\right)\Bigg], \\ f_2(y) &= \frac{e^{by/a}}{a}\Bigg[b(a - by)\psi_0\left(\frac{y}{a}\right) + (a - 2by)\left(d\psi_0\left(\frac{y}{a}\right) - a\psi_1\left(\frac{y}{a}\right)\right) \\ &\quad - yd^2\psi_0\left(\frac{y}{a}\right) + 2ayd\psi_1\left(\frac{y}{a}\right) - a^2y\psi_2\left(\frac{y}{a}\right)\Bigg], \\ f_3(y) &= \frac{e^{by/a}}{2}\left[b^2\psi_0\left(\frac{y}{a}\right) + 2bd\psi_0\left(\frac{y}{a}\right) + d^2\psi_0\left(\frac{y}{a}\right) - 2ab\psi_1\left(\frac{y}{a}\right) - 2ad\psi_1\left(\frac{y}{a}\right) + a^2\psi_2\left(\frac{y}{a}\right)\right], \\ &\quad y \in (-\infty, 0]. \end{aligned}$$

As a result, the functions f_i are determined by the boundary conditions (3) on the half-open interval $(-\infty, 0]$. For the resulting solution to belong to the class $C^3(Q)$, it is necessary that the function $u(t, x)$, together with its derivatives, is continuous in the domain Q. This is equivalent to the fact that the functions f_i, together with their derivatives of order ≤ 3, are continuous at zero; i.e.,

$$d^i f_1^{(0)}(0) = d^i f_1^{(1)}(0), \qquad i = 0, \ldots, 3, \tag{15}$$

$$d^i f_2^{(0)}(0) = d^i f_2^{(1)}(0), \qquad i = 0, \ldots, 3, \tag{16}$$

$$d^i f_3^{(0)}(0) = d^i f_3^{(1)}(0), \qquad i = 0, \ldots, 3. \tag{17}$$

However, these conditions are equivalent to the relations in Lemma 2. Indeed, relation (15) is equivalent to condition (7); relation (16), to condition (8); and relation (17), to condition (9). Therefore, if conditions (7)–(9) are satisfied, then the solution that we have found belongs to the class $C^3(\overline{Q})$. The same reasoning implies the necessity in Lemma 2. Indeed, if the problem has a solution, then, from the general form of the solution and from the initial and boundary conditions, we find that the functions f_i, $i = 1, 2, 3$, are defined in the same way on the interval $[0, l]$ and on the half-open interval $(-\infty, 0]$. Since this solution belongs to the class $C^3(\overline{Q})$, it follows that relations (15)–(17) hold. This implies relations (7)–(9). The proof of Lemma 2 is complete.

Next, consider the boundary value problem (2), (4), (5).

Lemma 3. *Suppose that the given functions satisfy the smoothness conditions* $\varphi_i \in C^{5-i}([0, l])$, $i = 0, 1, 2$, $\psi_j \in C^{5-j}([0, +\infty))$, $j = 0, 1$, *and* $\mu \in C^3([-l/a, +\infty))$. *Problem* (2), (4), (5) *is uniquely solvable in the class* $C^3(\overline{Q})$ *if and only if the following matching conditions are satisfied*:

$$\begin{aligned}
&\varphi_0(0) = \psi_0(0), \qquad d\varphi_0(0) = \psi_1(0),\\
&2a^2(e^{by}\mu(y))|_{y=-l/a} - l^2d^2(e^{by}\psi_0(y))|_{y=0} + 2ald(e^{by}(l\psi_1(y) + \psi_0(y)))|_{y=0}\\
&\qquad = a^2(l^2d^2\varphi_0(0) + 2ld\varphi_0(0) + 2\varphi_0(0)),\\
&6a^2d(e^{by}\mu(y))|_{y=-l/a} - 2l^2d^3(e^{by}\psi_0(y))|_{y=0} + 3ald^2(e^{by}(l\psi_1(y) + 2\psi_0(y)))|_{y=0}\\
&\qquad = d^3\varphi_0(0)a^3l^2 + 6a^2d(e^{by}(l\psi_1(y) + \psi_0(y)))|_{y=0},
\end{aligned} \tag{18}$$

$$\begin{aligned}
&\varphi_1(0) = d\psi_0(0), \qquad d\varphi_1(0) = d\psi_1(0),\\
&4a^2d(e^{by}\mu(y))|_{y=-l/a} - l^2d^3(e^{by}\psi_0(y))|_{y=0} + ald^2(e^{by}(l\psi_1(y) + 4\psi_0(y)))|_{y=0}\\
&\qquad - 4a^2d(e^{by}(l\psi_1(y) + \psi_0(y)))|_{y=0} = a^2l^2(ad^3\varphi_0(0) - bd^2\varphi_0(0) - d^2\varphi_1(0)),\\
&6a^2d^2(e^{by}\mu(y))|_{y=-l/a} - l^2d^4(e^{by}\psi_0(y))|_{y=0} + ald^3(e^{by}(l\psi_1(y) + 6\psi_0(y)))|_{y=0}\\
&\qquad - 6a^2d^2(e^{by}(l\psi_1(y) + \psi_0(y)))|_{y=0} = a^3l^2(ad^4\varphi_0(0) - bd^3\varphi_0(0) - d^3\varphi_1(0)),
\end{aligned} \tag{19}$$

$$\begin{aligned}
&\varphi_2(0) = d^2\psi_0(0), \qquad d\varphi_2(0) = d^2\psi_1(0),\\
&2ad^2(e^{by}\mu(y))|_{y=-l/a} - 2ad^2(e^{by}(l\psi_1(y) + \psi_0(y)))|_{y=0} + 2ld^3(e^{by}\psi_0(y))|_{y=0}\\
&\qquad = al^2(2b(d^2\varphi_1(0) - ad^3\varphi_0(0)) + a(ad^4\varphi_0(0) - 2d^3\varphi_1(0)) + b^2d^2\varphi_0(0) + d^2\varphi_2(0)),\\
&2ad^3(e^{by}\mu(y))|_{y=-l/a} - 2ad^3(e^{by}(\psi_0(y) + l\psi_1(y)))|_{y=0} + 2ld^4(e^{by}\psi_0(y))|_{y=0}\\
&\qquad = a^2l^2(2b(d^3\varphi_1(0) - ad^4\varphi_0(0)) + a(ad^5\varphi_0(0) - 2d^4\varphi_1(0)) + b^2d^3\varphi_0(0) + d^3\varphi_2(0)).
\end{aligned} \tag{20}$$

Proof. By analogy with the proof of Lemma 2, we substitute the general solution (6) into the boundary conditions (4) and obtain a system of differential equations for the functions $f_i(y)$, $i = 1, 2, 3$. By solving it, we obtain

$$\begin{aligned}
f_1(y) = \frac{e^{by/a}}{al^2}\Bigg(&al^2\psi_0\left(\frac{y}{a}\right) - y\left(bl^2\psi_0\left(\frac{y}{a}\right) + l^2d\psi_0\left(\frac{y}{a}\right) - al^2\psi_1\left(\frac{y}{a}\right)\right)\\
&+ y^2\left[ae^{-bl/a}\mu\left(\frac{y-l}{a}\right) + (bl-a)\psi_0\left(\frac{y}{a}\right) + ld\psi_0\left(\frac{y}{a}\right) - al\psi_1\left(\frac{y}{a}\right)\right]\Bigg),\\
f_2(y) = \frac{e^{by/a}}{l^2}\Bigg(&bl^2\psi_0\left(\frac{y}{a}\right) + l^2d\psi_0\left(\frac{y}{a}\right) - al^2\psi_1\left(\frac{y}{a}\right)\\
&- 2y\left[ae^{-bl/a}\mu\left(\frac{y-l}{a}\right) + (bl-a)\psi_0\left(\frac{y}{a}\right) + ld\psi_0\left(\frac{y}{a}\right) - al\psi_1\left(\frac{y}{a}\right)\right]\Bigg),\\
f_3(y) = \frac{ae^{by/a}}{l^2}&\left[ae^{-bl/a}\mu\left(\frac{y-l}{a}\right) + (bl-a)\psi_0\left(\frac{y}{a}\right) + ld\psi_0\left(\frac{y}{a}\right) - al\psi_1\left(\frac{y}{a}\right)\right].
\end{aligned}$$

As a result, by using the boundary conditions (4), we define the functions f_i, $i = 1, 2, 3$, on the half-open interval $(-\infty, 0]$. To ensure that the resulting solution belongs to the class $C^3(Q)$, we require that the function $u(t, x)$, together with corresponding derivatives, is continuous in the domain Q. This is equivalent to saying that the functions f_i, together with their derivatives of orders ≤ 3, are continuous at zero, i.e., that relations of the form (15)–(17) hold.

However, these conditions are equivalent to the matching conditions (18)–(20). Indeed, relation (15) is equivalent to condition (18); relation (16), to condition (19); and relation (17), to condition (20). Therefore, if conditions (18)–(20) are satisfied, then the solution that we have found belongs to the class $C^3(\overline{Q})$. The same reasoning implies the necessity in Lemma 3. Indeed, if the problem has a solution, then the general form of the solution and the initial and boundary conditions imply that the functions f_i, $i = 1, 2, 3$, are defined in a similar way on the interval $[0, l]$ and on the half-open interval $(-\infty, 0]$. Since this solution belongs to the class $C^3(\overline{Q})$, it follows that relations (15)–(17) hold, which implies relations (18)–(20). The proof of Lemma 3 is complete.

4. INHOMOGENEOUS EQUATION

Now consider the following Cauchy problem with homogeneous initial conditions for Eq. (1):

$$(\partial_t - a\partial_x + b)^3 v(t,x) = f(t,x), \qquad (t,x) \in Q, \tag{21}$$

$$\partial_t^i v(0,x) = 0, \qquad i = 0,1,2, \qquad x \in [0,l]. \tag{22}$$

Lemma 4. *If the function f belongs to $C^3(\overline{Q})$, then there exists a function $v(t,x)$ of the class $C^3(\overline{Q})$ that is a solution of problem* (21), (22).

Proof. Consider the following boundary value problem for the function $\omega(t,\tau,x)$:

$$\begin{aligned} &(\partial_t - a\partial_x + b)^3 \omega(t,\tau,x) = 0, \qquad (t,x) \in Q, \qquad \tau \in [0,+\infty), \\ &\partial_t^i \omega(0,\tau,x) = 0, \quad i = 0,1, \quad \partial_t^2 \omega(0,\tau,x) = f(\tau,x), \quad x \in [0,l], \quad \tau \in [0,+\infty). \end{aligned} \tag{23}$$

The general solution of Eq. (23) has the form

$$\omega(t,\tau,x) = e^{-bt} W_1(x+at,\tau) + e^{-bt} W_2(x+at,\tau)t + e^{-bt} W_3(x+at,\tau)t^2.$$

By substituting this solution into the initial conditions of the Cauchy problem (23), we determine the functions W_i on the interval $[0,l]$; namely,

$$\begin{aligned} &W_1(x,\tau) = 0, \\ &a\partial_x W_1(x,\tau) + W_2(x,\tau) = 0, \qquad x \in [0,l], \\ &-ab\partial_x W_1(x,\tau) + a^2\partial_x^2 W_1(x,\tau) + 2a\partial_x W_2(x,\tau) - bW_2(x,\tau) + 2W_3(x,\tau) = f(\tau,x). \end{aligned} \tag{24}$$

By solving system (24), we find the functions W_i, $i = 1, 2, 3$, on the interval $[0,l]$,

$$W_1^{(0)}(y,\tau) = 0, \qquad W_2^{(0)}(y,\tau) = 0, \qquad W_3^{(0)}(y,\tau) = \frac{1}{2} f(\tau,y).$$

We extend the functions $W_i^{(0)}$, $i = 1, 2, 3$, to the interval $(-\infty, 0]$ so as to obtain three functions W_i, $i = 1, 2, 3$, of the class $C^3((-\infty, l])$. Let

$$\begin{aligned} &\widetilde{W}_1(y,\tau) = 0, \qquad y \in (-\infty,0], \\ &\widetilde{W}_2(y,\tau) = 0, \qquad y \in (-\infty,0], \\ &\widetilde{W}_3(y,\tau) = \frac{1}{2}\left(f(\tau,0) + y\partial_y f(\tau,0) + \frac{y^2}{2}\partial_y^2 f(\tau,0) + \frac{y^3}{6}\partial_y^3 f(\tau,0) \right), \qquad y \in (-\infty,0]. \end{aligned}$$

We have thereby obtained a solution of the Cauchy problem in the class $C^3(\overline{Q})$, where the functions W_i are defined by the relations

$$W_i(y,\tau) = W_i^{(0)}(y,\tau), \quad y \in [0,l], \quad W_i(y,\tau) = \widetilde{W}_i(y,\tau), \quad y \in (-\infty,0].$$

We introduce a function $v(t,x)$ by the relation

$$v(t,x)=\int_0^t \omega(t-\tau,\tau,x)\,d\tau.$$

The function v belongs to the class $C^3(\overline{Q})$ and satisfies the initial conditions (22). Indeed,

$$v(0,x)=0,\qquad \partial_t v(0,x)=\omega(0,t,x)+\int_0^t \partial_{t-\tau}\omega(t-\tau,\tau,x)\,d\tau|_{t=0}=0,$$

$$\partial_t^2 v(0,x)=\partial_\tau\omega(0,t,x)+\partial_t\omega(0,t,x)+\int_0^t \partial_{t-\tau}^2\omega(t-\tau,\tau,x)\,d\tau|_{t=0}=0.$$

The function v is also a solution of Eq. (21), which can readily be justified by a straightforward substitution. The proof of Lemma 4 is complete.

Let $v(t,x)\in C^3(\overline{Q})$ be a solution of problem (21), (22). Then, along with the boundary value problem (1)–(3), consider the problem

$$(\partial_t-a\partial_x+b)^3\,\overline{u}(t,x)=0,\qquad (t,x)\in Q, \tag{25}$$

$$\partial_t^i\overline{u}(0,x)=\varphi_i(x),\qquad i=0,1,2,\qquad x\in[0,l], \tag{26}$$

$$\partial_x^i\overline{u}(t,0)=\psi_i(t)-\partial_x^i v(t,0)=\widetilde{\psi}_i(t),\qquad i=0,1,2,\qquad t\in[0,+\infty), \tag{27}$$

and, along with problem (1), (2), (4), consider problem (25), (26) with the boundary conditions

$$\begin{aligned}\partial_x^i\overline{u}(t,0)=\widetilde{\psi}_i(t)=\psi_i(t)-\partial_x^i v(t,0),\qquad i=0,1,\qquad t\in[0,+\infty),\\ \overline{u}(t,l)=\widetilde{\mu}(t)=\mu(t)-v(t,l),\qquad t\in[-l/a,+\infty).\end{aligned} \tag{28}$$

Lemma 5. *Problem* (1)–(3) *is uniquely solvable if and only if problem* (25)–(27) *is uniquely solvable*, *and problem* (1), (2), (4) *is uniquely solvable if and only if problem* (25), (26), (28) *is uniquely solvable.*

Proof. First, let us prove the first part of the lemma, dealing with problem (1)–(3). Assume that problem (1), (3) has a unique solution u. Then, by taking $\overline{u}=u-v$, we obtain a solution of problem (25)–(27). Assume that problem (25)–(27) has a solution $\overline{u}_1$ other than $\overline{u}$. Then the function $u_1=\overline{u}_1+v$ differs from the function u and is a solution of problem (1)–(3) as well. We have obtained a contradiction. Therefore, problem (25)–(27) is uniquely solvable.

Now let us prove the sufficiency. Let problem (25)–(27) be uniquely solvable, and let the function $\overline{u}$ be its solution. Then, obviously, the function $u=\overline{u}+v$ is a solution of problem (1)–(3). Assume that a function u_1 other than u is a solution of that problem as well. In this case, the function $\overline{u}_1=u_1-v$ differs from $\overline{u}$ and is a solution of problem (25)–(27). We have obtained a contradiction. We have thereby proved the first part of Lemma 5. The second part of the lemma, dealing with problem (1), (2), (4), can be proved in a similar way. The proof of the lemma is complete.

Theorem 1. *Suppose that the conditions* $\varphi_i\in C^{5-i}([0,l])$, $i=0,1,2$, $\psi_j\in C^{5-j}([0,+\infty))$, $j=0,1,2$, *and* $f\in C^3(\overline{Q})$ *are satisfied*; *then problem* (1)–(3) *is uniquely solvable in the class* $C^3(\overline{Q})$ *if and only if the following matching conditions hold*:

$$\begin{aligned}&\varphi_0(0)=\psi_0(0),\qquad d\varphi_0(0)=\psi_1(0),\qquad d^2\varphi_0(0)=\psi_2(0),\\ &a^3d^3\varphi_0(0)=b^3\psi_0(0)+3b^2(d\psi_0(0)-a\psi_1(0))\\ &\quad+3b(d^2\psi_0(0)-2ad\psi_1(0)+a^2\psi_2(0))+(d^3\psi_0(0)-f(0,0)-3ad^2\psi_1(0)+3a^2d\psi_2(0)),\\ &\varphi_1(0)=d\psi_0(0),\qquad d\varphi_1(0)=d\psi_1(0),\qquad d^2\varphi_1(0)=d\psi_2(0),\end{aligned} \tag{29}$$

$$\begin{aligned}
&a^4d^4\varphi_0(0) - 3a^3bd^3\varphi_0(0) + 3a^2b^2d^2\varphi_0(0) - ab^3d\varphi_0(0)\\
&= a^3d^3\varphi_1(0) + 3a^2d^2\psi_2(0) - a(3b^2d\psi_1(0) + 9bd^2\psi_1(0) + 5d^3\psi_1(0) - 5\partial_x f(0,0))\\
&\quad + 2(b^3d\psi_0(0) + 3b^2d^2\psi_0(0) + 3bd^3\psi_0(0) - 3bf(0,0)\\
&\quad + [d^4\psi_0(0) - \partial_t f(0,0) + 3bf(0,0) - 3a\partial_x f(0,0)]),
\end{aligned} \tag{30}$$

$$\begin{aligned}
&\varphi_2(0) = d^2\psi_0(0), \qquad d\varphi_2(0) = d^2\psi_1(0),\\
&a^3d^3\varphi_1(0) = a^2(d^2\varphi_2(0) + 2d^2\psi_2(0)) + b^3d\psi_0(0) + 3b^2d^2\psi_0(0) + [d^4\psi_0(0) - \partial_t f(0,0)\\
&\quad + 3bf(0,0) - 3a\partial_x f(0,0)] + 3b(a^2d^2\varphi_1(0) - 2ad^2\psi_1(0) + d^3\psi_0(0) - f(0,0))\\
&\quad - 3a(b^2d\psi_1(0) + d^3\psi_1(0) - \partial_x f(0,0)),\\
&a^4(3bd^4\varphi_0(0) + 2d^4\varphi_1(0)) + a^2(b^3d^2\varphi_0(0) + [d^3\psi_2(0) - \partial_x^2 f(0,0)]) + a(b^3d\psi_1(0)\\
&\quad - 3b[d^3\psi_1(0) - \partial_x f(0,0)] - 2[d^4\psi_1(0) - \partial_t\partial_x f(0,0) + 3b\partial_x f(0,0) - 3a\partial_x^2 f(0,0)])\\
&\quad + b^3d^2\psi_0(0) + 3b^2[d^3\psi_0(0) - f(0,0)] + 3b[d^4\psi_0(0) - \partial_t f(0,0) + 3bf(0,0) - 3a\partial_x f(0,0)]\\
&\quad + [d^5\psi_0(0) - \partial_t^2 f(0,0) - [6b^2f(0,0) - 12ab\partial_x f(0,0) + 6a^2\partial_x^2 f(0,0)]\\
&\quad + 3b\partial_t f(0,0) - 3a\partial_t\partial_x f(0,0)]\\
&= a^3(a^2d^5\varphi_0(0) + 3b^2d^3\varphi_0(0) + 3bd^3\varphi_1(0) + d^3\varphi_2(0)).
\end{aligned} \tag{31}$$

Proof. It follows from Lemmas 4 and 5 that problem (1)–(3) is uniquely solvable if and only if the homogeneous problem (25)–(27) is uniquely solvable. By Lemma 2, the latter problem is uniquely solvable only if the following matching conditions hold:

$$\begin{aligned}
&\varphi_0(0) = \widetilde{\psi}_0(0), \qquad d\varphi_0(0) = \widetilde{\psi}_1(0), \qquad d^2\varphi_0(0) = \widetilde{\psi}_2(0),\\
&a^3d^3\varphi_0(0) = b^3\widetilde{\psi}_0(0) + 3b^2(d\widetilde{\psi}_0(0) - a\widetilde{\psi}_1(0))\\
&\quad + 3b(d^2\widetilde{\psi}_0(0) - 2ad\widetilde{\psi}_1(0) + a^2\widetilde{\psi}_2(0)) + (d^3\widetilde{\psi}_0(0) - 3ad^2\widetilde{\psi}_1(0) + 3a^2d\widetilde{\psi}_2(0)),
\end{aligned} \tag{32}$$

$$\begin{aligned}
&\varphi_1(0) = \widetilde{\psi}_0'(0), \qquad d\varphi_1(0) = d\widetilde{\psi}_1(0), \qquad d^2\varphi_1(0) = d\widetilde{\psi}_2(0),\\
&a^4d^4\varphi_0(0) - 3a^3bd^3\varphi_0(0) + 3a^2b^2d^2\varphi_0(0) - ab^3d\varphi_0(0)\\
&= a^3d^3\varphi_1(0) + 3a^2d^2\widetilde{\psi}_2(0) - a(3b^2d\widetilde{\psi}_1(0) + 9bd^2\widetilde{\psi}_1(0) + 5d^3\widetilde{\psi}_1(0))\\
&\quad + 2(b^3d\widetilde{\psi}_0(0) + 3b^2d^2\widetilde{\psi}_0(0) + 3bd^3\widetilde{\psi}_0(0) + d^4\widetilde{\psi}_0(0)),
\end{aligned} \tag{33}$$

$$\begin{aligned}
&\varphi_2(0) = d^2\widetilde{\psi}_0(0), \qquad d\varphi_2(0) = d^2\widetilde{\psi}_1(0),\\
&a^3d^3\varphi_1(0) = a^2(d^2\varphi_2(0) + 2d^2\widetilde{\psi}_2(0)) + b^3d\widetilde{\psi}_0(0) + 3b^2d^2\widetilde{\psi}_0(0) + d^4\widetilde{\psi}_0(0)\\
&\quad + 3b(a^2d^2\varphi_1(0) - 2ad^2\widetilde{\psi}_1(0) + d^3\widetilde{\psi}_0(0)) - 3a(b^2d\widetilde{\psi}_1(0) + d^3\widetilde{\psi}_1(0)),\\
&a^4(3bd^4\varphi_0(0) + 2d^4\varphi_1(0)) + a^2(b^3d^2\varphi_0(0) + d^3\widetilde{\psi}_2(0)) + a(b^3d\widetilde{\psi}_1(0) - 3bd^3\widetilde{\psi}_1(0)\\
&\quad - 2d^4\widetilde{\psi}_1(0)) + b^3d^2\widetilde{\psi}_0(0) + 3b^2d^3\widetilde{\psi}_0(0) + 3bd^4\widetilde{\psi}_0(0) + d^5\widetilde{\psi}_0(0)\\
&= a^3(a^2d^5\varphi_0(0) + 3b^2d^3\varphi_0(0) + 3bd^3\varphi_1(0) + d^3\varphi_2(0)).
\end{aligned} \tag{34}$$

By using the relations

$$\widetilde{\psi}_i(t) = \psi_i(t) - \partial_x^i v(t,x), \qquad i = 0,1,2,$$

we obtain

$$d^j\widetilde{\psi}_i(t) = \partial^j\psi_i(t) - \partial_x^i\partial_t^j v(t,x), \qquad i = 0,1,2, \qquad j = 0,\dots,5-i.$$

Therefore,

$$\begin{aligned}
&d^j\widetilde{\psi}_i(0) = d^j\psi_i(0), \qquad j = 0,1,2,\\
&d^3\widetilde{\psi}_i(0) = d^3\psi_i(0) - \partial_x^i f(0,0), \quad d^4\widetilde{\psi}_i(0) = d^4\psi_i(0) - \partial_t\partial_x^i f(0,0) - \partial_t^3\partial_x^i\omega(0,0,0),\\
&d^5\widetilde{\psi}_i(0) = d^5\psi_i(0) - \partial_t^2\partial_x^i f(0,0) - \partial_t^3\partial_x^i\partial_\tau\omega(0,0,0) - \partial_t^4\partial_x^i\omega(0,0,0);
\end{aligned}$$

i.e.,

$$\begin{aligned} d^4\widetilde{\psi}_0(0) &= d^4\psi_0(0) - \partial_t f(0,0) - \partial_t^3\omega(0,0,0), \\ d^4\widetilde{\psi}_1(0) &= d^4\psi_1(0) - \partial_t\partial_x f(0,0) - \partial_t^3\partial_x\omega(0,0,0), \\ d^5\widetilde{\psi}_0(0) &= d^5\psi_0(0) - \partial_t^2 f(0,0) - \partial_t^3\partial_\tau\omega(0,0,0) - \partial_t^4\omega(0,0,0). \end{aligned} \tag{35}$$

Let us compute the derivatives of the function $\omega(t,\tau,x)$ at the point $(0,0,0)$,

$$\begin{aligned} \partial_t^3\omega(0,0,0) &= 3a\partial_x f(0,0) - 3bf(0,0), \\ \partial_t^3\partial_x\omega(0,0,0) &= 3a\partial_x^2 f(0,0) - 3b\partial_x f(0,0), \\ \partial_t^3\partial_\tau\omega(0,0,0) &= 3a\partial_t\partial_x f(0,0) - 3b\partial_t f(0,0), \\ \partial_t^4\omega(0,0,0) &= 6b^2 f(0,0) - 12ab\partial_x f(0,0) + 6a^2\partial_x^2 f(0,0). \end{aligned}$$

By substituting relations (35) into these derivatives, we obtain the values of the derivatives of the functions $\widetilde{\psi}_0$ and $\widetilde{\psi}_1$ at the point $t = 0$,

$$\begin{aligned} d^4\widetilde{\psi}_0(0) &= d^4\psi_0(0) - \partial_t f(0,0) + [3bf(0,0) - 3a\partial_x f(0,0)], \\ d^4\widetilde{\psi}_1(0) &= d^4\psi_1(0) - \partial_t\partial_x f(0,0) + [3b\partial_x f(0,0) - 3a\partial_x^2 f(0,0)], \\ d^5\widetilde{\psi}_0(0) &= d^5\psi_0(0) - \partial_t^2 f(0,0) + [3b\partial_t f(0,0) - 3a\partial_t\partial_x f(0,0)] \\ &\quad - [6b^2 f(0,0) - 12ab\partial_x f(0,0) + 6a^2\partial_x^2 f(0,0)]. \end{aligned}$$

After the substitution of the computed value of $d^j\widetilde{\psi}_i(0)$ into conditions (32)–(34), we obtain Eqs. (29)–(31). The proof of the theorem is complete.

Problem 1. In problem (1)–(3), the function φ_i, $i = 0,1,2$, can be defined not on the segment $[0,l]$ but on the entire half-line $[0,\infty)$. In this case, Eq. (1) should be considered in the plane quadrant $\overline{\Omega} = [0,\infty) \times [0,\infty)$. Therefore, we have a problem on the nonsmooth boundary $\partial\Omega$ of the domain $\Omega = (0,\infty) \times (0,\infty)$. The construction of the classical solution of problem (1)–(3) in the case of the domain Ω is essentially reproduced without any modifications. Let us state the definitive result for the classical solution of problem (1)–(3) in this case in the form of a theorem.

Theorem 2. *Assume that* $\varphi_i, \psi_i \in C^{5-j}([0,+\infty))$, $j = 0,1,2$, *and* $f \in C^3(\overline{\Omega})$; *then problem* (1)–(3) *is uniquely solvable in the class* $C^3(\overline{\Omega})$ *if and only if the matching conditions* (29)–(31) *are satisfied.*

The proof of Theorem 2 reproduces the proof of Theorem 1 without any important modifications.

Theorem 3. *Assume that* $\varphi_i \in C^{5-i}([0,l])$, $i = 0,1,2$, $\psi_j \in C^{5-j}([0,+\infty))$, $j = 0,1$, $\mu \in C^3([-l/a,+\infty))$, *and* $f \in C^3(\overline{Q})$; *then problem* (1), (2), (4) *is uniquely solvable in the class* $C^3(\overline{Q})$ *if and only if the following matching conditions are satisfied*:

$$\begin{aligned} &\varphi_0(0) = \psi_0(0), \qquad d\varphi_0(0) = \psi_1(0), \\ &2a^2(e^{by}\mu(y))|_{y=-l/a} - \int\limits_0^{-l/a} (l+a\tau)^2 e^{b\tau} f(\tau,-a\tau)\,d\tau - l^2 d^2(e^{by}\psi_0(y))|_{y=0} \\ &\quad + 2ald(e^{by}(l\psi_1(y) + \psi_0(y)))|_{y=0} = a^2(l^2 d^2\varphi_0(0) + 2ld\varphi_0(0) + 2\varphi_0(0)), \\ &6a^2 d(e^{by}\mu(y))|_{y=-l/a} - 3a \int\limits_0^{-l/a} (l+a\tau)^2 e^{b\tau}\partial_x f(\tau,-a\tau) - 2(l+a\tau)e^{b\tau} f(\tau,-a\tau)\,d\tau \\ &\quad - 2l^2 d^3(e^{by}\psi_0(y))|_{y=0} + 2l^2 f(0,0) + 3ald^2(e^{by}(l\psi_1(y) + 2\psi_0(y)))|_{y=0} \\ &= d^3\varphi_0(0)a^3 l^2 + 6a^2 d(e^{by}(l\psi_1(y) + \psi_0(y)))|_{y=0}, \\ &\varphi_1(0) = d\psi_0(0), \qquad d\varphi_1(0) = d\psi_1(0), \end{aligned}$$

$$4a^2d(e^{by}\mu(y))|_{y=-l/a} - 2a\int\limits_0^{-l/a} (l+a\tau)^2e^{b\tau}\partial_x f(\tau,-a\tau) - 2(l+a\tau)e^{b\tau}f(\tau,-a\tau)\,d\tau$$
$$- l^2d^3(e^{by}\psi_0(y))|_{y=0} + ald^2(e^{by}(l\psi_1(y)+4\psi_0(y)))|_{y=0} - 4a^2d(e^{by}(l\psi_1(y)+\psi_0(y)))|_{y=0}$$
$$= a^2l^2(ad^3\varphi_0(0) - bd^2\varphi_0(0) - d^2\varphi_1(0)) - l^2f(0,0),$$
$$6a^2d^2(e^{by}\mu(y))|_{y=-l/a} - l^2d^4(e^{by}\psi_0(y))|_{y=0} + l^2\partial_t f(0,0) + 3al^2\partial_x f(0,0) + bl^2f(0,0)$$
$$- 6a^2\int\limits_0^{-l/a} e^{b\tau}f(\tau,-a\tau) - 2(l+a\tau)e^{b\tau}\partial_x f(\tau,-a\tau) + \frac{(l+a\tau)^2}{2}e^{b\tau}\partial_x^2 f(\tau,-a\tau)\,d\tau$$
$$+ ald^3(e^{by}(l\psi_1(y)+6\psi_0(y)))|_{y=0} - 6alf(0,0) - al^2\partial_x f(0,0) - 6a^2d^2(e^{by}(l\psi_1(y)+\psi_0(y)))|_{y=0}$$
$$= a^3l^2(ad^4\varphi_0(0) - bd^3\varphi_0(0) - d^3\varphi_1(0)),$$
$$\varphi_2(0) = d^2\psi_0(0), \qquad d\varphi_2(0) = d^2\psi_1(0),$$
$$2ad^2(e^{by}\mu(y))|_{y=-l/a} - 2ad^2(e^{by}(l\psi_1(y)+\psi_0(y)))|_{y=0} + 2ld^3(e^{by}\psi_0(y))|_{y=0} - 2lf(0,0)$$
$$- 2a\int\limits_0^{-l/a} e^{b\tau}f(\tau,-a\tau) - 2(l+a\tau)e^{b\tau}\partial_x f(\tau,-a\tau) + \frac{(l+a\tau)^2}{2}e^{b\tau}\partial_x^2 f(\tau,-a\tau)\,d\tau$$
$$= al^2(2b(d^2\varphi_1(0) - ad^3\varphi_0(0)) + a(ad^4\varphi_0(0) - 2d^3\varphi_1(0)) + b^2d^2\varphi_0(0) + d^2\varphi_2(0)),$$
$$2ad^3(e^{by}\mu(y))|_{y=-l/a} + 2ld^4(e^{by}\psi_0(y))|_{y=0} - 2l\partial_t f(0,0) - 6al\partial_x f(0,0) - 2blf(0,0)$$
$$- 2ad^3(e^{by}(\psi_0(y)+l\psi_1(y)))|_{y=0} + 2a(f(0,0) + l\partial_x f(0,0) - e^{-bl/a}f(-l/a,l))$$
$$- a^2\int\limits_0^{-l/a} 6e^{b\tau}\partial_x f(\tau,-a\tau) - 6e^{b\tau}(l+a\tau)\partial_x^2 f(\tau,-a\tau) + e^{b\tau}(l+a\tau)^2\partial_x^3 f(\tau,-a\tau)\,d\tau$$
$$= a^2l^2(2b(d^3\varphi_1(0) - ad^4\varphi_0(0)) + a(ad^5\varphi_0(0) - 2d^4\varphi_1(0)) + b^2d^3\varphi_0(0) + d^3\varphi_2(0)).$$

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