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Control of the Deployment of an Orbital Tether System That Consists of Two Small Spacecraft

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Abstract—Control of an orbital tether system that consists of two small spacecraft has been considered. The proposed control laws are based on the modification of well-known programs for the deployment of tether system systems under the assumption that the masses of spacecraft and the tether are comparable in magnitude. To construct nominal deployment programs, we have developed a mathematical model of the motion of the given system in an orbital moving coordinate system taking into account the specific features of this problem. The performance of the proposed deployment programs is assessed by a mathematical model of the orbital tether system with distributed parameters written in the geocentric coordinate system. The test calculations involve a linear regulator that implements feedback on the tether length and velocity.

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STATEMENT OF THE PROBLEM

At present, the construction of orbital tether systems (OTSs) for different purposes and configurations has remained an interesting problem. There are dozens of space tether experiments that have been completed or planned [1–3]. The most responsible and complex stage of any space tether experiment is the OTS deployment, which is responsible for the success of the space operation as a whole. The stage of OTS deployment is a complex dynamical process that requires a comprehensive study of control laws and their implementation. In recent years, small spacecraft are finding ever increasing use due to constant improvement of onboard equipment. The deployment of OTSs with low end masses requires the development of new OTS deployment programs and the modification of existing programs.

To construct nominal tethering programs, we developed a mathematical model of the OTS motion in an orbital moving coordinate system taking into account the specific features of the given problem, i.e., the fact that the masses of end bodies and the tether are comparable. The mathematical model was designed for an inextensible tether in an orbital moving coordinate system. The model developed was used for the case of two nominal laws of OTS deployment that differ by the end state of the system, i.e., (1) in the vertical position and (2) deviated from is a parametric law that implements the dependence of the tether tension force on its length and rate. The control program is based on a modification of the well-known law of the optimum damping of OTS oscillations given in the classical monograph [1]. Other modifications of this law were used in many studies [4, 5], as well as in the real experiment described in [6]. The nominal program of OTS deployment with a tether deviation in the end state from the vertical position is sometimes used to construct a system with some angular velocity of rotation relative to its center of mass. In this case, for example, when the tether is broken, the end bodies acquire additional pulses that change the parameters of their orbital motion. Specifically, this maneuver was used in the YES2 real tether experiment [2] in order to increase the effective braking pulse when the descent capsule returns back from the orbit with the help of the OTS. These programs are typically designed using the performance criteria [4]; they are close to the relay control laws and consist of acceleration and deceleration phases. Clearly, this involves relatively large accelerations; therefore, it becomes of particular significance to take into account the tether mass, which increases the inertia of the system.

Supposedly, in the initial position, small spacecraft constitute a single entity and move along a circular Earth orbit. After separation, the small spacecraft move in the vicinity of the local vertical; here, their motion trajectory, the deployment time, and other parameters depend on the parameters of control laws.

The performance of control laws is assessed by the model of OTS motion with distributed parameters, where the tether is represented as a set of material points connected to one another by elastic links. These mathematical models were used in many studies (see, e.g., [7, 8]). It is assumed that, in each elementary segment, the tension force obeys Hooke's law of elasticity.

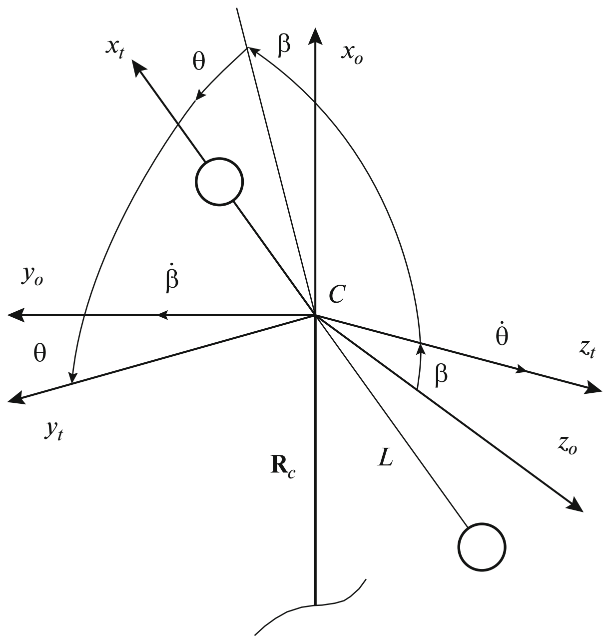

Fig. 1.

The model with distributed parameters is coupled with the equation for the control mechanism of the tether operating only for deceleration. The strength in the control mechanism is given according to the principle of feedback in terms of measurements of the tether length and rate. The spacecraft in the model with distributed parameters are treated as material points. If necessary, the rotation of end bodies can be taken into account with the help of existing methods [9]. To assess the performance of the nominal program, the so-called ideal control is used; i.e., the specific features of the control system (such as the discreteness of control, delay, and measurement errors) are disregarded.

MODEL FOR CONSTRUCTING NOMINAL PROGRAMS OF OTS DEPLOYMENT

The motion of the system is described using the following set of coordinate systems: $OXYZ$, $OX_oY_oZ_o$, $Cx_oy_oz_o$, and $Cx_ty_tz_t$. Here, $OXYZ$ and $OX_oY_oZ_o$ are orbital geocentric coordinate systems, $Cx_oy_oz_o$ is the orbital coordinate system associated with the OTS center of mass, and $Cx_ty_tz_t$ is the tether system of coordinates. The OX and OX_o axes are directed to the ascending node of the orbit and along the radius–vector of the OTS center of mass, the OZ and OZ_o axes coincide and are parallel to the vector of the kinetic moment of motion of the center of mass C of the system, and the axes of the $OX_oY_oZ_o$ and $Cx_oy_oz_o$ coordinate systems are parallel. The coordinate system $OXYZ$ is assumed to be fixed. The coordinate system $OX_oY_oZ_o$ is rotated relative to the fixed coordinate system $OXYZ$ with angular velocity $du/dt = \Omega$, where u is the argument of latitude; here, $\Omega = \text{const}$, $u = \Omega t$ for an angular orbit. The tether coordinate system $Cx_ty_tz_t$ was used earlier in [10]. In view of the specifics of the given problem, the Cx_t axis coincides with the vector of the tether tension force $\mathbf{T}$ and is directed from the center of mass C to body 1 (Fig. 1) if the tether length is $L \neq 0$. The position of the coordinate system $Cx_ty_tz_t$ relative to $Cx_oy_oz_o$ is determined by angles θ and β (Fig. 1). The transition matrix between coordinate systems is the same as in [10]. For $\beta=0$, the OTS is deployed in the orbital plane; in this case, the planes Cx_ty_t and Cx_oy_o coincide and θ is the angle of tether deviation from the local vertical drawn from the center of mass C of the system. The tether is deployed from body 1 with a mass of $m_1 = m_1^0 - \rho L$, where ρ is the linear density of the tether material and m_1^0 is the initial mass.

The equations of system motion are derived using the classical Lagrange method

$$\frac{d}{dt}\left(\frac{\partial T_c}{\partial \dot q_i}\right) - \frac{\partial T_c}{\partial q_i} = -\frac{\partial \Pi}{\partial q_i} + Q_i, \qquad (1)$$

where T_c and Π are the kinetic and potential energies of the system, respectively; q_i and $\dot q_i$ ($i = 1, 2, 3$) are the generalized coordinates and velocities, respectively; Q_i are the generalized nonpotential forces; and $q_1 = L$, $q_2 = \theta$, and $q_3 = \beta$.

The equations of motion are derived using the following obvious relations:

$$M\,\mathbf{R}_c = m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_t\mathbf{r}_t, \qquad (2)$$

$$\mathbf{r}_1 = \mathbf{R}_c - \frac{(m_1 - L\rho/2)}{M}\Delta\mathbf{r}, \qquad \mathbf{r}_2 = \mathbf{R}_c + \frac{(m_2^0 - L\rho/2)}{M}\Delta\mathbf{r}, \qquad (3)$$

where M is the center of mass of the system; $\mathbf{R}_c, \mathbf{r}_1, \mathbf{r}_2$ are the radius–vectors of the center of mass and end points; $m_t = L\rho$ is the tether mass; $\mathbf{r}_t = (\mathbf{r}_1 + \mathbf{r}_2)/2$ is the radius–vector of the center of mass of the tether; and $\Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$.

The kinetic energy in Eqs. (1) is the sum of the kinetic energies

$$T_c = \frac{1}{2}m_1\left(\dot x_1^2 + \dot y_1^2 + \dot z_1^2\right) + \frac{1}{2}m_2\left(\dot x_2^2 + \dot y_2^2 + \dot z_2^2\right) + T_t, \qquad (4)$$

where x_1, y_1, and z_1 and x_2, y_2, and z_2 are the coordinates of end bodies in the fixed coordinate system $OXYZ$ and T_t is the kinetic energy of the tether.

According to expressions (2)–(3), the coordinates of end bodies can be calculated as follows:

$$
\begin{aligned}
x_1 &= \left(R_c + L\frac{m_2 + L\rho / 2}{M}\cos\theta\cos\beta \right) \\
&\times \cos u - L\frac{m_2 + L\rho / 2}{M}\sin u, \\
y_1 &= \left(R_c + L\frac{m_2 + L\rho/2}{M}\cos\theta\cos\beta \right) \\
&\times \sin u + L\frac{m_2 + L\rho/2}{M}\cos u, \\
z_1 &= L\frac{m_2 + L\rho/2}{M}\cos\theta\sin\beta.
\end{aligned} \quad (5)
$$

$$
\begin{aligned}
x_2 &= \left(R_c - L\frac{m_1^0 - L\rho/2}{M}\cos\theta\cos\beta \right) \\
&\times \cos u + L\frac{m_1^0 - L\rho/2}{M}\sin u, \\
y_2 &= \left(R_c - L\frac{m_1^0 - L\rho/2}{M}\cos\theta\cos\beta \right) \\
&\times \sin u - L\frac{m_1^0 - L\rho/2}{M}\cos u, \\
z_2 &= -L\frac{m_1^0 - L\rho/2}{M}\cos\theta\sin\beta,
\end{aligned} \quad (6)
$$

where R_c is the magnitude of the radius–vector $\mathbf{R}_c$.

The equations are derived under the assumption that the center of mass C moves in a circular orbit, i.e., $R_c = \text{const}$ and $\Omega = \text{const}$. The validity of this assumption can be verified on specific examples for simulating the motion of OTS as a distributed system in the geocentric coordinate system.

The kinetic energy of the tether as any mechanical system is the sum of kinetic energy of the center of mass and kinetic energy of tether motion relative to its center of mass (T_r). Therefore, we have

$$T_t = \frac{1}{2}L\rho V_t^2 + T_r, \quad (7)$$

where V_t is the velocity of the center of mass of the tether.

The velocity vector of the center of mass of the tether has the following components:

$$\mathbf{V}_t = \mathbf{V}_c + \mathbf{V}_s + \mathbf{V}_r, \quad (8)$$

where $\mathbf{V}_c = \mathbf{\Omega R}_c$ is the vector of the center of mass of the system; $\mathbf{V}_s$ is the vector of the center of mass of the tether relative to the center of mass of the system due to the rotation of the tether coordinate system depending on the angular velocities $\dot\beta, \dot\theta, \Omega$; and $\mathbf{V}_r$ is the vector of the center of mass of the tether relative to the center of mass of the system due to changes in the tether length.

The components of $\mathbf{V}_c$ in the fixed coordinate system $OXYZ$ have the form

$$V_{cx} = -R_c\Omega\sin u, \quad V_{cy} = R_c\Omega\cos u, \quad V_{cz} = 0. \quad (9)$$

The components of $\mathbf{V}_s$ in the coordinate system $OXYZ$ are determined as follows:

$$\begin{pmatrix} V_{sx} \\ V_{sy} \\ V_{sz} \end{pmatrix} = D\begin{pmatrix} \omega_{sx} \\ \omega_{sy} \\ \omega_{sz} \end{pmatrix}\Delta, \quad (10)$$

where $D = D_u^T D_\beta^T D_\theta^T$ is the matrix of transition from the tether coordinate system $Cx_ty_tz_t$ to the fixed coordinate system $OXYZ$; D_u, D_β, D_θ are the elementary rotation matrices [10], (T) means matrix transposition; ω_{sx}, ω_{sy}, ω_{sz} are the projections of the vector of angular velocity of rotation of the tether system of coordinates $Cx_ty_tz_t$ relative to the coordinate system $OXYZ$ (the projections are determined in the $Cx_ty_tz_t$ system); and Δ is the quantity that determines the position of the center of mass of the tether relative to the system's center of mass.

The angular velocities ω_{sx}, ω_{sy}, ω_{sz} are determined from the following expressions

$$
\begin{aligned}
&\omega_{sx} = \dot\beta\sin\theta - \Omega\cos\theta\sin\beta, \\
&\omega_{sy} = \dot\beta\cos\theta + \Omega\sin\theta\sin\beta, \quad \omega_{sz} = \dot\theta + \Omega\cos\beta.
\end{aligned} \quad (11)
$$

The value of Δ is determined as follows:

$$\Delta = \frac{L}{2} - \frac{(m_2 + L\rho/2)}{M}L = \frac{m_1^0 - m_2 - L\rho}{2M}L. \quad (12)$$

The rate of change in the tether length has two components ($\dot L = \dot\Delta_1 + \dot\Delta_2$) as follows:

$$\dot\Delta_1 = \frac{(m_2 + L\rho)}{M}\dot L, \quad \dot\Delta_2 = \frac{(m_1^0 - L\rho)}{M}\dot L. \quad (13)$$

When the tether is released, the end points move away from each other and from the center of mass of the system, and the vectors of their relative velocities have opposite directions. Because the model of OTS motion is constructed for inextensible tether, all the tether points located on opposite sides of the center of mass of the system have the same velocity, which is equal to the velocity of end masses. Therefore, if the center of mass of the tether is between the center of mass of the system and mass m_2, the velocity V_r should be taken as $\dot\Delta_2$. Otherwise, we have $V_r = \dot\Delta_1$. Knowing that velocity V_r, which is always directed along the tether (in some direction), one can easily determine its components in the coordinate system $OXYZ$:

$$\begin{pmatrix} V_{rx} \\ V_{ry} \\ V_{rz} \end{pmatrix} = D \begin{pmatrix} -V_r \\ 0 \\ 0 \end{pmatrix}. \quad (14)$$

Here, the positive direction of V_r is taken to be the case when the center of mass of the tether is between the center of mass of the system and the point m_2; i.e., the velocity vector V_r is directed towards m_2 and, therefore, is opposite to the x_t axis (Fig. 1). However, it should be noted that, from the viewpoint of kinetic energy calculations, the final expressions obtained for $\mathbf{V}_t$ are the same for any position of the center of mass of the tether and, thus, the equations of motion are the same.

The kinetic energy of the tether T_r, as it rotates relative to its center of mass is

$$T_r = \frac{1}{2} J_t \left(\omega_{sy}^2 + \omega_{sz}^2\right), \quad (15)$$

where $J_t = \frac{1}{12}\rho L^3$ is the moment of inertia of the tether relative to its center of mass.

The set of expressions (5)–(15) describes the dependence of kinetic energy of OTS on generalized coordinates and velocities, which makes it possible to use expression (4) in the Lagrange equations.

To obtain generalized forces conditioned by the Newtonian central gravity field of the Earth, we use the following expressions for potential energy of each body involved in the system:

$$\Pi_1 = -\frac{K(m_1^0 - L\rho)}{r_1}, \quad \Pi_2 = -\frac{Km_2}{r_2}, \quad \Pi_t = \int_L d\Pi_t, \quad (16)$$

where K is the Earth's gravity parameter,

$$r_1 = \sqrt{x_1^2 + y_1^2 + z_1^2} = \sqrt{R_c^2 + \Delta_1^2 + 2\Delta_1 R_c \cos\theta\cos\beta},$$

$$r_2 = \sqrt{x_2^2 + y_2^2 + z_2^2} = \sqrt{R_c^2 + \Delta_2^2 - 2\Delta_2 R_c \cos\theta\cos\beta},$$

$$d\Pi_t = -\frac{K\rho dx}{r_t} = -\frac{K\rho dx}{\sqrt{R_c^2 + x^2 - 2xR_c\cos\theta\cos\beta}}.$$

Using the expression for the differential $d\Pi_t$, we write the potential energy of the tether as

$$\Pi_t = -K\rho \int_{-\Delta_1}^{\Delta_2} \frac{dx}{\sqrt{R_c^2 + x^2 - 2xR_c\cos\theta\cos\beta}}. \quad (17)$$

To approximately calculate the generalized gravity forces, we expand the functions $\Pi_1, \Pi_2, d\Pi_t$ in series with respect to the parameters Δ_1, Δ_2, x, keeping the terms up to the second order. Then, we obtain

$$\begin{gathered} \Pi_1 \approx -\frac{Km_1}{R_c} \\ \times\left[1 - \Delta_1\cos\theta\cos\beta + \frac{1}{2}\Delta_1^2\left(3\cos^2\theta\cos^2\beta - 1\right)\right], \\ \Pi_2 \approx -\frac{Km_2}{R_c} \\ \times\left[1 + \Delta_2\cos\theta\cos\beta + \frac{1}{2}\Delta_2^2\left(3\cos^2\theta\cos^2\beta - 1\right)\right], \\ d\Pi_t \approx -\frac{K\rho}{R_c} \\ \times\left[1 + x\cos\theta\cos\beta + \frac{1}{2}x^2\left(3\cos^2\theta\cos^2\beta - 1\right)\right]. \end{gathered} \quad (18)$$

Approximate expressions (18) can be used if $\Delta_{1,2}/R_c \ll 1$ and $x/R_c \ll 1$. Calculating integral (17), in view of expressions (18) and differentiating with respect to generalized coordinates, we obtain

$$\begin{gathered} Q_L = -\frac{\partial\Pi}{\partial L} = L\nu_e\Omega^2\left(3\cos^2\theta\cos^2\beta - 1\right), \\ Q_\theta = -\frac{\partial\Pi}{\partial\theta} = -\frac{3}{2}J_e\Omega^2\cos^2\beta\sin 2\theta, \\ Q_\beta = -\frac{\partial\Pi}{\partial\beta} = -\frac{3}{2}J_e\Omega^2\cos^2\theta\sin 2\beta, \end{gathered} \quad (19)$$

where $J_e = \dfrac{L^2\left(12m_1^0 m_2 - 8L\rho m_2 + 4L\rho m_1^0 - 3L^2\rho^2\right)}{12M}$,

$$\nu_e = \frac{\left(m_1^0 - L\rho\right)\left(m_2 + L\rho/2\right)}{M}.$$

Here, it should be noted that J_e is the moment of inertia of the system relative to its center of mass and the relation $\dfrac{dJ_e}{dL} = 2L\nu_e$ is true.

The system is influenced by not only potential gravity forces, but also nonpotential forces (e.g., the nominal tension force F_n at the point of tether release with work only on the possible shift δL).

Differentiating kinetic energy (4) in accordance with the Lagrange equations and taking into account potential (19) and nonpotential forces, we obtain a system of differential equations for the OTS motion as follows:

$$\begin{gathered} \ddot{L} = \frac{\nu_e}{M_e} L\left(\dot{\theta}^2 + 2\dot{\theta}\Omega\cos\beta\right. \\ + \dot{\beta}^2\cos^2\theta - \Omega^2\cos^2\theta\sin^2\beta \\ \left. + \Omega\dot{\beta}\sin\beta\sin 2\theta + 3\Omega^2\cos^2\theta\cos^2\beta\right) - \frac{F_n + R_e}{M_e}, \end{gathered} \quad (20)$$

$$\ddot{\theta} = -2\frac{\nu_e}{J_e}L\dot{L}\left(\dot{\theta} + \Omega\cos\beta\right) + \frac{1}{2}\Omega^2\sin 2\theta\sin^2\beta - \frac{1}{2}\dot{\beta}^2\sin 2\theta + 2\Omega\dot{\beta}\cos^2\theta\sin\beta - \frac{3}{2}\Omega^2\sin 2\theta\cos^2\beta, \quad (21)$$

$$\cos^2\theta\ddot{\beta} = -2\frac{\nu_e}{J_e}L\dot{L}\left(\dot{\beta}\cos^2\theta + \Omega\sin 2\theta\sin\beta\right) + \dot{\theta}\dot{\beta}\sin 2\theta - 2\Omega\dot{\theta}\cos^2\theta\sin\beta - 2L^2\Omega^2\cos^2\theta\sin 2\beta, \quad (22)$$

where

$$R_e = \rho\dot{L}\dot{\Delta} = \rho\dot{L}^2\frac{m_1^0 - m_2 - 2L\rho}{2M},$$

$$M_e = \frac{\left(m_1^0 - L\rho\right)\left(m_2 + L\rho\right)}{M}.$$

If the mass of the first body is $m_1^0 \gg m_2, \rho L$, we have $m_1^0 \approx M$ and, therefore, $M_e \approx m_2 + L\rho$, $\nu_e \approx m_2 + L\rho/2$, $J_e \approx L^2\left(m_2 + L\rho/3\right)$. Then, as a partial case, we obtain the equations given in [5], when the base spacecraft is a body of a large mass and moves along a fixed circular orbit. If this assumption is coupled with the consideration of the plane problem $\beta = \dot{\beta} = \ddot{\beta} = 0$, we obtain the equations given in [2]. The nominal trajectories of OTS deployment are often constructed from equations with a weightless tether (see, e.g., [4, 10]) under the following assumptions: $m_1^0 \gg m_2$, $\rho \approx 0$, which also follow from system (20)–(22). Equations (20)–(22) can be used not only in the OTS deployment from small spacecraft, but also for designing extended tether systems with a mass that is comparable to the masses of the end bodies.

ANALYSIS OF NOMINEE PROGRAMS OF OTS DEPLOYMENT IN THE VERTICAL POSITION

The control force in the tethering mechanism is often given as the sum as follows [2, 10]:

$$F = F_n + p_l\left[l - L(t)\right] + p_V\left[V_l - V(t)\right], \quad (23)$$

where $L(t)$ and $V(t)$ are program (nominal) dependences of the tether length and velocity on time, which corresponds to system (20)–(22); p_l, p_V are feedback coefficients; and l, V_l are the perturbed values of the tether length and velocity, respectively.

Equations (20)–(22) make it possible to correct the existing laws of OTS deployment in the vertical position. We consider programs of OTS deployment in the vertical position on the basis of the law [1], which ensures that the transverse oscillations in the tether system are damped. To correct this law, it is sufficient to consider Eq. (20) at $\dot{L} = \ddot{L} = 0$. Then, system (20)–(22) will have vertical equilibrium positions $\theta = 0$ and $\theta = \pi$ ($\beta = \dot{\beta} = \dot{\theta} = \dot{L} = 0$, $L = L_{\text{end}} \neq 0$) if the tension force is determined from the expression

$$F_n = \nu_e\Omega^2\left[a\left(L - L_{\text{end}}\right) + b\frac{V}{\Omega} + 3L_{\text{end}}\right], \quad (24)$$

where a, b are parameters of the law and L_{end} is the tether length in the end position of the system.

The stability analysis of the end equilibrium positions is performed in a standard way by constructing a linearized system. The linearized system has the following specific features: (1) The equations for deviations by the variables $\beta, \dot{\beta}$ are separated from the remaining equations of the system and have always purely imaginary eigenvalues;

(2) for $b = 0$ and $a > a_* > 0$, where a_* is some critical value, all the eigenvalues of the linearized system are purely imaginary;

(3) if $b > 0$ $\left(a > a_*\right)$, we have asymptotic stability for the plane motion of OTS $\left(\beta = \dot{\beta} = 0\right)$ in the linearized formulation;

(4) there is a critical value of the parameter b_*, such that the two eigenvalues become real and negative at $b > b_* > 0$.

These propositions are independent of the OTS parameters $\left(m_1^0, m_2, \rho\right)$, the end length of the tether $\left(L_{\text{end}}\right)$, and the height of the original circular orbit $\left(H\right)$. Here, we consider fairly high orbits for which the atmospheric influence can be disregarded during the deployment. In typical terminal cases, for a weightless tether $\left(\rho = 0\right)$ and a base spacecraft with a large mass $\left(m_2/m_1^0, \rho L/m_1^0 \to 0\right)$, we have $a_* = 3$. In other cases, the critical value satisfies $0 < a_* < 3$ and depends on the parameters $m_1^0, m_2, \rho, L_{\text{end}}, H$.

If we consider tether control mechanisms that only operate for deceleration and have no tether retraction, the parameter must be satisfy the condition $b > b_*$. In this case, the nonlinear model yields $L(t) \to L_{\text{end}}$ $\left(L(t) < L_{\text{end}}\right)$ and $\dot{L}(t) \to 0$ $\left(\dot{L}(t) > 0\right)$ as $t \to \infty$, which corresponds to the specific features of these mechanisms. Due to asymptotic stability, the OTS deployment can be completed if $L_{\text{end}} - L(t) < \delta_l$ and $\dot{L} < \delta_V$, where δ_l and δ_V are given tolerances.

The parameter b describes the nominal dependence for velocity $V(t)$. Therefore, the choice of b can ensure that

$$0 < V_{\min} \leq V \leq V_{\max}, \quad (25)$$

where the limiting values depend on capabilities of the control mechanism.

When the parameter b increases, the time of OTS deployment increases on the one hand and the maximum rate $\max_t \dot{L}$, decreases, which allows the upper limitation in inequality (25) to be satisfied. On the other hand, when $b \to b_*$ $\left(b > b_*\right)$, the deployment time decreases. The lower limitation in inequality (25) may be violated in the initial section of the OTS deployment (immediately after the separation of the small spacecraft). However, when the parameter b decreases $\left(b > b_*\right)$, this limitation can also be satisfied. Thus, the choice of the parameter b is based on a compromise ensures that inequality (25) is satisfied.

Another important limitation that should be taken into account with law (24) is the following limitation on the nominal tension force:

$$0 < F_{\min} \le F_n \le F_{\max}. \tag{26}$$

For an OTS that consists of small spacecraft, the upper limitation on the force is certainly satisfied due to the materials used. The lower limitation should eliminate the tether slacking; here, it is necessary to take into account the transient processes occurring during the operation of the control system because the control force is given as (23). An analysis indicated that the limitation $F_{\min} \le F_n$ in using law (24) can only occur in the initial section of the OTS deployment (immediately after the separation of the small spacecraft). To avoid this, a simple algorithm is used. If the force F_n determined by formula (24) is less than $F_{\min}$, it is taken that $F_n = F_{\min}$; i.e., the system is deployed with a constant tension force. During further deployment of the OTS, the tether length increases, which leads to an increased magnitude of F_n calculated according to (24). Therefore, after a relatively short time, the nominal tension force is again calculated from formula (24). Clearly, the use of this algorithm in the initial section of the OTS deployment cannot change the end state of the system determined according to law (24).

Here, it should be noted that, for the partial case considered in [5], when $m_2/m_1^0 \ll 1$ and $\nu_e = m_2 + \rho L/2$, the lower limitations in inequalities (25)–(26) can only be violated at relatively large tether lengths L_{end} (at least 20–30 km). For a system that consists of small spacecraft with masses comparable to the tether mass, the same limitations can also be violated for small values of L_{end} (a few kilometers). Therefore, in this case, the choice of the law parameters is a more difficult task.

In view of the above discussion, the construction of the nominal deployment program is formulated as a two-parameter problem of constrained optimization $\min_{a,b} J$, where J is some optimality criterion (e.g., the time of system deployment) under the constraints given above (25)–(26). The relative rate of separation of small aircrafts can be given or included in the set of chosen parameters because its value directly affects the lower limitations in inequalities (25)–(26).

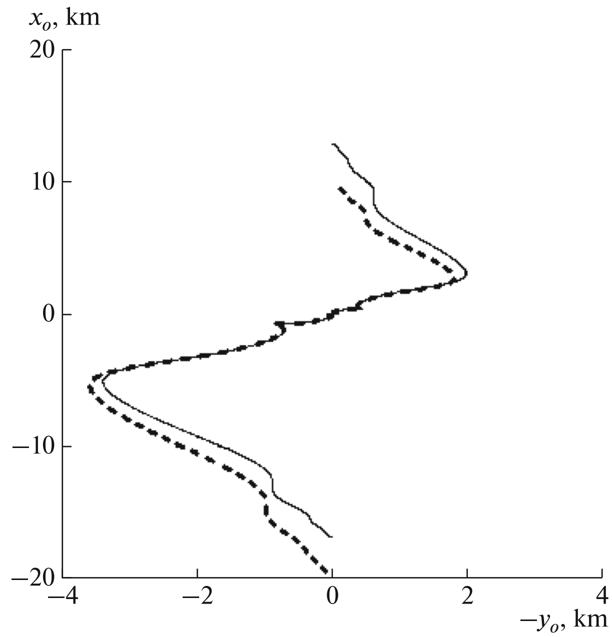

Fig. 2.

Mathematical model (20)–(22) makes it possible to correct the OTS deployment trajectories and nominal dependences $L(t)$, $V(t)$, $F_n(t)$, which enters into expression (23) to calculate the required control force in the tethering mechanism. As an example, Fig. 2 shows the trajectories of end bodies relative to the center of mass for a deployment length of L_{end}=30 km with the following initial data: $m_1^0 = 20$ kg, $m_2 = 10$ kg, $\rho = 0.2$ kg/km, $H = 1000$ km, and $a = b = 4$. The axis of ordinates coincides with the local vertical drawn through the center of mass of the system. The initial relative rate of separation ($\dot{L}(0) = 2.5$ m/s) is directed along the local vertical. The OTS deployment time corresponds to approximately three turns of the center of mass of the system around the Earth and provides 0.2% of the relative error by the tether length. Figure 2 shows the trajectories of end bodies with weighted (solid line) and weightless (dashed line) tethers. The positions of end bodies calculated by various models differ (in terms of distance) by almost 3 km. In this example, the tether mass is 6 kg, which is comparable to m_2 and constitutes around one-third of the mass m_1^0 of spacecraft that releases the tether.

If the specific features of the OTS deployment with small spacecraft are disregarded, this can lead to large errors in the estimate of the nominal tension force of the tether F_n. For simpler models, the error in calculating F_n is largely associated with the fact that the coefficient ν_e in expression (24) is estimated incor-

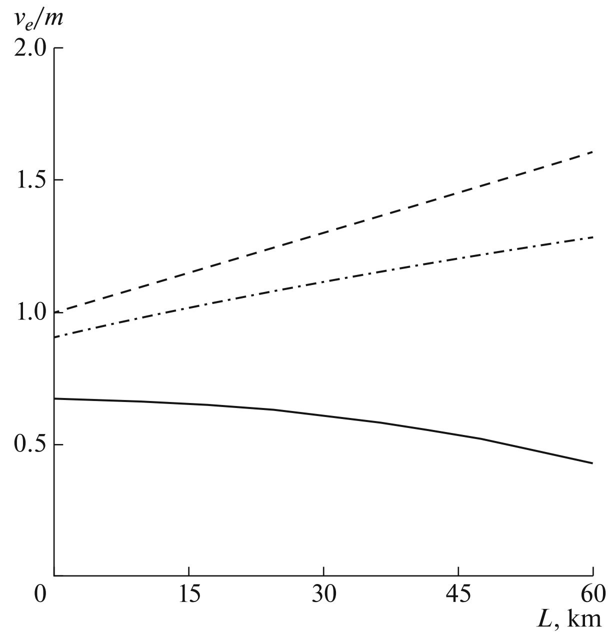

Fig. 3.

rectly. Figure 3 compares the estimates of ν_e for different models. Since the coefficient ν_e has a unit of mass, the plots show the dimensionless parameter ν_e/m_2 as a function of the tether length. The value of the ratio ν_e/m_2 provides an approximate estimate for the tether tension calculated by different models compared with the tension force found in the simplest case of a weightless tether $\rho \to 0$ and $m_2/m_1^0 \to 0$; then, $\nu_e \to m_2$ [1, 4]. The upper dashed line corresponds to the model described in [2, 5] for $m_2/m_1^0 \to 0$; therefore, $\nu_e \to m_2 + \rho L/2$. The solid line corresponds to the case of $\rho = 0.2$ kg/km, $m_1^0 = 20$ kg, and $m_2 = 10$ kg. The dash-dotted line corresponds to $m_1^0 = 100$ kg, $m_2 = 10$ kg.

Thus, the mathematical model given by Eqs. (20)–(22) and the resulting program of tether release (24) make it possible to correct the existing laws of OTS deployment in the vertical position and obtain nominal dependences that take into account the specificity of the given problem.

NOMINAL PROGRAM FOR QUICK DEPLOYMENT OF OTS WITH SMALL SPACECRAFT

The OTS programs that ensure the system motion after the tether release with some angular velocity of rotation relative to the center of mass are normally constructed using the criteria of fast operation [4]. In this case, the use of the Pontryagin maximum principle leads to relay control laws [4]. Therefore, in this case, the OTS deployment is divided into two phases, i.e., acceleration and deceleration. The programs of quick OTS deployment differ in their dynamics because they are characterized by relatively large accelerations. Therefore, the use of the tether mass in the construction of programs of OTS deployment with small spacecraft becomes of special importance.

To estimate the tether mass effect, we consider a fairly simple program of rapid deployment of OTS [4] obtained from using the Pontryagin maximum principle:

$$F_n(t) = \begin{cases} F_{\min}, \text{ if } t < t_n \\ F_{\max}, \text{ if } t \geq t_n \end{cases}, \tag{27}$$

where $F_{\min}, F_{\max}, t_n$ are the law parameters. This law of deployment (with an appropriate choice of its parameters) provides the deployment of OTS with a length of L_{end} and a tether deviation in the end state from the vertical with a zero end relative velocity.

As an example of the use of law (27), we consider an OTS deployment with the same characteristics as described above. In the initial state, the center of mass of the system moves along a circular orbit of a height of 1000 km. The bodies in the system have the following characteristics: $m_1^0 = 20$ kg, $m_2 = 10$ kg, and $\rho = 0.2$ kg/km. The end length of the tether is $L_{\text{end}} = 30$ km. Program (25) with the parameters $F_{\min} = 0.02$ H, $F_{\max} = 0.434$ H, and $t_n = 4730$ s allows the OTS to be deployed at a given length with a zero end relative velocity. The deployment time is 1.5 h. The law parameters were chosen from model (20)–(22) taking into account the tether mass (in the end state, the total mass of the deployed tether is 6 kg). The calculation of the deployment process with the same law and the same parameters by the model with a weightless tether $(\rho = 0)$ is characterized by noticeable differences in the dependences $L(t)$ and $V(t)$, which describe the nominal motion of the system (Fig. 4). Figure 4a shows the function $V(t)$ obtained by the model for weighted (solid line) and weightless (dashed line) tethers. The rate difference at the end of the OTS deployment is almost 2.3 m/s, which is comparable to the initial separation rate. Here, the end relative rate becomes negative, which is unacceptable in this case. The difference in functions $V(t)$ leads to a difference in dependences $L(t)$; here, the difference $\Delta L(t)$ reaches 1.5 km (Fig. 4b). The positions of end bodies obtained by various models differ (in terms of distance) by almost 3.5 km.

The parameters of law (25) are chosen by solving boundary value problems, which ensure the given end conditions for the system motion using nonlinear programming methods. It follows from the above example that, if the masses of end bodies and the tether are comparable in magnitude, this requires the use of more complex models, such as (20)–(22), taking into account the tether weight.

ASSESSMENT OF THE IMPLEMENTATION OF PROGRAMS OF DEPLOYMENT OF OTS WITH SMALL SPACECRAFTS

The implementation of programs of the deployment of OTS with small spacecraft and the validity of assumptions are assessed by a discrete model of motion, which treats the tether as a set of n material points. Each material point (including the end points) is affected by gravity forces in the central Newtonian field, inertial forces, and tether tension forces. The tension forces are calculated by Hooke's law taking into account that the mechanical links are one-sided (the tether is not affected by compressive forces). The aerodynamic forces and dissipative forces in the tether are disregarded. If the number of points n is large enough, this model describes the dynamics of OTS as a system with distributed parameters.

Here, we describe only some characteristic features of the discrete model because similar mathematical models have been used in many studies (see, e.g., [5, 7, 8]).

The motion of a tether system is described in the geocentric fixed coordinate system *OXYZ* as the following set of differential equations:

$$\frac{d\mathbf{r}_k}{dt} = \mathbf{V}_k, \quad m_k \frac{d\mathbf{V}_k}{dt} = \mathbf{G}_k + \mathbf{T}_k - \mathbf{T}_{k+1}, \tag{28}$$

where $\mathbf{r}_k$, $\mathbf{V}_k$, and m_k are the radius–vector, velocity, and mass of the kth material point; $\mathbf{G}_k$ is the gravity force; $\mathbf{T}_k$ is the tether tension force acting between the kth and $(k+1)$th points and applied to the kth point; and $k = 1, 2, ..., n$. In system (28), the tether is released from a small spacecraft with mass m_1. During the system deployment, the number of material points n that describe the tether increases. Thus, the second end body (another small spacecraft) is a material point of mass m_n.

System (28) is coupled with equations that approximately take into account the dynamics of the control mechanism operation. These equations can be written as [10]

$$m_i \frac{dV_l}{dt} = T_1 - F, \quad \frac{dl}{dt} = V_l, \tag{29}$$

where the coefficient m_i describes the inertia of the control mechanism; l is the length of the tether released from the control mechanism; V_l is the tether release rate; T_1 is the tension force in the first phase, which starts with the spacecraft that releases the tether; and F is the control force in deceleration mechanism (23). The change in inertia of m_i during the OTS deployment is ignored.

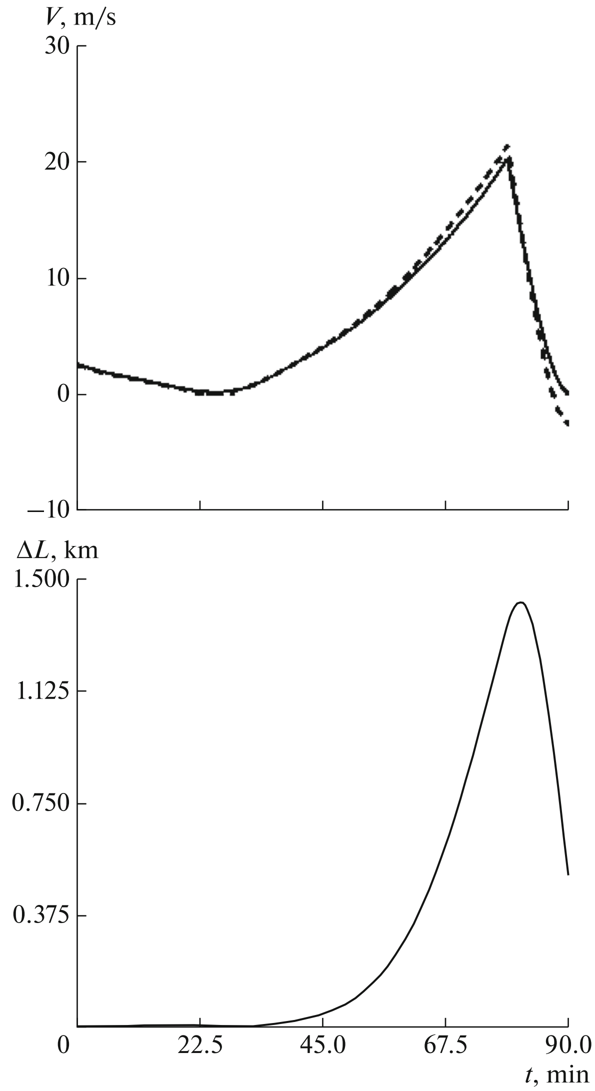

Fig. 4.

The tension force is calculated by Hooke's law taking into account that the mechanical link is one-sided as follows:

$$\mathbf{T}_k = c\lambda_k(\gamma_k - 1)\frac{\Delta\mathbf{r}_k}{\Delta r_k}, \tag{30}$$

where $\Delta\mathbf{r}_k = \mathbf{r}_{k+1} - \mathbf{r}_k$, $\gamma_k = \Delta r_k / \Delta L_{0k}$, ΔL_{0k} is the undeformed length of the tether segment, c is the coefficient of tether stiffness, and $\lambda_k = \begin{cases} 1, \text{ if } & \Delta r_k \geq \Delta L_{0k} \\ 0, \text{ if } & \Delta r_k < \Delta L_{0k} \end{cases}$.

For the numerical simulation of system (28)–(29), an important point is the algorithm of the introduction of a new point [5], which is necessary for the OTS deployment process. The new material point is introduced at a time when the undeformed length of the first section of the tether, measured from the spacecraft that releases the tether, becomes higher than

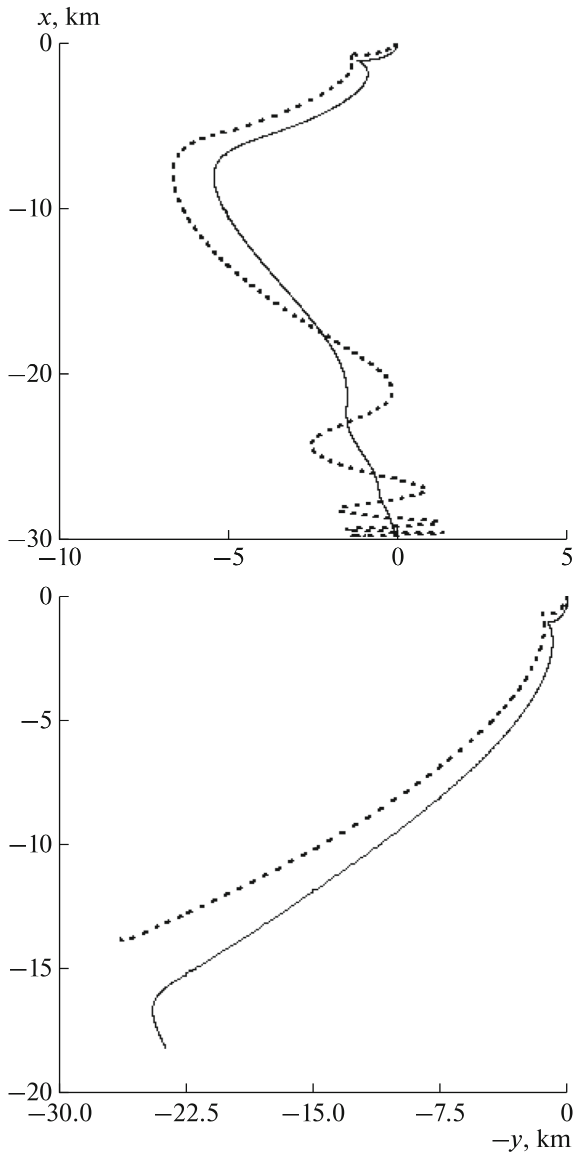

Fig. 5.

$L_{end}/(n-1)$. Here, n is the total number of points when the tether is fully released according to the nominal control law. The undeformed length of the section is calculated by Eqs. (29), which excludes the tether sections that have already been formed. The position of a new point is determined from the condition that the tension forces on both sides obtained by formula (30) are equal; here, the magnitude of these forces must be equal to the tension force at the same section before the introduction of the new point. The velocity vector of the new point relative to the spacecraft (m_1) that releases the tether is determined from the known relative velocity of the nearest point through appropriate proportions. Finally, the conservation law for the motion of the system is used to correct the velocity components of the base spacecraft (m_1).

Equations (28)–(29) make it possible to assess how the OTS deployment is affected by some factors that were not taken into account in the construction of nominal trajectories of the system motion. These factors include the following: (1) the expandability of the tether, (2) the inertia of the control mechanism, and (3) the perturbed motion of the center of mass of the system. Mathematical model (28)–(29) can also be used for the preliminary choice of feedback coefficients p_l, p_V, which enter into the expression for control force (23). However, this choice will not take into account the disturbances that arise during the implementation of specific control systems; the discreteness of control actions and individual elements of control systems, the delay, the measurement errors, etc. However, the quality operation of this ideal control is a necessary condition that must be met at the preliminary stage of the control system construction.

The numerical calculations using the discrete mathematical model of OTS motion (28)–(29) were performed for the above-mentioned nominal deployment programs and for the same initial data. The tether rigidity is $c = 7070$ N. The minimum control force is 0.02 N and the inertia of the control mechanism is 0.3 kg. The problem of optimal choice of the feedback coefficient was not solved because it requires separate consideration. Therefore, the feedback coefficients were taken to be constant ($p_l = 0.243$, $p_V = 7.824$) in accordance with the results of [5, 11], which considers OTS deployment programs similar to program (24), but for the above-mentioned special cases.

The simulation of the OTS deployment (L_{end} = 30 km) by the discrete model in accordance with program (24) indicated that, if the separation of small spacecraft is ideal (the relative separation rate coincides with as the local vertical), the final deployment errors are slight, i.e., $\Delta L < 10$ m, $\Delta V < 0.1$ m/s. The inertia of the control mechanism in a fairly wide range $m_i \in [0.2, 1]$ kg has almost no effect on the magnitude of errors. The errors in the separation of small spacecraft affect the transient processes in the control system differently. The change in the relative separation rate in a fairly wide range $\dot{L}(0) = 2.5 \pm 0.5$ m/s does not add up to control errors. The highest effect on the accuracy of OTS reduction to a given state is based on the errors associated with the direction of the relative separation rate of small spacecraft. For example, for the given initial data, if the error in the direction of separation is $\Delta\theta(0) = -20°$, the control errors become increased. Specifically, the maximum (by trajectory) error along the tether length $\max_t \Delta L$ grows up to 1.5 km. In addition, the finally deployed system is not in the vertical position and oscillates relative to the center of mass with some amplitude. This effect is illustrated in Fig. 5a, which shows the perturbed trajectory (dashed line) of the end body m_n relative to a small spacecraft that releases the tether (the origin of coordinates). The unperturbed trajectory of the same body is shown by a solid line. The resulting perturbation cannot be compensated for by control because the control force is always directed along the tether.

Since the program of quick tether release (27) is characterized by relatively large accelerations and the end position of OTS is not an equilibrium state and, in this case, the simulation of the OTS deployment ($L_{\text{end}} = 30$ km) by discrete model (28)–(29) leads to different results compared to the previous program. Even with the ideal separation of small spacecraft, the above-mentioned program leads to maximum control errors $\max_t \Delta L \approx 1.7$ km, $\max_t \Delta V \approx 0.6$ m/s, although the same errors when the deployment is completed are close to zero. However, the large errors arising during the OTS deployment lead to a difference in the position of the end point relative to its nominal position (1.2 km). The effect of object separation errors on control defects is qualitatively similar to the previous case. The deployment trajectory is especially sensitive to the initial errors in the direction of the separation of small spacecraft. Figure 5b shows the trajectories of a smaller spacecraft (m_n) relative to a spacecraft with control mechanism for the case when the error along the separation direction is $\Delta\theta(0) = -20^{\circ}$. The solid line denotes the nominal trajectory and the dashed line denotes the perturbed trajectory. The error in the end state of the smaller spacecraft relative to the base spacecraft reaches 5.2 km. The authors did not manage to significantly reduce the control errors by varying the feedback coefficients in expression (23).

The construction of nominal programs of OTS deployment assumed that center of mass of the system moves along a fixed circular orbit. In these cases, the verification of this assumption by the model with distributed parameters shows that the deviations in the orbit height ($H = 1000$ km) at the end tether length ($L_{\text{end}} = 30$ km) do not exceed 16 km.

When simulating the motion with discrete model (28)–(29) for the OTS with the above-mentioned characteristics, it will suffice to specify the length of the segment at a level of 1–3 km. A further increase in the number of points hardly changed the simulation results.

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