

Predicting the Parameters of the Orientation of the Earth in Problems of Navigation Taking into Account the Phenomenon of the Development of Irregularity in the Earth's Rotation

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Abstract—This article discusses the application of a previously proposed [1] methodology for predicting the Earth's orientation parameters (EOPs) that provides high accuracy as a result of optimizing a special procedure of processing historical data using the least-mean-squares method. The results of investigating the accuracy characteristics of the obtained EOP estimations are presented in relation with a predictive task in the interval from 2019 to 2022, when, for the first time in the history of observing the Earth's daily rotation, a change was recorded in the difference trend between Universal Time and Coordinated Universal Time was recorded, caused by the length of day decreasing. The influence of the length of the day trend changing and related problems on the accuracy of EOP prediction in various navigation tasks using classical polynomials describing EOP evolution is discussed. A comparative analysis of the EOP prediction made by the International Earth Rotation and Reference Systems Service for a similar time period is carried out.

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INTRODUCTION

The fundamental problem of determining and predicting the parameters of the Earth's rotation (PERs), including pole shift x_p and y_p , and the nonuniformity of diurnal rotation in the form of the difference between UT1 (Universal Time) and UTC (Coordinated Universal Time), has been repeatedly discussed in a significant number of works [2–5] due to its exceptional importance in various applications related to navigation, geodynamics, and space geodesy. This problem attracted the attention of the authors in the context of solving the problem of increasing the accuracy and autonomy of the operation of GLONASS [6, 7]. In particular, papers [8, 9] describe an information technology for processing measurement data between spacecraft (SCs) and ground stations in order to refine the forecast values of the parameters of the Earth's rotation onboard navigation spacecraft. The implementation of this technology in the future may lead to getting rid of the procedure for regularly uploading data on the real evolution of the parameters of the Earth's rotation to the spacecraft of the orbital group (OG). Within the framework of the technology under discussion, a significant role is given to a priori analysis of the dependences that describe the motion of the pole, the irregularity of the Earth's rotation, and their contribution to the equivalent error of the pseudorange to the consumer (EEPC), as well as the method for pre-

dicting the ERR based on the available a posteriori data, which is discussed below in this article.

In [1], an approach was developed for choosing optimal polynomial coefficients from the point of view of forecast accuracy achieved over a long historical interval, describing the evolution of the PERs. A study was conducted that showed that, depending on the length of the required forecasting interval, it is necessary not only to change the form of the approximated function (which would most closely correspond to the dynamics of the discussed parameters on a given segment), but also the value of the approximation interval, on the basis of data processing of which the required values of the coefficients are the calculated polynomials that are used. The result of this work and analysis of the results of the calculations performed for a long time interval (about 10 years, from 2009 to 2019) was a set of parameters that include, for each of the required forecasting segments (for example, from 5 to 90 days), its own recommended type of function and value of the duration of the approximation interval. Guaranteed levels of accuracy in forecasting each of the parameters of the Earth's rotation were obtained within the framework of the proposed forecasting technique based on the approach developed by the authors. It should be emphasized that, due to the high uncertainty in the evolution of the uneven rotation of the Earth, it is this parameter among the three (x_p , y_p , and nonuniform diurnal rotation) is that is the worst

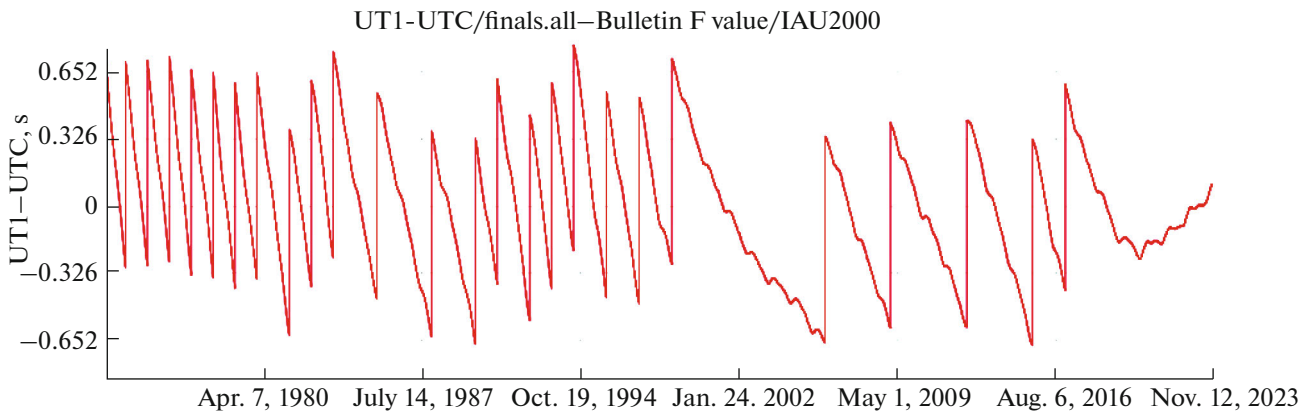


Fig. 1. Dependence of difference between UT1 and UTC time scales.

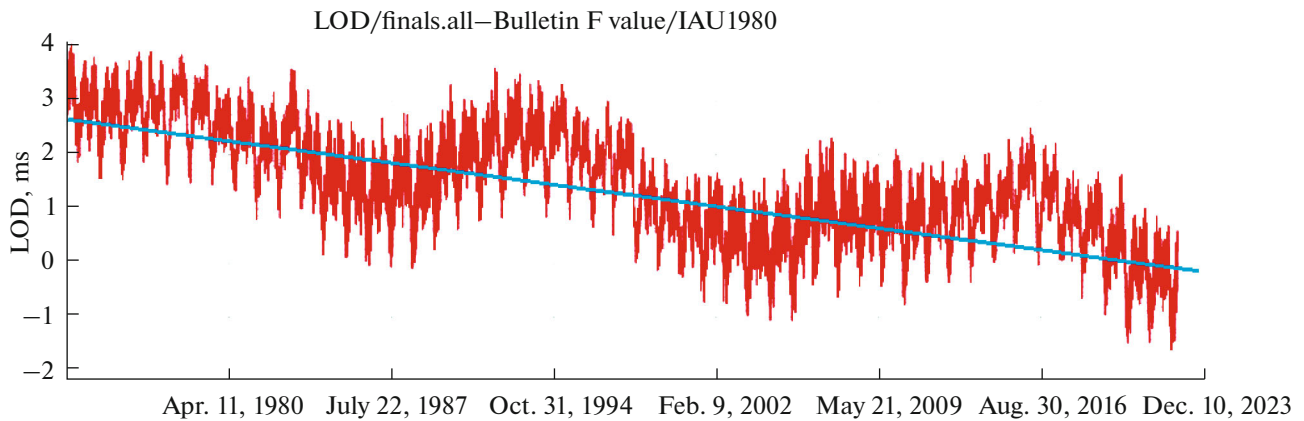


Fig. 2. LOD dependence.

predicted and, as a result, introduces the largest error in the final forecast of the evolution of PERs and EEP. As a result, the change in the trend of the UT1–UTC dependence (Fig. 1 https://www.iers.org/IERS/EN/Home/home_node.html) in recent years may, at first glance, cast doubt on the possibility of using the proposed types of polynomials and approximation intervals, since, despite the fact that they were determined for a long historical interval (10 years), the discussed period, from 2019 to 2022, was not considered.

The foregoing leads to the conclusion that, in the interests of verifying the correctness and possibly improving one of the components of the developed information technology that is “responsible” for forecasting the PERs, it is an important task to analyze the accuracy characteristics of the forecast in relation to the selected types of approximated functions and the lengths of the approximation segments on new time intervals. Before proceeding to the discussion of the results obtained, it is necessary to make the following reservation regarding the “anomalous” evolution of UT1–UTC (DUT): if we study not only the graph of

this dependence directly, but also the historical graph of the evolution of the length of the day (LOD), then it is easy to see the sinusoidal trend characteristic of the LOD dependence multiplied with a straight line. Here, the straight line approximating the linear trend has a reverse slope and crosses the zero (nominal) mark of the length of the day approximately in the period of 2022–2023 (Fig. 2 [10]).

Thus, analyzing Fig. 2, we can conclude that the second derivative of the function describing the trend in the average length of the day is negative over the entire observation period. In other words, the length of the day has been decreasing for decades, while initially, at the time of the start of observation of the Earth’s rotation, this value was greater than the nominal value. This fact gives grounds to suppose that the behavior of both the LOD and DUT, which is actually the integral of the LOD (excluding added seconds), cannot be considered anomalous. As a consequence, it can be supposed that the dependences approximating their evolution, as well as the previously obtained values of the parameters of the forecasting procedure, are optimal in terms of forecasting accuracy over a long

time interval within the framework of the methodology that was proposed in [1]. Below, this circumstance is verified by determining the statistical characteristics of the accuracy of forecasting PERs in the new time interval of 2019–2022 using the previously obtained parameters of the forecasting procedure.

MODELS, INPUT DATA, AND ALGORITHMS

Before proceeding to the development of the described technology in the new time period from 2019 to 2022 compared to the one studied in [1], let us briefly recall the essence of the research procedure and the chosen forecasting technique that were implemented in the corresponding software (SW). The research procedure contained the following steps:

- choosing the models for approximating the PER series;
- iterative enumeration of the models used for PER approximation;
- enumeration of the values of the lengths of approximation intervals and forecasting intervals;
- estimation of polynomial coefficients based on the use of the least-squares method (LSM) and available a posteriori data;
- construction of forecast series of PERs for various lengths of forecasting intervals; and
- assessment of the accuracy of the obtained predictive values of the PERs.

The estimation of the accuracy of the resulting forecast is carried out by comparing the forecast series of the PERs obtained using the created software with the a posteriori data of the long-term series of the PERs from IERS published in C04 bulletins. The considered models describing the evolution of the PERs included linear and polynomial types of functions. For comparison, other models were also used, which were then excluded from consideration due to unacceptable result accuracy. As a result of the procedure described above, tables of correspondence between the minimum forecast error and the main parameters of the forecast procedure are formed: the type of the function being approximated and the interval of approximation of a posteriori data.

We emphasize that the choice of the type of polynomial model used to describe the evolution of the PERs has not yet been finally decided. There are a number of approaches in which the harmonics of the polynomials describing the PERs are supplemented by various deterministic analytical series that take into account tides, the influence of the Moon and the Sun, etc. In this article, the question of the expediency of choosing one or another polynomial model is not discussed. In addition, in [1], an analysis was already carried out and estimates of the forecast accuracy were obtained based on the polynomial models used (1) that are given in a number of documents affiliated with IERS and the National Geospatial-Intelligence Agency (NGA):

$$\begin{aligned}
 x_p &= A + B(T - T_a) + \sum_{n=1}^2 C_n \sin\left(\frac{2\pi(T - T_a)}{P_n}\right) + \sum_{n=1}^2 D_n \cos\left(\frac{2\pi(T - T_a)}{P_n}\right) \\
 y_p &= E + F(T - T_a) + \sum_{n=1}^2 G_n \sin\left(\frac{2\pi(T - T_a)}{Q_n}\right) + \sum_{n=1}^2 H_n \cos\left(\frac{2\pi(T - T_a)}{Q_n}\right) \\
 UT1 - UTC &= I + J(T - T_b) + \sum_{n=3}^4 K_n \sin\left(\frac{2\pi(T - T_b)}{R_n}\right) + \sum_{n=3}^4 L_n \cos\left(\frac{2\pi(T - T_b)}{R_n}\right),
 \end{aligned} \tag{1}$$

where $A, B, C_n, D_n, E, F, G_n, H_n, I, J, K_n, L_n, R_1, R_2, T_a, T_b$ are coefficients that were subject to estimation when processing a posteriori data by the least-squares method; T is the date for which a forecast is required; P_1, Q_1, R_3 is the length of the sidereal year; P_2, Q_2 is the Chandler cycle; R_4 is the semi-annual cycle; R_1 is the Lunar cycle; and R_2 is the semilunar cycle (all in days). Since at the moment constants K and L numbered 1 and 2 in the NGA methodological recommendations, starting from 2005, are set to zero, and instead of them the values corresponding to the zonal tide models are calculated, there are no corresponding harmonics in (1). However, as studies have shown [1], the use of a priori calculated values instead of the obtained estimates of coefficients K and L in the presented series leads to a deterioration in the accuracy of the forecast. In this

regard, the aforementioned coefficients were also to be determined in the course of research, and the summation in (1) was carried out for $n = 1, 4$. The research procedure and all the details are described in more detail in [1]. Ultimately, the reasons for choosing model (1) are, first, the relative simplicity of its computational implementation, including when processing large amounts of data (there is no need to separately calculate long deterministic series to take into account the motion of the Moon–Sun), and, second, as is shown below, the correspondence of the terms of the model spectral analysis results. First of all, we are talking about DUT as the most problematic parameter for forecasting. It is known that the DUT value (Fig. 1) is the LOD development integral (Fig. 2) minus added seconds. Spectral analysis of the LOD time dependence confirms the presence of exactly the

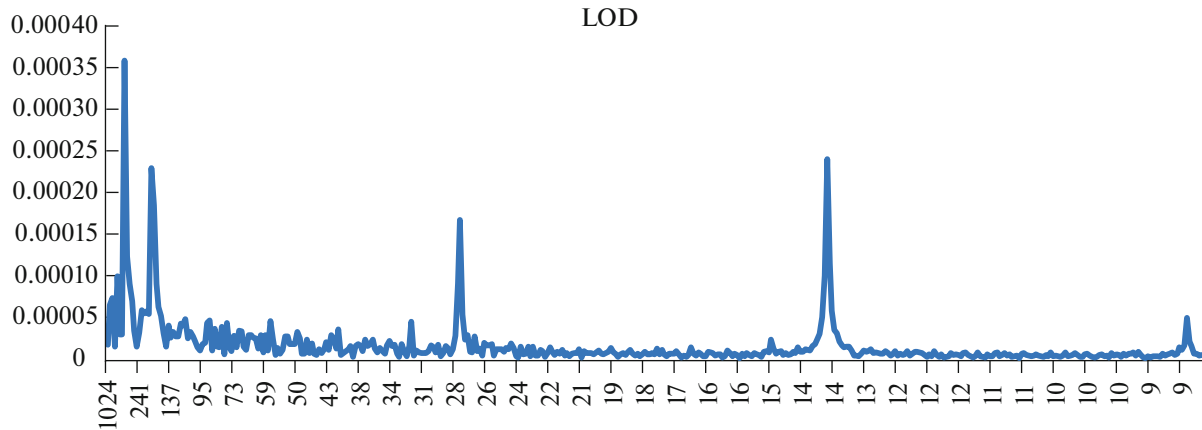


Fig. 3. Amplitude of the harmonics of the Fourier series constructed from the length of the time series for the LOD depending on the period expressed in days.

same harmonics that are used in relation (1) to describe the uneven rotation of the Earth, namely, annual, semi-annual, lunar, and semilunar (Fig. 3).

Let us present the final table of results, which includes the obtained values of the forecast errors guaranteed by the probability level of 0.95, the corresponding form of the function, and the interval of approximation obtained by processing the evolution of the PER over the interval from 2009 to 2019. This table was given earlier in [1] and is used as the initial data for assessing the accuracy of forecasting in the interval of 2019–2022.

Results of Forecasting Using the Previously Developed Methodology

A comparative analysis is given below of the resulting accuracy of the forecast for the forecast for the

period of 2019–2022 in comparison with the data from Table 1. It should be noted that, despite the potentially better results that could be obtained with some changes in the parameters of the forecast procedure, all approximation intervals and the form of the function were chosen in accordance with Table 1, since otherwise this study would be meaningless. Together with the previously determined parameters, the following results were obtained (Table 2).

Figure 4 shows a comparative description of the results of using the parameters declared in Table 1 when forecasting the PERs in the 2019–2022 period and in the period from 2009 to 2019.

The results shown in Fig. 4 demonstrate the achievement of a similar level of accuracy in forecasting over short periods (up to 20–30 days), as well as a higher accuracy in predicting the pole coordinates

Table 1. Error values for development estimations x_p, y_p , and (mas) and DUT (ms) at a confidence level of 0.95 for the conducted studies

PER	Parameter	Forecast interval (days)						
		5	10	15	20	30	60	90
xp	Minimum value on the class of approximable functions	5.3	11.3	16.9	22.3	32	46	54
xp	Type of function/interval of approximation	1/2	1/2–8	1/6–16	1/7–14	p/680–705	p/660–680	p/735–749
yp	Minimum value on the class of approximable functions	3.8	7.2	11.3	15.7	26.5	45	55
yp	Type of function/interval of approximation	1/2–3	1/6–9	1/8–11	1/10–11	1/6–11	p/665–686	p/665–686
DUT	Minimum value on the class of approximable functions	2.4	5.9	8.6	11	14	25	38
DUT	Type of function/interval of approximation	1/2	1/2	1/2–3 or p/315–330	1/11–26 or p/300–320	p/310–325	p/315–329	p/300–320

Note: “1” means preferred use of a linear relationship for approximation; “p” is polynomial (1). The number of days indicated by “/” must be used as the value of the required approximation interval to minimize the forecast error on a specific time interval.

Comparison of EOP prediction errors (mas/ms) depending on the prediction day and time interval

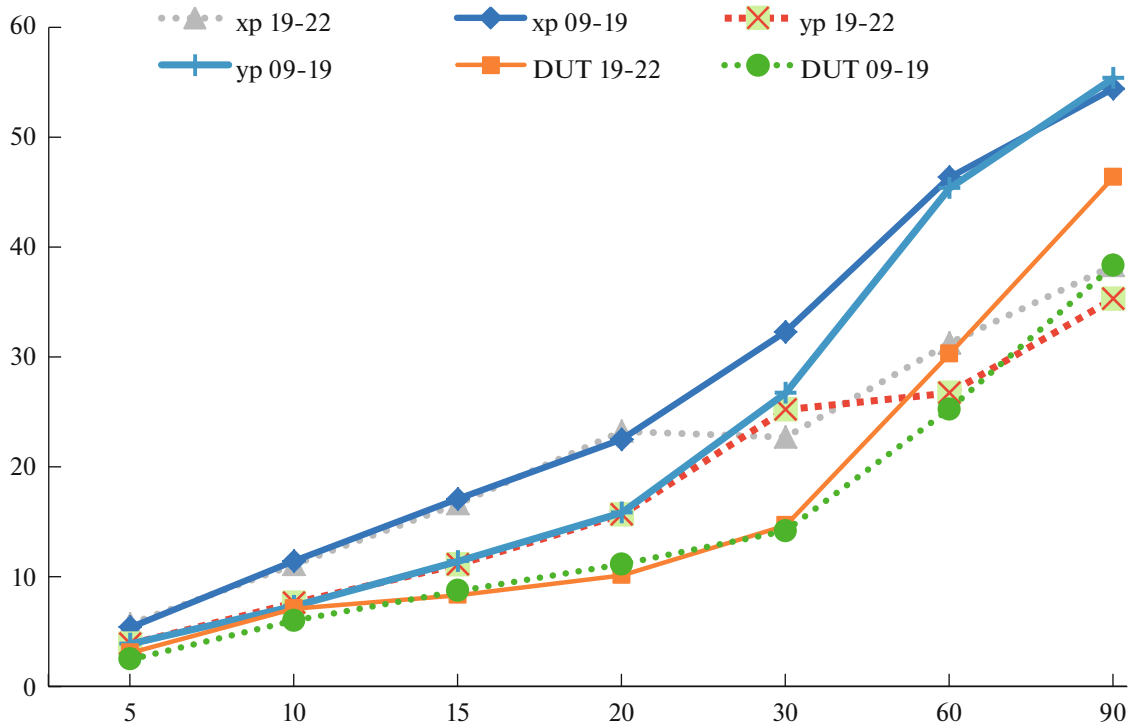


Fig. 4. PER forecasting errors.

Table 2. Guaranteed 0.95 probability values of PER forecast errors

Processing interval	Parameter/approximation method	Forecast day						
		5	10	15	20	30	60	90
2019–2022	$x_p/“1”$	5.6	11	16.5	23			
2019–2022	$x_p/“p”$			18	20	22.5	31	38
2019–2022	$x_p/\text{combination}$	5.6	11	16.5	23	22.5	31	38
2019–2022	$y_p/“1”$	3.8	7.5	11	15.5	25		
2019–2022	$y_p/“p”$				15	17.5	26.5	35
2019–2022	$y_p/\text{combination}$	3.8	7.5	11	15.5	25	26.5	35
2019–2022	DUT/“1”	3	7	9	11.5	18		
2019–2022	DUT/“n”			8.2	10	14.5	30	46
2019–2022	DUT/combination	3	7	8.2	10	14.5	30	46
2019–2022	DUT/“1”*	3	6	8.6	11.5	18		
2019–2022	DUT/“n”*			7.8	9.7	13.5	27.5	45
2019–2022	$y_p/“p”$ *						24	33
2009–2019	$x_p/\text{combination}$	5.3	11.3	16.9	22.3	32	46	54
2009–2019	$y_p/\text{combination}$	3.8	7.2	11.3	15.7	26.5	45	55
2009–2019	DUT/combination	2.4	5.9	8.6	11	14	25	38

Note. * These rows present the results obtained by selecting other approximation intervals that give better results (the amplitude of the change in the interval was in some cases only from 10 to 30 days). The values presented under the symbol * are given to assess the relative improvement in the accuracy of forecasting when choosing the best approximation conditions instead of those declared earlier.

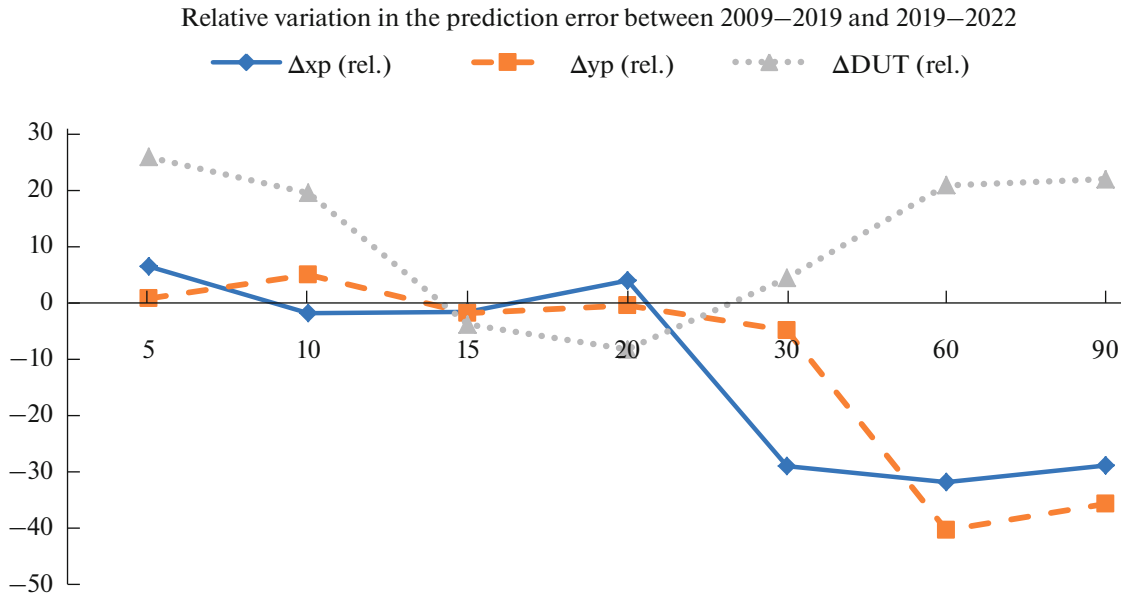


Fig. 5. Relative change in PER forecasting accuracy.

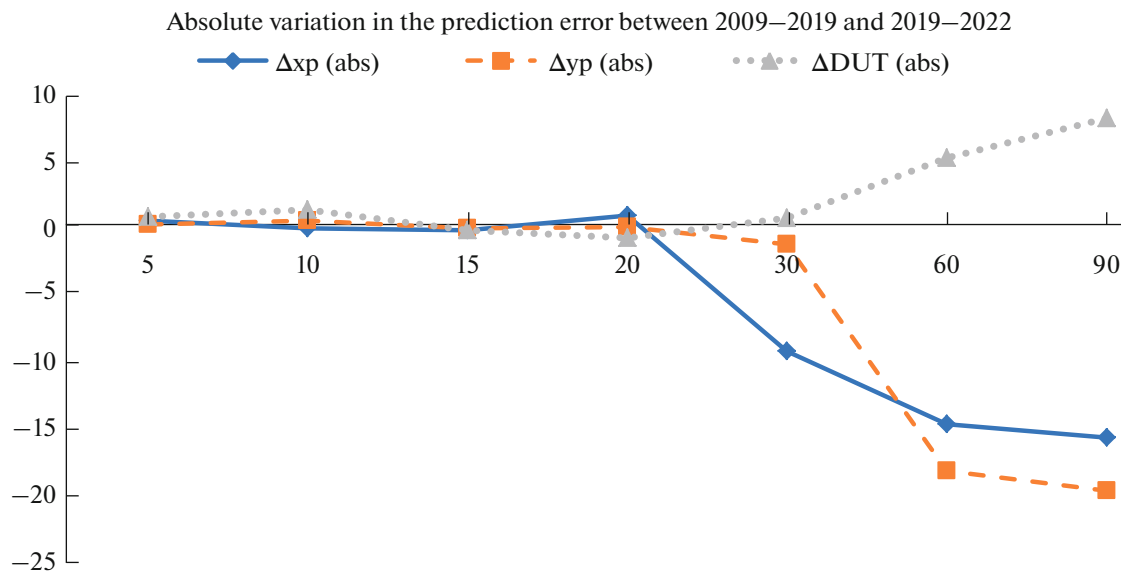


Fig. 6. Absolute change in PER forecasting accuracy.

shift over long intervals—from 30 days and more. At the same time, the accuracy of forecasting the irregularity of the Earth’s rotation over short intervals has parity between 2009–2019 and 2019–2022, while over long intervals it has deteriorated quite significantly (Figs. 5 and 6).

It should be noted that the choice of new intervals (see Table 2 with “*”) did not allow achieving the same accuracy of predicting the uneven rotation of the Earth, which was obtained earlier for the period 2009–2019; namely, when choosing the length of the

approximation interval, the gain in forecast accuracy reached only 1–3 ms; that is, it is not enough to maintain the previous characteristics of the forecast accuracy of the PERs. This fact indicates either the unacceptability of the model used (1), or the need to refine the developed forecasting methodology, or anomalies in the DUT trend in the considered 3-year period of 2019–2022. To determine the influence of the DUT trend for the forecasting process it is advisable, as shown below, to refer to the results of the forecasting of PERs performed by the IERS.

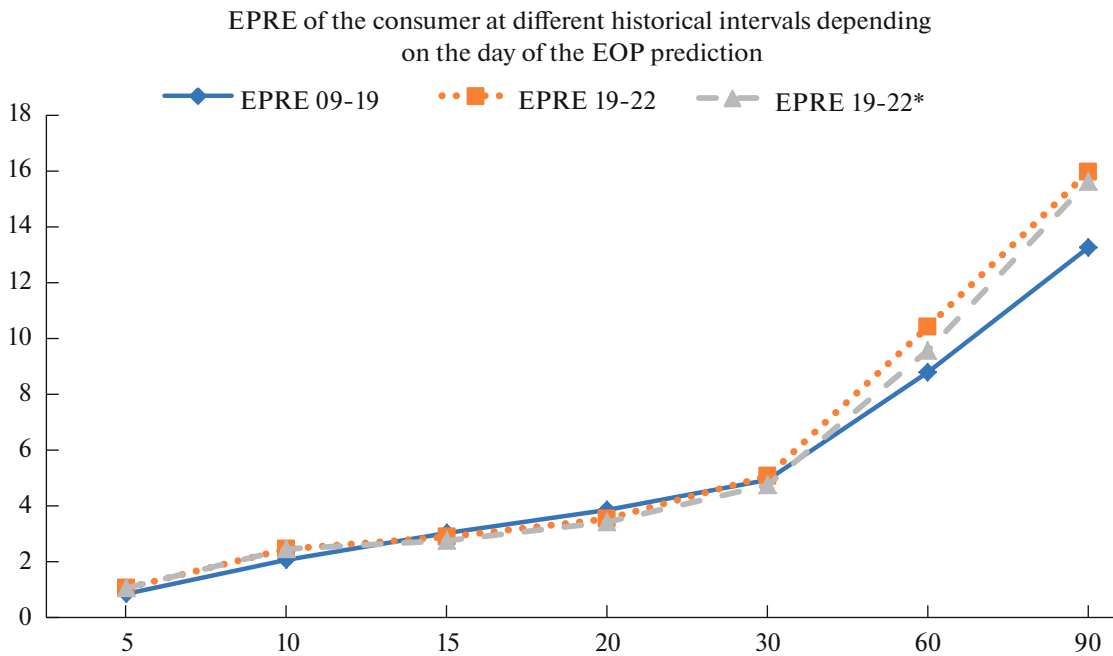


Fig. 7. Dependence of the EPPDP on the duration of the forecast of PERs.

At the same time, the results of the pole-shift forecast have improved significantly in absolute and relative terms. Taking into account this phenomenon, the analysis of the contribution of the PER error to the equivalent pseudorange error is considered separately (Fig. 7).

Figure 7 shows some deterioration of the EPPDP due to the more significant contribution of the unevenness of the Earth's rotation, which, using the settings obtained earlier, is about 3 m after 3 months of forecasting. This delta decreases to 2.3 m as the approximation intervals change. At the same time, during the month, the discrepancy in the EPPDP between the intervals of 2009–2019 and 2019–2022 is less than 1 m, which indicates the relative stability of the results obtained for a new time interval according to the previously developed methodology and values of the parameters of the forecast procedure [1].

In order to find out the reasons for the deterioration in the accuracy of the forecast of the uneven rotation of the Earth using the previously developed methodology, we will use the data provided by the IERS, since, as the analysis of all available data shows, the IERS has been shown to have the most impressive results in the field of geodynamics research: IERS forecasts are more accurate than those obtained using the technique developed by the authors for 90 days, and the DUT prediction error for IERS is less than that obtained by the authors by 10 ms (~40%). However, the forecasting technology, the type of relations used, and other parameters for solving the IERS data forecasting problem are the intellectual property of this organization, despite the significant number of

indirectly affiliated works. Thus, the IERS forecast can only be used, as in this case, through the results published in various bulletins of this organization. In other words, the presence of anomalies in the DUT trend can be determined by evaluating the accuracy of the forecast in the bulletins.

Estimation of IERS Forecast Accuracy and Analysis of DUT Predictability Character

To evaluate the accuracy of the forecasting of the PERs presented in the time series published by the IERS, a corresponding comparison was made, which consisted of comparing the available data from various sources. The data from the daily A bulletins containing rows with the corresponding forecast series of 1-year-long forecast series of the PERs were used as the predicted experimental values of the PERs. IERS posterior data from C04 bulletins were used as reference. The overall prediction results are shown below in the analysis color chart.

Figure 8 in different colors shows the change in the accuracy of DUT forecasting depending on the length of the forecast interval and the considered historical segment for which the forecast is being constructed. It is easy to notice a significant deterioration in the accuracy of the forecast (an increase in the error from 20 to 50 ms), which falls exactly on a point in the discussed interval from 2019 to 2022. For this interval (highlighted in Fig. 8), when constructing a forecast series with a length of 30 days or more, the IERS shows an almost doubling in errors relative to the standard. Figure 9 shows the dependences of the DUT forecast error, guaranteed at the level of 0.95, depending on the length

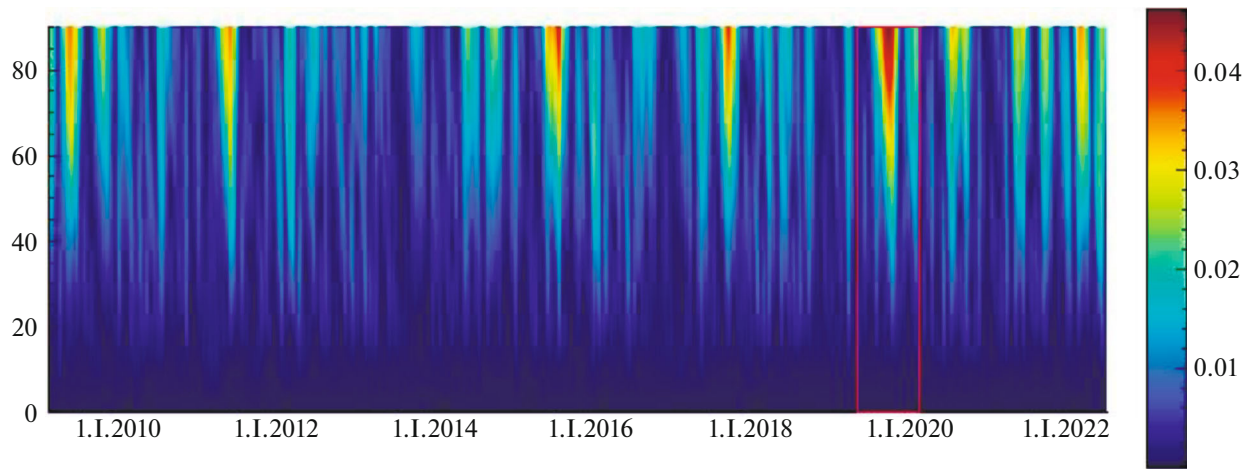


Fig. 8. Diagram of the error level of the IERS PER forecast.

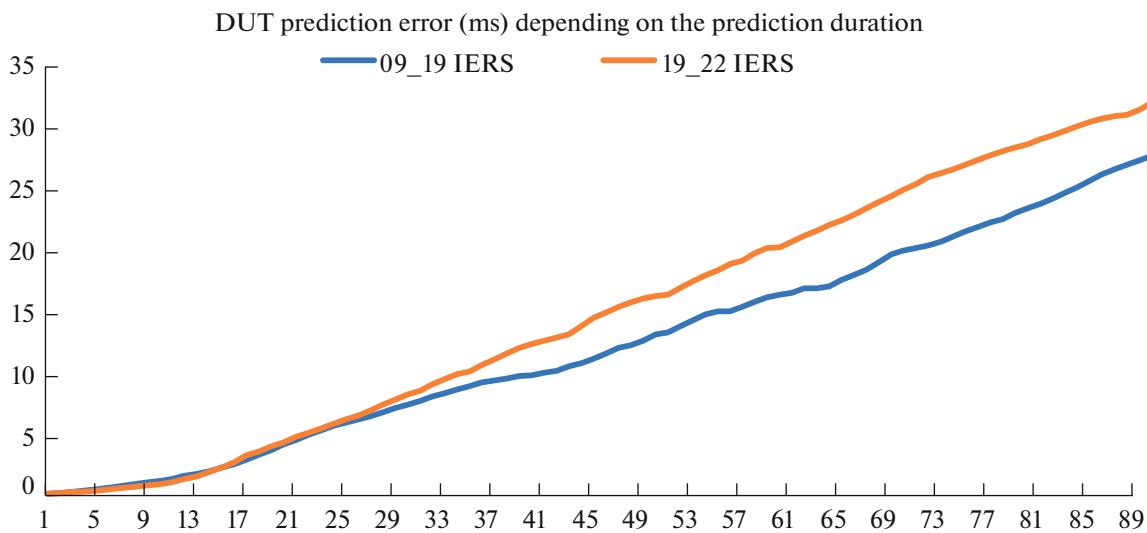


Fig. 9. Comparison of DUT forecast by IERS A bulletins.

of the forecast interval constructed from the IERS A bulletins for the historical intervals of 2009–2019 and 2019–2022. It is noticeable that, for the date range of 2019–2022 discussed in the article, the growth of errors in the forecast of the PERs for the length of the interval of a month or more up to 20–30% is typical in comparison with similar errors obtained for the range of 2009–2019, despite the fact that the first interval is shorter than the second by about three times. This fact indicates the presence of anomalies in the usual trend of DUT development dependence.

CONCLUSIONS

Analysis of the research results presented in this article allows the following conclusions to be drawn:

—the method proposed by the authors of forecasting PERs demonstrates, as shown in this article, stability and predictability of the results;

—the developed method for optimizing the forecasting procedure based on the selection of the type of function being approximated and the length of the approximation interval depending on the duration of the forecasting segment as a whole remains operable on new data and gives results comparable in terms of forecast accuracy for future periods;

—the accuracy of the forecasting of the PERs at future intervals for the proposed parameters of the methodology turns out to be close to the maximum achievable accuracy provided by re-selecting the parameters of the forecasting procedure for a specific time interval;

—when using the parameters of the method obtained in [1] at intervals of up to 2 months, there is no deterioration in the accuracy of forecasting the PERs, which confirms the possibility of using the proposed forecasting method in the future;

—the phenomenon of anomalies in a UT1–UTC trend takes place, which is confirmed by the deteriora-

tion of the forecasting accuracy starting from the second month of the forecast in its value in the 2019–2022 period compared to previous periods for any possible approximation parameters and the deterioration in the accuracy of forecasting provided by the IERS; and

—despite the problems with the DUT prediction accuracy, the corresponding EPPDP, taking into account the error in forecasting the PERs, remains within the boundaries close to the previous range of values [1] with a difference of no more than 3 m relative to its previously obtained estimates, and for short intervals (up to a month) of no more than a meter.

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