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## COMPLEX SYSTEMS BIOPHYSICS

# A Correlational Analysis of Electroencephalograms Based on the Modeling of Biopotentials of the Cerebral Cortex

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**Abstract**—Our previous study on the quantitative nonlinear analysis of integral equations of the averaged membrane potentials in excitatory (the EEG analogue) and inhibitory neurons of the neocortex has shown that the characteristic equation has a set of oscillating solutions with negative decrements in the stability region. We have shown that an electroencephalogram can be represented as a convolution of harmonic functions with negative decrements and discrete (uniformly discontinuous) white Gaussian noise. We have suggested methods of decrement calculation in encephalograms using correlation functions and tested them on both modeled processes with preset parameters and actual encephalograms of rats and mice. Investigation of decrements and amplitude-frequency parameters potentially increases the capacity of spectral correlation analysis of electroencephalograms and expands the results of mathematical processing of brain signals.

*Keywords:* modeling, cerebral cortex, electroencephalogram

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In our previous studies [1–5], we investigated the model of the development of rhythmic processes in the cerebral cortex, which was based on the integral equations of averaged membrane potentials of excitatory and inhibitory neocortical neurons. The qualitative nonlinear analysis of these equations revealed that the stability region included a set of oscillating solutions with negative decrements. Therefore, the solutions of a linearized system represent a sum of convolutions of external signals with corresponding original functions. Discrete white Gaussian noise used as an afferent input in the awake resting state caused the appearance of model electroencephalogram (EEG) represented by the real part of a convolution of centered discrete white noise and harmonic functions with negative decrements. In [6], EEG $\xi(t)$ was represented as a convolution of a single harmonic function. In [7], the autocorrelation function of a signal that resulted from the sum of two independent processes was discussed and decrements were evaluated by power spectra. In this study, we summarize the results of previous studies. Signal behavior as a convolution of both independent processes and the same discrete white Gaussian noise with the sum of harmonic functions, as well as the behavior of two signals with correlated discrete white Gaussian noises, are discussed.

## RESULTS AND DISCUSSION

An EEG consists of electrical waves of a certain frequency. Averaging of EEG power spectra from several consecutive epochs of analysis usually results in one or two wide peaks in different ranges of delta- (0.5–4.0 Hz), theta- (4–8 Hz), and alpha-rhythm (8–13 Hz) in animals and alpha- and beta-rhythms (13–30 Hz) in humans. We modeled the human EEG as a real part of a sum of convolutions of damped harmonic functions and discrete white Gaussian noise with the sampling rate $\Delta t$. The behavior of a signal consisting of a sum of main EEG rhythms is described via the equation:

$$\xi(t) = \mathrm{Re}x,\ x(t) = \sum_{l=1}^{m} x_l,$$

where $x_l(t) = \int_0^t q(t')k_l \exp(z_l \times (t - t'))dt'$, $z_l = \alpha_l + j\omega_l$, $x_l$ is the EEG rhythm created by the discrete (uniformly discontinuous) white Gaussian noise $q(t)$ with the sample rate $\Delta t$ and the variance $\sigma_q^2$. In the discrete case:

$$x_l^i = k_l \Delta t \sum^{I} q_j \exp(z_l(i - j)\Delta t),$$

$$x_l^{i+1} = s_l x_l^i + k_l \Delta t q_{i+1},\quad s_l = \exp(z_l \Delta t).$$

Therefore, the signal $x_l^i$ is represented as $x_l^i = k_l t(q_i + s_l q_{i-1} + s_l^2 q_{i-2} + \ldots)$. Then,

$$x_l^{i+n} = k_l \Delta t \left(q_{i-1} + s_l q_{i+n-1} + s_l^2 q_{i+n-2} + \ldots\right) + s_l^n \left(q_i + s_l q_{i-1} + s_l^2 q_{i-2} + \ldots\right).$$

If the rhythms are independent, the averaged autocorrelation function is as follows:

$$R_x(\tau) = \overline{\sum_l x_l^{i*} \sum_l x_l^{i+n}} = \sum_l \overline{x_l^{i*} x_l^{i+n}} = \sum_l k_l^2 r_{x_l}(\tau) = \sum_l k_l^2 \frac{\Delta t^2 \sigma_q^2 \exp(z_l \tau)}{1 - |\exp(2 z_l \Delta t)|}$$

and

$$R_\xi(\tau) = \sum_l \overline{\xi_l^{i*} \xi_l^{i+n}} = \sum_l k_l^2 r_{\xi_l}(\tau) = \sum_l \frac{\Delta t^2}{2} \times \mathrm{Re}\left[\exp(z_l \tau)\left(\frac{1}{1 - \exp(2\alpha_l \Delta t)} + \frac{1}{1 - \exp(2 z_l \Delta t)}\right)\right]\sigma_q^2.$$

The estimation of the normalized autocorrelation function of such signal, which is represented by a sum of independent functions, is defined as:

$$r_\xi(\tau) = \frac{\sum k_i^2 r_i(\tau)}{\sum k_i^2 r_i(0)},$$

where

$$r_i(\tau) = \mathrm{Re}\left[\exp(z_i \tau)\left(\frac{1}{1 - \exp(2\alpha_i \Delta t)} + \frac{1}{1 - \exp(2 z_i \Delta t)}\right)\right].$$

Now, we will consider dependent rhythms that originate from the same discrete Gaussian noise $q(t)$. Then:

$$R_x(\tau) = \overline{\sum_l x_l^{i*} \sum_l x_l^{i+n}} = \Delta t^2 \sigma_q^2 \sum_{l=1} k_l \exp(z_l \tau) \sum_{\nu=1}^{m} \frac{k_\nu^*}{1 - \exp\left(z_l + z_\nu^*\right)\Delta t},$$

$$R_\xi(\tau) = \overline{\mathrm{Re}\sum_l x_l^i \sum_l x_l^{i+n}} = \Delta t^2 \sigma_q^2 \sum_{\nu=1}^{m} k_\nu \left(\mathrm{Re}\, s_\nu^n + \mathrm{Re}\, s_\nu^{n+1} + \mathrm{Re}\, s_l^2 \,\mathrm{Re}\, s_\nu^{n+2} + \ldots\right)$$
$$= \frac{\Delta t^2 \sigma_q^2}{2} \sum_{l=1}^{m} k_l \,\mathrm{Re}\left\{\exp(z_l \tau) \sum_{\nu=1}^{m} k_\nu \left[\frac{1}{1 - \exp(z_l + z_\nu)\Delta t} + \frac{1}{1 - \exp(2\alpha_\nu + z_l - z_\nu)\Delta t}\right]\right\}.$$

In this case, the estimation of the normalized autocorrelation function consists of the following functions:

$$r_\xi(\tau) = \frac{\sum k_l r_l(\tau)}{\sum k_l r_l(0)},$$

$$r_l(\tau) = \mathrm{Re}\left\{\exp(z_l \tau) \sum_{\nu=1}^{m} k_\nu A_\nu^l(z_l, z_\nu)\right\},$$

$$A_\nu^l = \frac{1}{1 - \exp(z_l + z_\nu)\Delta t} + \frac{1}{1 - \exp(2\alpha_\nu + z_l - z_\nu)\Delta t}.$$

Let us consider the behavior of two correlated signals; the second process is represented as: $\eta(t) = \mathrm{Re}\sum_{l=1}^{m} y_l$, where $y_l(t) = \int_0^t p(t') k_l^y \times \exp\left(p_l^y \times (t - t')\right) dt'$, $z_l^y = \alpha_l^y + j\omega_t^y$, and $p(t)$ is the discrete (uniformly discontinuous) white Gaussian noise, which correlates with $q(t)$ and has the same variance. We modeled it as $p(t) = \left(q(t) + \beta q_p(t)\right)/\sqrt{l + \beta^2}$ with the correlation coefficient $\rho = l/\sqrt{l + \beta^2}$. Then

$$y_l^{i+n} = k_l^y \Delta t \left(p_{i+n} + s_l^y p_{i+n-1} + \ldots + \left(s_l^y\right)^n\right) \times \left(p_i + s_l^y p_{i-1} + \left(s_l^y\right)^2 p_{i-2} + \ldots\right),$$

and if the number of independent rhythms in both processes is even, the cross-correlation function is represented as:

$$R_{xy}(\tau) = \overline{\sum_l x_l^{i*} \sum_l y_l^{i+n}} = \Delta t^2 \rho \sigma_q^2 \sum_{l=1}^{m} k_l^y \exp\left(z_l^y \tau\right) \frac{k_\nu^x}{1 - \exp\left(z_l^y + z_\nu^{x*}\right)\Delta t}.$$

Similarly,

$$R_{yx}(\tau) = \overline{\sum_l y_l^{i*} \sum_l y_l^{i+n}} = \Delta t^2 \rho \sigma_q^2 \sum_{l=1}^{m} k_l^y \exp\left(z_l^y \tau\right) \frac{k_\nu^{y*}}{1 - \exp\left(z_l^x + z_\nu^{y*}\right)\Delta t}.$$

For the real parts, the average cross-correlation function is as follows:

$$R_{\xi\eta}(\tau) = \overline{\operatorname{Re} \sum_l x_l^i \operatorname{Re} \sum_l y_l^{i+n}} = \Delta t^2 \rho \sigma_{qp}^2 \sum_{l=1}^{m} k_l^x k_l^y \left(\operatorname{Re}\left(s_l^y\right)^n + \operatorname{Re} s_l^x \operatorname{Re}\left(s_l^y\right)^{n+1} + \operatorname{Re}\left(s_l^x\right)^2 \operatorname{Re}\left(s_l^y\right)^{n+2} + \ldots\right)$$

$$= \frac{\Delta t^2 \rho \sigma_{qp}^2}{2} \sum_{l=1}^{m} k_l^y k_l^x \operatorname{Re}\left\{\exp\left(z_l^y \tau\right)\left[\frac{1}{1 - \exp\left(\left(z_\nu^x + z_l^y\right)\Delta t\right)} + \frac{1}{1 - \exp\left(\left(2\alpha_\nu^x + z_l^x - z_\nu^x\right)\Delta t\right)}\right]\right\}.$$

If the rhythms are independent and originate from the same white Gaussian noise, then:

$$R_{yx}(\tau) = \overline{\sum_l x_l^{i*} \sum_l y_l^{i+n}}$$

$$= \Delta t^2 \rho \sigma_q^2 \sum_{l=1}^{m} k_l^y \exp\left(z_l^y \tau\right) \sum_{l=1}^{m} \frac{k_\nu^{y*}}{1 - \exp\left(z_l^x + z_\nu^{y*}\right)\Delta t},$$

$$R_{xy}(\tau) = \overline{\sum_l y_l^{i*} \sum_l x_l^{i+n}} = \Delta t^2 \rho \sigma_q^2 \sum_{l=1}^{m} k_l^x \exp\left(z_l^x \tau\right) \times \sum_{\nu=1}^{m} \frac{k_\nu^{y*}}{1 - \exp\left(z_l^x + z_\nu^{y*}\right)\Delta t},$$

$$R_{\xi\eta}(\tau) = \overline{\operatorname{Re} \sum_\nu x_\nu^i \operatorname{Re} \sum_l y_l^{i+n}} = \Delta t^2 \rho \sigma_{qp}^2 \sum_{\nu=1}^{m_x} k_\nu^x \sum_{l=1}^{m_y} k_l^y \left(\operatorname{Re}\left(s_l^y\right)^n + \operatorname{Re} s_\nu^x \operatorname{Re}\left(s_l^y\right)^{n+1} + \operatorname{Re}\left(s_\nu^x\right)^2 \operatorname{Re}\left(s_l^y\right)^{n+2} + \ldots\right)$$

$$= \frac{\Delta t^2 \rho \sigma_{qp}^2}{2} \sum_{l=1}^{m_y} k_l^y \operatorname{Re}\left\{\exp\left(z_l^y \tau\right) \sum_{\nu=1}^{m_x} k_\nu^x \left[\frac{1}{1 - \exp\left(\left(z_\nu^x + z_l^y\right)\Delta t\right)} + \frac{1}{1 - \exp\left(\left(2\alpha_\nu^x + z_l^x - z_\nu^x\right)\Delta t\right)}\right]\right\}.$$

These equations show that one branch of the cross-correlation function represents oscillations of the signal $x$, and the other are the signal $y$, which is accurate within the coefficients depending on both processes.

In contrast to [7], we simplified the model to calculate the decrements: the coefficients that depend on decrements and frequencies were included in the factors in front of the exponents: $R_\xi(\tau) = \frac{\sum k_I \operatorname{Re}[\exp(z_i\tau)]}{\sum k_i}$. Figure 1 shows the curves of a single harmonic function of the difference between the maximum of the spectral power and the frequency of the model depending on the frequency at different values of the damping decrement (Fig. 1a) and the dependency of the decrement on the spectral maximum at various frequencies (Fig. 1b). The graph shows that in this model, the frequency of the spectral power maximum is always lower than the one described by the model. Therefore, this model is more resistant to the initial approximations at the selection of parameters for further iterations, since it reduces the dependency of solutions on the initial conditions.

A half-hour EEG recording from a mouse 20 min after apomorphine administration was used for the verification of this method. Normalized autocorrelation functions of 1 s (analyzed epoch 4 s) were calculated and averaged; the average power spectrum was defined using the Parzen smoothing function in the range of 0–20 Hz; extremums and inflection points, which could cover the presence of rhythms, were found. We limited the analysis with four harmonic functions. The frequencies and decrements of the model EEG were determined using the least-squares method depending on the number of functions. It was critical to select initial parameters for the subsequent iterations. The intuitive formula was used as initial values of decrements and frequencies. The model parameters were selected for the frequency and spectral maximum equal to the sum of all maximums or spectral values in the selected inflection points; x-axis intersection of the model autocorrelation function was used to determine the initial values of coefficients in this model. The program then searched through all parameters at a preset rate and calculated the minimal sum of squares of the difference between the model autocorrelation function and averaged real function. These parameters were the initial values for the Newton-Kantorovich method. Partial derivatives with respect to the parameters of sum of squares of differences of the modeled autocorrelation function and the real function were set equal to zero. The system of nonlinear equations was solved using the Newton-Kantorovich method via the Jacobian matrix. Figure 2 shows examples of experimental and modeled auto-

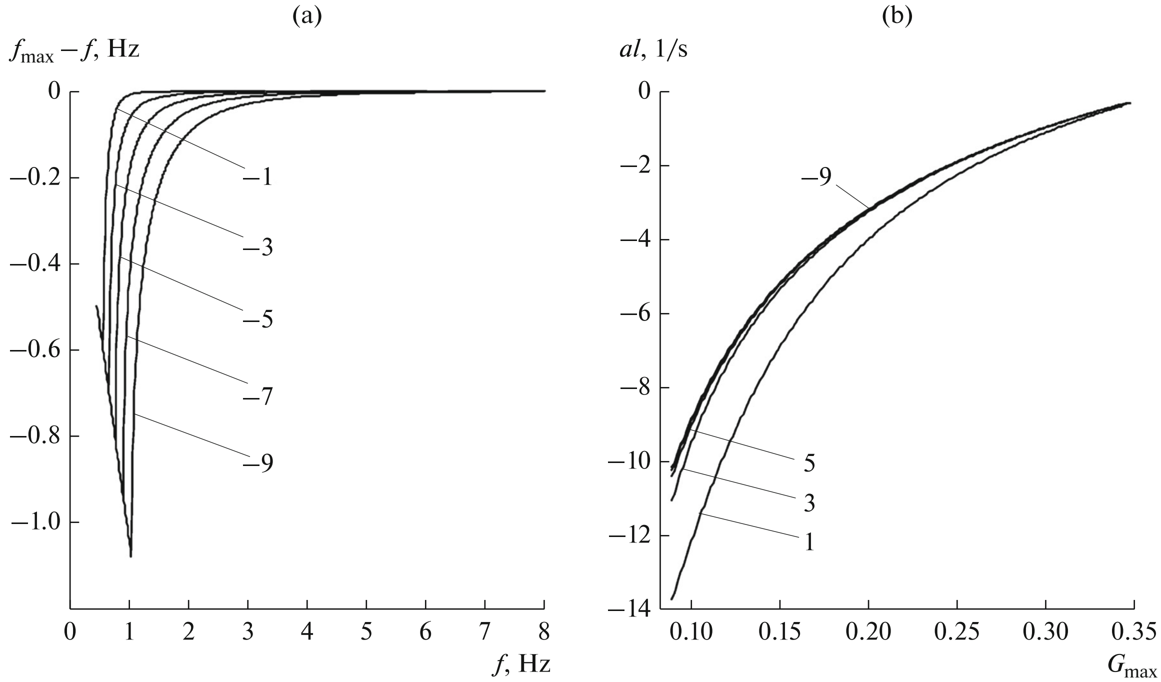


**Fig. 1.** The dependency of the difference between frequencies of spectral power maximums and the model frequency on the frequency at various damping decrements, which are marked with arrows (a); and the dependency of the decrement on the value of the spectral power maximum at various frequencies, which are also marked with arrows (b) for a selected harmonic function. On (a): the linear dependency at the frequencies below 1 Hz indicated the absence of the spectral maximum.

correlation functions. Figure 2d shows a single spectral power maximum at the frequency of 1.8 Hz and inflection points at the frequencies 3.0, 5.17, 6.63, 9.08, 10.79, and 12.92 Hz. The 3.0, 6.63, and 12.93 Hz frequencies were chosen. The results are presented in the table. Figure 2e shows two spectral peaks at 1.5 and 7.1 Hz and inflection points at 9.84, 12.74, and 15.19 Hz. There are also two peaks at 1.7 and 7.1 Hz and inflection points at 10.02, 12.64, and 14.95 Hz. These results were preliminarily presented at the 5th Congress of Russian Biophysicists in 2015 [8].

## CONCLUSIONS

Taking both the points of the prominent spectral power maximums and inflection points of the averaged spectral curve into account helps to reduce the discrepancy between the theoretical and experimental

The coefficients, frequencies, and decrements determined by the experimental curves presented in Fig. 2

| | $k_1$ | $f_1$ | $\alpha_1$ | $k_2$ | $f_2$ | $\alpha_2$ | $k_3$ | $f_3$ | $\alpha_3$ | $k_4$ | $f_4$ | $\alpha_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| a, d | 1 | 1.645 | –4.609 | 0.653 | 3.23 | –5.868 | 1.257 | 7.202 | –17.681 | 0.085 | 13.77 | –3.827 |
| b, e | 1 | 1.972 | –8.435 | 0.131 | 6.986 | 2.54 | 0.568 | 7.917 | –13.73 | 0.07 | 12.74 | –5.225 |
| c, f | 1 | 2.109 | –8.871 | 0.734 | 7.13 | –6.377 | 0.187 | 10.46 | –7.923 | 0.074 | 13.14 | –5.996 |

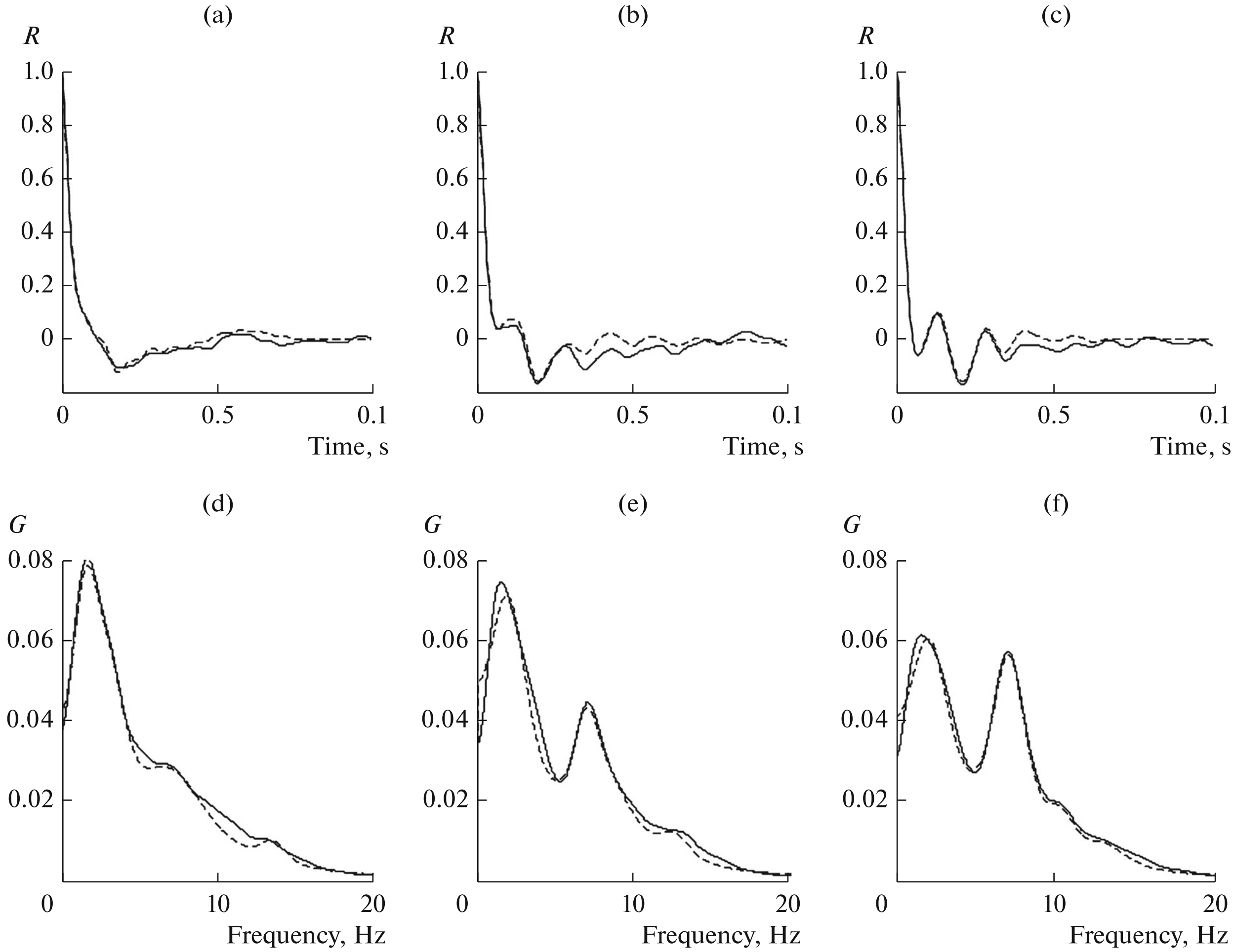


**Fig. 2.** Examples of experimental averaged autocorrelation functions (a–c) and their power spectra (d–f). Results of averaging of 40 analyzed epochs is shown. The dotted lines represent the results of the search of theoretical autocorrelation functions using the least-squares method.

autocorrelation functions. Power spectra are also described more accurately. This method helps to discover hidden frequencies in the alpha- and beta-ranges and investigate the dynamics of decrements together with the frequency-amplitude parameters.

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