ISSN 0005-1179, Automation and Remote Control, 2022, Vol. 83, No. 7, pp. 1093–1107. © Pleiades Publishing, Ltd., 2022.
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Coordination of Collective Actions by Using the Stackelberg Strategy

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Received November 14, 2021; revised March 4, 2022; accepted March 31, 2022

Abstract—The paper deals with a theoretical study of the coordination of actions of members of a self-managed team using the Stackelberg strategy aimed at increasing their individual gains. It is assumed that the team creates a total income that increases with the growth of efforts made by each agent and obeys the diminishing return law. The unique Nash equilibrium that exists under the conditions of complete autonomy of all agents is Pareto inefficient. It is shown that for the transition to a Pareto-preferential outcome it suffices to form a small group (coalition) in the team whose members trust each other and are not prone to opportunistic behavior. Following a coalition strategy aimed at achieving the maximum coalition gain, the coalition members increase their efforts; this leads to an increase in the total income. Conditions are found under which the coalition can use the Stackelberg leadership strategy. It is shown that the Stackelberg equilibrium outcome dominates in the sense of Pareto over Nash equilibrium outcomes both in noncooperative and coalitional games.

Keywords: collective action, coordination, Nash equilibrium, Stackelberg equilibrium, Pareto efficiency, coalition

DOI: 10.1134/S0005117922070062

1. INTRODUCTION

The article examines the activities of a self-managed team that creates a common income as a result of the application of individual efforts by its members. The goal of each agent is to maximize his/her own individual payoff.

The main source of problems of collective actions is rooted in the selfish aspirations of agents that lead to a mismatch between individual and collective optima under the conditions of the diminishing return law. The consequence of agents independently choosing the amount of efforts is a Nash equilibrium that is achieved in a Pareto inefficient outcome, which is clearly illustrated by the model known as the “prisoners’ dilemma” [1, 2]. The same factors create the moral hazard problem described in the Bengt Holmström model [3], as well as in the incomplete contract models due to Grossman–Hart–Moore [4, 5] and Tirole–Furubotn–Richter [6, vol. 1, pp. 50–54; 7, pp. 293–301]. Note that, as a rule, incomplete contract models consider the interaction of only two agents and simply do not reflect many problems of collective actions.

We assume that in a collective, viewed as a large group of agents, a small group can be formed—a coalition capable of coordinating the efforts of its members. The purpose of this paper is to study the possibility of forming a coalition using the Stackelberg leadership strategy to improve the efficiency of the collective activity.

Recall that initially the Cournot [8] and Stackelberg [9] models were developed to describe a duopoly, but later they were extended to an arbitrary number of firms [10,11] and to multicommodity markets [12]. It is well known that the Stackelberg model is based on rejecting the symmetry underlying the Cournot model. In the Stackelberg model, one of the competing firms is assumed to play the role of a leader who takes the first step and the other, the role of a follower. The follower chooses his/her strategy taking into account the leader's strategy, known to him/her, considering it to be given. Preliminarily, the leader incorporates the competitor's response curve known to him/her into his/her profit function, as a result of which his/her profit becomes a function only of the strategy he/she chooses, and he/she only has to find a strategy that maximizes his/her profit.

We note the papers [13, 14], which analyze the possibilities of achieving Stackelberg equilibrium by an oligopoly in cases where agents do not have reliable information about the size of marginal costs of competitors or their choice. The Geraskin oligopoly model [15] examines the dependence of the payoffs of agents in the Stackelberg equilibrium on the form of cost functions. In the papers by Germeier [16], Gorelov [17], and Gubko and Novikov [18], the Stackelberg strategy is analyzed in terms of models of a hierarchical system of the Principal–agent type. The system is represented by two players, one of whom plays the role of the Principal, limiting the set of possible actions of the second (passive) player by his/her first move. The problem is usually solved in general form, and the Stackelberg strategy is analyzed as one possible option. The payoff of the passive agent obviously depends only on his/her own choice.

When extending the Stackelberg model developed for an oligopolistic market to collective actions, one should take into account the fundamental difference between a team whose goal is to produce a common good and an oligopoly. In particular, collective action is characterized by relations of cooperation, while oligopoly is characterized by relations of competition. Accordingly, each member of the team is interested in the high activity of his/her partners, while any firm in an oligopoly, on the contrary, benefits from the low activity of competitors.

Novikov [19] considers a class of problems of minimizing the costs of a team performing work in a given volume under various assumptions regarding the hierarchy of the agent perceptions of each other types, the volume of work performed, the presence or absence of a control Principal, etc. Despite a certain similarity with other problems of this class, the problem dealt with in the present paper has significant differences. In particular, here we are looking not for an optimal distribution of the amount of work between members of the team, which minimizes the total costs, but for incentives for each agent to increase the level of his/her efforts (and hence costs) relative to the amount of effort corresponding to the Nash equilibrium outcome.

The model under consideration assumes the absence of any asymmetry in the distribution of information; the total income and individual payoff functions, as well as the composition of the coalition and the desire of its members to maximize the coalition payoff and of each noncooperative agent to maximize his/her individual payoff, are common knowledge. The way some agents influence the choice of others is quite specific, is devoid of any elements of directive (administrative) control, and is based solely on the property of complementarity of efforts and common knowledge of the agents. The payoff of any agent explicitly depends on the choice of each team member.

The papers previously published by the present authors show that if a coalition is formed in a team to implement a strategy aimed at maximizing the coalition gain, *an outcome is achieved that is Pareto dominating the inefficient Nash equilibrium achieved in a noncooperative game* [20, 21].

If the income function ensures the complementarity of the efforts of the team members, i.e., the property of the marginal income with respect to the efforts of an agent to increase with the growth of the efforts applied by another agent, then this positive effect is formed not only due to an increase in the equilibrium values of the efforts of the members of the coalition, but also due to an increase in the efforts of other agents, not included in the coalition.

This possibility, generated by a positive relationship between the efforts of the team members, has already been subjected to theoretical study. For example, in the model of a two-agent team proposed by Huck and Rey-Biel [22], the utility of the follower grows as the gap between the volume of efforts carried out by him/her and the leader decreases. In the paper by Gervais and Goldstein [23], Pareto improvement is associated with an inadequate assessment of his/her own efforts by one of the agents, i.e., with an actually irrational behavior. In these papers, Stackelberg equilibrium is not achieved. The Stackelberg strategy is fully implemented in the collective model proposed by Kim [24]. In his paper, in contrast to the model in the present article, the leader is actually appointed by the Principal, who is able to influence the agent incentives by the terms of the contract.

It is important to point out that coalition members can have a stimulating effect on the amount of efforts of noncooperative agents only by the amount of their own efforts and only when noncooperative agents choose the amount of their efforts based on reliable information about the values of efforts of coalition members [25]. In a sequential two-period game, noncooperative agents exercise their efforts only after so do the coalition members and hence receive this information directly by observing the efforts of the coalition members. It should be noted that the description of collective actions within the framework of a sequential game may not always be adequate; for example, this is the case if the activity of the team is associated with a technological process requiring a simultaneous application of efforts by all members of the team.

In the case of a simultaneous game, noncooperative agents can obtain information about the efforts that coalition members apply only on the basis of a message from the coalition itself or certain preliminary actions of the coalition that leave no room for doubt about its intentions. Common knowledge about the amount of efforts that the members of the coalition carry out in a simultaneous game is a necessary and sufficient condition for rational agents to successfully coordinate efforts [26]. When this condition is satisfied, the coalition can optimize the amounts of efforts by its members, determining them by the backward induction method, i.e., applying the Stackelberg strategy, with delivering this information to noncooperative agents before the start of the game.

However, a simple declaration of the coalition about its intentions may not be enough in a simultaneous game to achieve the same outcome that corresponds to the Stackelberg equilibrium in a two-period game. The point is that in a simultaneous game, as will be shown below, the coalition gain reaches its maximum at a lower value of effort by coalition members than in a two-period game at the same level of efforts by noncooperative agents. Therefore, if at least one of the noncooperative agents doubts the intention of the coalition members to carry out their efforts in the promised amount, then he/she will carry out his/her efforts in an amount insufficient to achieve the Stackelberg equilibrium. As a result, not only the maximum of the coalition gain will not be reached, but also the maximum profit of those noncooperative agents who believe the promises of the coalition and carry out their efforts in amounts corresponding to the Stackelberg equilibrium. Since each member of the team possesses this knowledge, the outcome that corresponds to the Stackelberg equilibrium in a two-period game may turn out to be unattainable in a simultaneous game.

The coalition can avoid this kind of uncertainty by paying a deposit or entering into an appropriate contract or, alternatively, by investing in an appropriate specific resource the best alternative use of which is less profitable than the promised amounts of efforts. In a simultaneous game, it is this action that actually constitutes the first step of the coalition that is necessary to reliably achieve an outcome that coincides with the Stackelberg equilibrium in a two-period game. This first step in the form of performing one of the listed actions is equivalent to direct observation of the efforts of coalition members.

In the present paper, we consider the general case of an inseparable income function and a linear cost function. It will be shown that the positive dependence of the optimal strategy of each autonomous agent on the contributions of the coalition members creates the preconditions for the coalition to use the Stackelberg leadership strategy. To implement this strategy successfully, the coalition members must be sure that all other agents adhere to the follower strategy, i.e., determine their optimal efforts as a function of the efforts by the coalition members. The result is a Stackelberg equilibrium achieved in an outcome that dominates not only the inefficient Nash equilibrium in a coalition-free game but also the equilibrium outcome in a game in which the coalition is a single player, without using the Stackelberg strategy but employing the Cournot strategy.

2. BASIC MODEL

By n we denote the number of individuals that make up the collective, in which, through the implementation of individual efforts, the total income $D = D(\sigma_1, \ldots, \sigma_n)$ is created, where $\sigma_1, \ldots, \sigma_n$ are their monetary equivalents. We assume that the following conditions are satisfied for all $\sigma_i \in (0, \infty)$, where $i = 1, \ldots, n$:

1. The amount of income increases as the applied efforts grow,

$$\frac{\partial D}{\partial \sigma_i} > 0. \tag{1}$$

2. For the payoff functions to have a single maximum, the income function is strictly concave. It follows from this condition that the income function satisfies the diminishing returns law,

$$\frac{\partial^2 D}{\partial \sigma_i^2} < 0. \tag{2}$$

3. The following conditions are satisfied for the solution not to go to zero or infinity:

$$\lim_{\sigma_i \to 0} \frac{\partial D}{\partial \sigma_i} = \infty, \quad \lim_{\sigma_i \to \infty} \frac{\partial D}{\partial \sigma_i} = 0. \tag{3}$$

4. The efforts by each agent positively impact the value of the marginal income for the efforts by any other team member,

$$\frac{\partial^2 D}{\partial \sigma_i \partial \sigma_k} > 0 \quad \text{for} \quad i \neq k. \tag{4}$$

We will assume that the income and payoff functions are common knowledge for all team members, and the amounts of efforts made are observable for them after being fully implemented. At the ex ante stage, the team establishes a distribution rule for the future expected total income D, according to which agent i owns a relative share of α_i, $0 < \alpha_i < 1$, $\sum_{i=1}^{n} \alpha_i = 1$. In an unstructured team, each individual autonomously chooses the amount of effort he/she makes pursuing the goal of maximizing his/her payoff,

$$U_k = \alpha_k D(\sigma_k, \sigma_{-k}) - \sigma_k, \quad k = 1, \ldots, n, \tag{5}$$

where σ_{-k} are the values of the efforts by all team members except for individual k.

It was proved in [27] that in a noncooperative game where the payoffs of agents are given by formulas (5), for any set α_k there exists a unique Nash equilibrium N determined from the first-order maximum conditions for the functions U_k, i.e., the system of equations

$$\alpha_k \frac{\partial D}{\partial \sigma_k} = 1, \quad k = 1, \ldots, n. \tag{6}$$

We use the following notation: σ_k^N, $k = 1, \ldots, n$, is a solution of system (6), D^N is the total income value, U^N is the total payoff of all team members, and U_k^N is the individual payoff of agent k. It was shown in [27] that this equilibrium outcome is not Pareto efficient, because there are Pareto-preferred states to the right of it, i.e., for $\sigma_i > \sigma_i^N$ (if at least two agents make additional investment).

It is beneficial for each agent that all other team members increase their efforts above the equilibrium level determined by system (6), but at the same time, such an increase in the volume of one's own efforts *is unprofitable for him/her* (owing to the diminishing return law). Therefore, a rational agent will increase his/her efforts only if he/she is sure that at least one of its partners does the same. In other words, to achieve any Pareto-preferred outcome, the efforts of at least two members of the team must be coordinated. Therefore, under the condition of autonomy of all the members, the transition from the Nash equilibrium to any outcome that dominates it contradicts the principle of individual rationality. Thus, an unstructured team consisting of autonomous egotistical agents is doomed to be trapped in an inefficient Nash equilibrium.[1]

An important characteristic of collective action is the existence of an outcome corresponding to a social optimum. Here, as in the economic theory of contracts, the social optimum is defined as the outcome in which the maximum value is achieved by the cumulative gain of the entire team, equal to the difference between the total income and the sum of costs of all team members,

$$U = \sum_{i=1}^{n} U_i = D - \sum_{i=1}^{n} \sigma_i. \tag{7}$$

The first-order maximum conditions for the function U have the form of the system of equations

$$\frac{\partial D}{\partial \sigma_i} = 1, \quad i = 1, \ldots, n. \tag{8}$$

We use the following notation: σ_k^P, $k = 1, \ldots, n$, is a solution of system (8), D^P is the amount of total income, U^P is the amount of total payoff of all team members, and U_k^P is the amount of individual payoff of agent k.

This outcome of the game, which we denote by P, will be Pareto optimal[2] when the distribution of total income in the team meets the following conditions:

$$U_k^P \geq U_k^N, \quad \text{where} \quad k = 1, \ldots, n. \tag{9}$$

[1] This conclusion is completely consistent with the conclusion obtained in Holmström paper [3], in the "prisoners' dilemma" model, and in incomplete contract models [4–7].

[2] To make sure that the total payoff U is higher in the outcome P than in the outcome N, it suffices to refer to the gradient of the function U. At the Nash equilibrium point N, each coordinate of $\operatorname{grad} U$ is greater than zero; it follows that the function U reaches higher values at effort levels that exceed the equilibrium.

Since, according to conditions (1)–(2), we have $U^P = \sum_{k=1}^n U_k^P > \sum_{k=1}^n U_k^N = U^N$, it follows that there exists a distribution of income among the members of the collective in which the socially optimal outcome is Pareto optimal.

3. STIMULATING EFFECT OF THE COALITION STRATEGY

Now we will show that the formation of a coalition in the team and the implementation of the coalition strategy makes it possible to overcome the Nash equilibrium. Let us assume that a coalition has been formed in the team whose members are able to coordinate the amounts of their efforts in order to achieve the maximum coalition payoff. The issues of coalition formation and stability are a separate problem, partially covered by the present authors in [28], and therefore are not considered here. Here we will assume that coordination between coalition members is based on trust, since it is in this case that the transaction costs of coordination are minimal. In addition, we believe that the desire of the coalition to maximize the coalition payoff is common knowledge.

Denote the coalition by C. Noncooperative (not included in the coalition) agents form the set NC. We use the following notation: $[\sigma_i]$ with $i \in C$ is the tuple of the amounts of effort by coalition members, $[\sigma_j]$ with $j \in NC$ is the tuple of the amounts of effort by noncooperative agents, and $\alpha_C = \sum_{i \in C} \alpha_i$ is the relative share of the coalition in the total income D. We write an expression for the coalition payoff in the form

$$U_C = \alpha_C D\big([\sigma_i], [\sigma_j]\big) - \sum_{i \in C} \sigma_i, \quad i \in C, \quad j \in NC. \tag{10}$$

The amounts of efforts by the coalition members for which their total payoff reaches a maximum is determined by the system of equations

$$\frac{\partial D}{\partial \sigma_i} = \frac{1}{\alpha_C}, \quad i \in C. \tag{11}$$

The noncooperative agents choose the amount of their efforts from the conditions of the maximum of their own individual payoffs, i.e., from equations similar to (6),

$$\frac{\partial D}{\partial \sigma_j} = \frac{1}{\alpha_j}, \quad j \in NC. \tag{12}$$

The game resulting from the formation of the coalition C differs from the original noncooperative game in that one aggregated player represented by the coalition C participates in it along with autonomous (noncooperative) players $j \in NC$. The union of systems (11) and (12) has a unique solution, and accordingly, the new game has a unique equilibrium outcome, which we denote by $\hat{C}$. The values of the efforts by the agents in this outcome will be denoted by $\sigma_i^{\hat{C}}$ and $\sigma_j^{\hat{C}}$, $i \in C$, $j \in NC$, the total income by $D^{\hat{C}}$, and the payoffs of the coalition and the agents by $U_C^{\hat{C}}$, $U_i^{\hat{C}}$, and $U_j^{\hat{C}}$, respectively.

Let us show that the outcome $\hat{C}$ is Pareto dominant over the outcome N and that the following inequalities hold:

$$\sigma_i^{\hat{C}} > \sigma_i^N, \quad i \in C; \quad \sigma_j^{\hat{C}} > \sigma_j^N, \quad j \in NC; \tag{13}$$

$$U_j^{\hat{C}} > U_j^N, \quad j \in NC; \quad U_C^{\hat{C}} > U_C^N = \sum_{i \in C} U_i^N. \tag{14}$$

Let us compare the joint solution of systems (11) and (12), which determines the outcome $\hat{C}$, with the solution of system (6), which determines the outcome N. We can assume that system (11)–(12) is formed from system (6) by replacing the quantities α_i on the right-hand sides of the equations with $i \in C$ by the quantity $\alpha_C = \sum_{i\in C} \alpha_i$ exceeding them. Since, according to (2), the solution of system (6) depends positively on the quantities α_k, we see that this change leads to higher values of σ_i corresponding to the solution of systems (11)–(12).[3]

The left-hand sides of Eqs. (12) are functions of the effort σ_j by noncooperative agents and the effort σ_i by coalition members. For any fixed values of σ_i, system (12) has a unique solution with respect to the variables σ_j with $j \in NC$, which determines the values of these variables as functions of the variables σ_i with $i \in C$. We denote these solutions by

$$\sigma_j = R_j\big([\sigma_i]\big), \quad i \in C, \quad j \in NC. \tag{15}$$

The functions (15) are the *response functions* of noncooperative agents to the amounts of efforts by coalition members known to them. By virtue of inequalities (4), the left-hand sides of Eqs. (12) are increasing functions of the variables σ_i with $i \in C$. Therefore, the solutions of these equations, i.e., the response functions $\sigma_j = R_j([\sigma_i])$, increase in all variables σ_i with $i \in C$ as well.

According to condition (1), it follows from inequalities (13) that $D^{\hat{C}} > D^N$. Since each payoff function (either U_j or U_C) has a single maximum, we see that their maxima in the outcome $\hat{C}$ exceed the corresponding maxima achieved in the outcome N; i.e., inequalities (14) hold true. It can be assumed that the mutual trust relations connecting the members of the coalition are capable of ensuring a conflict-free distribution of the coalition gain between them in order to ensure individual rationality conditions for all members of the coalition, and therefore, the inequality $U_C^{\hat{C}} > U_C^N$ is consistent with the inequalities

$$U_i^{\hat{C}} > U_i^N, \quad i \in C. \tag{16}$$

Thus, the *implementation of a coalition strategy leads to an outcome that is Pareto dominant over the Nash equilibrium achieved in a noncooperative game*.

4. STACKELBERG STRATEGY

The Stackelberg model of the oligopolistic market assumes that the leader makes the first move: he/she is either the first to implement his/her strategy (for example, produces a certain volume of products) or defiantly makes a nonrefundable investment that convinces competitors that the leader has chosen a certain strategy. Only under this condition will competitors believe in his/her choice and take it into account when choosing their optimal strategies.

In the model in the present paper, it is assumed that the coalition, as was mentioned above, has convinced the noncooperative agents by its first move that the members of the coalition would make efforts in the amount $[\sigma_i]$, $i \in C$. Before the game begins, the coalition finds the response functions $\sigma_j = R_j([\sigma_i])$ from Eqs. (12). In the space of values σ_k $(k = 1, \dots, n)$, the system of these functions determines a response surface at each point of which conditions (12) for the maximum payoffs of noncooperative agents are satisfied. On the response surface, the amount of total income is a function of the respective efforts by the coalition members,

$$D\Big([\sigma_i], R_j\big([\sigma_i]\big)\Big), \quad i \in C, \quad j \in NC. \tag{17}$$

[3] A rigorous proof of the theorem on the increase of all variables that form the solution of system (6) with a decrease in the value of at least one of the parameters that represent its right-hand sides is given in [25].

Thus, being a Stackelberg leader, the coalition incorporates the response functions of noncooperative agents into its net income function, which takes the form

$$U_C = \alpha_C D\Big([\sigma_i], R_j\big([\sigma_i]\big)\Big) - \sum_{i\in C}\sigma_i \tag{18}$$

with maximum conditions in the form

$$\alpha_C D'_{\sigma_i} - 1 = 0, \quad i \in C, \tag{19}$$

where D'_{σ_i} is the total derivative with respect to σ_i of the function (18) taken on the response surface,

$$D'_{\sigma_i} = \frac{\partial D}{\partial \sigma_i} + \sum_{j\in NC}\frac{\partial D}{\partial \sigma_j}\frac{\partial R_j}{\partial \sigma_i}, \quad i \in C. \tag{20}$$

In view of (12), the expression (20) for the derivatives acquires the form

$$D'_{\sigma_i} = \frac{\partial D}{\partial \sigma_i} + \sum_{j\in NC}\frac{1}{\alpha_j}\frac{\partial R_j}{\partial \sigma_i}. \tag{21}$$

In view of (21), Eqs. (19) transform into

$$\frac{\partial D}{\partial \sigma_i} + \sum_{j\in NC}\frac{1}{\alpha_j}\frac{\partial R_j}{\partial \sigma_i} = \frac{1}{\alpha_C}, \quad i \in C. \tag{22}$$

We denote the game outcome corresponding to the Stackelberg solution by $\hat{S}$, the amounts of efforts by coalition members in this outcome by $\sigma_i^{\hat{S}}$, $i \in C$, and $\sigma_j^{\hat{S}}$, $j \in NC$, the total income by $D^{\hat{S}}$, and the values of payoffs of the coalition and agents by $U_C^{\hat{S}}$, $U_i^{\hat{S}}$, and $U_j^{\hat{S}}$, respectively. Let us compare the results of the outcomes $\hat{S}$ and $\hat{C}$.

By virtue of Eqs. (12), the outcome $\hat{C}$, just as the outcome $\hat{S}$, corresponds to a point on the response surface. Therefore, both the income D and the marginal income $\frac{\partial D}{\partial \sigma_i}$ are functions of the variables σ_i, $i \in C$. Let us denote the value of the marginal income $\frac{\partial D}{\partial \sigma_i}$ at the point $[\sigma_i^{\hat{S}}]$ by $\frac{\partial D}{\partial \sigma_i}(\hat{S})$ and that at the point $[\sigma_i^{\hat{C}}]$ by $\frac{\partial D}{\partial \sigma_i}(\hat{C})$. Then, according to Eqs. (11), Eqs. (22) can be written as

$$\frac{\partial D}{\partial \sigma_i}(\hat{S}) + \sum_{j\in NC}\frac{1}{\alpha_j}\frac{\partial R_j}{\partial \sigma_i} = \frac{\partial D}{\partial \sigma_i}(\hat{C}), \quad i \in C. \tag{23}$$

Since the functions $R_j([\sigma_i])$ are increasing in all variables, it follows that relations (23) imply the inequalities

$$\frac{\partial D}{\partial \sigma_i}(\hat{S}) < \frac{\partial D}{\partial \sigma_i}(\hat{C}), \quad i \in C; \tag{24}$$

hence, according to conditions (2), we have the inequalities

$$\sigma_i^{\hat{S}} > \sigma_i^{\hat{C}}. \tag{25}$$

Inequalities (25) and the fact that the $R_j([\sigma_i])$ are increasing imply the inequalities

$$\sigma_j^{\hat{S}} > \sigma_j^{\hat{C}}, \quad j \in NC. \tag{26}$$

Thus, we can state the first conclusion. When passing from the outcome $\hat{C}$—the only Nash equilibrium in a game in which the coalition C and noncooperative agents maximize their net incomes based on symmetrical expectations for all other players—to the outcome $\hat{S}$ achieved in the case where the coalition C applies the Stackelberg leadership strategy, the values of the efforts by all players, both the coalition members and the noncooperative agents, increase.

Let us compare the net income of the coalition in the outcomes $\hat{S}$ and $\hat{C}$. If the coalition uses the Stackelberg strategy, it means that it maximizes its payoff U_C on the response surface given by Eqs. (15). Hence Eqs. (22) determine the point of maximum of the net income of the coalition on the response surface corresponding to the outcome $\hat{S}$. The point corresponding to the outcome $\hat{C}$ also lies on the response surface, but, according to inequalities (25) and (26), its coordinates do not coincide with the coordinates of the maximum of the function U_C on the response surface. Therefore, owing to the uniqueness of the maximum, we have the inequality

$$U_C^{\hat{S}} > U_C^{\hat{C}}. \tag{27}$$

Since, according to the assumption introduced earlier, the members of the coalition are able to make a proportional division of the total payoff of the coalition among themselves, we see that the following inequalities are a consequence of (27):

$$U_i^{\hat{S}} > U_i^{\hat{C}}, \quad i \in C. \tag{28}$$

Let us compare the payoffs of noncooperative agents in the outcomes $\hat{S}$ and $\hat{C}$. On the response surface, the net income functions of the noncooperative agents have the form

$$U_j = \alpha_j D\Big([\sigma_i], R_j\big([\sigma_i]\big)\Big) - R_j\big([\sigma_i]\big), \quad i \in C, \quad j \in NC. \tag{29}$$

Let us proceed to the derivative $(U_j)'_{\sigma_i} = \alpha_j D'_{\sigma_i} - (R_j)'_{\sigma_i}$, which, in view of (21), acquires the form

$$(U_j)'_{\sigma_i} = \alpha_j \frac{\partial D}{\partial \sigma_i} + \sum_{\substack{k \in NC, \\ k \neq j}} \frac{\alpha_j}{\alpha_k} \frac{\partial R_k}{\partial \sigma_i}, \quad i \in C, \quad j \in NC. \tag{30}$$

All terms on the right-hand side in (30) are positive for any $j \in NC$, and consequently, $(U_j)'_{\sigma_i} > 0$.

Thus, we arrive at the second conclusion. If a coalition C occupies the Stackelberg leadership position and all noncooperative agents take the positions of followers, then the payoffs of all noncooperative agents increase on the response surface with respect to all independent variables σ_i. Accordingly, (25) implies that

$$U_j^{\hat{S}} > U_j^{\hat{C}}, \quad j \in NC. \tag{31}$$

Inequalities (28) and (31) allow one to draw the third conclusion. The outcome $\hat{S}$—the equilibrium in a game in which the coalition C takes the position of the Stackelberg leader and all noncooperative agents take the positions of followers—Pareto dominates the outcome $\hat{C}$—the Nash equilibrium in a game in which the coalition C and noncooperative agents maximize their payoffs based on symmetrical expectations for all other players.

Now let us proceed to the consideration of the reason why coalitions in a simultaneous game need to send a preliminary information signal that can completely convince noncooperative agents that all members of the coalition will carry out their efforts in the amount of $\sigma_i^{\hat{S}}$, $i \in C$. If the team faces a sequential game in which noncooperative agents apply their efforts only after the members of the coalition, then the coalition finds the amount of efforts of its members as a result of solving the problem of finding a *conditional maximum* of its payoff function (18) under condition (19). The corresponding system of equations acquires the form (22) with the solution $\sigma_i = \sigma_i^{\hat{S}}$ and $\sigma_j = \sigma_j^{\hat{S}}$, $i \in C$, $j \in NC$. In this case, it is unprofitable for any of the agents to deviate from this decision in their choice.

In the simultaneous game, the coalition chooses the level of efforts of its members based on the assumption that all noncooperative agents will certainly carry out their efforts in the amount of $\sigma_j^{\hat{S}}$. In this case, it solves the problem of finding an *unconditional maximum* and finds the effort level Eqs. (11) of its members with $\sigma_j = \sigma_j^{\hat{S}}$, $j \in NC$. A comparison of (22) with (11) shows that the partial derivatives $\frac{\partial D}{\partial \sigma_i}$ in Eqs. (22) are lower than in Eqs. (11). This means that the values of $\sigma_i^{\hat{S}}$, which are solutions of Eqs. (22), exceed the values of σ_i, which are solutions of Eqs. (11), although the amounts of efforts by noncooperative agents are the same in the solutions of both systems. That is why, in a simultaneous game, for all members of the coalition, if they are sure that all noncooperative agents will certainly carry out their efforts in the amount of $\sigma_j = \sigma_j^{\hat{S}}$, it is profitable to carry out their efforts in volumes lower than in the first period of the sequential game. Accordingly, in a simultaneous game, in order to achieve the same outcome that is achieved in a sequential game as a result of the Stackelberg strategy, the coalition is forced to take the corresponding first step, convincing all noncooperative agents that its members will make efforts in the amount of $\sigma_i^{\hat{S}}$.

5. CONCLUSIONS

Based on the models presented above, we arrive at the following conclusions.

1. If a small group (coalition) is formed in the team whose members strive to increase the coalition gain, then the team is able to get out of the trap of inefficient Nash equilibrium where the unstructured team falls. A consequence of the coalition strategy is an increase in the individual payoffs of all members of the team relative to their values in the Nash equilibrium achieved in a noncooperative game.
2. If the size of the marginal income of noncooperative members of the team increases with the increase in the efforts of coalition members, then the coalition strategy has a stimulating effect on noncooperative agents as a result of which the volume of their efforts also increases. The existence of such a dependence is a necessary prerequisite for the implementation of the strategy developed by Stackelberg in the duopoly model.
3. The incorporation of the response functions of noncooperative agents by the coalition into its net income function creates an opportunity for the team to achieve a new equilibrium $\hat{S}$ that is Pareto dominant over the equilibrium $\hat{C}$ achieved in a coalition game in which the coalition and noncooperative agents maximize their payoffs independently.
4. Achieving a Stackelberg equilibrium outcome $\hat{S}$ is possible only if certain mutual expectations exist on the part of both the coalition members and the noncooperative agents. These expectations should come from a common knowledge of the payoff functions, the existence of a coalition, its composition, and the amount of efforts that its members are committed to make.
5. When the assumptions accepted in this model are satisfied, the hierarchical structure of a team in which the coalition occupies the position of the Stackelberg leader turns out to be more efficient than a nonhierarchical coalition structure.

APPENDIX

Let us consider a demonstration example of the results obtained above. Let the total income function have the form

$$D = \lambda \prod_{i=1}^{n} \sigma_i^a, \tag{A.1}$$

where $\lambda > 0$ and $0 < a < 1/n$. The function (A.1) satisfies all of conditions (1)–(4). We assume that all the team members have equal income shares, $\alpha_i = 1/n$.

1. First, consider a noncooperative game in which each member of the team strives to maximize his/her individual payoff,

$$U_i = \lambda/n \prod_{j=1}^{n} \sigma_j^a - \sigma_i \to \max_{\sigma_i > 0}, \quad i = 1, \ldots, n. \tag{A.2}$$

Since the income function (A.1) satisfies the constant elasticity conditions

$$\frac{\sigma_i}{D} \frac{\partial D}{\partial \sigma_i} = a, \tag{A.3}$$

the conditions (8) of maximizing the individual payoff (A.2) can be written in the form

$$\sigma_i = \frac{aD}{n}, \quad i = 1, \ldots, n. \tag{A.4}$$

Having substituted the expressions for efforts (A.4) into (A.1), we obtain an equation for D from which we find the value of the total income in a Nash equilibrium,

$$D^N = \left(\lambda \left(a/n\right)^{an}\right)^{1/(1-an)}. \tag{A.5}$$

Using (A.5), (A.4), and (A.2), we find the amounts of efforts and payoffs,

$$\sigma_i^N = aD^N/n; \quad U_i^N = D^N(1-a)/n; \quad i = 1, \ldots, n. \tag{A.6}$$

For $\lambda = 12 \times 10^3$, $n = 100$, and $a = 1/120$, we obtain

$$\begin{gathered} D^N = \left(12 \times 10^3 \left(\frac{0.01}{120}\right)^{5/6}\right)^6 = 12 \cdot 10^3; \\ \sigma_i^N = 1.0; \quad U_i^N = 119; \quad i = 1, \ldots, 100. \end{gathered} \tag{A.7}$$

2. Let us proceed to the consideration of a coalition game. Let the coalition consist of the first m members of the team. Equation (10) for the efforts by a coalition member can be written as

$$\sigma_i = \frac{am}{n} D, \quad i = 1, \ldots, m, \tag{A.8}$$

and Eq. (11) for the efforts by the noncooperative agents, in the form

$$\sigma_j = \frac{aD}{n}, \quad j = m+1, \ldots, n. \tag{A.9}$$

Substituting the expressions for the efforts from (A.8) and (A.9) into the income function $D = \lambda \prod_{i=1}^{m} \sigma_i^a \prod_{j=m+1}^{n} \sigma_j^a$, we obtain an equation for D,

$$D = \lambda (a/n)^{an} m^{am} \left(D\right)^{an},$$

from which we find an expression for the value of the total income in the equilibrium outcome $\hat{C}$,

$$D^{\hat{C}} = \big(\lambda(a/n)^{an} m^{am}\big)^{\frac{1}{1-an}}. \tag{A.10}$$

Using (A.8), (A.9), and (A.10), we obtain the following expressions for the amounts of efforts:

$$\begin{aligned} \sigma_i^{\hat{C}} &= (\lambda a/n)^{\frac{1}{1-an}} m^{\frac{1-a(n-m)}{1-an}}, \qquad i \in 1, \dots, m; \\ \sigma_j^{\hat{C}} &= (\lambda a/n)^{\frac{1}{1-an}} m^{\frac{am}{1-an}}, \qquad j = m+1, \dots, n. \end{aligned} \tag{A.11}$$

Using (A.2), (A.10), and (A.11), we obtain the following expressions for the values of individual payoffs:

$$\begin{aligned} U_i^{\hat{C}} &= D^{\hat{C}}(1-am)/n; \\ U_j^{\hat{C}} &= D^{\hat{C}}(1-a)/n. \end{aligned} \tag{A.12}$$

For $n = 100$, $a = 1/120$, and $m = 10$, we find the following ratios of the parameter values in the outcomes $\hat{C}$ and N:

$$\begin{aligned} \frac{D^{\hat{C}}}{D^N} &= m^{\frac{am}{1-an}} = \sqrt{10}; \\ \frac{\sigma_i^{\hat{C}}}{\sigma_i^N} &= m^{\frac{1-a(n-m)}{1-an}} = 10 \cdot \sqrt{10}; \\ \frac{\sigma_j^{\hat{C}}}{\sigma_j^N} &= m^{\frac{am}{1-an}} = \sqrt{10}; \end{aligned} \tag{A.13}$$

$$\begin{aligned} \frac{U_i^{\hat{C}}}{U_i^N} &= \frac{D^{\hat{C}}(1-am)}{D^N(1-a)} = \sqrt{10} \cdot \frac{110}{119} \approx 2.9; \\ \frac{U_j^{\hat{C}}}{U_j^N} &= \frac{D^{\hat{C}}}{D^N} = \sqrt{10} \approx 3.2; \qquad i \in 1, \dots, m, \quad j = m+1, \dots, n. \end{aligned} \tag{A.14}$$

As we see, the transition from the equilibrium N to the equilibrium $\hat{C}$ leads to an increase in the individual payoffs of all members of the team.

3. Assume that the coalition C has taken the position of a Stackelberg leader and that the noncooperative agents have taken the positions of followers. It follows from (A.9) that $aD = n\sigma_j$ with $j \in NC$. Substituting the resulting expression for D into (A.1), we obtain an equation for σ_j by solving which we find the values of the efforts by the noncooperative agents as a function of the efforts by the coalition members; i.e., we obtain the response functions

$$\sigma_j = R_j\big([\sigma_i]\big) = (\lambda a/n)^{\frac{1}{1-a(n-m)}} \prod_{k=1}^{m} \sigma_k^{\frac{a}{1-a(n-m)}}, \quad j \in NC. \tag{A.15}$$

Since the agents are identical, it follows that the partial derivatives of these functions can be written as

$$\frac{\partial R_j}{\partial \sigma_i} = \frac{a}{1-a(n-m)} (\lambda a/n)^{\frac{1}{1-a(n-m)}} \sigma_k^{\frac{an-1}{1-a(n-m)}}, \quad j \in NC, \quad k, i \in C. \tag{A.16}$$

The partial derivatives of the income function in the variables σ_i can be found from Eqs. (A.3) and (A.1),

$$\frac{\partial D}{\partial \sigma_i} = \frac{aD}{\sigma_i} = a\lambda \sigma_i^{am-1} \sigma_j^{a(n-m)}, \quad j \in NC, \quad i \in C. \tag{A.17}$$

Taking into account the the agents being identical and substituting the expressions for σ_j from (A.15) into (A.17), we obtain

$$\frac{\partial D}{\partial \sigma_i} = (\lambda a/n)^{\frac{1}{1-a(n-m)}} n \sigma_i^{\frac{an-1}{1-a(n-m)}}, \quad i \in C. \tag{A.18}$$

We substitute the expressions for the derivatives from (A.16) and (A.18) into Eqs. (22). As a result of the transformation of the corresponding equation, we obtain

$$\frac{1}{\alpha_C} = \frac{1}{1-a(n-m)} \cdot (\lambda a/n)^{\frac{1}{1-a(n-m)}} \cdot n \cdot \sigma_i^{\frac{an-1}{1-a(n-m)}}, \quad i \in C. \tag{A.19}$$

Solving Eq. (A.19) for σ_i, we find expressions for the amount of efforts by coalition members in the outcome $\hat{S}$,

$$\sigma_i^{\hat{S}} = \left(\frac{\lambda a}{n}\right)^{\frac{1}{1-an}} \left(\frac{m}{1-a(n-m)}\right)^{\frac{1-a(n-m)}{1-an}}. \tag{A.20}$$

Using (A.20) and (A.11), we obtain an expression for the ratio of the efforts by the coalition members in the outcomes $\hat{S}$ and $\hat{C}$,

$$\frac{\sigma_i^{\hat{S}}}{\sigma_i^{\hat{C}}} = \left(\frac{1}{1-a(n-m)}\right)^{\frac{1-a(n-m)}{1-an}}, \quad i \in C. \tag{A.21}$$

It follows from Eqs. (12), properties (A.3), and the form of the function (A.1) that

$$\sigma_j^{\hat{S}} = \frac{aD^S}{n} = \frac{\lambda a}{n} \left(\sigma_i^{\hat{S}}\right)^{am} \left(\sigma_j^{\hat{S}}\right)^{a(n-m)}, \quad j \in NC. \tag{A.22}$$

Substituting the expression for $\sigma_i^{\hat{S}}$ from (A.20) into the right-hand side of (A.22), we obtain an equation for $\sigma_j^{\hat{S}}$, by solving which we find

$$\sigma_j^{\hat{S}} = \left(\frac{\lambda a}{n}\right)^{\frac{1}{1-an}} \left(\frac{m}{1-a(n-m)}\right)^{\frac{am}{1-an}}. \tag{A.23}$$

Relations (A.20) and (A.23) imply the following expression for the ratio of efforts by a coalition member and a noncooperative agent:

$$\frac{\sigma_i^{\hat{S}}}{\sigma_j^{\hat{S}}} = \frac{m}{1-a(n-m)}, \quad i \in C, \quad j \in NC. \tag{A.24}$$

For the case of $n = 100$, $a = 1/120$, and $m = 10$, it follows from (A.21) and (A.24) that $\sigma_i^{\hat{S}}/\sigma_i^{\hat{C}} = 4^{1.5} = 8$ and $\sigma_i^{\hat{S}}/\sigma_j^{\hat{S}} = 40$. Since $U_j = \frac{1}{n}D - \sigma_j = \frac{\sigma_j}{a} - \sigma_j = \sigma_j \frac{1-a}{a}$, in view of (A.9) and (A.22) we obtain

$$\frac{D^{\hat{S}}}{D^{\hat{C}}} = \frac{U_j^{\hat{S}}}{U_j^{\hat{C}}} = \frac{\sigma_j^{\hat{S}}}{\sigma_j^{\hat{C}}} = \left(\frac{1}{1-a(n-m)}\right)^{\frac{am}{1-an}} = \left(\frac{120}{30}\right)^{\frac{1}{2}} = 2, \quad j \in NC.$$

Since

$$U_i^{\hat{S}} = U_j^{\hat{S}} + \sigma_j^{\hat{S}} - \sigma_i^{\hat{S}} = \sigma_j^{\hat{S}}\left(\frac{1-a}{a} + 1\right) - \sigma_i^{\hat{S}} = \sigma_j^{\hat{S}}\left(\frac{1}{a} - \frac{\sigma_i^{\hat{S}}}{\sigma_j^{\hat{S}}}\right),$$

in view of (A.12) and (A.9) we obtain

$$\frac{U_i^{\hat{S}}}{U_i^{\hat{C}}} = \frac{a\sigma_j^{\hat{S}}}{\sigma_j^{\hat{C}}(1-am)}\left(\frac{1}{a} - \frac{\sigma_i^{\hat{S}}}{\sigma_j^{\hat{S}}}\right) = \frac{16}{11} \approx 1.45.$$

Thus, as a result of the transition from the outcome $\hat{C}$ to the Stackelberg equilibrium outcome $\hat{S}$, noncooperative agents increase their efforts by two times and coalition members by eight times. The payoffs of noncooperative agents are doubled, and the payoffs of coalition members are increased by 45%.

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This paper was recommended for publication by M.V. Gubko, a member of the Editorial Board