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Semi-Empirical Model of the Combustion Wave in a Gas Suspension of Magnesium Particles

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Abstract: A physicomathematical model within the framework of the approach of mechanics of reacting heterogeneous media is proposed to describe the combustion wave in a mixture of a gas and fine magnesium particles. The model is verified on the basis of dependences of the limiting temperature of ignition and combustion wave velocity on the radius and volume concentration of particles. It is guaranteed that the model is valid in the range of particle radii from 7.5 to 35 μm and in the range of volume concentrations of particles $(1.2–2.4) \cdot 10^{-4}$.

Keywords: gas suspension, ignition, combustion, mathematical modeling, combustion wave structure.

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INTRODUCTION

The problem of the physicomathematical description of the combustion wave in gas suspensions is of considerable interest owing to various issues of industrial dust fire safety. In particular, one of the criteria characterizing the fire hazard of gas suspensions, in addition to the ignition delay time, is the velocity of propagation of the combustion wave over the mixture or, in a broader sense, the dependence of these parameters on the initial parameters of the mixture: pressure, volume concentration of particles, particle radius, and, possibly, some other parameters. In our previous publications [1, 2], we proposed a discrete-continuum model for the description of combustion of magnesium particles in a mixture with a gaseous oxidizer with due allowance for the pre-flame oxidation process. In this mathematical model, the heat sources (particles) are discrete and have small, but finite sizes. Therefore, the particle radius is one of the initial parameters. The global kinetic scheme of oxidation of a single particle allows one to describe some characteristics of the flame in an ensemble of particles. Based on this scheme, it was demonstrated that the area of applicability of the discrete-continuum model for aerosuspensions of magnesium particles is extended if the pre-flame oxidation reaction is taken into account. In particular, reasonable agreement was obtained between the calculated and experimental data on the flame velocity as a function of the size and initial mass concentration of particles. It should be noted, however, that this model, which described the experiments [3] in a certain range of initial parameters of the mixture, is rather complicated in implementation because it is a system of ordinary differential equations for the temperature of N particles. This means that the model becomes computationally expensive at large values of N. Moreover, the gas temperature was described by the thermal distributed (one-dimensional unsteady) mathematical model with the corresponding adjoint boundary conditions, which further increases the computational time. Nevertheless, an advantage of this model should be also noted: it guarantees stability of the resultant combustion wave in the form of the limiting solution in calculations at large times and some reasonable values of the problem parameters.

It seems to be useful to construct a simple phenomenological model of the examined phenomenon, which would allow one to determine the steady structure

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of the combustion wave based on experimental data and to perform calculations in an extended range of parameters of the mixture as compared to experimental investigations. This model does not describe the process of initiation, which requires solving the problem in an unsteady formulation and is planned for further studies.

FORMULATION OF THE PROBLEM OF IGNITION

Let us consider a fine magnesium particle located in an oxidizer medium with a temperature T_c. The equations that describes ignition in the Semenov model of the thermal explosion has the following form (see, e.g., [4]):

$$mc_p\frac{dT}{dt} = -Sk(T-T_c) + S\rho_{\text{ox}}qKf(T_m,T)\exp\left(-\frac{E}{RT}\right). \quad (1)$$

Here $m = (4/3)\pi r_p^3\rho_p$, c_p, and T are the mass, specific heat, and temperature of the particle, respectively, $k = \lambda_1\text{Nu}/2r_p$ is the heat transfer coefficient, λ_1 is the thermal conductivity of the gas, r_p is the particle radius, Nu is the Nusselt number, $S = 4\pi r_p^2$ is the particle surface area, ρ_p and ρ_{ox} are the densities of the particle and oxide film, respectively, $q = 4.9\cdot10^7$ J/kg is the thermal effect of the reaction normalized to the oxide mass [1], T_m is a certain limiting temperature, K is the preexponent (kinetic constant), E is the activation energy, and $R = 8.3144$ J/(mole$\cdot$K) is the universal gas constant. The function $f(T_m,T)$ is equal to $(T_m-T)/T_0$ if thermal deceleration of the oxidation reaction at high temperatures is taken into account (model 1) and to unity otherwise (model 2), and T_0 is the initial temperature of the particle:

$$T(0) = T_0. \quad (2)$$

Thus, determination of the thermal dynamics of a particle heated in a gas with a temperature $T_c > T_0$ reduces to solving the Cauchy problem (1), (2).

Let us introduce the time scale $t_0 = r_p/K$ and dimensionless variables

$$\bar T = \frac{T}{T_0},\ \ \bar T_c = \frac{T_c}{T_0},\ \ \bar T_m = \frac{T_m}{T_0},\qquad \bar t = \frac{t}{t_0},\ \ \bar E = \frac{E}{RT_0},\ \ \bar q = \frac{q}{c_pT_0}. \quad (3)$$

Then, Eqs. (1) and (2) can be presented in the following form (hereinafter, the bar over the dimensionless variables is omitted):

$$\frac{dT}{dt} = 3\xi q\left[-\alpha(T-T_c) + f(T_m,T)\exp\left(-\frac{E}{T}\right)\right], \quad (4)$$

$$T(0) = 1.$$

In these equations, we have

$$\xi = \frac{\rho_{\text{ox}}}{\rho_p},\ \ \alpha = \frac{\lambda_1\text{Nu}T_0}{2\rho_p r_p\xi qK}. \quad (5)$$

Ignition Manifold

Let us perform calculations by model 1. To study the types of solutions of the Cauchy problem (4), we use the methods of the elementary catastrophe theory. For this purpose, we equate the right side of the differential equations (4) and its derivative with respect to T to zero:

$$-\alpha(T-T_c) + (T_m-T)\exp\left(-\frac{E}{T}\right) = 0, \quad (6)$$

$$-\alpha - \exp\left(-\frac{E}{T}\right) + (T_m-T)\exp\left(-\frac{E}{T}\right)\frac{E}{T^2} = 0. \quad (7)$$

Equations (6) and (7) yield the equation for determining steady points of Eq. (4)

$$T^2 + \frac{E(T_m+T_c)}{T_c-T_m-E}\,T - \frac{ET_cT_m}{T_c-T_m-E} = 0, \quad (8)$$

whence we find twice degenerate critical points in the plane (α, T):

$$T^{\pm} = \frac{E(T_m+T_c)}{2(E+T_m-T_c)} \times\left[1 \pm \sqrt{1-\frac{4T_cT_m(E+T_m-T_c)}{E(T_m+T_c)^2}}\right], \quad (9)$$

$$\alpha^{\pm} = \frac{T_m-T^{\pm}}{T^{\pm}-T_c}\exp\left(-\frac{E}{T}\right). \quad (10)$$

It should be noted that Eq. (9) yields the constraint on the minimum value of the activation energy

$$E \geqslant E_{\min} = \frac{4T_cT_m}{T_m-T_c}. \quad (11)$$

If we assume that $T_m = nT_c$ $(n > 1)$, then it follows from Eq. (11) that the value of $E_{\min}$ decreases with increasing n or, which is the same, with increasing T_m; in the limit, we obtain $\underset{T_m\to\infty}{E_{\min}} = 4T_c$.

Table 1. Limiting temperature of ignition T_c as a function of the particle radius r_p

r_p, μm	T_c, K
7.5	1053
10	1030
15	990
20	960
25	940
30	920
35	912

Determination of Kinetic Constants

The dependence of the limiting temperature of the medium (i.e., minimum temperature at which ignition occurs) on the particle radius is known from experiments. These data borrowed from [5–8] are summarized in Table 1.

At the limit of ignition, Eq. (9) yields the approximate relation for the critical temperature

$$T^- = L(1 + h\varepsilon), \tag{12}$$

where

$$L = \frac{bg}{2}, \quad h = \frac{a}{g}\left(\frac{2c}{b^2\sqrt{d}} - g\right), \quad a = T_m - T_c,$$

$$b = T_m + T_c, \quad c = T_m T_c, \quad d = 1 - 4c/b^2,$$

$$g = 1 - \sqrt{d}, \quad \varepsilon = 1/E \ll 1.$$

Using Eq. (12), we obtain the following expression from Eq. (10):

$$\alpha^- = \frac{a - T_c h\varepsilon}{T_c h\varepsilon} \exp\left(-\frac{E}{L}\right) \exp\left(\frac{h}{L}\right). \tag{13}$$

Then, taking into account Eq. (5), we obtain the following relation at the limit of ignition:

$$\frac{\lambda_1 \mathrm{Nu} T_0}{2\rho_{\mathrm{ox}} r_p q K} = \frac{a - T_c h\varepsilon}{T_c h\varepsilon} \exp\left(-\frac{E}{L}\right) \exp\left(\frac{h}{L}\right). \tag{14}$$

Using the experimental dependence $T_c(r_p)$, we write Eq. (14) at two points (r_{pi}, T_{ci}), $i = 1, 2$; as a result, we obtain the formula for estimating the activation energy:

$$E = \frac{1}{L_2^{-1} - L_1^{-1}} \times \ln\left[\frac{T_{c1} h_1}{T_{c2} h_2} \frac{a_2}{a_1} \frac{r_{p2} \exp(h_1/L_2)}{r_{p1} \exp(h_1/L_1)}\right]. \tag{15}$$

After that, using Eq. (14), we can estimate the pre-exponent:

Table 2. Values of the kinetic constants E and K for fine magnesium particles for different values of the factor n

n	E, J/kg	$E_{\min}$, J/kg	K, m/s
1.5	$5.54 \cdot 10^6$	$4.32 \cdot 10^6$	1.47
1.75	$4.59 \cdot 10^6$	$3.36 \cdot 10^6$	$8.32 \cdot 10^{-2}$
2	$3.92 \cdot 10^6$	$2.88 \cdot 10^6$	$2.7 \cdot 10^{-2}$
2.5	$2.98 \cdot 10^6$	$2.40 \cdot 10^6$	$7.06 \cdot 10^{-4}$
3	$2.31 \cdot 10^6$	$2.16 \cdot 10^6$	$1.06 \cdot 10^{-4}$
3.5	$1.79 \cdot 10^6$	$2.02 \cdot 10^6$	$2.59 \cdot 10^{-5}$

$$K = \frac{\lambda_1 \mathrm{Nu} T_0}{2\rho_{\mathrm{ox}} r_{p1} q} \frac{T_{c1} h_1 \varepsilon}{a_1 - T_{c1} h_1 \varepsilon} \times \exp\left(\frac{E}{L_1}\right) \exp\left(-\frac{h_1}{L_1}\right). \tag{16}$$

In accordance with the data in Table 1, we take $T_{c1} = 1053$ K, $r_{p1} = 7.5$ μm, $T_{c2} = 912$ K, and $r_{p2} = 35$ μm. The values of E and K estimated by Eqs. (15) and (16) with these parameters and the values of $E_{\min}$ obtained by Eq. (11) for different values of $T_m = nT_c$ $(n > 1)$ for fine magnesium particles are listed in Table 2. It is seen that the constraint $E > E_{\min}$ is not satisfied at $n > 3$. Thus, the resultant approximate estimate is valid at $n \leqslant 3$.

The kinetic parameters estimated in [4] within the framework of model 2 are $E = 4.3 \cdot 10^6$ J/kg and $K = 0.103$ m/s. Thus, as is seen from Table 2, for instance, at $n = 1.75$, there is only a minor difference in the activation energy values estimated by models 1 and 2 ($\approx$6%), whereas the estimates of the pre-exponent differ by 20%.

After the values of the constants E and K are found, Eq. (14) can be considered as an expression that defines the dependence of the limiting temperature of ignition T_c on the particle radius r_p. Figure 1 shows the experimental data on the dependence $T_c(r_p)$ [5–10] and the approximate values calculated by the approximate formula (14) for different values of n. It is seen that the experimental data have a certain scatter as usually, and the approximate formula (14) at $n = 2$ adequately describes the experimental data [5–9]. The scatter of the experimental data may be caused by physicochemical properties of particles and by other random factors. In the case considered here, it is important to describe some observable parameter, in particular, this dependence of the limiting temperature on the particle radius.

Ignition Delay Time

Based on the data of Table 2, we take $E = 3.92 \cdot 10^6$ J/kg, $K = 2.7 \cdot 10^{-2}$ m/s, and

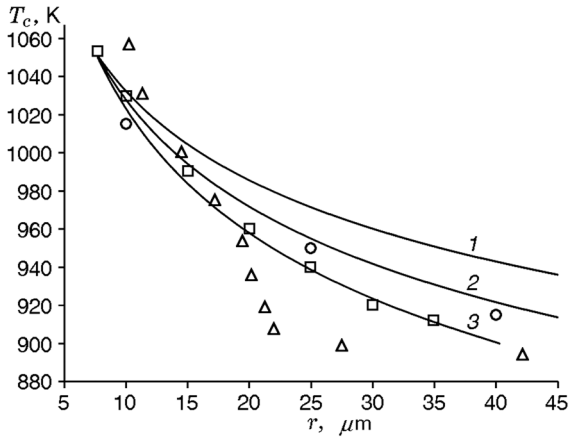

Fig. 1. Limiting temperature of ignition versus the particle radius: the points show the experimental data of [5–8] (□), [9] (○), and [10] (△); the curves are the approximations by Eq. (14) for $n = 1.5$ (*1*), 1.75 (*2*), and 2 (*3*).

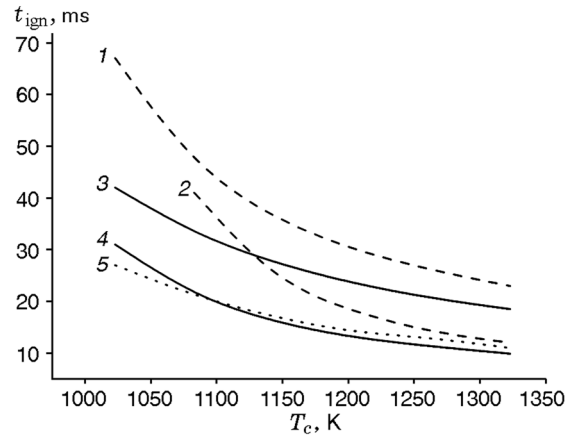

Fig. 2. Ignition delay time versus the ambient temperature and the Nusselt number for a particle with a radius of 22 μm: Nu = 2 (*1* and *3*), 5 (*2* and *4*), and 2.1 (*5*); data [4] obtained by model 2 (*1* and *2*), data of the present work calculated by model 1 (*3* and *4*), and data [11] (*5*).

$n = 2$ and study the ignition delay time as a function of the ambient temperature and the Nusselt number, $t_{\text{ign}} = t_{\text{ign}}(T_c, \text{Nu})$, for a particle with a fixed radius $r_p = 22$ μm. Figure 2 shows the results calculated by model 1 in this work (solid curves) and by model 2 in [4] (dashed curves) for Nu = 2 and 5 and for $T_c = 1023$–1323 K. It is seen that the results qualitatively coincide with each other; as the ambient temperature increases, the results predicted by the two models become closer to each other. Figure 2 also shows the calculated induction time of a magnesium particle of the same size in air as a function of the ambient temperature, which were obtained in [11] for Nu = 2.1 (dotted curve). It should be noted that there is a certain difference in the ignition delay times calculated by the models in [4, 11] and in this work, especially at comparatively moderate temperatures (cf. curves *1*, *3*, and *5*). The values of the ignition delay time obtained in this work with the new estimate of the kinetic parameters occupy an intermediate position between the data of [4, 11]. In what follows, we use the values of E and K found above.

IGNITION–COMBUSTION WAVE

Stage of Ignition in the Wave Structure

To describe chemical conversions of a cloud of particles in the ignition and combustion wave, we use the equations of conservation of energy of the gas and condensed phases. It should be noted that the energy equation in the case of averaging has a factor N (the number of particles in a unit volume of the mixture) at each term. After dividing by this factor, we obtain a standard equation of the theory of the thermal explosion of a single particle. In the gas energy equation, we neglect the term that describes heat transfer between the phases and the heat possibly released in the gas phase due to chemical transformations in the particle. The volume concentration of particles is assumed to be small, which gives grounds for justification of the assumptions put forward above to some extent. The problem is considered in a coordinate system moving together with the combustion wave. Let us introduce a self-similar variable $\eta = x - Dt$, where $D > 0$ is the wave propagation velocity, and write the equation for the ambient temperature T_1 [12]

$$\lambda_1 \frac{d^2 T_1}{d\eta^2} + \rho_1 c_1 D \frac{dT_1}{d\eta} = 0, \tag{17}$$

where λ_1, ρ_1, and c_1 are the thermal conductivity, density, and specific heat of the gas, respectively. The solution of Eq. (17) satisfying the conditions

$$T_1(\infty) = T_0, \quad T_1(\eta_{\text{ign}}) = T_{1\text{ign}},$$

where T_0 is the initial equilibrium temperature of the mixture and $T_{1\text{ign}}$ is a certain (conventional) intermediate temperature, has the form

$$T_1 = T_0 + (T_{1\text{ign}} - T_0) \exp\left[-\frac{\rho_1 c_1 D}{\lambda_1}(\eta - \eta_{\text{ign}})\right]. \tag{18}$$

Passing to the dimensionless variables

$$\bar{\eta} = \bar{x} - \bar{D}\bar{t}, \quad \bar{x} = \frac{x}{r_p}, \quad \bar{\eta} = \frac{\eta}{r_p}, \quad \bar{D} = \frac{Dt_0}{r_p}, \quad \bar{t} = \frac{t}{t_0}$$

and using the notations $T = T_2$ and $T_c = T_1$ in Eq. (4), we obtain the equation of the particle energy at the stage of ignition

$$\frac{dT_2}{d\eta} = -\frac{3\xi q}{D} \times \left[-\alpha(T_2 - T_1) + (T_m - T_2)\exp\left(-\frac{E}{T_2}\right)\right], \quad (19)$$

where, in view of Eq. (18), we have

$$T_1 = 1 + (T_{1\text{ign}} - 1)\exp[A_1(\eta - \eta_{\text{ign}})], \qquad A_1 = -\frac{\rho_1 c_1 D r_p}{\lambda_1}. \quad (20)$$

Equation (19) is a non-self-similar equation of the modified Semenov theory of the thermal explosion owing to the presence of a term that depends on the self-similar variable η. Generally speaking, this could make us expect a more complicated behavior of the solutions of the corresponding Cauchy problem for Eq. (19) than that in the case of a constant value of the ambient temperature of the medium around the particle. Such a situation requires an additional analytical study.

In the numerical calculation of the Cauchy problem for Eq. (19), which is a mathematical model of the ignition process in the combustion wave, the infinitely distant point is shifted to a finite point η_0 determined in accordance to Eq. (20) from the condition $T_1(\eta_0) = 1+\varepsilon$ (ε is a certain small number):

$$\eta_0 = \frac{1}{A_1}\ln\frac{\varepsilon}{T_{\text{ign}} - 1} + \eta_{\text{ign}}, \quad (21)$$

and η_{ign} is found from the solution of problem (4) of determining the ignition delay time of particles.

Stage of Combustion in the Wave Structure

Let us use the structural form of our previously developed model [13, 14] to describe combustion of a metal particle. The equations of energy conservation and particle burnout take the form

$$\frac{N}{V} m_p c_2 \frac{dT_2}{dt} = -\frac{N}{V} Sk(T_2 - T_1) + \frac{N}{V} S\delta q_b \rho_3 \frac{T_b - T_2}{T_0}\left|\frac{dr_p}{dt}\right|, \quad (22)$$

$$\frac{dr_p}{dt} = -\frac{1}{r_p}\frac{K_1}{\tau_b}\exp\left(-\frac{E}{RT_2}\right). \quad (23)$$

Equations (22) and (23) are written in the Lagrangian variables. Here $V = V_1 + V_2 = 1$ is the unit volume of the mixture, V_1 is the gas volume, V_2 is the volume of particles, K_1 is the pre-exponent [which differs from that used previously in Eq. (1)], τ_b is the particle burnout relaxation time, $T_b > T_m$ is the particle burning temperature, and $0 < \delta \leqslant 1$ is a parameter characterizing the fraction of heat transferred to the particle. Passing to the self-similar variable and dimensionless variables and applying certain transformations, we obtain the equations

$$\frac{dT_2}{d\eta} = \frac{3}{2}\,\text{Nu}\frac{T_2 - T_1}{Dr_p^2} - \frac{3}{2}\,\text{Nu}\frac{\delta}{\beta_1}\frac{T_b - T_2}{Dr_p^2}\exp\left(-\frac{E}{T_2}\right), \quad (24)$$

$$\frac{dr_p}{d\eta} = \frac{\text{Nu}}{2}\frac{\rho_2 c_2 T_0}{\rho_3 q_b \beta_1}\frac{1}{Dr_p}\exp\left(-\frac{E}{T_2}\right), \quad (25)$$

where

$$\beta_1 = \frac{\text{Nu}}{2}\frac{\lambda_1 T_0 \tau_b}{\rho_3 q_b K_1}. \quad (26)$$

Equation (17), which describes the change in the gas temperature at the stage of combustion, can be written in this region with allowance for heat exchange between the phases:

$$\lambda_1\frac{d^2T_1}{d\eta^2} + \rho_1 c_1 D\frac{dT_1}{d\eta} = Sk\frac{N}{V}(T_2 - T_1) + (1-\delta)\frac{N}{V}Sq_b\rho_3\frac{T_b - T_2}{T_0}\left|\frac{dr_p}{dt}\right|. \quad (27)$$

As $S\dfrac{N}{V} = \dfrac{3m_2}{r_p}$, where $m_2 = \dfrac{4}{3}\pi r_p^3 N\dfrac{1}{V}$ is the volume concentration of particles, Eq. (27) in the dimensionless variables transforms to

$$\frac{d^2T_1}{d\eta^2} - A_1\frac{dT_1}{d\eta} = \frac{3}{2}\,\text{Nu}m_{20}r_p\left[(T_2 - T_1) - (1-\delta)(T_b - T_2)\frac{1}{D\beta_1}\exp\left(-\frac{E}{T_2}\right)\right], \quad (28)$$

where $m_{20} = \dfrac{4}{3}\pi r_{p0}^3 N\dfrac{1}{V}$ is the initial volume concentration of particles and r_{p0} is the initial radius of particles.

System of Eqs. (24), (25), and (28) with the internal boundary conditions

$$\eta = \eta_{\text{ign}}: \quad T_1 = T_{1\text{ign}},\ \ T_2 = T_{2\text{ign}}, \quad \frac{dT_1}{d\eta} = A_1(T_{1\text{ign}} - 1),\ \ r_p = 1 \quad (29)$$

describes the stage of combustion of a cloud of particles. The values of the sought functions in conditions (29) are taken from the boundary of the region of ignition, i.e., they are obtained by solving problem (19), (20) on the interval $[\eta_0, \eta_{\text{ign}}]$. The third condition in Eqs. (29) corresponds to equality of the heat fluxes from the region of combustion to the region of ignition.

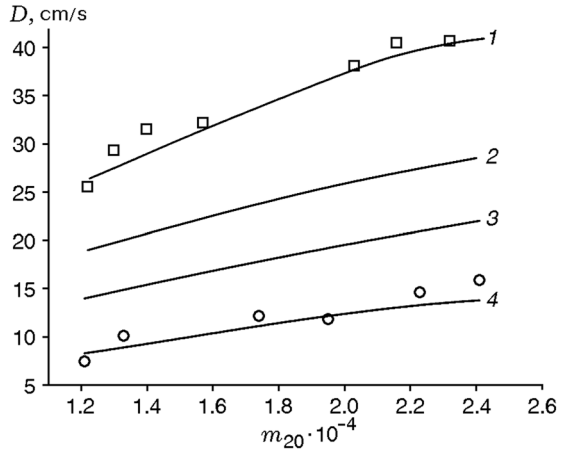

Fig. 3. Combustion velocity versus the particle size and volume concentration: the points are the experimental data [3] for particles with radii of 7.5 and 35 μm (□ and ○, respectively); the curves show the results calculated by the approximation formula (30) for particles with radii of 7.5 (*1*), 15 (*2*), 22 (*3*), and 35 μm (*4*).

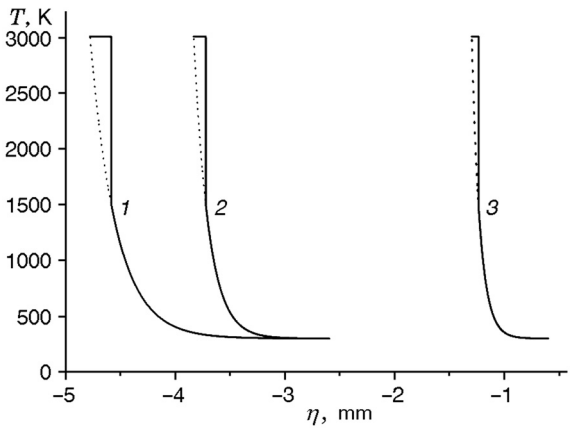

Fig. 4. Distributions of phase temperatures along the combustion wave front: the solid and dotted curves show the data for the particles and the gas, respectively; the particle radius is 35 (*1*), 22 (*2*), or 7.5 μm (*3*).

Problem (24), (25), (28), (29) is calculated until the conditions $T_1 \approx T_2 \approx T_b$ and $\lim r_p \to r_{pk}$ are satisfied (r_{pk} is the particle radius at the end of the combustion region, which has a certain small value). This overdetermined boundary-value problem is correctly solved as an eigenvalue problem. The eigenvalues are the unknown velocity of the combustion wave D and the length of the combustion region with a state of thermal equilibrium and particle burnout at the end of it.

DISCUSSION OF RESULTS OF COMBUSTION WAVE CALCULATIONS

Comparisons with Experiments

Experimental data on the combustion velocity of a dust cloud of magnesium particles as a function of the particle size ($r_p = 7.5$ and 35 μm) and the volume concentration of particles m_{20} in air can be found in [3]. For our numerical data to be consistent with these experiments, the dependence of the bifurcation parameter $\beta_1 = \beta_1(r_p, m_{20})$ is chosen in the form

$$\beta_1 = a \cosh \frac{\pi(r_p - x_c)}{w_x} \cosh \frac{\pi(m_{20} - y_c)}{w_y} + b, \quad (30)$$

where

$$a = 2.1647 \cdot 10^{-6}, \quad x_c = 10.0908,$$
$$w_x = 52.1217, \quad y_c = 2.5518 \cdot 10^{-4},$$
$$w_y = 2.2952 \cdot 10^{-4}, \quad b = 1.1249 \cdot 10^{-6}.$$

The results for the combustion wave structure calculated by Eq. (30) are plotted in Fig. 3, which shows the combustion velocity as a function of the particle radius and the volume concentration of particles in the gas suspension. It is seen that the calculations with the pre-exponent K_1 chosen on the basis of Eq. (30) adequately describes the experimental data [3] (curves *1* and *4*) and predicts reasonable values of the combustion velocity for intermediate values of the particle radii (curves *2* and *3*).

Distributions of the Phase Temperatures over the Spatial Coordinate

After verification of this semi-empirical model, it is possible to determine the spatial distributions of the temperatures of the phases in the air suspension, which are shown in Fig. 4. The combustion wave has the following structure. Magnesium particles ignite on the thermal background of the gas heated due to heat conduction from the combustion region. The temperature of these particles is close to the gas-phase temperature. When the process of high-temperature oxidation of particles in the combustion zone begins, the particle temperature rapidly relaxes to the temperature T_b; in this region, the gas is heated by the energy of heat release due to combustion until the gas and particle temperatures become equal to each other. A typical temperature profile is formed with a smooth increase in the particle temperature in the ignition region with a subsequent drastic decrease to a certain finite value, followed by a further drastic increase in temperature and by subsequent combustion at a constant value of tem-

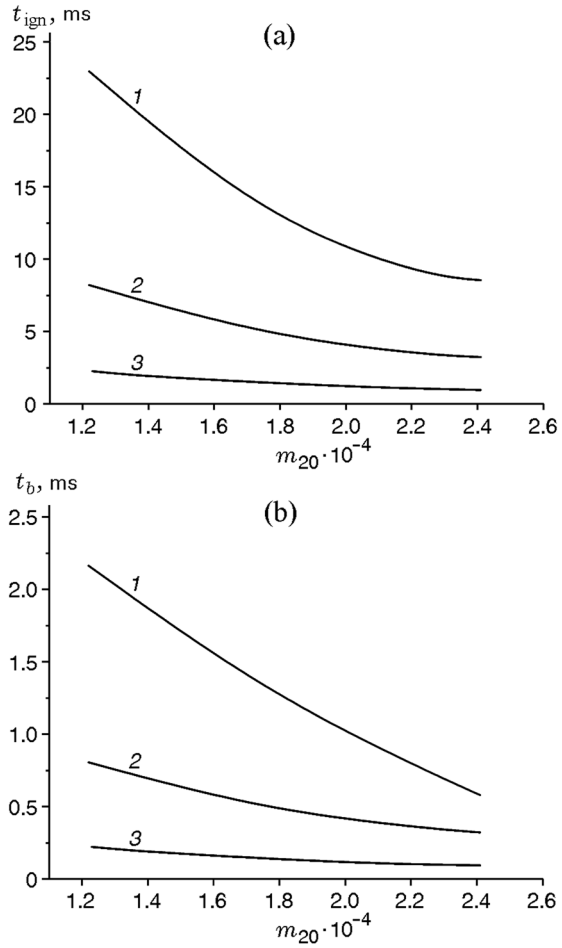

Fig. 5. Times of ignition (a) and burning (b) versus the volume concentration and size of particles: the particle radius is 35 (*1*), 22 (*2*), or 7.5 μm (*3*).

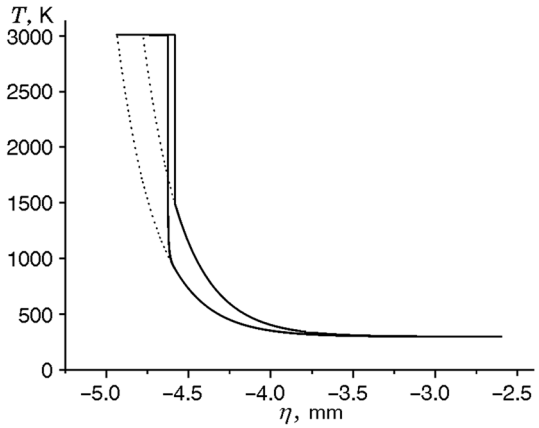

Fig. 6. Distributions of the phase temperatures in the combustion wave structure at $T_{1\text{ign}} = 1500$ (upper curve) and 912 K (lower curve): the solid and dotted curves show the results for the particles ($r_p = 35$ μm) and the gas, respectively.

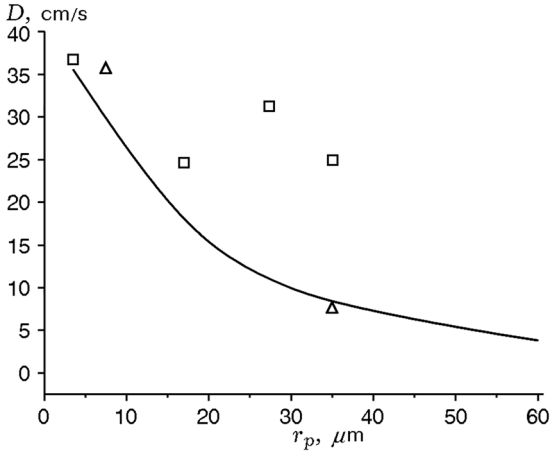

Fig. 7. Combustion wave velocity versus the particle radius: the points are the experimental data [15] (□) and [3] (△), and the curve shows the results of the present calculations.

perature. For stable propagation of this regime, there should be a thermal "piston" on the left boundary with an appropriate boundary condition imposed there.

It should be also noted that the main part of the ignition and combustion wave structure in terms of its length is caused by a slow increase in the particle temperature in the ignition region; the combustion process is much faster in this mathematical model and the combustion region is several-fold smaller.

Figure 5 shows the times of ignition t_{ign} and burning t_b for different values of the volume concentration and radius of particles. It should be noted that the length of the combustion wave structure in this model is mainly caused by the ignition process. The time needed for particle burning turned out to be much shorter than the ignition time for two reasons. First, the ignition delay time correlates with the data of the mathematical experiment on the ignition delay time of single particles [4]. Second, there is a correlation with the experiments [3] on the combustion wave velocity as a function of the volume concentration of magnesium particles in the air suspension.

The influence of the parameter $T_{1\text{ign}}$ (intermediate temperature at the end of the ignition wave front) is illustrated in Fig. 6 ($r_p = 35$ μm). It is seen that the ignition region length does not experience any significant changes with a reasonable increase in this parameter (see the upper and lower curves in the ignition region). The length of the combustion region changes to a greater extent. Moreover, as the particle radius increases, the influence of the parameter $T_{1\text{ign}}$ on the

spatial characteristics of the combustion wave becomes slightly more pronounced.

A comparison of the results obtained by using our mathematical model and the experimental data [15] on the combustion wave velocity as a function of the particle radius (Fig. 7) shows that our model provides an adequate description of the experiments [3], but there are considerable differences with the results [15].

CONCLUSIONS

A simple physicomathematical model of mechanics of reacting heterogeneous media is proposed to describe the structure of the combustion wave in a gas suspension of magnesium particles in an oxidizing medium. The model is verified on the basis of the dependences of the limiting ignition temperature and the combustion wave velocity as functions of the radius and volume concentration of particles. The validity of the model is guaranteed in the range of particle radii from 7.5 to 35 μm and in the range of volume concentrations of particles $(1.2-2.4)\cdot 10^{-4}$.

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