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SENSORS AND SYSTEMS

Intelligent Control Systems and Fuzzy Controllers. I. Fuzzy Models, Logical-Linguistic and Analytical Regulators[1]

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Abstract—The problems of control systems intellectualization are observed. The necessity of intellectualization of a wide range of systems and control methods is proved. The hierarchy of levels of intellectual control observed and comparison analysis of different artificial intelligence devices given. Importance of target setting's automation problems' solving in control systems is pointed out, as well as intellectualization of anthropocentric systems, including the ones based on fuzzy logic and case-based reasoning. The logical-linguistic and analytical, fuzzy controllers are considered, based on fuzzy logics of Zadeh, implication of Mamdani and Lukasiewicz. An overview of the Mamdani-type controllers, controllers based on TS-model is provided. The conditions of optimality and stability of control systems with Mamdani fuzzy controllers are analyzed. The Sugeno dynamic models and the ANFIS adaptive models and the methods of learning developed on the basis of fuzzy controllers are considered.

Keywords: intelligent control, logical-linguistic controllers, stability conditions, the Sugeno and Mamdani controllers, TS-model, membership function, fuzzification, defuzzification

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1. INTRODUCTION

Artificial intelligence (AI) as a field of research and development has emerged and is evolving in parallel with the development of automatic control theory, starting around the 1950s, with major applications first in computing and computer science, and later in control automation [1]. The first commercial and industrial AI applications date back to the 1980s [2]. AI reached some level of stability and maturity during this period.

The important factor capable to lead today to reconsideration reached and to new rises of the theory and practice of AI, is sharp increase in opportunities of the computer equipment, including by hardware implementation of logical and other means of AI.

The intellectual control system is understood as the set of technical means and the software integrated by information process working in interaction with the person (group of people) or is autonomous, capable on the basis of data on the environment and own status with knowledge and motivation to synthesize the purpose, to make a decision on action and to find rational ways of achievement of the goal [3].

[1] The article provides an extended review on the materials of the plenary report presented at the XII Intelligent Systems Symposium-2016, Moscow, Russia, October 5–7, 2016.

At present, there remains an increased interest in the science and practice of control in integrating classical automatic control methods with AI methods and AI applications in the field of controlling complex, weakly formalizable objects and processes, particularly in cases where information, system status, control quality criteria, and control objectives themselves change over time, are unclear and sometimes contradictory.

The review examines the hierarchy of levels of intellectual control and provides a comparative analysis of various AI tools.

Due to the fact that over the past decade there has been an avalanche-like increase in the number of theoretical and applied research in the field of fuzzy controllers, the main attention in the article is given to a review of the most important achievements in this field, although even in it, unfortunately, it is not possible to carry out this review with completeness free of taste preferences of the authors.

2. GENERAL QUESTIONS OF INTELLECTUALIZATION OF CONTROL SYSTEMS

Successful solution of problems of ensuring technological independence of the state in the sphere of development and application of complex technical and other objects of civil and military purpose depends significantly on efficiency of created systems and control technologies. Adequate theory and control technologies are needed taking into account the possible shortage of certain (depending on applications) required resources: information, time, energy, financial, material, personnel, etc.

Known accidents and disasters in transport, industry, energy and other areas are often attributed to the so-called "human factor" (HF), including the overload of operators. Often HF occurs due to insufficient quality of the control system design, in particular as a result of emergency situations of uncontrollability. Errors due to HF, as well as exhaustion of technical resources of control facilities and systems, often observed in the conditions of modern Russia, strongly require guaranteed reliability and quality of control, including improvement of design, operation and modernization capabilities of control.

Methods and technologies are required to evaluate and ensure optimum control systems, their functional and operational reliability, operability, fault tolerance and persistence in the following conditions:

- insufficient a priori information about the control object and its external environment, including in counteraction conditions;
- a large number of difficult to take into account factors of non-stability and their subjective nature;
- degradation (due to failures, accidents) or the need for targeted reconfiguration (recovery or development control).

With the expansion of its functional load, control systems become significantly more complex. Among the factors of complexity of modern and promising control systems are:

- multilevel of control, heterogeneity of the description of subsystems by quantitative and qualitative models, different scales of processes by space and time, multi-mode, multi-link, decentralization and networking, general structural complexity of modern control systems and controlled objects,
- presence of uncontrolled coordinate-parametric, structural, regular and singular effects, including active counteraction in the conflict environment,
- application of deterministic and probabilistic models of description of uncertainty of information about the state vector and parameters of the system, about properties of measurement errors and external environment,

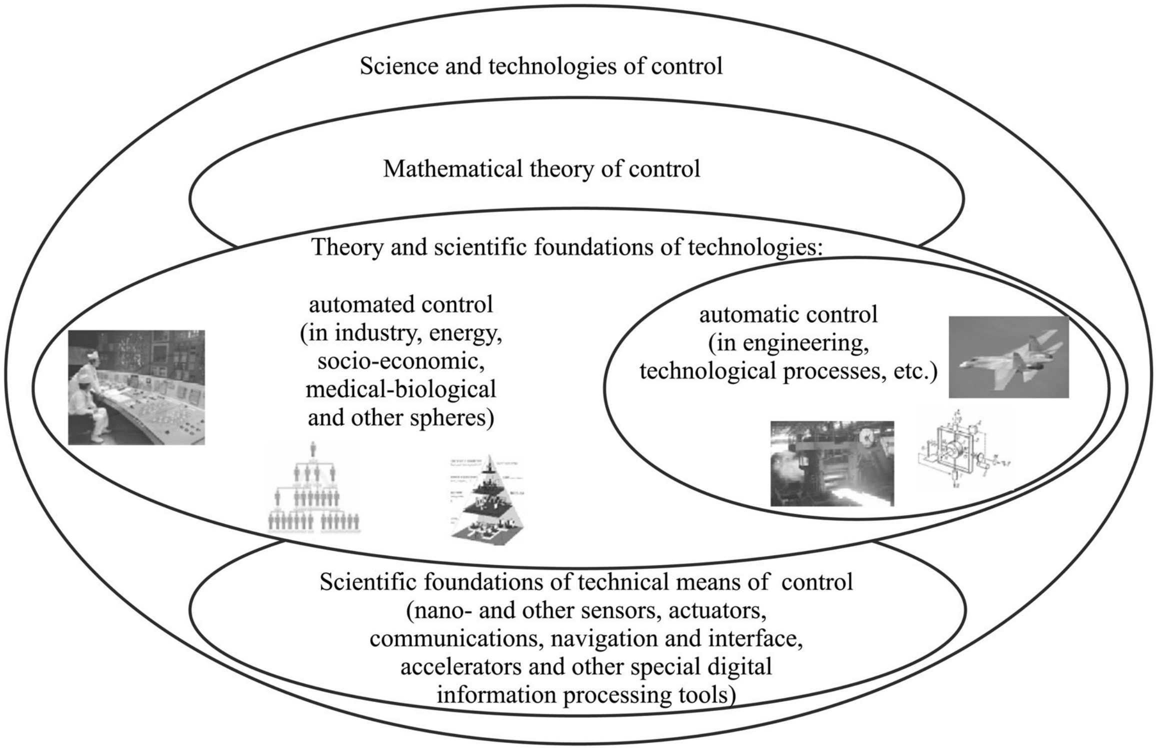

Fig. 1. An expanded structure of control science and technology.

- nonlinearity, distribution of parameters, presence of effects of delay in control or object and pulse actions, high dimension, etc.

The overall structure of control science and technology is shown in Fig. 1.

Adaptive, robust, predictive control techniques and others developed in control theory are designed to account for the undersimulation of dynamics by obtaining missing information at the learning stage or in real time. The use of AI tools enhances the capacity to control complex systems by covering tasks with unknown or already unfair quantitative models from some point in operation, as well as tasks in which quantitative models can be inferior in efficiency to AI models (as in action planning tasks) or can be used in conjunction with AI models [1].

A variety of artificial intelligence tools—neural network, evolutionary, logical and others—can be used for the tasks of action planning and control in general. Each of these classes has its own advantages and disadvantages, especially in view of real-time requirements, and implements the upper levels of heterogeneous control of complex systems (Fig. 2).

Intensive development of technical systems and technological processes (network interaction, miniaturization of sensors, actuators, computers, increase of their speed, etc.) puts new requirements to modern control systems and opens new opportunities both at the level of built-in control systems of different scale, and at the level of group interaction of decentralized multi-agent systems.

Research and development on transition from the robots functioning in the uncertain environment, but with the interface from the operator (supervisory UAVs), to intelligent robots is up-to-date. At the same time it is necessary to reduce the cost of robots on the basis of the modular principle of their construction and miniaturization, to solve problems of sensitivity, formation of models of the external environment, problems of achieving the goal of control of the robot team and expansion of the sphere of application. Even in agriculture and road construction, radical transformation of standards requires robots with precision navigation and intelligent control.

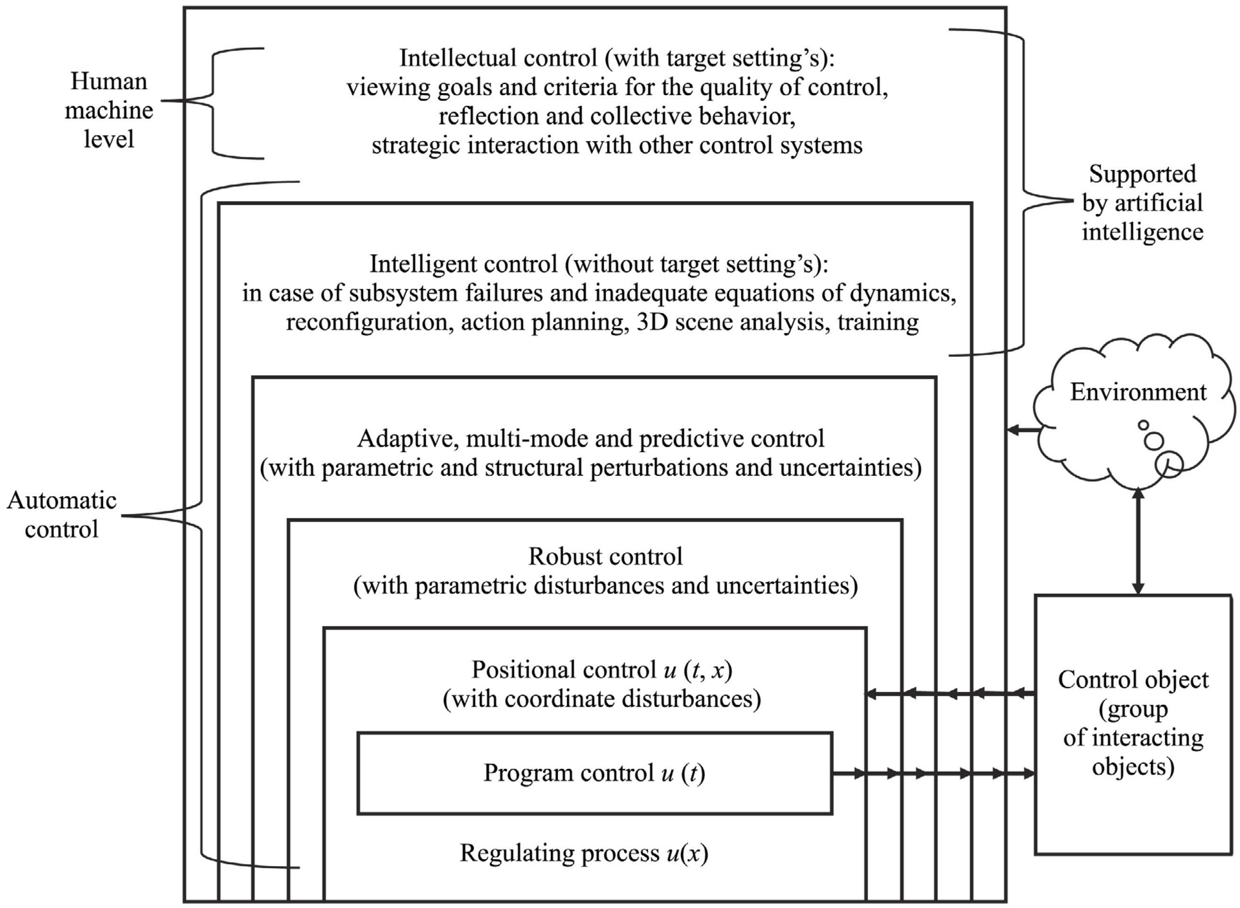

Fig. 2. Heterogeneous control of complex systems.

Examples of critical processes and intelligent control facilities are large-scale electrical infrastructure systems. At the same time, the unsustainable structure of the electricity grid and generating capacities, insufficient energy saving in the sphere of electricity consumption, technological and commercial losses in electrical networks, technological backwardness and high degree of equipment wear, the high level of monopolization of energy markets, as well as the vulnerability of the UEPS to terrorist and cybernetic threats and much more, require the development of models of complex infrastructure dynamic systems and the creation of efficient and highly reliable systems of intelligent control of active-adaptive networks (smart-grid) [4–6].

Control on the basis of logical-reactive (production) knowledge in so-called expert, recommending and operational-consulting systems needs to be strengthened with new capabilities:

- organization of interaction with other control intellectualization tools (artificial neural networks, genetic algorithms) and adaptive, robust and predictive control algorithms;
- leveling the complexity of the interface of logical control systems with the external physical world by combining the methods of symbolic and multimedia presentation and knowledge processing;
- operations with partially formalized and natural language texts;
- abductive and inductive replenishment of knowledge;
- integration of ontologies and quantitatively-qualitative models of different subject areas characterizing the problem situation.

Some typical advantages and disadvantages of artificial intelligence are presented in Table 1.

Table 1. Comparison of intelligent control tools

AI tools	Advantages	Disadvantages
Neural network (neuro-reactive)	1. Applicable in multifactorial problems with poor formalizability of patterns. 2. High degree of parallelism and speed. 3. Learning ability.	1. The need for training information—a representative set of examples of "input-output" ("more likely—the eye than the brain"). 2. Slow learning.
Evolutionary (genetic)	High degree of parallelism and speed.	1. A priori unknown effectiveness in the application. 2. "Rather self-organization of the natural elements than the creative process."
Logic-reactive (production)	1. The naturalness of the rules ("if-then"). 2. The possibility of representing declarative procedural knowledge.	1. The complexity of the performance of large product bases, lack of structurability. 2. The difficulty of ensuring correctness. 3. Incomplete languages regarding first-order expressiveness.
Object-oriented (frames,...)	1. Good structured. 2. High performance mechanisms of inheritance of properties, defaults, etc.	1. The complexity of programming (avoiding the ideals of AI). 2. Lack of expressiveness.
Logical	1. High expressive power. 2. Correctness. 3. High complexity of offline tasks.	1. Inadequate performance, traditional applications—offline. 2. The insolubility of rich logics. 3. The failure of one logic.
Object-logical	Combining the benefits of object-oriented and logical models.	1. The disadvantages of logical models. 2. The complexity of programming.
Multi-agent	Taking into account reflection, self-organization.	Correctness requires theory.

Different combinations of the individual AI tools are possible. For example, [7] neuro-reactive, logic-reactive (production) and logical level of intelligent control are combined. The latter processes a wider layer of knowledge, while the first two support "reasonable" behavior by providing the simplest heuristically conditioned responses of the control system to changes in the environment or control object. The logic-reactive level with small, but sometimes numerous "if-then" rules especially needs to be verified. In the case of production rules of the Boolean type with constructive semantics, verification of the knowledge base can be reduced to a dynamic analysis of automaton networks. This analysis is further simplified by the class of automata that are monotonic in state by applying the method of transferring the properties of mathematical models [8].

An important task of AI remains the automatic assessment of the lack of relevance of knowledge, as not only the shortage, but also the excess of information lead to the degradation of intellectual control systems.

Current advances in intellectual management consist in automating the search for ways to achieve externally defined goals, while advances in automating targeting functions and revising control quality criteria are insufficient. It has also now been realized that improving only the "hardware component" of the man-machine systems being developed is not sufficient to achieve the desired dramatic improvement in their efficiency. It is possible to achieve this in the creation of anthropocentric systems by directing the efforts of designers and scientists to improve the intellectual component of the "system-forming core" of the system—the set of algorithms of built-in digital computers and algorithms of operator activity, called "on-board intelligence" [8, 9].

And first of all, this on-board intelligence is in demand in aviation, especially for typical combat situations of fighter aircraft—the type of situations that is characterized by the most aggressive external situation and severe time restrictions on the work of the crew. On-board intelligence is a functionally integral complex aimed at performing all tasks of the aircraft [9]. Scientific and technological advances in this area will also be useful in other AI applications in a multi-criteria, uncertainty and risk environment to improve the quality of control in an operator's information overload environment, limited time and stress.

The development of practically significant on-board operational-consulting expert systems, including on the basis of fuzzy logic and case-law reasoning by analogy, has entered the practical stage of creating their models and prototypes. They are being extensively developed in the world in the interest of the creation of manned combat aircraft of Generation 4++ and Generation 5, as well as combat unmanned aerial vehicles, and their separate fragments are already appearing on modernized fighter aircraft of Generation 4++.

In foreign developments, primarily on board the new US fighter F-22, F-35, modernized aircraft F-16, F-15, F/A-18 and helicopters, it is planned to place a number of onboard intellectual systems for solving tactical problems [9]. The results of research work, the improvement of on-board computers, the information control field of aircraft cabins, and airborne complexes of aircraft make it possible for next-generation aircraft/helicopters to develop and implement on-board computer systems of a new type that will be able to solve tactical problems (operational assignment of the current flight target and the choice of a rational way to achieve this goal), which in the aircraft of past generations were decided only by the crew.

In the following, the issues of intelligent automatic control systems in the form of fuzzy regulators and their combination with other artificial intelligence tools will be discussed in more detail. Note that the first regulators developed in Greece in the 3rd century BC, partly can be considered as fuzzy regulators, described linguistically with logical operations. And today a huge number of practical applications of fuzzy control systems in industry, transport, energy, oil and gas sector, metallurgy, medicine, other industries and household appliances are noted in Japan, China, USA, Germany, France, England, Russia and other countries.

We will consider four main types of regulators: logical-linguistic, analytical, trained and proportional-integral-differential (PID) fuzzy regulators [1, 7, 11–17]. Since the information about them is not systematized and scattered over many publications, we will give their analysis to help the specialist navigate in this area.

3. BASIC CONCEPTS AND DEFINITIONS

Consider such basic concepts as fuzzy sets and some operations on them, a linguistic variable and a fuzzy relation. We start with a description of the fuzzy input X and output Y sets of the generalized fuzzy controller and control system.

A fuzzy set X on a universal set $\mathbf{X} = \{x : x_{\min} \le x \le x_{\max}\}$ is an ordered collection of pairs [17]

$$X = \{x, X(x)\}, \quad x \in \mathbf{X},$$

where $X(x)$ is the membership function of x to X that maps $\mathbf{X}$ to the interval $[0, 1]$.

On fuzzy sets X_1 and X_2, the operations of union are defined (connectives "or," "otherwise")

$$(X_1 \cup X_2)(x) = X_1(x) \vee X_2(x) = \max(X_1(x), X_2(x))$$

and intersections (connective "and")

$$(X_1 \cap X_2)(x) = X_1(x) \wedge X_2(x) = \min(X_1(x), X_2(x)).$$

A linguistic variable is defined by the triple $(x, T_x, \mathbf{X})$, in which x is the name of the variable, $T_x = \left\{T_x^1, T_x^2, \ldots, T_x^k\right\}$ is the term is the set of linguistic values or terms $T_x^l, l = \overline{1, k}$, with the corresponding membership functions $T_x^l(x)$ defined on the universal set $\mathbf{X}$.

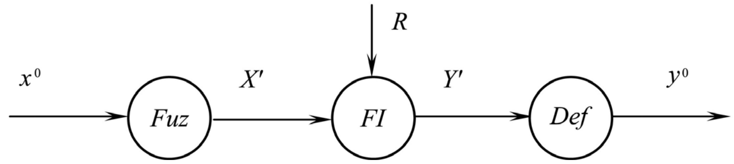

Fig. 3. Transforming of the input in the fuzzy controller.

The fuzzy relation R on the Cartesian product of the sets $\mathbf{X} \times \mathbf{Y} = \{(x, y) : x \in \mathbf{X}, y \in \mathbf{Y}\}$ is a fuzzy set in $\mathbf{X} \times \mathbf{Y}$ with the membership function $R(x, y)$ that characterizes the degree of compatibility of the pair x, y with R. If x, y are points, that is, $x \in \mathbf{X} = \{x^1, \ldots, x^k\}$, $y \in \mathbf{Y} = \{y^1, \ldots, y^s\}$, then the relation is a matrix with elements $R(x^l, y^r), l = \overline{1, k}, r = \overline{1, s}$ [17].

In fuzzy PID controllers, the fuzzy input set E characterizes some generalized error e on the universal set $\mathbf{E} = \{e : e_{\min} \le e \le e_{\max}\}$ and can be determined by an ordered collection of pairs [10]

$$E = \{e, E(e)\}, \quad e \in \mathbf{E},$$

where $E(e)$ is the membership function of e to E, mapping $\mathbf{E}$ into the interval $[0, 1]$.

The output fuzzy set U of the PID controller characterizes the control u on the universal set $\mathbf{U} = \{u : u_{\min} \le u \le u_{\max}\}$ and can be defined as

$$U = \{u, U(u)\}, \quad u \in \mathbf{U}.$$

By analogy with the above, it is possible to specify the linguistic values of the variables e and u, as well as the fuzzy relation R on the Cartesian product $\mathbf{E} \times \mathbf{U} = \{(e, u) : e \in \mathbf{E}, u \in \mathbf{U}\}$.

4. LOGICAL-LINGUISTIC CONTROLLERS (LLC)

Such fuzzy controllers contain fuzzy sets, logical operations of union, intersection and composition with linguistic values of variables, a fuzzy relation formed by one or more logical operations, and a rule for deduction a fuzzy output with a known input. The first LLCs [18–20] had a very strong influence on subsequent research in the field of fuzzy control systems and deserve to first outline the basic principles of their construction, and then show how these principles are implemented in one of the controllers.

Let us consider the principles of constructing a logical-linguistic controller using the example of a simple, generalized controller with one input x (usually a control error) and one output y (regulatory or control action) connected by fuzzy rules:

$$\begin{aligned}
&R^1: \text{ if } x \text{ is } X^1, \text{ then } y \text{ is } Y^1, \text{ otherwise} \\
&R^2: \text{ if } x \text{ is } X^2, \text{ then } y \text{ is } Y^2, \text{ otherwise} \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&R^n: \text{ if } x \text{ is } X^n, \text{ then } y \text{ is } Y^n,
\end{aligned} \tag{1}$$

containing fuzzy sets $X^\theta \in T_x$ and $Y^\theta \in T_y$.

In one way or another, the LLC's functioning algorithm contains procedures for transforming (fazzification *Fuz*) the measured value x^0 of the variable x into the linguistic X', the fuzzy inference *FI* of the linguistic output Y' by the known input X' and the set of rules $R = \{R^1, \ldots, R^n\}$, and transformations (defazzification *Def*) of the linguistic value of the output Y' into real y^0 (Fig. 3).

An input measurable variable x with value x^0 corresponds the so-called "degenerate" fuzzy set X' with membership function

$$X'(x) = \begin{cases} 1, & \text{if} \quad x = x^0 \\ 0, & \text{if} \quad x \neq x^0, \end{cases}$$

where x^0 is the point called the singleton of the set X'. Let us write an expression of fuzzy inference for LLC defined by a set of rules (1):

$$\begin{array}{l} R^1: \text{ if } x \text{ is } X^1, \text{ then } y \text{ is } Y^1, \text{ otherwise} \\ R^2: \text{ if } x \text{ is } X^2, \text{ then } y \text{ is } Y^2, \text{ otherwise} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ R^n: \text{ if } x \text{ is } X^n, \text{ then } y \text{ is } Y^n, \\ \underline{x \text{ is } X'} \\ y \text{ is } Y'. \end{array} \tag{2}$$

The truth values of the statements "x is X^θ," "y is Y^θ" and "x is X'" in the rules (1) and the premise of the conclusion of expression (2) are determined by the value of the corresponding membership functions $X^\theta(x), Y^\theta(y)$ and $X'(x)$ for $x \in \mathbf{X}, y \in \mathbf{Y}$.

Each rule R^θ is a fuzzy implication

$$R^\theta: \text{ if } x \text{ is } X^\theta, \text{ then } y \text{ is } Y^\theta = X^\theta \to Y^\theta.$$

LLC uses the Zadeh maximin composition as the conclusion procedure for Y' [17]

$$Y'(y) = \left(X' \circ R\right)(y) = \bigvee_{x \in \mathbf{X}} (X'(x) \wedge R(x, y)), \tag{3}$$

where

$$R(x, y) = \bigvee_{\theta=1}^{n} R^\theta(x, y) = \bigvee_{\theta=1}^{n} X^\theta(x) \wedge Y^\theta(y).$$

At the point x^0, expression (3) after substituting $X'(x^0) = 1$ takes the form

$$Y'(y) = \bigvee_{\theta=1}^{n} R\left(x^0, y\right) = \bigvee_{\theta=1}^{n} \left(X\left(x^0\right) \wedge Y(y)\right). \tag{4}$$

The output value y^0 can be determined by maximizing

$$y^0 = \max_{y \in Y} Y'(y) \tag{5}$$

or computing the "center of gravity" of the membership function $Y'(y)$

$$\int_{y_{\min}}^{y^0} Y'(y)dy = \int_{y^0}^{y_{\max}} Y'(y)dy. \tag{6}$$

The most famous and often cited LLC, designed to control a steam engine [18], has four inputs (x_1 is the pressure error equal to the deviation of the current from the set value; x_2 is the speed error, defined as x_1; x_3 is the rate of change of x_1; x_4 is the speed changes of x_2) and two output (y_1 is the change in heat; y_2 is the change in vapor pressure) variables.

The ranges of change of input $\mathbf{X}_1, \ldots, \mathbf{X}_4$ and output $\mathbf{Y}_1$ variables $x_1, \ldots, x_4, y_1$ break into seven intervals with the following linguistic values: *PB* is a positive big; *PM* is a positive mean; *PS* is a positive small; *NO* is null; *NS* is a negative small; *NM* is a negative mean; *NB* is a negative big. The range of change $\mathbf{Y}_2$ of the output variable y_2 consists of five intervals with the linguistic values *NB*, *NS*, *NO*, *PS*, *PB* defined on them.

These linguistic values form two term sets $T_1 = \{NB, NM, NS, NO, PS, PM, PB\}$ and $T_2 = \{NB, NS, NO, PS, PB\}$.

The fuzzy controller consists of two sets of rules ($j = 1, 2$), which determine the change in heat y_1 and pressure y_2

$$\begin{array}{c} R_j^1: \text{ if } x_1 \text{ is } X_{1j}^1 \text{ and } x_2 \text{ is } X_{2j}^1 \text{ and } \ldots \text{ and } x_4 \text{ is } X_{4j}^1, \text{ then } y_j \text{ is } Y_j^1, \text{ otherwise} \\ R_j^2: \text{ if } x_1 \text{ is } X_{1j}^2 \text{ and } x_2 \text{ is } X_{2j}^2 \text{ and } \ldots \text{ and } x_4 \text{ is } X_{4j}^2, \text{ then } y_j \text{ is } Y_j^2, \text{ otherwise} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ R_j^{n_j}: \text{ if } x_1 \text{ is } X_{1j}^{n_j} \text{ and } x_2 \text{ is } X_{2j}^{n_j} \text{ and } \ldots \text{ and } x_4 \text{ is } X_{4j}^{n_j}, \text{ then } y_j \text{ is } Y_j^{n_j}, \\ \underline{\quad x_1 \text{ is } X'_{1j} \text{ and } x_2 \text{ is } X'_{2j} \text{ and } \ldots \text{ and } x_4 \text{ is } X'_{4j}, \quad} \\ y_j \text{ is } Y'_j. \end{array} \tag{7}$$

The statements "x_i is $X_{ij}^{\theta_j}$" and "x_i is X'_{ij}" in the premise of the expression (7) with truth values given by the corresponding membership functions $X_{ij}^{\theta_j}(x)$ and $X'_{ij}(x)$, $i = \overline{1,4}$, $j = 1, 2$, $\theta_j = \overline{1,n}$ are combined by a logical connective "and" that implements the intersection operation. Then the truth of the left side of the θ_jth rule is defined as

$$X_{1j}^{\theta_j}(x_1) \wedge X_{2j}^{\theta_j}(x_2) \wedge \ldots \wedge X_{4j}^{\theta_j}(x_4), \tag{8}$$

and the truth of the premise is defined as

$$X'_{1j}(x_1) \wedge X'_{2j}(x_2) \wedge \ldots \wedge X'_{4j}(x_4), \quad j = 1, 2. \tag{9}$$

The expression of maximin composition (3) will take the form

$$Y'_j(y_j) = \bigvee_{\substack{x_1 \in \mathbf{X}_1 \\ \cdots\cdots \\ x_4 \in \mathbf{X}_4}} \left(\left[X'_1(x_1) \wedge X'_2(x_2) \wedge \ldots \wedge X'_4(x_4) \right] \wedge R_j\left(x_1, \ldots, x_4, y_j\right) \right), \tag{10}$$

where

$$R_i(x_1, \ldots, x_4, y_j) = \bigvee_{\theta_j=1}^{n_j} \left(X_{1j}^{\theta_j}(x_1) \wedge X_{2j}^{\theta_j}(x_2) \wedge \ldots \wedge X_{4j}^{\theta_j}(x_4) \wedge Y_j^{\theta_j}(y_j) \right).$$

Since $x_1^0, \ldots x_4^0$ are singletons of sets X'_i, then after substitution $X'_i(x_i^0) = 1$, $i = \overline{1,4}$ in (10) we get a Mamdani implication

$$\begin{gathered} Y'_j(y_j) = R_j\left(x_1^0, \ldots, x_4^0, y_j\right) \\ = \bigvee_{\theta_j=1}^{n_j} \left(X_{1j}^{\theta_j}(x_1) \wedge X_{2j}^{\theta_j}(x_2) \wedge \ldots \wedge X_{4j}^{\theta_j}(x_4) \wedge Y_j^{\theta_j}(y_j) \right), \quad j = 1, 2. \end{gathered} \tag{11}$$

The actual output values of y_1^0 and y_2^0 are determined on the basis of the found membership functions $Y'_1(y_1)$ and $Y'_2(y_2)$ using relations (5) and (6).

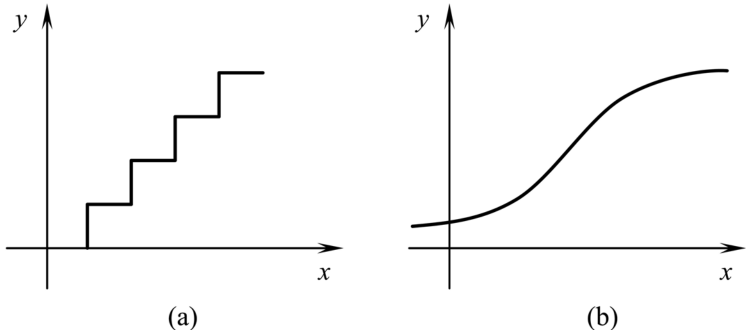

Fig. 4. Static characteristics of fuzzy controllers with the implications of (a) Mamdani and (b) Lukasiewicz.

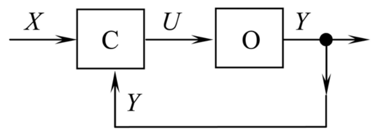

Fig. 5. The scheme of the closed system.

The LLC with implication (11) considered above is called the Mamdani regulator. If in (4) we take $R(x, y) = 1$, then the Mamdani controller will have a static characteristic of the multi-position relay (Fig. 4a), in which the linearity and continuity of output y relative to input x are violated.

Attempts to eliminate these shortcomings were made in [21–25] and consisted in using Lukasiewicz's implication as a fuzzy relationship $R(x, y)$ in (3)

$$R_L(x, y) = 1 \wedge [1 - X(x) + Y(y)]. \tag{12}$$

Indeed, if we take $R_L(x, y) = 1$, then the implication (12) in expression (4) with one input

$$Y'(y) = \mathop{\mathrm{V}}_{\theta_j=1}^{n} R_L^{\theta}(x, y)$$

allows to obtain a more advanced LLC, which has a static characteristic of a linear function with saturation (Fig. 4b).

However, controllers and fuzzy control systems using Zadeh implication have found much greater application. A large number of studies have been devoted to them, in which controllers and control systems are represented by fuzzy differential

$$\dot{X}(t) = X(t) \circ R \tag{13}$$

and difference equations

$$X_{t+1} = X_t \circ R. \tag{14}$$

The first publications [26–30] analyzed the stability and controllability of fuzzy dynamical systems of the type (13) and (14). For these purposes, Lyapunov functions [26, 27] and stability estimation methods based on such specific concepts of fuzzy sets as the energy of the fuzzy set X_t and fuzzy ratio R, peak characteristics of fuzzy sets and measures of their proximity were used [28–30].

The main drawback of the proposed approaches is the lack of specific recommendations on the selection or synthesis of fuzzy controllers and control systems with certain dynamic properties (controllability, stability and quality of regulatory processes).

Table 2. Fuzzy object operator

$F_{OU} =$

U \ Y	*NB*	*NM*	*NS*	*ZE*	*PS*	*PM*	*PB*
NB	*ZE*	*NS*	*NS*	*NM*	*NB*	*NM*	*NS*
NM	*PB*	*ZE*	*NB*	*NB*	*NB*	*NM*	*NB*
NS	*NS*	*NS*	*ZE*	*NM*	*NM*	*NB*	*NB*
ZE	*PB*	*PB*	*PB*	*ZE*	*PS*	*PS*	*PS*
PS	*NM*	*ZE*	*PS*	*NB*	*PB*	*PM*	*PB*
PM	*NM*	*NS*	*ZE*	*PS*	*ZE*	*NS*	*NB*
PB	*NB*	*NB*	*NB*	*ZE*	*PB*	*PB*	*PB*

$\dot{Y}$

Table 3. Fuzzy closed system operator

$F^* =$

X \ Y	*NB*	*NM*	*NS*	*ZE*	*PS*	*PM*	*PB*
NB	*ZE*	*NS*	*NS*	*NM*	*NB*	*NB*	*NB*
NM	*PS*	*ZE*	*NS*	*NS*	*NM*	*NB*	*NB*
NS	*PS*	*PS*	*ZE*	*NS*	*NS*	*NM*	*NB*
ZE	*PB*	*PM*	*PS*	*ZE*	*NS*	*NM*	*NB*
PS	*PB*	*PM*	*PS*	*PS*	*ZE*	*NS*	*NM*
PM	*PB*	*PB*	*PM*	*PS*	*PS*	*ZE*	*NS*
PB	*PB*	*PB*	*PB*	*PM*	*PS*	*PS*	*ZE*

$\dot{Y}$

The first attempt to synthesize LLC, optimal in the sense of a minimum control error, was made in a closed control system (Fig. 5) based on the fuzzy operators of the object and the optimal closed system specified by the tables (see Table 2 and Table 3) [31].

For the sake of compactness of presentation, we present in analytical form the tabular operators of the control object O:

$$\dot{Y} = F_{OU}(Y, U), \tag{15}$$

optimal closed system:

$$\dot{Y} = F^*(Y, X) \tag{16}$$

and synthesized controller P:

$$U = F_{PX}(X, Y), \tag{17}$$

in which the linguistic variables characterizing setpoint X, output Y and its speed $\dot{Y}$, control U and disturbance W take values from the term set $T = \{NB, NM, NS, ZE, PS, PM, PB\}$. The operator of the object (15) is built on the basis of the results of research of its static and dynamic characteristics. For the tabular operator of the object (15), it is easy to obtain the inverse operator F_{OU}^{-1} with respect to the control U

$$U = F_{OU}^{-1}\left(\dot{Y}, Y\right), \tag{18}$$

and the operator F^* of an optimal closed system can be obtained from the graph of linguistic dynamics (Fig. 6) and the following heuristic considerations.

Points $1, 2, \ldots, 7$ on the graph characterize the equality of the linguistic values of setpoint X and output Y, as well as the minimum output speed $\dot{Y} = ZE$, which prevents overshoot. As the derivation between X and Y increases, i.e., the control error increases, the output speed $\dot{Y}$, directed

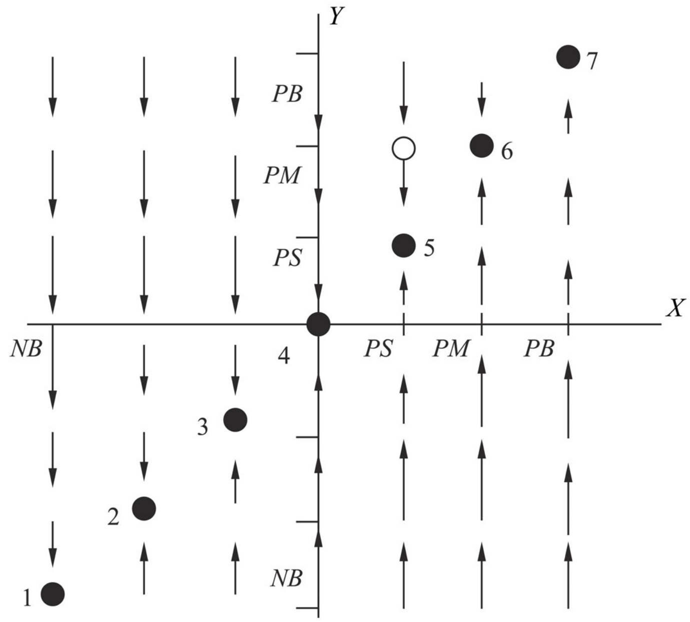

Fig. 6. Graph of linguistic dynamics.

towards one of these points, should increase. The direction and size of the arrows $\dot{Y}$ correspond to the accepted linguistic values.

For example, at point "○" of the graph of linguistic dynamics of an optimal closed system one data set of a tabular operator $F^*: X = PS, Y = PM, \dot{Y} = NS$ and the corresponding rule:

$$\text{if } X = PS \text{ and } Y = PM, \text{ then } \dot{Y} := NS$$

are defined.

Now let's formulate the problem of synthesis of the optimal fuzzy controller.

For all linguistic values of setpoint X and output Y, using the optimal closed-loop system operator (16) and the inverse object operator (18), determine the control U, i.e., the triples $\langle U, X, Y \rangle$ that form the controller operator (17).

Consider the procedure for determining the control U^* in the triple $\langle X^*, Y^*, U^* \rangle$ for $X^* = NS$, $Y^* = PM$. Substituting $X^* = NS$ and $Y^* = PM$ into Table 3, the operator of an optimal closed system gives $\dot{Y}^* = NM$. For $Y^* = PM$ and $\dot{Y}^* = NM$, from Table 2 we obtain $U^* = NM$, i.e., we implement the inverse operator of the object and determine the desired triple $\langle NS, PM, NM \rangle$.

In the general case, the inverse operator F_{OU}^{-1} is not single-valued. The optimal regulator operator found is not uniquely defined, which significantly reduces the practical value of this approach to LLC synthesis.

Further development of the methodology of synthesis of the tabular operator of the regulator was obtained in the work [32].

Based on the static characteristics and transient functions of the first order aperiodic link, forming operators of the object via control channels F_{OU} and perturbation F_{OW} (Fig. 7), as well as the qualitative description of the control process, it was possible to synthesize the fuzzy controller acting when changing the setpoint X

$$F'_p = F_X \cup F_Y \cup F_E$$

and a compensator eliminating the effect of the disturbance W on the output

$$F_K = F_W \cup F_Y \cup F_E.$$

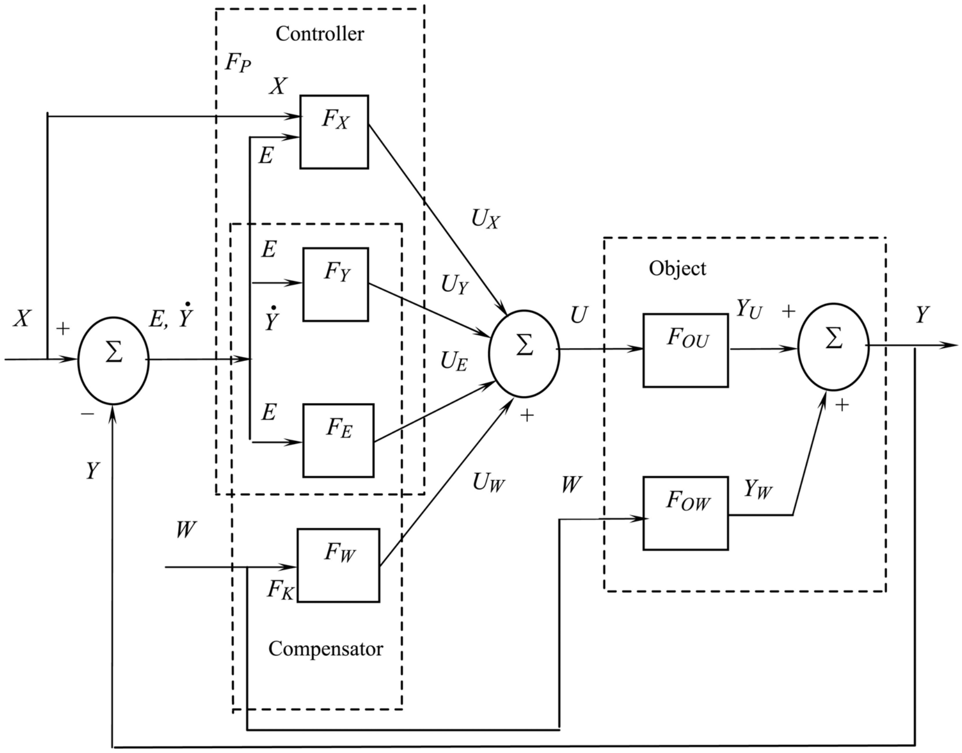

Fig. 7. Control system diagram with fuzzy controller and compensator.

Here ∪ is the operation of combining components that implement the three phases of control.

In the first phase of control $U_X = F_X(X, E)$ of the controller, with a significant change in the setpoint X, the value U_X is set to the limit. As soon as the output value Y reaches a certain neighborhood of setpoint X, a control action is selected from the static characteristic of the channel U–Y, in which the steady-state value of the output becomes close to the task.

The control U_W or the output of the component $U_W = F_W(W)$ of the compensator F_K is formed on the basis of two principles of fuzzy invariance.

Based on the static characteristic of the channel W–Y, a possible reaction of the output Y_W to a disturbance W is estimated and the control U_W is determined, which causes a change in the output Y, which is equal in magnitude and opposite in sign to the value of Y_W. Thus, it is possible to provide partial compensation of the disturbance or independence (invariance) of the output Y from the disturbance W.

A more complete compensation can be achieved by choosing a control U_W at which the rate of change of Y_U will be equal in magnitude and opposite in the direction of speed Y_W.

The F_Y component is used to eliminate overshoot, and the F_E component is used to eliminate static error. Based on the proposed principles of forming the components of the controller and compensator, methods for the synthesis of tabular operators F_X, F_W, F_Y, F_E were developed, which made it possible to ensure the required quality of temperature control at the output of the acetone pyrolysis furnace [32]. Close approaches to the synthesis of a tabular fuzzy controller were proposed for controlling a distillation unit [33] and other chemical objects [34].

The main disadvantages of LLC table type include their limited dimension. As for the subjectivity of the choice of intervals and the corresponding values of linguistic variables, it is precisely it that in anthropocentric applications of onboard intelligence provides the naturalness and intellectuality of the "man-machine" interface, due to which it is possible to combine the most powerful aspects of the operator and the machine. This is demonstrated, for example, by technologies of case-based reasoning systems as "reinforcing" or "non-alternative" technologies [1, 8, 9] in such difficult problems as goal-setting automation problems, especially in hard real-time conditions.

Let us note one important advantage of all LLCs. As previously mentioned, LLC is similar to a multi-position relay, in which the response levels are selected taking into account the properties of the control object. Thus, it is possible to significantly compensate the effect of non-linearity of the object, which significantly worsens the operation of control systems with linear P, PI, and PID controllers.

5. ANALYTICAL CONTROLLERS

The subjectivity of the choice of intervals and linguistic variables and the associated decrease in the quality of control can be largely eliminated in the so-called analytical fuzzy controllers (AFC) and control systems, the performance of which is ensured by well-known analytical and numerical methods of parametric identification, analysis and synthesis of linear and nonlinear systems involving fuzzy dynamic models.

A special place is occupied by the so-called fuzzy model of Takagi and Sugeno or the TS-model [35]. In this model, first, by analytical methods [36], and then in specific modeling and control problems (as a controller), its high approximation abilities were demonstrated. The fuzzy TS-model consists of a set of production rules containing linear difference equations in the right side [35]

$$\begin{gathered} \text{if } y(t-1) \text{ is } Y_1^\theta,\ \ldots,\ y(t-r) \text{ is } Y_r^\theta, \\ x(t) \text{ is } X_0^\theta,\ \ldots,\ x(t-s) \text{ is } X_r^\theta, \\ \text{then } y^\theta(t) = a_0^\theta + \sum_{k=1}^{s} a_k^\theta y(t-k) + \sum_{l=0}^{s} b_l^\theta x(t-l), \quad \theta = \overline{1,n}, \end{gathered} \tag{19}$$

where
$\boldsymbol{a}^\theta = \left(a_0^\theta, a_1^\theta, \ldots, a_r^\theta\right)$, $\boldsymbol{b}^\theta = \left(b_0^\theta, b_1^\theta, \ldots, b_s^\theta\right)$ are vectors of adjustable parameters;
$\boldsymbol{y}(t-r) = (1, y(t-1), \ldots, y(t-r))$ is a state vector;
$\boldsymbol{x}(t-s) = (x(t), x(t-1), \ldots, x(t-s))$ is an input vector;
$Y_1^\theta, \ldots, Y_r^\theta; X_0^\theta, \ldots, X_r^\theta$ are the fuzzy sets.

Expression (19) can be greatly simplified by redesignating the input variables

$$(u_0(t), u_1(t), \ldots, u_m(t)) = (1, y(t-1), \ldots, y(t-r)), x(t), x(t-1), \ldots, x(t-s)),$$

the coefficients of the difference equation

$$\left(c_0^\theta, c_1^\theta, \ldots, c_m^\theta\right) = \left(a_0^\theta, a_1^\theta, \ldots, a_r^\theta, b_1^\theta, \ldots, b_s^\theta\right),$$

and membership functions

$$\left(U_1^\theta(u_1(t)), \ldots, U_m^\theta(u_m(t))\right) = \left(Y_1^\theta(y(t-1)), \ldots, Y_r^\theta(y(t-r)), X_0^\theta(x(t)), \ldots, X_s^\theta(x(t-s))\right),$$

where $m = r + s + 1$.

The analytical form of the fuzzy model (19), designed to calculate the output $\hat{y}(t)$, has the form

$$\hat{y}(t) = \boldsymbol{c}^T \tilde{\boldsymbol{u}}(t), \tag{20}$$

where

$\boldsymbol{c} = (c_0^1, \dots, c_0^n, \dots, c_m^1, \dots, c_m^n)$ is a vector of specified parameters;
$\tilde{\boldsymbol{u}}^T(t) = \Big(u_0(t)\beta^1(t), \dots, u_0(t)\beta^\theta(t), \dots, u_m(t)\beta^1(t), \dots, u_m(t)\beta^n(t)\Big)$ is an extended input vector;
$\beta^\theta(t) = \dfrac{U_1^\theta(u_1(t)) \otimes \ldots \otimes U_m^\theta(u_m(t))}{\sum\limits_{\theta=1}^{N} \left(U_1^\theta(u_1(t)) \otimes \ldots \otimes U_m^\theta(u_m(t))\right)}$ is a fuzzy function, where $\otimes$ is the minimization or product operation.

At given in the initial moment $t = 0$ the vector $\boldsymbol{c}(0) = 0$, the correction matrix $Q(0)$ of size $nm \times nm$, and the values $u(t)$ at times $t = \overline{1, N}$, the vector of parameters $\boldsymbol{c}(t)$ is calculated using the well-known multistep least squares method [36]:

$$\boldsymbol{c}(t) = \boldsymbol{c}(t-1) + Q(t)\tilde{\boldsymbol{u}}(t)\left[y(t) - \boldsymbol{c}^T(t-1)\tilde{\boldsymbol{u}}(t)\right], \tag{21}$$

$$\begin{gathered} Q(t) = Q(t-1) - \frac{Q(t-1)\tilde{\boldsymbol{u}}(t)\tilde{\boldsymbol{u}}^T(t)Q(t-1)}{1 + \tilde{\boldsymbol{u}}^T(t)H(t-1)\tilde{\boldsymbol{u}}(t)}, \\ Q(0) = \gamma I, \quad \gamma \gg 1, \end{gathered} \tag{22}$$

where I is the unit diagonal matrix.

The complete identification algorithm, in addition to algorithm (21), (22), also contains identification algorithms for the number of rules n, order r, s of the difference equation and parameters $\boldsymbol{d}$ of membership functions [37–39].

The introduction of the TS-model had a great influence on the subsequent development of the theory of fuzzy control systems.

Firstly, among fuzzy models, for the first time, the use of traditional parametric identification became legitimate for this model.

Secondly, despite the presence on the right side of the rules of linear difference equations, in the TS-model, by clarifying the parameters $\boldsymbol{c}$, the order r, s and increasing the number of rules n, nonlinear dynamic processes can be described with very high accuracy.

Third, the averaging properties of the inference mechanism y and the specific form of membership functions make the TS-model less sensitive to disturbances and measuring inaccuracy.

Fourth, being a non-linear and continuous function of input variables and parameters, the TS-model provides wide possibilities for the analytical study of the stability of non-linear systems with its presence and their subsequent training in order to obtain the required quality of transient processes.

For a closed control system with a fuzzy controller based on model (19), the problem of stability and its quantitative assessment is also relevant.

In the spirit of the classical representation of linear systems, Tanaka and Sugeno [40] proposed a fuzzy block (Fig. 8)—a dynamic object described by a fuzzy difference model (19) in vector form:

$$\begin{gathered} R^i: \text{ if } \boldsymbol{y}(t) \text{ is } \mathbf{Y}^i \text{ and } \boldsymbol{x}(t) \text{ is } \mathbf{X}^i, \\ \text{then } y^i(t+1) = a_0^i + \sum_{k=1}^{r} a_k^i y(t-k+1) + \sum_{l=0}^{S} b_l x(t-l+1), \end{gathered} \tag{23}$$

where

$$\begin{gathered} \boldsymbol{y}(t) = [y(t), y(t-1), \dots, y(t-r+1]^T \\ \boldsymbol{x}(t) = [x(t), x(t-1), \dots, x(t-s+1)]^T, \\ \mathbf{Y}^i = \left[Y_1^i, \dots, Y_r^i\right], \\ \mathbf{X}^i = \left[X_1^i, \dots, X_s^i\right]; \end{gathered}$$

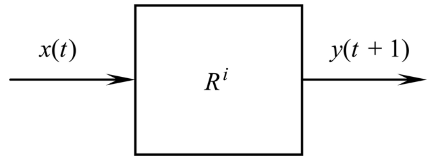

Fig. 8. Fuzzy block (open-loop system).

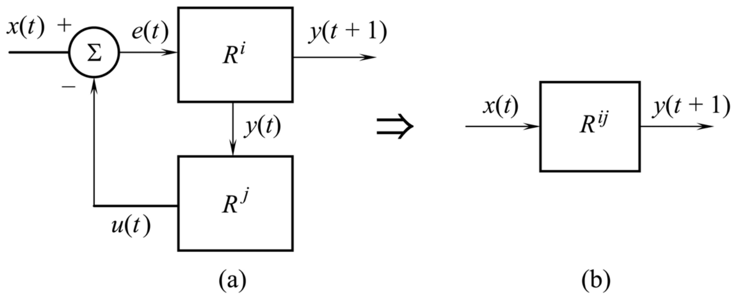

Fig. 9. (a) A feedback connection and (b) a general representation of a feedback link in the form of an open system.

r, s—order of the difference equation;

$$\boldsymbol{y}(t) \text{ is } \mathbf{Y}^I \;\Rightarrow\; y(t) \text{ is } Y_1^i \text{ and } \ldots \text{ and } y(t-r+1) \text{ is } Y_r^i.$$

Various connections (parallel and feedback) are formed from such blocks and their mathematical models are derived.

For example, a feedback connection (Fig. 9) containing object blocks

$$R^i: \text{ if } \boldsymbol{y}(t) \text{ is } \mathbf{Y}_1^i \text{ and } \boldsymbol{e}(t) \text{ is } \mathbf{E}_1^i,$$
$$\text{then } y^i(t+1) = a_{10}^i + \sum_{k=1}^{r} a_{1k}^i y(t-k+1) + \sum_{l=0}^{S} b_{1l} e(t-l+1)$$

and controller

$$\begin{gathered} R_2^i: \text{ if } \boldsymbol{y}(t) \text{ is } \mathbf{Y}_2^j \text{ and } \boldsymbol{e}(t) \text{ is } \mathbf{E}_2^i, \\ \text{then } u^j(t) = a_{20}^j + \sum_{k=1}^{r} a_{2k}^j y(t-k+1), \end{gathered} \tag{24}$$

is equivalent to the block:

$$R^{ij}: \text{ if } \boldsymbol{y}(t) \text{ is } \mathbf{Y}^{ij} \text{ and } \boldsymbol{e}(t) \text{ is } \mathbf{E}^{ij},$$
$$\text{then } y^{ij}(t+1) = a_{10}^i + b_1^i a_{20}^j + b_1 x(t) + \sum_{k=1}^{r} \left(a_{1k} - b_1^i a_{2k}^j\right) y(t-k+1),$$

where $i = 1, 2, \ldots, n_1$, $j = 1, 2, \ldots, n_2$;

$$\boldsymbol{e}(t) = \left[x(t) - u(t), x(t-1) - u(t-1), \ldots, x(t-m+1) - u(t-m+1)\right]^T;$$
$$\mathbf{Y}^{ij} = \left(Y_1^i \bigcap Y_2^j\right), \quad \mathbf{E}^{ij} = \left(E_1^i \bigcap E_2^j\right).$$

The analytical estimates of the stability of fuzzy systems (23) and (24) are derived using the Lyapunov method based on the equation of free motion:

$$\begin{gathered} R^i: \text{ if } \boldsymbol{y}(t) \text{ is } \mathbf{Y}_1^i \text{ and } \boldsymbol{y}(t-r+1) \text{ is } \mathbf{Y}_r^i, \\ \text{then } y^i(t+1) = a_1^i y(t) + \ldots + a_r^i y(t-r+1), \quad i = \overline{1, n}, \end{gathered} \tag{25}$$

the right-hand side of which can be written in matrix form: $A_i\boldsymbol{y}(t)$, where

$$\boldsymbol{y}(t) = [y(t), y(t-1), \ldots, y(t-r+1)]^T,$$

$$A_i = \begin{bmatrix} a_1^i & a_2^i & \ldots & a_{r-1}^i & a_r^i \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 1 & 0 \end{bmatrix}.$$

It was shown in [40–42] that the fuzzy system (25) represented by the calculated dependence

$$y(t+1) = \frac{\sum\limits_{i=1}^{n} w^i A_i \boldsymbol{y}(t)}{\sum\limits_{i=1}^{n} w^i},$$

is asymptotically stable globally if for all subsystems there exists a positive definite matrix B such that

$$A_i^T B A_i - B < 0, \quad \forall i \in \{1, 2, \ldots, n\}. \tag{26}$$

The validity of estimate (26) was confirmed only for the simplest proportional controller.

A close approach to stability analysis, based on Lyapunov methods, was developed in [43] for a fuzzy system in the state space

$$\begin{gathered} R^i: \text{ if } \boldsymbol{y}_1(t) \text{ is } \mathbf{Y}_1^i, \ldots, \boldsymbol{y}_r(t) \text{ is } \mathbf{Y}_r^i, \text{ then } x(t) \text{ is } X^i, \text{ then} \\ y_1(t+1) = a_{11}^i y_1(t) + a_{12}^i y_2(t) + \ldots + a_{1r}^i y_r(t) + b_1^i x(t), \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ y_r(t+1) = a_{r1}^i y_1(t) + a_{r2}^i y_2(t) + \ldots + a_{rr}^i y_r(t) + b_r^i x(t), \end{gathered}$$

and an analytical estimate of the stability of a closed system with a proportional controller is obtained. To achieve stability, it is proposed to refine the parameters a_{jl}^i and the gain factor of the regulator using the gradient method.

Similar approaches to the stability analysis of fuzzy systems using the Lyapunov methods, followed by the synthesis of controllers, are described in [44–59]. The limitations of the Lyapunov method are obvious: it allows to realize the stability of the control system only by the simplest proportional controllers and does not give recommendations on how to achieve the required quality of transients. Functions for maintaining the quality of transients in fuzzy control systems can be provided by fuzzy trained regulators and control systems.

6. CONCLUSIONS

The increasing complexity of man-made systems requires guaranteed efficiency in controlling them using different control intellectualization methods. The article considers the hierarchy of control levels with a comparative analysis of different AI tools. The urgency of solving the problems of goal-setting automation in control systems, as well as the intellectualization of hard real-time anthropocentric systems, is noted.

The review of the most important achievements in the field of fuzzy controllers is given. They are of great interest to researchers and developers of control systems and, first of all, due to the fact that they are applicable in poorly formalized areas of applications, they remain operational under conditions of perturbation and measurement errors, and they quickly take into account and adjust to changing operating conditions, increasing the quality of control. The work on fuzzy controllers that are beyond the scope of this review, as a rule, is one or another version of the development of the logical-linguistic and analytical controllers considered above.

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