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The Threshold Characteristics of the Retinohypothalamic Tract in the Regulation of Human Circadian Activity Rhythms by Solar Radiation

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Abstract—A discrete threshold model was shown to be suitable for studying and describing unconscious reactions of the human body and, in particular, characteristics of the retinohypothalamic tract, which mediates the regulation of human circadian activity rhythms by solar radiation.

Keywords: visual system, humans, threshold, perception, continuous threshold model, discrete threshold model, unconscious reactions

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The human visual system includes an evolutionarily ancient tract that regulates the endocrine system; a retinohypothalamic tract, which mediates the regulation of human circadian activity by solar radiation and is a component of the first tract; and a visual perception tract, which arose more recently in evolution.

A necessary condition for these sensory systems to receive and process information about the environment is that the energy characteristics of optical solar radiation exceed a certain threshold. The threshold is related not only to the perception conditions and the properties of the sensory system, but also to the character of a particular task that the sensory system should solve.

Threshold models of sensory systems have been studied most comprehensively in the context of their application to the visual perception tract of the human visual system.

Little attention has been paid to the threshold characteristics of the tract that regulates the human neuroendocrine system and, in particular, the retinohypothalamic tract, which mediates the regulation of human circadian activity by solar radiation.

There are currently two groups of threshold theories and threshold models constructed on their basis. One group of theories and models includes discrete threshold models, which are based on the discrete threshold theory developed [1, 2] by the founder of psychophysics, Fechner [3]. The theory [3] and models constructed on its basis are distinguished by assuming that a constant lower threshold limit of external physical effects is a characteristic of a sensory system so that the human body does not respond to signals that are below the limit. The constant limit is thought to be an inherent threshold of the sensory system.

The second group of threshold theories and models includes the so-called continuous models, which originate from [4] and are based on the statistical decision theory. The model [4] assumes that body responses vary continuously down to infinitely small values of external triggering factors. The threshold is thought to be an operating characteristic in the theories and models of the group and is introduced as an empirical parameter obtained via measurements according to certain rules.

The current threshold theories accept the hypothesis that thresholds exist, but are divided over their interpretation and often oppose each other.

The two groups coexist because the both of two threshold concepts receive support from different experiments. However, the discrete and continuous threshold theories have been shown to be noncontradictory and integral in more recent studies that employed the mathematical apparatus of statistical decision theory as a common approach to describe both types of models [5, 6].

To determine the threshold type for the retinohypothalamic tract that mediates the regulation of human circadian activity by solar radiation, the most common and illustrative one-dimensional models have been applied to perception processes (with the example of solving a signal detection task). The set included a model [4] that combines a group of continuous threshold models and Fechner's threshold model [3], which is a typical discrete threshold model. The

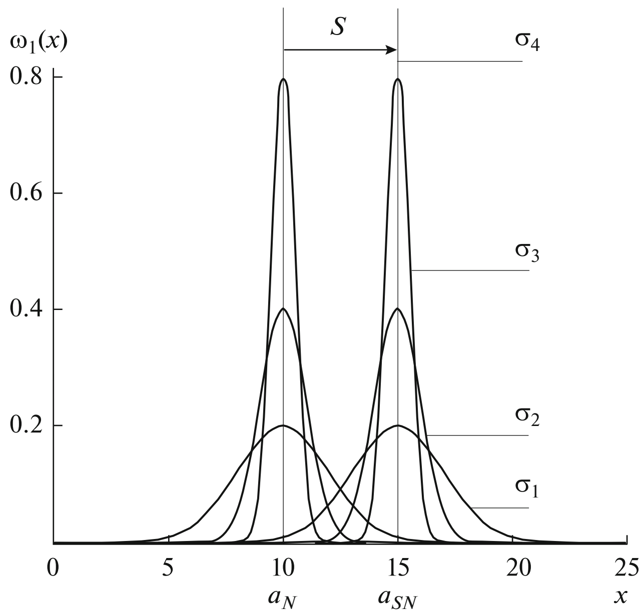

Fig. 1. Probability density functions constructed for exposure to Gaussian noise alone (on the left) or an additive mixture of Gaussian noise N and the deterministic signal S (on the right). $\sigma_1 > \sigma_2 > \sigma_3 > \sigma_4 = 0$.

signal was assumed to be presented only once for greater certainty.

In the model in [4], the following equations describe the one-dimensional conditional probability density function for exposure to Gaussian noise N alone or an additive mixture of noise N and the deterministic signal S (i.e., $S + N$):

$$\omega_1(x|N) = \frac{1}{\sigma_N\sqrt{2\pi}}\exp\left[-\frac{(x-a_N)^2}{2\sigma_N^2}\right], \quad (1)$$

$$\omega_1(x|SN) = \frac{1}{\sigma_{SN}\sqrt{2\pi}}\exp\left[-\frac{(x-a_{SN})^2}{2\sigma_{SN}^2}\right], \quad (2)$$

where σ_N and σ_{SN} are, respectively, the root-mean-square deviations of Gaussian noise and the signal–noise combination $S + N$ ($\sigma_N = \sigma_{SN}$ in the case of the deterministic signal S) and a_N and a_{SN} are the conditional mathematical expectations of noise and the additive signal–noise mixture ($a_N \le a_{SN}$).

Gaussian noise N in Eqs. (1) and (2) is an additive mixture of Gaussian noise of the object space and intrinsic Gaussian noise of the visual system [5].

The phenomena that are described by Eqs. (1) and (2) occur in a certain limited sensory space [1, 2, 7]. Both immobile and mobile boundaries are possible for the sensory space in a general case. The space boundaries can only be reflecting in nature because an absorbing character of at least part of the sensory space boundary would violate the normalization conditions of the probability density functions (1) and (2) [7]. An absorbing boundary leads to an irreversible decrease of the probability densities (1) and (2), which is equivalent of termination of brain activity and death of the brain.

A mathematical model of intrinsic noise with immobile reflecting boundaries was described in [7]. The same model was used to describe the image of external space noise in the sensory space.

The problem is addressed below with the use of a particular case with a zero number of probability density reflections from infinitely distant boundaries of the sensory space. The case directly follows from the model in [7].

In this case, a structurally simple equation can be taken, for example, from [8] to describe the likelihood ratio $\lambda(x)$, which provides a measure to evaluate how the information contained in the external influence x affects decision making about whether a signal is present or absent. The ratio is Eq. (2) divided by Eq. (1):

$$\lambda(x) = \exp\frac{\left[(x-a_N)^2 - (x-a_{SN})^2\right]}{2\sigma^2}. \quad (3)$$

When an external influence triggers the first conscious and probably nonverbal response in a subject, the visual system is concluded to have responded to the external influence. The following decision function is used in this case to decide whether the signal S is present or absent in the mixture SN:

$$\begin{cases}\lambda(x) \ge \lambda_{\text{thr}} \to y \text{ (signal present)}\\ \lambda(x) < \lambda_{\text{thr}} \to n \text{ (signal absent)}\end{cases}, \quad (4)$$

where λ_{thr} is the threshold likelihood ratio. Its numerical value is determined using decision-making criteria that are based on human cognitive activities.

Because $\lambda(x)$ is an increasing monotonic function in Eq. (3), only one value of the operating threshold x_{thr} corresponds to the threshold likelihood ratio λ_{thr}:

$$x_{\text{thr}} = \frac{\sigma^2 \ln\lambda_{\text{thr}}}{a_{SN} - a_N} + \frac{a_{SN} + a_N}{2}. \quad (5)$$

Plots constructed for functions (1)–(3) and (5) with regard to rule (4) are shown in Figs. 1 and 2.

It follows from Eq. (5) that the λ_{thr} value is the only factor that determines the operating threshold x_{thr} on the axis of optic influences at the constant root-mean-square deviations $\sigma_N = \sigma_{SN} = \sigma$ and constant mathematical expectations a_N and a_{SN}. The λ_{thr} value corresponds to the particular decision making criterion that is used to solve a particular problem.

The criteria are determined by the goal of problem solving in the model. The goal determines the decision function. In turn, the function forms the decision rule, which makes it possible to choose a particular decision according to Eq. (4) for each external influence.

Equations (1)–(5) are used to obtain the probability characteristics for the signal detection process

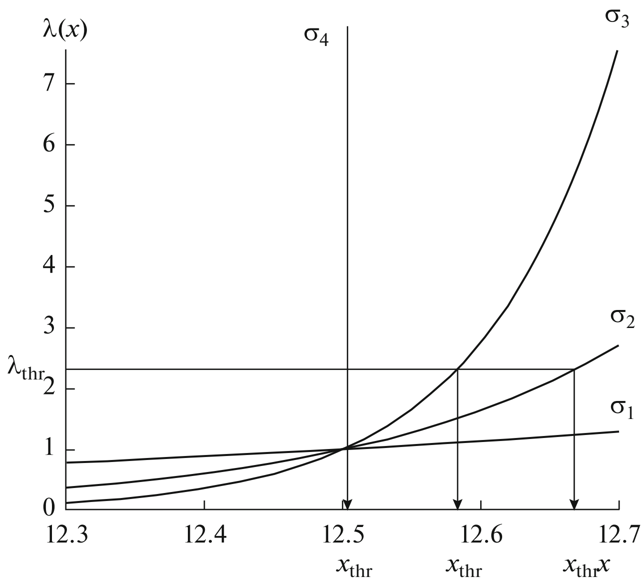

Fig. 2. The behavior of the likelihood ratio function in the vicinity of $x = 12.5$ at different σ values: $\sigma_1 > \sigma_2 > \sigma_3$ in the continuous threshold model and $\sigma_4 = 0$ in the discrete threshold model.

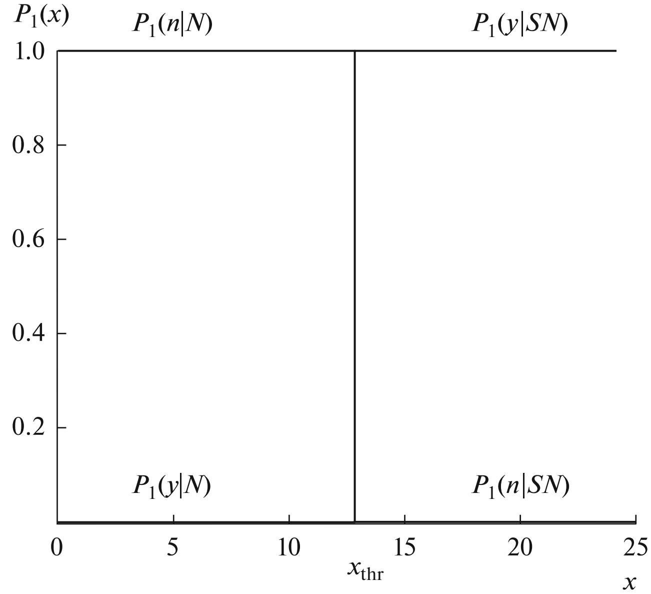

Fig. 3. Conditional integral distribution functions of noise and the mixture of the signal and noise in the discrete threshold model.

with a preset decision criterion and the corresponding threshold likelihood ratio λ_{thr} or operating threshold x_{thr}.

The probability $P_1(y|SN)$ that the response y will occur in the presence of the signal (correct detection), the probability $P_1(y|N)$ that the response y will occur in the absence of the signal (false alarm), the probability $P_1(n|SN)$ of a lack of the response in the presence of the signal (missed signal), and the probability $P_1(n|N)$ of the lack of a response in the absence of the signal (correct nondetection) have the following forms:

$$P_1(y|SN) = \frac{1}{\sigma\sqrt{2\pi}}\int_{x_{\text{thr}}}^{\infty}\exp\left[-\frac{(x-a_{SN})^2}{2\sigma^2}\right]dx, \quad (6)$$

$$P_1(y|N) = \frac{1}{\sigma\sqrt{2\pi}}\int_{x_{\text{thr}}}^{\infty}\exp\left[-\frac{(x-a_{N})^2}{2\sigma^2}\right]dx, \quad (7)$$

$$P_1(n|SN) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x_{\text{thr}}}\exp\left[-\frac{(x-a_{SN})^2}{2\sigma^2}\right]dx, \quad (8)$$

$$P_1(n|N) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x_{\text{thr}}}\exp\left[-\frac{(x-a_{N})^2}{2\sigma^2}\right]dx. \quad (9)$$

As immediately follows from the above, the continuous model of sensory system thresholds is correct when human cognitive activity, which is provided by intelligence-related mechanisms, is involved in perceiving external influences. Noise from the space of objects and intrinsic noise of the sensory system affect the perception processes in this case. This primarily applies to vision processes, which imply that various criteria are developed and used to evaluate human activity while a visual (objective) scheme of the outer world is constructed in the human mind. This similarly applies to evaluation, registration, and subsequent verbalization of the relationships that occur between various parameters in the scheme of the world.

The threshold model [3] postulates that events are of a deterministic nature and that the sensory system has its own specific (fixed) threshold x_{thr}.

A primary unconscious reaction arises when the energy of optical radiation exceeds a certain threshold, x_{thr}. In the discrete threshold model of a sensory system, the response to an external influence is described by the following decision rule:

$$\begin{cases} x \ge x_{\text{thr}} \rightarrow y \text{ (response present)} \\ x < x_{\text{thr}} \rightarrow n \text{ (response absent)} \end{cases}. \quad (10)$$

Responses described by rule (10) occur at a conditional probability equal to 1. This is possible only in the case where the one-dimensional conditional integral distribution functions of noise and the additive mixture of the signal and noise are Heaviside functions [9–11]:

$$P_1(x^*|N) = 1_+(x^* - a_N) = \begin{cases} 1 \text{ for } x^* \ge a_N \\ 0 \text{ for } x^* < a_N \end{cases}, \quad (11)$$

$$P_1(x^*|SN) = 1_+(x^* - a_{SN}) = \begin{cases} 1 \text{ for } x^* \ge a_{SN} \\ 0 \text{ for } x^* < a_{SN} \end{cases}. \quad (12)$$

The derivatives of the conditional probability functions (11) and (12) describe the conditional probability density functions for noise alone and the mixture of the signal and noise in the following form:

$$\omega_1(x|N) = \delta(x - a_N), \tag{13}$$

$$\omega_1(x|SN) = \delta(x - a_{SN}), \tag{14}$$

where $\delta(\cdot)$ is the Dirac δ function [12].

The likelihood ratio function is formally of the following shape in this case:

$$\lambda(x) = \frac{\delta(x - a_{SN})}{\delta(x - a_N)}. \tag{15}$$

As with the model in [4], probability characteristics of the process are determined by integrals with Eqs. (13) and (14).

Using the integration rules with δ functions, the following equations can be obtained for the conditional probabilities $P_1(y|SN)$, $P_1(y|N)$, $P_1(n|SN)$, and $P_1(n|N)$:

$$P_1(y|SN) = \int_{x_{\text{thr}}}^{\infty} \delta(x - a_{SN})dx = 1, \tag{16}$$

$$P_1(y|N) = \int_{x_{\text{thr}}}^{\infty} \delta(x - a_N)dx = 0, \ddot{x} \tag{17}$$

$$P_1(n|SN) = \int_{-\infty}^{x_{\text{thr}}} \delta(x - a_{SN})dx = 0, \tag{18}$$

$$P_1(n|N) = \int_{-\infty}^{x_{\text{thr}}} \delta(x - a_N)dx = 1. \tag{19}$$

As is seen from Eqs. (16)–(19), the resulting numerical values of the conditional probabilities fully agree with the axiomatics of Fechner's discrete threshold model.

When $\sigma \to 0$ in the continuous threshold model [4], the probability-based description given in Eqs. (1), (2), and (6)–(9) changes to a deterministic description presented in terms of probability. After application of the well-known relationship [13]

$$\lim_{\sigma \to 0} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{(x - a)^2}{2\sigma^2}\right] = \delta(x - a), \tag{20}$$

Eqs. (1) and (2) coincide with Eqs. (13) and (14) and Eqs. (6)–(9), with Eqs. (16)–(19).

It follows immediately from the above coincidences that Fechner's threshold model [3] is a result of degeneration (at $\sigma \to 0$) of the continuous threshold model [4].

As an example, Fig. 1 shows how functions (1) and (2) change at the arbitrarily chosen values $a_N = 10$ and $a_{SN} = 15$, that is, at the deterministic signal $S = 5$, when $\sigma \neq 0$ and $\sigma = 0$.

Figure 2 shows the respective changes in the likelihood ratio function $\lambda(x)$ (5) at $a_N = 10$ and $a_{SN} = 15$ (the deterministic signal $S = 5$) when $\sigma \neq 0$ and $\sigma = 0$.

In the continuous threshold model, the above criteria are set to solve a particular problem, which determines the decision function. In turn, the decision function forms the decision rule, which makes it possible to choose a particular solution in response to every influence according to Eq. (4).

It follows from Fig. 2 that different values are possible for the operating threshold x_{thr} in the continuous threshold model at a fixed decision criterion and the corresponding fixed likelihood ratio λ_{thr}. On the other hand, the same values are possible for the threshold x_{thr} at different decision criteria, that is, at different values of the likelihood ratio λ_{thr}.

Another direct consequence of the above reasoning is that conscious intellectual human activity is an essential prerequisite to the setting and formulation of a particular problem, the choice of the goal of problem solving, the choice of the decision function, and the design of the decision rule that makes it possible to choose a particular solution in response to each influence according to Eq. (4).

The condition that cognitive activity is mandatory apparently makes the continuous threshold model unsuitable for describing the regulation of unconscious and uncontrollable biological processes and, in particular, describing and modeling the regulation of human circadian rhythms by solar radiation.

As follows from Fig. 2, the function $\lambda(x)$ is singular at $\sigma = 0$. This means that conscious cognitive activity exerts no effect on biological processes in the human body because the goal, decision function, and decision rule do not form in this case, nor is a solution chosen to solve the problem of regulating circadian activity. The singular character of the function $\lambda(x)$ clearly indicates that the threshold x_{thr} value is independent of any value of the function $\lambda(x)$ in the point of singularity. According to the example considered (Fig. 2), the above applies to the representation of the discrete threshold model in terms of a continuous threshold model, which always considers the case of $S \neq 0$. $S = 5$ in Fig. 2. When $S \to 0$, the function $\delta(x - a_{SN})$ in Eq. (14) tends to the δ function $\delta(x - a_N)$ in Eq. (13). In this case, the intrinsic threshold x_{thr} of the discrete threshold model coincides with the position of the function $\delta(x - a_N)$, which describes the deterministic noise. It is clear that the goal, decision function, and decision rule are not determined in this case, as in the above case of $S \neq 0$, nor a solution is chosen for the problem of regulating circadian activity. It is also clear that the singular character of the function $\lambda(x)$ indicates again that the value of the threshold x_{thr} is independent of any value of the function $\lambda(x)$ in the point of singularity and that x_{thr} occurs in a constant position on the x axis, that is, in the position of deterministic noise; i.e., $x_{thr} = \delta(x - a_N)$.

The conditional integral distribution functions of noise alone and the mixture of noise and the signal have the following shapes in this case:

$$P_1(y|SN) = \int_{x_{thr}}^{\infty} \delta(x - x_{thr})dx = 1, \quad (21)$$

at $x \geq x_{thr}$ (correct detection),

$$P_1(y|N) = \int_{x_{thr}}^{\infty} \delta(x - x_{thr})dx = 0, \quad (22)$$

at $x < x_{thr}$ (false alarm),

$$P_1(n|SN) = \int_{-\infty}^{x_{thr}} \delta(x - x_{thr})dx = 0,$$

at $x \geq x_{thr}$ (missed signal), and

$$P_1(n|N) = \int_{-\infty}^{x_{thr}} \delta(x - x_{thr})dx = 1, \quad (23)$$

at $x < x_{thr}$ (correct nondetection).

The functions exactly correspond to the discrete threshold theory and model [3].

Figure 3 shows the plots of relationships (21)–(24) in the case where $\delta(x - a_{SN}) \to \delta(x - a_N)$, that is, $S \to 0$.

To conclude, the discrete threshold model describes unconscious processes and events that occur in biological systems of the human body. This model should be employed in describing and modeling the regulation of human circadian activity by solar radiation.

This conclusion makes it possible to use the well-known characteristics that have been obtained for optic radiation receptors of the eye retina with the discrete threshold model.

COMPLIANCE WITH ETHICAL STANDARDS

Conflict of interests. The author declares that he has no conflict of interest.

This article does not contain any studies involving animals or human subjects performed by the author.

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