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REVIEWS

# Spectral and Modal Methods for Studying Stability and Control of Electric Power Systems

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**Abstract**—The paper provides an overview of spectral and modal analysis methods for studying the stability of electric power systems (EPSs) and their control. Consideration is given to theoretical grounds of the methods and to the experience of their application for detecting the heterogeneity of the systems' structure, identifying the coherency of generators' motion, simplifying the mathematical model of the dynamics of EPSs, assessing their small-signal stability, and selecting the control actions to ensure it. The analysis of sub-Gramians for studying the EPS stability and other new directions in the development of the modal approach are discussed.

*Keywords*: electric power systems, spectral analysis, modal analysis, stability assessment, control, model order reduction, sub-Gramians

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## 1. INTRODUCTION

Electric power systems (EPSs) are human-made sophisticated technological facilities that include thousands of power generators interconnected for joint operation in an electric network; their rotors under normal conditions rotate at the same (synchronous) angular speed. The EPS stability under small and large disturbances has been and remains one of the most focal problems since the moment of constructing the first electric power system. This problem is especially significant for the sizeable present-day EPSs due to the emergence of new factors, such as irregular generation by renewable power sources, demand response of consumers, use of efficiently controlled devices based on power electronics in the electric network and among consumers. All these factors considerably deteriorate the properties of EPSs in terms of their stability. Furthermore, due to heterogeneity of the network structure, there remain bottlenecks (cutsets or groups of ties of the same directions with limited transfer capability), especially in the weakened network structure in post-emergency and maintenance conditions of large EPSs. The heterogeneity of the network structure stems from the presence in it of subsystems with strong internal ties and weak ties and buses between these subsystems (see Section 3). More extensive use of distributed generation units in EPSs, including the units based on renewable energy with low rotor inertia and the units connected to the network via back-to-back blocks, aggravates the situation as they reduce the system's inertia and raise the risk of its stability loss. Violation of the stability of a complex system can lead to the cascade development of an emergency process with a massive disruption of the power supply to consumers and severe consequences for the system [1, 2] et al.

Fundamental studies on the analysis and ensuring the stability of EPS, modeling the system and its elements in dynamic conditions were started as early as in the 1930–40s by A.A. Gorev, R. Park, P.S. Zhdanov, E.V. Kimbark, A.S. Lebedev, and others [3–5] et al. The results obtained for EPS took into account fundamental methods of mathematical stability theory for dynamic systems presented in the works of A.A. Lyapunov, J. Sylvester, and others [6, 7] et al. One of the most important directions of studies in this field is the problem of EPS stability "in the small," historically called in the electric power industry "steady-state stability."

There are several types of EPS steady-state stability. As applied to approaches used for complex EPS considered in this paper, the angle stability (i.e., relative rotor angles of synchronous machines) and voltage stability in the buses of the electric network are of great interest [3–5] et al. When studying the steady-state stability of EPS at small disturbances (stability "in small"), a classical approach of the mathematical theory of dynamic system stability is used. It involves the linearization of a nonlinear system in the proximity of its equilibrium, which allows the use of a broad range of rigorous methods for assessing the stability of linear dynamic systems. The paper discusses attempts to extend these methods to the analysis of nonlinear effects.

Studies of weak EPS disturbances were initially focused on the analysis of aperiodic steady-state stability using the criterion of sign reversal of the Jacobi matrix determinant, which is equal to the free term of the characteristic polynomial. It was believed that oscillating electromechanical processes should be damped by Automatic Voltage Regulator (AVR) or Power System Stabilizers (PSSs). However, practice demonstrated many cases of weakly damped oscillations and self-oscillation due to the occurrence of specific off-design circuit-mode situations for which the adjustment of the control coefficients of the AVR and PSS was not designed. This fact stimulated the development of more accurate methods for the analysis of EPS steady-state stability based on A.A. Lyapunov theory of stability "in the small," which was, in turn, stipulated by the efficient methods designed to determine the roots of a high-order characteristic equation (eigenvalues of the matrix of a linear dynamic system) [8] et al., which form the basis for the spectral analysis. Modal analysis, as applied to EPS, was first studied in [9, 10].

In a broad sense, both the spectral and modal analyses imply the study of properties of dynamic systems in terms of frequencies and related values such as energies, eigenvalues, and eigenvectors. The term "spectral" has a more general mathematical sense, but as applied to the methods for assessment of steady-state stability of EPS the terms "modal" and "spectral" are rather close and are related to the analysis of the location of eigenvalues of the matrix of a linearized system on the complex plane. For the convenience of presentation, spectral methods in this review are assumed to include earlier methods referred to as measuring methods that analyze the position of a spectrum of the matrix of a linearized dynamic system on the complex plane. Modal methods are assumed to include methods related to the identification of specific oscillation modes using the so-called "participation factors," which allow these oscillation modes to be associated with specific system state variables. Methods of spectral and modal analysis are rapidly developing and improving. According to the estimates, up to a hundred of articles devoted to developing these methods are published annually worldwide, and most of them are related to EPS stability problems.

This paper provides an overview of the most typical results of the application of spectral and modal analysis methods for solving various problems of analyzing the structural properties of EPS, modeling the dynamics of these systems, studying their stability, and controlling them. These are mainly recent results, but earlier results are also reported as appropriate. Section 2 briefly discusses the theoretical grounds of spectral and modal methods. Application of those methods covers practically all the stages of steady-state stability studies of complex EPS, namely, investigation of the studied facility, its properties, primarily the property of the system's structure heterogeneity (Section 3); identification of coherent motion of generators in the electromechanical transients in EPS, and simplification (equivalenting) of mathematical models of coherent clusters of generators (Section 4); assessment of steady-state stability of EPS and selection of control actions to ensure it (Section 5); analysis of sub-Gramians when studying the EPS stability, and other new directions in developing the modal approach (Section 6). Section 7 (Conclusion) summarizes the results of the review.

Potential readers of the review are mainly electric power engineers who are well aware of the structure and properties of large EPS and problems of their stability. The review can also be of interest to control theory specialists having some knowledge and understanding of complex EPS stability problems. Such a dual focus of the review determined its structure and content. Authors consider it as a particular compromise in presenting the material for those two groups of specialists.

The review focuses on the large complex EPSs characterized by heterogeneity of the power network structure that manifests itself in the presence of subsystems with strong couplings inside them and weak couplings and buses between subsystems (see Section 3). In these EPSs, such specific phenomena as inter-area oscillations and the coherency of disturbed motion of generator groups arise, which are significant for ensuring their stability.

## 2. THEORETICAL BACKGROUND

A general mathematical model of EPS for studying electromechanical transient processes can be represented by a joint system of nonlinear differential and algebraic equations [11],

$$\begin{aligned} \dot{y} &= f(y, z, v), \\ 0 &= g(y, z, v), \end{aligned} \tag{1}$$

where $y$ is a vector of the system's state variables of dimensionality $n$; $z$ is a vector of dependable variables of dimensionality $m$; $v$ is a control vector of dimensionality $l$; $f(\cdot)$ and $g(\cdot)$ are nonlinear vector-functions whose form is determined by models of synchronous machines, an electric network and loads.

Linearization of the system of Eqs. (1) in the neighborhood of a steady-state equilibrium point $(y_0, z_0)$ results in linear algebraic differential equations in the augmentations $\Delta y = y - y_0$, $\Delta z = z - z_0$, $\Delta v = v - v_0$:

$$\begin{aligned} \Delta y &= (F_y | F_z) \begin{pmatrix} \Delta y \\ \Delta z \end{pmatrix} + F_v \Delta v, \\ 0 &= (G_y | G_z) \begin{pmatrix} \Delta y \\ \Delta z \end{pmatrix} + G_v \Delta v. \end{aligned} \tag{2}$$

The system of Eqs. (2), provided that the Jacobi matrix is invertible, transforms to the Cauchy form:

$$\dot{x} = Ax + Bu, \tag{3}$$

where $x = \Delta y$ is a state vector of dimensionality $n$; $u = \Delta v$ is a control vector of dimensionality $l$.

The following so-called *classical model* of the system's dynamics is often considered when solving different problems of EPS stability

$$T_i \ddot{\delta}_i(t) + d_i \dot{\delta}(t) = a_i - \sum_{j=0}^{N} b_{ij} \sin\left(\delta_i(t) - \delta_j(t)\right), \quad i = \overline{1, N}, \tag{4}$$

where $\delta_i$ is a rotor angle of generator $i$ relative to a synchronous axis; $T_i$, $d_i$ are the inertia constant and damping factor of the $i$th rotor; $a_i$ is a mechanical power generated by a turbine of the unit; $b_{ij} = E_i E_j s_{ij}$, where $E_i$, $E_j$ are EMFs determined by the flux linkages of magnetic fields generated by excitation windings of synchronous machines $i$ and $j$, respectively, and $s_{ij}$ is the admittance of branches between generators $i$ and $j$, respectively.

The classical model of EPS dynamics is usually used for solving the auxiliary problems of spectral and modal analysis (see, for example, Section 4 of the review relative to identification of coherency of generators' motion, and simplifications of EPS models). Such auxiliary problems are solved for systems distant from the studied subsystem of a large extended EPS, where a classical model of the system's dynamics reflects the behavior of generators with acceptable accuracy. Identification of the studied subsystem is a separate problem that depends on the nature of the problem solved. As the investigated subsystem, for example, a geographically separated area of the EPS can be considered. Generators of the studied subsystem are represented by a sufficiently detailed model of their dynamics considering the impact of the main affecting factors, primarily of excitation and speed controllers. The choice of the representative model of generators of the studied subsystem is an independent task. Therefore, in papers devoted to studies on EPS stability, the problems of justification of models of generators' dynamics are usually not considered in detail, often being limited to their general representation of the form (1).

Linearization of the system of differential Eqs. (4) in the neighborhood of the system's equilibrium point produces a linear model of EPS dynamics in the space of states that in the general form is described as

$$\dot{x}(t) = Ax(t), \tag{5}$$

where $x$ is a vector of EPS state of dimensionality $N$; $N$ is the number of synchronous machines in the system; $A$ is a real matrix of fixed factors of dimensionality $N \times N$.

When solving some problems related to the EPS stability (see, for example, Section 3) using methods of spectral analysis, consideration is given to the eigenvalues and eigenvectors of the Jacobi matrix of the equations of the steady-state (equilibrium state), which in the general form are represented as

$$\begin{aligned} 0 &= f(y, z, v), \\ 0 &= g(y, z, v). \end{aligned} \tag{6}$$

One of the most popular methods for studying the stability of EPS models of the form (3) and (5) under small disturbances is a modal analysis based on calculating the spectrum of the dynamic matrix $A$, i.e., the set of its eigenvalues $\lambda_i$

$$\Lambda(A) = \left\{ \lambda_i : \det(\lambda_i I_N - A) = 0,\ i = \overline{1, N} \right\}, \tag{7}$$

where $I_N$ denotes an identity matrix of dimensionality $N \times N$. Eigenvalues in (7) determine the frequency of oscillations and damping factors of modes characterizing linear system behavior (hence the term "modal analysis"). Particularly, if all the eigenvalues $\lambda_i$, $i = \overline{1, N}$ have negative real parts, i.e.,

$$\operatorname{Re} \lambda_i < 0, \quad i = \overline{1, N}, \tag{8}$$

then EPS models (3) or (5) are statically (asymptotically) stable. The right and left eigenvectors $v_i$ and $w_i$ of matrix $A$ that correspond to the eigenvalue $\lambda_i$ are determined by expressions

$$Av_i = \lambda_i v_i, \quad w_i^{\mathrm{T}} A = \lambda_i w_i^{\mathrm{T}}, \quad v_i \neq 0, \quad w_i \neq 0. \tag{9}$$

These vectors allow one to associate the eigenmodes of the system with the corresponding state variables.

At the initial stage of development, spectral analysis was considered mainly as a set of measuring methods and computational algorithms for quickly finding specific groups of eigenvalues and eigenvectors of the system that are interesting from the standpoint of specific applications. In the electric power industry, research has focused primarily on slow and poorly damped oscillations that can cause a loss of steady-state stability. These oscillations may occur between one or several machines and the remaining system ("local oscillations" at a frequency of 1 to 2 Hz) or between large clusters of generators ("inter-area oscillations" at a frequency of 0.1 to 0.5 Hz) [12, 13].

Classical *methods for measuring* the spectral characteristics are based on direct Fourier transformation or the assessment of a correlation function (Shuster periodograms; modified periodograms; Bartlett, Welch, Blackman–Tukey methods.) Amplitude, energy, the number of oscillations, and other signal parameters can be used for measurements. More complicated methods of dynamic measurements for assessing spectral characteristics of the system include Prony analysis, Yule–Walker and Berg's methods, a moving-average model, wavelet analysis, neural networks, and genetic algorithms [14, 15] et al.

*Computational methods* of modal analysis assume that the dynamic matrix of the system is already known, and it is necessary to offer efficient methods for computation of eigenvalues and eigenvectors in the specified part of the spectrum. The matrix itself, as a rule, has a large dimensionality, it can be degenerate or ill-conditioned, and can have a sparse structure. Among the well-known and well-proven methods for isolating critical modes, we can mention the QR method, method of simultaneous iterations, Lanczos method, modified Arnoldi method, and their various modifications [16–20] et al. More recent approaches include the method for calculating spectrum of dominant poles [21] and the method of matrix signum-functions [22].

Eigenvalues have a simple conceptual interpretation, but analysis of eigenvectors raises certain problems. Eigenvectors do not allow a unique interpretation of the relationship between corresponding modes and state variables, since these vectors depend on the choice of the measurement units of variables. In [12, 13], a new framework of *selective modal analysis* (SMA) for linear systems was proposed, which made it possible to establish an unambiguous relationship between modes and state variables using the so-called 'participation factors' (coefficients) that do not depend on the measurement units used. The SMA allowed an accurate identification of elements in the system's structure associated with specific eigenmodes in the dynamics of the system behavior. The participation factors (PFs) and generalized participations are defined as [23]:

$$p_{ki} = v_i^k w_i^k \quad \text{and} \quad p_{kil} = v_i^k w_i^l, \tag{10}$$

where $v_i^k$ and $w_i^l$ denote the $k$th and $l$th components of the $i$th right and left eigenvectors of matrix $A$ in (9). Eigenvectors are assumed to be normalized, i.e.,

$$w_i^{\mathrm{T}} v_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{else.} \end{cases} \tag{11}$$

For linear systems, PFs determine relative contributions of system eigenmodes to the dynamics of the state variables evolution, i.e.,

$$x_k(t) = \sum_{i=1}^{N} p_{ki} x_k(0) e^{\lambda_i t} + \sum_{i,l=1;\, l\neq k}^{N} p_{kil} x_l(0)\ e^{\lambda_i t}. \tag{12}$$

Since the end of the 20th century, the modal analysis has been developed in several directions. First of all, the concept of PFs has found new areas of application. Today, these indicators are widely used in the power industry and other areas for stability analysis [12, 13, 24], simplification of models [25], placement of measurement and control devices in the network [26], and clustering [27]. Some of these applications will be discussed in more detail in the subsequent sections of the review.

Second, the conceptual interpretation of PFs has been broadened. Their relation to the sensitivity of eigenvalues [23], modal controllability and observability [28], and modal mobility [29] has been established. According to (12), PFs determine the dynamics of the state variable $x_k(t)$ if and only if $x_l(0) = 0$, $l \neq k$, i.e., under a specially selected initial condition. In [30], it was shown that such an assumption might lead to counterintuitive results, and an alternative method of averaging over an uncertain set of initial conditions was proposed. According to this approach, the original definition of PF (10) was retained for the analysis of the participation of "modes in states." For the analysis of participation of "states in modes," an alternative definition of PF (or SIMPF) was proposed. Subsequently, similar concepts of SIMPF were considered for dynamic nonlinear systems [31] and systems described by algebraic equations such as power flow Eqs. (6) [24].

Finally, active efforts were undertaken to extend the modal analysis to the case of nonlinear models. Attempts to account for nonlinear effects and inter-modal interactions within the modal analysis developed mainly in two directions. Approaches, in which a model of the system is assumed to be known, consider the second-order and higher-order terms in the Taylor expansion approximating the dynamic system. These methods most often use Poincaré normal forms [31–33]. A study in [34] shows that accounting for higher-order terms may be significant when studying inter-area oscillations in stressed EPS following large disturbances. The main idea of the Poincaré method is to choose a nonlinear change of variables in the form of a polynomial so that in the transformed system, the terms of the second order (and, possibly, of higher orders) in the Taylor expansion disappear. The main disadvantage of this approach is that it requires solving a highly nonlinear numerical problem and using computationally expensive algorithms. Therefore, alternative approaches suggest evaluating PFs directly from measurements. For example, this can be done using *the extended dynamic mode decomposition* [35] or *Koopman mode decomposition* [36]. These methods are measurement dependent and require careful testing in practical applications.

Along with the modal analysis, another conceptual method for stability analysis is associated with the names of James Sylvester and Alexander Lyapunov. At the end of the 19th century they discovered and investigated the properties of matrix equations of Sylvester and Lyapunov [6, 7]. Today these equations play an essential role in many areas of modern control theory, including the study of the stability of linear and nonlinear systems, robust and optimal control. Lyapunov proved that the asymptotic steady-state stability (8) of linear systems (3) or (5) is equivalent to the condition that for any positively defined matrix $Q$ $(Q > 0)$ there exists a positively defined solution $P$ $(P > 0)$ of the algebraic matrix Lyapunov equation [7, 37] et al.

$$A^{\mathrm{T}} P + PA = -Q, \quad Q = Q^{\mathrm{T}} > 0. \tag{13}$$

In this case, the spectrum of stable matrix $A$ satisfies the following estimate [38]:

$$\min_{i=1,\ldots,N} |\mathrm{Re}\{\lambda_i\}| \leq -\frac{1}{2\|P\|}. \tag{14}$$

Thus, computing the solution $P$ to Eq. (13) can assess the degree of system stability without computing its spectrum. Furthermore, the matrix $P$ in (13) can be represented as an integral with matrix exponents of matrix $A$

$$P = \int_0^\infty e^{A^{\mathrm{T}} t} Q e^{At} dt. \tag{15}$$

Let us further assume that system (3) is observed by the output signal

$$s(t) = Cx(t). \tag{16}$$

The Gramians of controllability $P_C$ and observability $P_O$ are usually used for the analysis of system (3) when $Q = BB^{\mathrm{T}} > 0$ and $Q = C^{\mathrm{T}}C > 0$, respectively, are selected as a positive definite matrix in Eq. (13). In general, we can say that the observability Gramian characterizes the system stability in terms of limiting the energy of its output signal, and the controllability Gramian characterizes the stability of the system in terms of its asymptotic stability to random disturbances of the input signal. For a stable linear dynamic system, Gramians are closely related to the squared $H_2$-norm of its transfer function or its impulse response. The physical interpretation of these quantities is that they determine the signal energy gain in the system, averaged over time or frequency.

In the middle of the 20th century, a serious breakthrough was achieved in developing efficient methods for solving the matrix equations and, in particular, Lyapunov and Sylvester equations. Orthogonalization methods have been developed, among which the methods of Bartels-Stewart and Golub–Nash should be mentioned [39]. The first work on calculating the solutions of the Lyapunov and Sylvester equations in the form of integrals of their matrix resolvent in the complex plane was published in the USSR by M.G. Krein [40]. D.K. Faddeev developed a spectral expansion of the resolvent of the dynamics matrix into the Faddeev series [41]. In [38, 42], the solvability of the Sylvester–Lyapunov–Krein matrix equations, dichotomy and bundle of their spectra, as well as computational problems of solving these equations were investigated. In [43], the structural properties of Gramians are discussed. In [44], methods for solving continuous and discrete matrix equations of Lyapunov and Sylvester based on the transformation of the dynamics matrix to the Jordan normal form were proposed. Computational methods for solving the matrix equations of more complex types and larger dimensionality have been intensively developed over the last 30 years (see References in the recent review [45]).

The conceptual interpretation of the Lyapunov method also developed. In particular, the interpretation of Gramians for Eq. (13), based on the "energy" concept, is generally preserved for the time-varying linear systems with the replacement of the matrix exponent $e^{At}$ in (15) by the fundamental solution $\Phi(0, t)$ to the homogeneous equation $\dot{x} = A(t)x$ [46, 47]. The concept of Gramians was later generalized and interpreted for generalized Lyapunov equations describing the properties of deterministic bilinear and stochastic linear systems, and was named energy functional [48, 49].

Spectral properties of Gramians and energy functionals were effectively used in model order reduction (MOR) methods. Among them, we note the method of balanced truncation [50], the method of using cross-Gramians [51], and their various modifications (see the survey [52]). In the monograph [53], devoted to the approximation of large dynamic systems, singular expansions of infinite controllability and observability Gramians were obtained based on the transformation of the dynamics matrix to a diagonal form. A more general form of spectral decompositions of Lyapunov functions into components corresponding to individual eigenvalues and their paired combinations was offered in [54–56]. Each term in these spectral decompositions was called *a sub-Gramian*. Sub-Gramians allow one to evaluate the interactions between the eigenmodes of the system. They also provide a framework for combining ideas of modal analysis with Lyapunov stability analysis. A more detailed description of this method and its application for stability analysis of power systems will be presented in Section 6 of the review.

## 3. STUDY OF HETEROGENEITIES IN THE ELECTRIC POWER SYSTEMS

The structure of an electric network in the complex EPS is always heterogeneous. Heterogeneity of structure is a fundamental property of complex EPS, as well as of other systems having a complex structure. It is essential to identify this heterogeneity, to assess it, and use it for EPS modeling, investigation, and controlling [57–59].

During its operation, the EPS is subjected to perturbations and responds to them by changing the system state variables. This response is determined by perturbation amplitude, its location, and properties of the system itself. Perturbations localized in different places of EPS cause a more noticeable response of state variables in the same buses and ties of the system. The elements, in which the state variables are most responsive to perturbations occurring in the network, are called *sensors*. The heterogeneities that cause the occurrence of sensors is determined by topology and parameters of the network structure.

The elements of the network where a change in the parameters lead to the greatest response of sensors to perturbations are called *weak points*. Weak points include weak ties and cutsets, whose resistance change leads to substantial changes in the values of the sensor's state variables. Weak points also include weak buses if the fixed voltage in them produces a similar effect. With the growth of load in the network, the state variables in the weak buses are the first to achieve critical values from the standpoint of EPS voltage stability disturbance. Angle stability violation and cascade propagation of emergencies occur in weak ties and cutsets [57, 58].

Let us consider (based on [60]) the methods for detecting sensors and weak points in large EPS using, in the general case, the linearized equations of power balance in the buses that relate changes in the magnitudes $\Delta U$ and phases $\Delta\delta$ of nodal voltages to changes in active $\Delta P$ and reactive $\Delta Q$ loads in the form of

$$\begin{pmatrix} \Delta\delta \\ \Delta U \end{pmatrix} = J^{-1} \begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix}, \tag{17}$$

where $J$ is a square Jacobi matrix

$$J = \begin{pmatrix} \partial P/\partial\delta & \partial P/\partial U \\ \partial Q/\partial\delta & \partial Q/\partial U \end{pmatrix}. \tag{18}$$

For detecting the sensors using the Jacobi matrix, we can use its singular decomposition [61]

$$J = V\Sigma W^{\mathrm{T}} = \sum_{i=1}^{n} v_i \sigma_i w_i^{\mathrm{T}}, \tag{19}$$

where $W = (w_1, \ldots, w_n)$ and $V = (v_1, \ldots, v_n)$ are orthogonal matrices of size $n \times n$ each, whose $i$th columns are $i$th left and $i$th right orthonormalized singular vectors; $\Sigma = \mathrm{diag}(\sigma_1, \cdots, \sigma_n)$ is a diagonal matrix of singular values. Given (19), Eq. (17) may be represented as

$$\begin{pmatrix} \Delta\delta \\ \Delta U \end{pmatrix} = \left( \sum_{i=1}^{n} v_i w_i^{\mathrm{T}} / \sigma_i \right) \begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix}. \tag{20}$$

If for ascending singular values $\sigma_1 \leq \sigma_2 \leq \cdots \leq \sigma_n$, the first of them is significantly less than the others, then, other conditions being equal, the first term of the sum in (20) makes the largest contribution to the changes in the voltage magnitudes and phases. Hence, (20) can be represented as

$$\begin{pmatrix} \Delta\delta \\ \Delta U \end{pmatrix} = \begin{pmatrix} \Delta\delta \\ \Delta U \end{pmatrix}^{(1)} + \sum_{i=2}^{n} \varepsilon_i = \left( v_1 w_1^{\mathrm{T}} / \sigma_1 \right) \begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} + \sum_{i=2}^{n} \varepsilon_i, \tag{21}$$

where $\sum_{i=2}^{n} \varepsilon_i$ is an error in determining the voltages due to the neglect of $n-1$ terms in (20).

The greater the difference between the first singular value from the others, the smaller the error due to the neglect of $n-1$ terms, and the more grounds we have to make conclusions about the behavior of state variables using only the first term associated with $\sigma_1$.

Under the above conditions, it is evident that the maximum changes in voltage magnitudes and phases with changes in the loads will occur at the buses corresponding to maximum components of the first right singular vector,

$$\begin{pmatrix} \Delta\delta \\ \Delta U \end{pmatrix}^{(1)} = v_1 \Delta S^{(1)} = \left(v_1 w_1^{\mathrm{T}}/\sigma_1\right) \begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix}. \tag{22}$$

The introduction of the scalar value $\Delta S^{(1)}$, called in [57] *the first generalized disturbance*, allows us to assert that maximum changes in voltage magnitudes and phases will occur at the sensor nodes.

The assessment of electric network heterogeneity is associated with the detection of *weak points*, which mainly determine the sensitivity of EPS. Weak nodes and ties, when considering the factors invariant to the operating conditions, can be found by studying the following derivatives:

$$\frac{\partial \sigma_1}{\partial \gamma_{si}} = \pm v_{i1}^2, \tag{23}$$

$$\frac{\partial \sigma_1}{\partial \gamma_{ij}} = \left(v_{i1} + v_{j1}\right)^2, \tag{24}$$

where $\gamma_{ij}$ and $\gamma_{si}$ are admittances of the line $ij$ and shunt at the node $i$. A double index $ij$ here and further will denote values associated with the tie between buses $i$ and $j$ (power and voltage losses, etc.).

Changes in the active and reactive power flow in line $ij$ $\Delta P_{ij} = \left(\partial P_{ij}/\partial \delta_{ij}\right)\Delta\delta_{ij}$ and $\Delta Q_{ij} = \left(\partial Q_{ij}/\partial U_{ij}\right)\Delta U_{ij}$ depend largely on the values $\Delta\delta_{ij}$ and $\Delta U_{ij}$, which can be found from the difference between the components of the first right singular vector $(v_{\delta i1} - v_{\delta j1})$ and $(v_{Ui1} - v_{Uj1})$ corresponding to phases and magnitudes of nodal voltages in buses $i$ and $j$, and can also be used as indicators of the weak tie. The greater the change in the $\Delta\delta_{ij}$ and $\Delta U_{ij}$ values with an increase in power flow through the tie, the sooner the limit of the transferred power is reached in it, and the sooner the degeneration of the Jacobi matrix occurs.

Tie $ij$ can be called 'weak' if its admittance change causes a maximum change in the minimum singular value $\sigma_1$:

$$\frac{\partial \sigma_1}{\partial \gamma_{ij}} = w_1^{\mathrm{T}} \frac{\partial J}{\partial \gamma_{ij}} v_1 = \left(w_{1\delta}, w_{1U}\right) \begin{pmatrix} \dfrac{\partial^2 P}{\partial \delta\, \partial \gamma_{ij}} & \dfrac{\partial^2 P}{\partial U\, \partial \gamma_{ij}} \\ \dfrac{\partial^2 Q}{\partial \delta\, \partial \gamma_{ij}} & \dfrac{\partial^2 Q}{\partial U\, \partial \gamma_{ij}} \end{pmatrix} \begin{pmatrix} v_{1\delta} \\ v_{1U} \end{pmatrix}. \tag{25}$$

From Eq. (25), it follows that the minimum singular value is influenced not only by the admittance of ties, but also by the current state variables. The weakening of the ties and the corresponding deterioration of the Jacobi matrix condition associated with the changes in the EPS operation (e.g., heavy load that can cause violation of angular and voltage steady-state stability), can be prevented by appropriate control actions using FACTS (Flexible Alternating Current Transmission System), power storage systems and other tools.

Thus, the above approach makes it possible to reasonably detect weak ties in the EPS electric network and thereby identify strongly related subsystems interconnected into a joint EPS structure. As noted above, violations of the steady-state EPS stability and cascading emergencies in the case of changes in operating conditions will occur primarily in the weak ties and less likely in strong ties within strongly connected subsystems. Therefore, EPS steady-state stability should be studied primarily in relation to weak ties.

Structural heterogeneity of EPS determines the specific motion of the system generators in the transient electromechanical process, namely, their *coherent motion* (coherency condition of generators' motion is represented below by Eq. (26)). Coherency of generators' motion is an objective factor for simplifying the mathematical model of EPS dynamics by aggregating the generators of strongly connected subsystems. Replacing a group of generators with one equivalent generator introduces an error in the mathematical model of EPS dynamics due to the neglect of inter-machine oscillations. The more coherent the motion of aggregated generators, the smaller the error. Therefore, the development of methods for detecting the coherency of the EPS generator's motion is required.

To detect the coherency of EPS generators motion in the strongly connected subsystems, aggregate the coherent groups of generators, assess the EPS steady-state stability with respect to weak ties and to identify control actions to ensure stability, one can use the methods of modal analysis, which will be discussed further in Sections 4 and 5.

## 4. DETECTION OF GENERATOR'S MOTION COHERENCY AND SIMPLIFICATION OF EPS MODELS

When analyzing the stability of complex multi-machine EPSs and justifying measures to ensure their stability, the investigator faces two contradictory problems: on the one hand, there is an obvious need for detailed modeling of the elements (primarily synchronous generators) and system's structure to take into account in the model all the factors influencing system's stability; on the other hand, the resulting mathematical model of a large EPS in the form of a joint system of nonlinear differential and algebraic equations of the form (1) becomes immense and often overwhelming for studies, especially under conditions of time shortage for justifying control actions in the cycle of on-line EPS control, when a large amount of calculations is needed to assess the steady-state stability of a complex EPS under various conditions and select control actions to ensure system stability. As a result, there is a need to validly simplify the mathematical model of EPS dynamics [24, 59, 62, 63] et al. A critical step in simplifying the mathematical model is to identify the coherency of generators' motion in the dynamic process and to represent each of the coherent groups of generators by one equivalent generator.

The coherency of the motion of the generators $i$ and $j$ (until recently, the equivalent term "phasing-in" was used in Russian literature) is defined as

$$\delta_i(t) - \delta_j(t) = \text{const}, \tag{26}$$

where notations correspond to those in (4).

Initially, the problem of identifying the coherency of generators' motion was solved using approximate, often empirical features and criteria. The introduction of the concepts of *local and global coherency* of generators' motion [64] was a step forward. Local coherency is conditioned both by disturbances and structural properties of the subsystem, whose generators are tested for motion coherency; global coherency is conditioned only by structural properties, i.e., it is invariant to disturbances. Structural properties of the subsystem reflect the heterogeneity of the EPS structure, i.e., strong ties in the subsystem conditioning the coherency of its generators motion, and weak ties of this subsystem with the rest of the system (see Section 3) [58, 59, 65].

The development of this approach is associated with the following transformation of coordinates [3, 66]:

$$\{\delta_i,\ \delta_j\} \longrightarrow \{\delta_{ic},\ \delta_{jc}, \delta_c\}, \tag{27}$$

where $\delta_{ic}$ is the rotor angle of the generator $i$ with respect to the center of the subsystem inertia $\delta_c$. It has been shown that the centers of inertia of coherent subsystems move slowly, and the coordinates $\delta_{ic}$ move fast [62, 67, 68]. On this basis, the concept of *slow coherency* was introduced whose estimate is invariant to disturbances [69, 70].

Let us consider a method for revealing the coherency of motion from [71] based on selective modal analysis of intersystem oscillations. In this case, intersystem oscillations are determined by the participation factors (10). We denote their matrix following [72] as

$$PF = [pf_1, pf_2, \dots, pf_n], \tag{28}$$

where

$$pf_i = \begin{bmatrix} v_i^1 w_i^1 \\ v_i^2 w_i^2 \\ \vdots \\ v_i^n w_i^n \end{bmatrix}, \forall\ i \in [1, \dots, n]. \tag{29}$$

The absolute values of participation factors related to the rotor angles of synchronous generators in the classical EPS model (4) ranked according to their values are denoted in (30) as $PF_\delta^{\text{sort}}$. The final minimum number $j$ of generators satisfying the requirement stated in (30) is determined for each mode, where $g$ is the total number of generators:

$$\frac{\sum\limits_{i=1}^{j} PF_\delta^{\text{sort}}(i)}{\sum\limits_{i=1}^{g} PF_\delta^{\text{sort}}(i)} > c, \quad \forall\ j \in [1, g]. \tag{30}$$

The value $c$ in [71] is estimated by experts and equals 0.9. Modes with the higher value of index $j$ are candidates for intersystem oscillations.

After identifying intersystem modes, the groups (clusters) of generators are formed according to the following criteria:

—coherency of generators' motion in the cluster;

—electrical proximity of generators in the cluster.

An algorithm for coherency identification works at a given number of clusters. Clusters are formed following the criterion of minimizing the sum of squared distances of generators from the cluster centroid [73].

As a result, two clusters are formed for each intersystem mode. Two clusters are needed to take into account mutual oscillations of modes. According to conventional assumption in the cluster analysis, the number of clusters is specified, and the clustering algorithm selects the objects and assigns them to a specific cluster. If there are problems with the clusterization of eigenvectors into two independent clusters (i.e., the grouping is not realistic), smaller clusters are formed by combining them by two clusters for each intersystem mode.

It may be that the coherent generators in the cluster are located in different geographic areas. In order to avoid such situations, the electrical proximity of the generators in the cluster is analyzed by the ratio of the self and mutual admittances of the generator nodes in the classical EPS model. If the generators in the cluster are located in different geographic areas, the considered cluster is split into the appropriate number of clusters.

A dynamic equivalent of a coherent group of generators (an equivalent generator) is determined by various methods, for example, by dynamic REI method [74].

Consider some other typical approaches to identification of generator's motion coherency and development of simplified models of EPS subsystems. First of all, let us consider a review of possible methods for constructing the dynamic EPS equivalents [75] prepared by the IEEE PES Working Group, and a review [76].

In [75], selective modal analysis of the linearized part of EPS selected for simplification is applied to the formation of low-frequency equivalents, which also allows for identifying the coherency of generators' motion. Specific features of the approach are investigated while maintaining the system's structure. An equivalent model of a reduced network should adequately reflect its response; an appropriate optimization procedure is used for its tuning.

In [76], the EPS is initially divided into the investigated and external subsystems, and an equivalent model is considered for the external subsystem. For this subsystem, the problem of identifying the coherence of the motion of generators is solved. Among the methods proposed for identifying coherency, selective modal analysis is considered in a form close to that described above. For constructing the external equivalent, various approaches are used, one of them is based on the use of the participation factors considered in the selective modal analysis as follows [77, 78].

A generator with the highest participation factor is selected in the coherent group, and it is assumed that this generator most reflects the dynamic characteristics of the coherent group. The remaining components of the participation matrix are neglected. The equivalent constant of the rotor inertia of the equivalent generator is determined by the formula

$$T_{eq} = \sum_{j \epsilon G_k} h_j T_j, \tag{31}$$

$$h_j = P_{kj} / \sum_{j \epsilon G_k} P_{kj}, \tag{32}$$

where $G_k$ is the set of generators in the cluster $k$; $P_{kj}$ is the power of generator $j$ in group $k$.

Reference [79] presents a method for synchronous modal equivalenting for dynamic equivalents that preserve the system's structure. In the system, the studied subsystem and one or several external subsystems are selected for which the coherent groups of generators are identified based on the slow coherency assessment using selective modal analysis. For each external subsystem, one generator with the highest participation factor is described by the original model. The dynamics of the remaining generators of external subsystems is represented by a linearized model in the state space.

Reference [80] proposes an assessment of the slow coherency of EPS generators' motion based on the intersystem modal characteristics and hierarchical ascending classification. Intersystem modal characteristics are obtained using Taylor–Fourier transform that integrates Taylor and Fourier subspaces into a generic space. As a result, Taylor–Fourier transform forms a specific filter that, by spectral decomposition of oscillating modes, selects modes least sensitive to the presence of noise. A hierarchical ascending classification uses the Elbow method for the identification of coherent groups of generators.

The paper [81] proposes a method for the identification of coherent groups of generators that is based on singular value decomposition (SVD) of the EPS matrix. In this method, the vector of the generator's rotor angles of high dimensionality is projected onto a low-dimensional subspace. As a result, subspace coefficients are obtained as key indicators of coherency. Then the coherency is identified by clustering the obtained coefficients using the $k$-means method. Reducing the dimension of the problem for the clustering algorithm by the SVD method accelerates the coherency identification. Since the proposed method uses real-time data on the generators' rotor angles, it has good prospects to implement real-time coherency identification.

Reference [82] considers the problem of forming a dynamic equivalent model for a section of a distribution network using the Prony analysis and nonlinear optimization by the least square method. The Prony analysis operates with a linear sum of exponential functions of the eigenvalues of the system. It is used for initial estimates of the model's parameters represented by the dependence of $Y$ on $X$, which is further optimized by the nonlinear least square method. As a result of optimization, amplitude, phase, frequency, and damping coefficient are determined as parameters of the identified model represented by the transfer function.

As can be seen from the results presented in this section, various modifications of the selective modal analysis method, along with other methods, are widely used both for identifying coherent groups of generators and for determining the parameters of simplified models external to the studied subsystem.

## 5. STUDIES OF EPS STABILITY AND CONTROL

From condition (7), it follows that assessment of steady-state stability of EPS consists in checking this condition for the considered state of the system. When choosing controls to provide EPS stability, the task is to ensure the fulfillment of condition (7) by control actions. At the same time, the selected control actions should ensure good damping of oscillations caused by disturbances. Selective modal analysis methods proved to be an efficient tool for accomplishing these two tasks. Nevertheless, the mathematical techniques of modal analysis for solving the EPS stability problems are still under development. In this section, which is far from being complete, we present some results obtained in this field.

Consider Refs. [83–89], characterizing the revival of interest in modal analysis for studying the EPS stability among Russian electric power specialists.

Reference [83] addresses the quadratic problem of eigenvalues as applied to various EPS problems. Relations (7) and (9) represent the so-called standard eigenvalue problem. In contrast, the quadratic eigenvalue problem (QEP) studies the properties of $n \times n$ matrix polynomials of the second degree (quadratic matrix pencils),

$$Q(\lambda) = \lambda^2 M + \lambda C + K. \tag{33}$$

QEP consists in determining the complex scalars $\lambda$ and non-zero complex vectors $v$ and $w$ satisfying the algebraic equations

$$Q(\lambda)\, v = 0, \quad w^{\mathrm{T}} Q(\lambda) = 0. \tag{34}$$

The use of QEP provides certain advantages for stability analysis of linear dynamic systems compared to solving a standard problem of eigenvalues. Reference [83] gives some examples of using QEP to solve the following electric power problems: the study of complex EPS stability "in the small"; stability analysis of shaft trains of powerful turbine generators; analysis of seismic stability of arched dams of hydropower plants; extrapolation of EPS behavior; and EPS state estimation. The research focuses on the problems of linearization and numerical methods for solving QEP. The concepts of the EPS spectrum and the pseudo spectrum are discussed. The concept of the spectrum is defined in Section 2 when discussing expression (7). The theory of pseudo spectra of numerical matrices places emphasis on changes in the location of eigenvalues of the original matrix under the influence of perturbations.

Reference [84] is devoted to the assessment of the influence of disturbances on EPS stability. The authors note that the elements of matrix $A$ in the models of real EPS of the form (5) are obtained from physical observations and measurements, or they are calculated based on the design parameters of the system's components, and therefore they contain errors. As a consequence, a real matrix of the linearized EPS model looks like $A + \Delta$, and the model itself can be written as

$$\dot{x}(t) = (A + \Delta)\, x(t). \tag{35}$$

Here $\Delta \epsilon R^{n\times n}$ is a matrix of unknown disturbances that, in addition to the factors listed above, also contains real disturbances.

The pseudospectra allow solving two problems: 1) Is it possible to determine how the eigenvalues of the matrix $A$ change, and whether the EPS described by the model (35) remains asymptotically stable if some characteristics of the perturbation matrix $\Delta$ are known (e.g., maximum values of magnitudes or matrix norm); 2) how large the perturbation $\Delta$ can be so that the EPS described by the model (35) remains asymptotically stable.

Reference [85] considers the problems of nonlinear modal interaction in EPS. It is shown that an effective tool for the analysis of nonlinear modal interaction is the method of Poincaré–Dulac normal forms that allows through a non-degenerate nonlinear transformation to linearize the original nonlinear EPS model in the absence of strong resonance of oscillation modes. The authors conclude that the EPS models obtained using normal forms can be used for the system's stability analysis and synthesis of automatic controllers (e.g., system stabilizers) by modal methods.

Reference [86] is devoted to studying the advantages of the modal method for EPS stability analysis. The authors note that the possibilities of the modal approach are extensive and that it is constantly actively developing. It enables efficient identification of oscillations in EPS that are critical from the standpoint of possible loss of stability. The modal approach allows using a variety of methods for synthesizing control laws.

Reference [87], in a sense, generalizes the results obtained in [83–86]. It is proposed to use the norm of the matrix exponent as a generalized transient function of the disturbed EPS motion. Necessary and sufficient conditions for the existence of static stability margins are determined based on the concepts of stability radius and pseudospectrum of the Jacobi matrix. The paper demonstrates the capabilities and advantages of the combined modal and linear-quadratic approaches for the synthesis of centralized and decentralized control, as well as the prospects for the analysis of nonlinear oscillations and ensuring the dynamic EPS stability.

The paper [88] considers the problem of choosing a coordinated emergency control of FACTS devices and traditional discrete units of an emergency shutdown for generators and loads to ensure the dynamic stability of EPS. In this case, the optimization of the settings of the proportional-integral FACTS controller ensures acceptable damping of oscillations in the system, which is regulated by the position of eigenvalues of the matrix of the linearized EPS model. The minimum emergency undersupply of electricity to consumers is considered as an optimization criterion.

We also note a joint Russian-Italian study of slow inter-system oscillations in a super-large energy interconnection represented by jointly operating interconnected EPSs (IEPSs) of continental Europe and interconnections of the former USSR [89]. A specific feature of the linearized model of this super interconnection was the representation in this model of modern elements, such as power transmission lines and DC links, FACTS devices, and system stabilizers. The studies have shown that weakly damped and low-frequency oscillation modes may occur within the IEPS of continental Europe and former USSR; they can also arise between zones of different IEPS in the scenario of joint operation of two AC (alternating current) IEPSs. The use of FACTS devices with the appropriate control allows damping these oscillations.

Research in [90] proposes a general nonlinear modal representation of large-scale EPS. The authors demonstrate the capabilities of the methodology of normal forms of a vector field for modeling nonlinear systems. To correctly represent the EPS behavior, its oscillations, and interaction of modes, the researchers use the so-called method of modal series in the form of Taylor series having polynomial nonlinearities, which characterize responses of a nonlinear system through the relations in a closed form. This method develops the concepts of the theory of linear systems for understanding and analyzing the nonlinear systems, as well as for designing systems for their control.

As a result of the volatility of the electricity market and growing energy demand, modern EPSs are forced to operate ever closer to the limits of their stability. This tendency makes a system more vulnerable and increases the risks of stability loss. A common practice of stability and emergency control when planning the operation and conducting contingency analysis includes identification and assessment of critical oscillations and corresponding network cutsets that are dangerous in terms of stability loss (weak cutsets, see Section 3). Critical oscillations are determined based on the analysis of eigenvalues, and eigenvectors and matrices of current and voltage sensitivity to corresponding changes in the state variables are used to find critical lines and buses (sensitivity analysis). A case study of a sensitivity analysis methodology for an emergency control system in case of unexpected inter-area oscillations is described in [91] along with the results of testing this system in a large real EPS in China. The analysis of the sensitivities to eigenmodes also allows identifying the location of centers and "corridors" of critical oscillations on the graph of the electric network, which are dangerous for the system stability [92].

The EPS stability is traditionally estimated using off-line numerical integration of its model for different scenarios of emergencies, operating conditions, and different network topologies. Such calculations, both on-line and off-line, can be notably accelerated with the help of selective modal analysis (SMA) that allows reducing the EPS model dimensionality [12, 13] (see Section 4) in the iteration process. Generators for the reduced model are selected at each iteration based on the value of the corresponding participation factors. For example, in [93], the SMA allows a reduction of the EPS model to such an extent that the method of linear matrix inequalities (LMI) can be applied to it, and a robust stabilizer can be constructed to suppress inter-area oscillations.

The more intensive use of transfer capabilities of transmission lines in recent decades aggravates the problem of voltage instability, which develops into voltage collapse. For voltage stability analysis, the participation factors (PFs) were defined for a system of power flow algebraic equations as indicators that measure the contribution of a critical mode of power flow Jacobian to the system state variable [94] (see (17)). Besides, for the power flow problem, both the "mode in the states" PFs and the "states in the modes" PFs were introduced [24]. Moreover, "states in the modes" PFs (or SIMPFs) depend on the state variables in the network buses and on commissioned active and reactive power on the lines. Therefore, SIMPFs directly indicate the active power to be added and reactive power to be compensated for increasing the system's voltage stability.

Stability "in the small" is traditionally ensured by optimal placement of power system stabilizers (PSSs) and tuning their control signals. In the last decades, the FACTS controllers have also been recognized as a useful tool for damping weakly attenuating low-frequency electromechanical oscillations, which usually occur in EPS with long transmission lines under heavy loads. It is well known, however, that the damping effect of FACTS controllers highly depends on their location and the tuning of their control systems. These tasks can be successfully solved by modal analysis methods. Apparently, for the first time, the method of participation factors was applied to determine the optimal placement of PSSs on the electric network graph in [95]. In [96], the participation factors for critical modes are used to determine the optimal placement of static var compensators (SVCs) for voltage stabilization. A similar problem for the optimal placement of SVCs and the choice of control signals in EPS is addressed in [97]. An extensive literature review on the application of modal analysis to determine the optimal location and control tuning of FACTS and PSS controllers is given in [26, Section 3.1].

A specific type of control in EPS is related to islanding the large interconnected electric networks into weakly related areas that are more efficient and easier to control (see also Section 3 of this paper). Such islanding is based on graph theory and logically leads to the spectral clustering method [98, 99]. Network islanding can, as a rule, ensure flexible, distributed and adaptive control of a power supply system in a large area within a concept of smart grids. This concept makes it possible to represent at the same time different clusterization levels and establish the relationship between the parameters of the resulting graph models. Organization and visual representation of a hierarchical structure of the network rely on *dendrograms*, i.e., a graphical method for representing the results of hierarchical clusterization, which shows the degree of proximity of individual energy facilities and clusters, and graphically demonstrates the sequence of their interconnection or islanding. Dendrograms store information on further islanding (or interconnections) of a network into smaller or larger segments. The spectral clustering method uses eigenvalues and eigenvectors of the Laplace matrix of the electric network, which describes its electric connectivity and power flows. The normalized coordinates of the dendrogram geographically relate the vertices and edges of the graph to the electric network structure. In [99], spectral clustering technologies were demonstrated on the examples of the test IEEE 118-bus model and the EPS model of Poland.

The problem of constructing a low-order EPS model for tuning the parameters of system stabilizers in large EPSs is considered in [100]. For solving this problem, the authors implement the optimization algorithms applied for a simplified EPS model and models with uncertain poles. The method of constructing a simplified model uses the solution of the high-dimensional Lyapunov matrix equation based on the extended Krylov subspace method, as well as the sparse Jacobi matrices of the linearized model of the system. Reference [101] discusses creating a global damping controller for damping inter-area oscillations in large continental EPSs. The proposed approach uses wide-area monitoring systems (WAMS) for measuring the power system states and the integration of the frequency control systems available in some areas of EPS into a global controller. A central controller is designed following the Linear Quadratic Gaussian (LQG) architecture of a multi-connected controller, which coordinates the frequency controllers of individual power regions. For their tuning, the authors use an optimization based on the principles of artificial intelligence, namely, Particle Swarm Optimization. It should be noted that other approaches to solving this problem are known in the literature, ranging from an adaptive robust PID controller to sophisticated $H_2/H_{\inf}$ controllers [102].

The work [103] analyzes the stability of torsional sub-synchronous oscillations of the generators in the leading electric network caused by the connection of wind farms to it. It is shown that the cause of weakly damped and even unstable torsional oscillations of generators' turbines is the resonant interaction between the modes of individual generators of the leading network and the modes of generators of wind farms. The paper proposes a criterion for assessing the risk of oscillation stability loss, which is based on calculating the residues of the transfer functions of the interconnected EPS.

In [55], a method of spectral decompositions of Gramians was applied for the analysis of EPS steady-state stability. A simple case study of a two-area EPS with four power plants shows that the norms of the spectral decomposition terms allow identification of both local electromechanical modes and instabilities caused by inter-area oscillations. The paper [104] is devoted to analyzing EPS stability based on spectral decomposition of the squared $H_2$-norm of the system transfer function. The behavior of individual terms in the decomposition allows identification and localization of stability violation threat at an early stage. In a numerical test experiment, it was shown that weakly damped low-frequency oscillations are the result of interaction between the local subsystem of Russky Island and the continental power system. Such oscillations can be especially dangerous for the development of a cascade accident and create beatings with a characteristic difference frequency.

## 6. A METHOD OF THE SPECTRAL DECOMPOSITION OF LYAPUNOV FUNCTIONS IN THE EPS STUDIES

This section discusses the method of spectral decompositions of Lyapunov functions that was proposed in [54, 55], and its application to studying the dynamic behavior of EPS. The interest in this method is due to its good prospects to become the basis for a natural combination of two principal methodologies in the study of EPS stability, namely, modal analysis and Lyapunov methods.

Consider the dynamical system (3) or (5) and the algebraic Lyapunov equation (13) used for the analysis of its steady-state condition. To facilitate further presentation, we will assume that the system matrix $A$ has a simple spectrum $\sigma(A)$. Let us define matrix residues $R_i$ as coefficients in the expansion of the resolvent of matrix $A$:

$$(Is - A)^{-1} = \frac{R_1}{s-\lambda_1} + \frac{R_2}{s-\lambda_2} + \cdots + \frac{R_n}{s-\lambda_n}. \tag{36}$$

If $\lambda_i^* + \lambda_j \neq 0$ for all $\lambda_i, \lambda_j \in \sigma(A)$ then for any matrix $Q$ there is a unique solution to Lyapunov equation (13) that can be represented in the form of [105]:

$$P = \sum_{i=1}^{n} \tilde{P}_i = \sum_{i,j=1}^{n} P_{ij}, \quad \tilde{P}_i = \sum_{j=1}^{n} P_{ij}, \tag{37}$$

$$\tilde{P}_i = -\Big\{ R_i^* Q (\lambda_i^*\ I + A)^{-1} \Big\}_{Herm}, \quad P_{ij} = \left\{ \frac{-1}{\lambda_i^* + \lambda_j} R_i^* Q R_j \right\}_{Herm}, \tag{38}$$

where $\{\cdots\}_{Herm}$ is a Hermitian part of the matrix, $R_i$ and $R_j$ are the matrix residues defined in (36) and corresponding to the eigenvalues $\lambda = \lambda_i$ and $\lambda = \lambda_j$, respectively. Each term $\tilde{P}_i$ or $P_{ij}$ in decompositions (37), (38) is called *a sub-Gramian*. It characterizes the contribution of corresponding eigenmodes or their pairs to the system's energy variation determined by the corresponding Gramian at the infinite time interval. In particular, the norms of sub-Gramians increase if the frequencies of the corresponding oscillating modes are getting close. Thus, the proposed decompositions offer an opportunity for a quantitative assessment of resonance modal interactions occurring in the system.

Spectral decompositions of the form (37) for Eq. (13) were generalized in [56] to a wider class of solutions of matrix equations of M.G. Krein, which includes continuous and discrete Lyapunov and Sylvester equations as special cases. In [105], these decompositions were also extended to generalized Lyapunov equations of the form:

$$A^{\mathrm{T}} P + PA + \sum_{\gamma=1}^{m} N_\gamma^{\mathrm{T}} P N_\gamma = -Q, \quad \text{where} \ \ Q = Q^{\mathrm{T}} > 0. \tag{39}$$

These equations characterize the controllability and observability of the state vector of a deterministic bilinear system [48], and matrices $N_\gamma \in \mathbb{R}^{N \times N}$ take into account the bilinear components. The same Eqs. (39) arise in the stability analysis and stabilization of stochastic linear systems [49, 106].

In [104, 107], the method of sub-Gramians was applied to study the stability of linear continuous dynamic systems operating near the stability boundary. A practical method for analyzing the small-signal stability of EPS is the identification of dominant weakly stable modes and construction of asymptotic models of sub-Gramians for this group of modes. Asymptotic expressions for sub-Gramians have been obtained in the presence of one, two, or three dominant weakly stable modes. The developed method was applied to analyze the steady-state stability of a model of a real EPS at Russky Island. The test experiment confirmed the possibility of using sub-Gramians to identify the resonant interaction between weakly stable natural oscillations in a system operating near its stability boundary.

In [108], the authors obtained spectral decompositions for the solutions of Lyapunov and Sylvester differential equations, taking into account non-zero initial conditions. These solutions are called *finite Gramians*. In contrast to the spectral decompositions of infinite Gramians in (37), these decompositions depend on time and allow analyzing the stability of non-stationary systems, as well as the development of instability in a system that has lost stability. A numerical experiment with a model of the EPS of Russky Island showed that the proposed decompositions could predict the time interval within which slow transient processes will lead the system to a loss of stability. Thus, the spectral analysis of finite Gramians makes it possible to forecast the risk for a given time interval and assess the EPS stability in the case of slow transient processes.

A problem of developing the principles and algorithms of the intelligent immune system for monitoring the EPS steady-state stability based on the methods of associative search, multi-agent control, and spectral decompositions of Gramians was formulated and solved in [109]. The main idea of this approach is the formation of the current discrete dynamic model using the methods of associative search, technological archives, and intelligent data analysis. Then the risk assessment of the stability loss is formed using the spectral decompositions of Gramians for the considered model. A virtual analyzer of the stability loss risk is implemented using modern methods of identification and advanced technologies for data processing.

The paper [110] formulated and solved the problem of the spectral decomposition of the solution of Lyapunov matrix equations for discrete bilinear dynamic systems. This solution was obtained using an iterative procedure proposed in [111] that solves the linear Lyapunov matrix equation at each iteration. The Gramian of a bilinear system, which is a solution to the generalized Lyapunov equation, can be represented as the Volterra matrix series. The formulas of the spectral decomposition of the iterative process for computing the controllability and observability Gramians of discrete bilinear systems were derived. The obtained spectral decompositions allow one to implement the procedures of balanced truncation in the problem of simplification of a mathematical model of a bilinear system taking into account the spectral properties of the linear approximation of a dynamics matrix, as well as to calculate energy functionals using the proposed iterative procedures.

In [112], for the first time, sub-Gramians were associated with the system state variables and compared with the corresponding participation factors. In a numerical experiment, participation factors and sub-Gramians were applied for the selective modal analysis of the IEEE 57-bus test model. It was noted that the most influential modes in terms of participation factors are not always important in terms of their participation in amplifying the energy of perturbations in the system, and vice versa. This observation opens up the possibility of a new direction in the modal analysis, which would include Lyapunov stability principles based on the sub-Gramians method [113]. The first results of using the sub-Gramian method confirmed the good prospects of its application for the stability analysis of EPS.

## 7. CONCLUSION

The paper provides an overview of the general development of spectral and modal methods for assessing the stability of dynamic systems, as well as their application for monitoring and controlling the stability of large complex EPS. Practical problems of EPS monitoring are considered, such as identifying critical inter-area oscillations, structure heterogeneity, weak nodes and lines in the network, studying the coherency of generators' motion, and simplifying EPS models. The main problems of stability control in EPS were also analyzed. These are the synthesis and tuning of automatic controllers, their optimum location in the network for suppressing the inter-area oscillations, coordination of emergency control between different devices, voltage stability control by compensating active and reactive power in the lines, and optimal islanding of the EPS in emergencies. Along with traditional approaches based on the linearized models of the system, the review also presents their extensions that take into account different nonlinear effects. In particular, a quadratic problem of eigenvalues and a pseudo spectrum of a dynamics matrix, a method of Poincaré normal forms, and spectral decompositions of Lyapunov functions in the modal analysis are considered.

Materials cited in this paper demonstrate the intensive development of methods for modal analysis of dynamic systems in general and their active application and improvement to solve the problems of analyzing the structural properties of large EPS, simplifying their dynamics models, assessing their steady-state stability, and their control. The presented review of the latest developments in modal analysis methods and their extension to nonlinear systems shows the potential for an effective solution to the stability problems of complex EPSs.

In this review, large EPSs are taken as the principal object for applying the spectral and modal methods. At the same time, mini and microsystems formed by distribution networks with a voltage of 0.4 kV and $6 - 10 - 20 - 35$ kV are beyond the scope of the presented review. These systems operating independently or jointly with large EPSs have a specific structure and properties that generate specific problems of ensuring their stability [1, 114] et al. Analysis of these problems and related methods is the subject of a separate study.

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