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Adaptive Quaternion-Based Quadrotor Control System

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Abstract—In this paper we propose the quaternion-based control system for quadrotor. Adaptive scheme for thrust coefficients identification, based on speed-gradient method, is designed. Proofs of stability are provided, as well the results of numerical simulations. In existing theoretical works, Euler angles are often used as coordinates for describing quadrotor's coordinates. Equations using those coordinates, however, have a singularity, which prevents their use near certain points. We use quaternions instead, which have no such restrictions. The process of discovering PID-regulator coefficients is known to be tedious, error-prone and specific for each quadcopter. We propose a control scheme in which most of the parameters are physical values, and the rest do not depend on the quadcopter and can be found once for the whole class of the flying machines. An identification algorithm for obtaining physical parameters is also described. MATLAB modelling is used to test and confirm the performance of the proposed scheme.

Keywords: UAV, quadrotor, adaptive control, speed-gradient method, quaternions

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1. INTRODUCTION

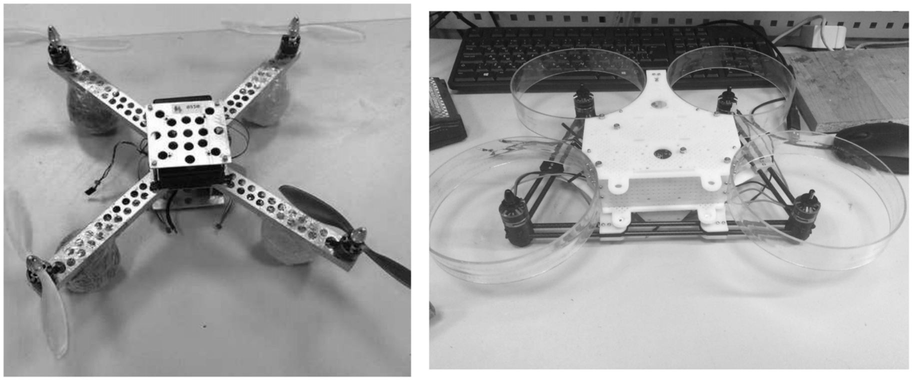
Recently, unmanned aerial vehicles, in particular the most accessible of them—quadrotors, are beginning to play an increasing role in society. In recent years, their popularity has been ensured by the simplicity of the design and the appearance of a large number of different control systems, many of which are open-source programs.

There are many problems that can be solved in connection with quadrotors, including scientific ones. For example, the works [10, 11, 24] demonstrate the training of robots in effective flight paths, the works [7, 23]—cooperative behavior, [25, 26, 28]—navigation using cameras and RGBD sensors, and [2, 22]—tracing the trajectory by non-linear regulators. Evaluation of the orientation of such machines is usually carried out by modifications of a complementary filter [17, 18] or by an extended Kalman filter [14], and an overview of the mathematical models used to describe a quadrotor is given in [8].

A review of the main existing solutions to control real quadrotors can be found in [16]. However, there are two common properties inherent in these systems: they all work on PID-controllers, and they all use Euler angles (otherwise known as Krylov angles) as state variables.

The problem of using Euler angles is that this method has a singularity, which means that in some areas of the orientation space it will not work. For example, none of the publicly available systems can make a fully controlled flip, without disabling the main controller. Unlike Euler angles, quaternions do not have similar problems, although they are rarely used to create control systems for quadrotors. An example of a system completely based on quaternions may be found in [9].

The problem with the PID-controller is that you need to re-adjust the coefficients for each particular robot. Special instructions on how to do this are attached to open-source programs,



(a) (b)

Fig. 1. Test self-made quadrotors: (a) robot Quadracon (FlyMaple); (b) robot Bond (TRIK).

but this approach still reduces the efficiency of the system, since it does not allow to achieve the optimal parameters. Examples of the synthesis of a PID-controller for a quadrotor can be found in [13, 19, 21].

In connection with these problems, a stabilization system was created, which uses quaternions as a state variable for the synthesis of the controller and an idea similar to linearization by feedback. The result is a controller, most of the parameters of which are exactly the physical parameters of a particular quadrotor, and the remaining coefficients do not depend on the robot and can be selected once for the entire class of machines.

Some of the physical parameters of a particular robot are often difficult to measure, or they can change over time right in flight. Therefore, the system acquires a particularly great value, which allows identifying unknown parameters and using them in the stabilization system. Existing adaptive systems, for example [1, 20, 29], rely on Euler angles and, in general, standard PID-controllers.

This work consists of four parts. The first defines the mathematical model that used. The second part dealing with the stabilization system and the proof of its stability. The third part provides a system for the adaptation and identification of quadrotor thrust coefficients, which are key for stable flight. The last part describes the conducted modeling of the overall system.

It should also be noted that the stabilization system was tested on real, self-made quadrotors (Fig. 1), one of which worked on the FlyMap controller, the second on the TRIK controller (for more on this Russian development, see www.trikset.com, [27]).

2. MATHEMATICAL MODEL OF QUADROTOR

2.1. Designations

Let q be a quaternion. Denote by q_w the scalar part of q, and by q_v—the vector part. Then

$$q = (q_w, q_v) = (q_w, q_x, q_y, q_z).$$

Let r and s be two vectors in $\mathbb{R}^3$. Their scalar product is denoted as $\langle r, s\rangle$, and the vector one as $r \times s$. The product of quaternions $a = (a_w, a_v)$ and $b = (b_w, b_v)$ can be written as

$$a * b = (a_w b_w - \langle a_v, b_v\rangle, \quad a_w b_v + b_w a_v + a_v \times b_v)\,.$$

By multiplying the quaternion a by the vector r, we mean the quaternion product $a * (0, r)$.

The quaternion conjugate to q will be denoted as q^*. If $\|q\| \equiv 1$, then $q * q^* = q^* * q = (1, 0, 0, 0)$.

2.2. Quaternion of Quadrotor Rotation

Suppose we have an absolute (terrestrial) coordinate system XYZ (where XY is a horizontal plane, and the Z axis is directed upward, against gravity). Then for the system $X'Y'Z'$ associated with the quadrotor, there is a single axis u and a single angle $\phi \in [0, \pi)$ such that if you turn the hatched system around the axis u by the angle ϕ counterclockwise, the original system will be obtained. Let (u_x, u_y, u_z) be the unit vector in the system XYZ, which is the guiding for axis u. Then the quaternion describing the rotation of $X'Y'Z'$ to the position of XYZ is called

$$q = \left(\cos\frac{\phi}{2}, u_x \sin\frac{\phi}{2}, u_y \sin\frac{\phi}{2}, u_z \sin\frac{\phi}{2}\right).$$

Note that $\|q\| \equiv 1$ for any u and ϕ. Now let the vector v' be given in the system $X'Y'Z'$. For its representation v in the system XYZ, the equality $v = q * v' * q^*$ will be satisfied.

If the system $X'Y'Z'$ rotates with respect to the vector of angular velocity ω, then the law of changing the quaternion of rotation q will be follows:

$$\dot{q} = \frac{1}{2} q * \omega. \tag{1}$$

This equation uses the coordinates of the vector ω in the system associated with the quadrotor. Thus, q and ω are part of the state vector of a dynamical system describing a quadrotor. In addition, we can measure ω using gyroscopes mounted on the robot, and use it to estimate q. The derivation of Eq. (1), as well as additional information on quaternion arithmetic, can be found in [4].

2.3. Model

In addition to the Eq. (1), let's introduce two equations:

$$I\dot{\omega} = \begin{pmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ \xi(F_1 + F_3 - F_4 - F_2) \end{pmatrix} - \omega \times I\omega, \tag{2}$$

$$\ddot{H} = \frac{1}{m}\left(1 - 2q_x^2 - 2q_y^2\right)(F_1 + F_2 + F_3 + F_4) - g. \tag{3}$$

The Eq. (2) is the Newton–Euler equation for a rotating body (see [3]). Here F_i is the thrust force (N) created by the ith propeller, I is the matrix of inertia moments (kg $\times$ m^2), assumed to be diagonal, L is the distance from the center of mass to the propellers (m), and ξ is the ratio of the thrust force of the propeller with the reactive moment (m). The correspondence between the propeller numbers and their positions and directions of rotation can be seen in Fig. 2.

Equation (3) describes the quadrotor dynamics along the vertical axis Z directed against gravity (movement in the horizontal plane XY is omitted for simplicity, since in this paper we do not observe or attempt to control the position of the robot in this plane). Here H is the coordinate of the robot along the Z axis (m), m is the mass of the robot (kg), g is the gravitational acceleration (m/s^2). The multiplier $\left(1 - 2q_x^2 - 2q_y^2\right)$ is equal to the cosine of the angle between the Z and Z' axes and determines how much the total thrust of the quadrotor affects its acceleration in height. For convenience, this multiplier will be denoted by k_{mod}.

The thrust force F_i is assumed to depend quadratically on the speed of rotation of the propeller (see [15]), which in turn can be considered linearly dependent on the voltage applied to the ith motor. Imagine that the control u_i is the voltage on the ith motor, while given on a scale from 0 to 1,

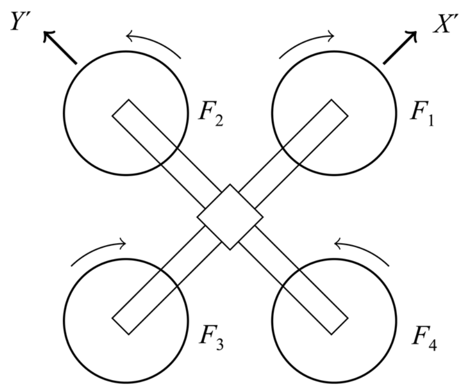


Fig. 2. A schematic image of a quadrotor.

where 0 is the absence of voltage, and 1 is the maximum voltage to the motor. Then it is possible to enter the coefficient of proportionality of the square of the voltage and the thrust force K_i, which will also have the meaning of the maximum thrust value of the ith propeller. If $F_i = K_i u_i^2$, then we can also replace the control variable by introducing $U_i = u_i^2$. Then the relation between the thrust power and the control is assumed to be linear with the proportionality coefficient K_i. Estimation of the coefficient K_i is considered in section 5. Section 4 implies that the values of K_i are known, and, moreover, for simplicity of presentation, they are equal to a single coefficient K.

3. STABILIZATION SYSTEM

3.1. Orientation Stabilization

The main idea of the control design lies in the creation of two successive controllers: first for the quaternion of rotation, and then for the angular velocity. If the control goal is the coincidence of the reference systems XYZ and $X'Y'Z'$ (i.e., static hovering), then the fulfillment of the goal means that the q quaternion has the value $(1, 0, 0, 0)$. Then we define the relevant quaternion q_d, which will be the control target. In the simplest case, its value will be $(1, 0, 0, 0)$, however, it is possible to use any other values to implement more complex movements, up to and including flips.

Algorithm.

(1) Define the desired derivative of the quaternion:

$$\tau_d = k_p(q_d - \langle q_d, q\rangle\, q),$$

where $k_p > 0$ is the gain, and $\langle q_d, q\rangle$ is scalar product of quaternions as vectors in $\mathbb{R}^4$.

(2) By inverting Eq. (1), we obtain the target angular velocity:

$$\omega_d = 2q^* * \tau_d.$$

(3) Determine the desired angular acceleration:

$$\rho_d = k_d(\omega_d - \omega),$$

where $k_d > 0$ is the gain.

(4) By inverting Eq. (2), we obtain the target moment:

$$M_d = I\rho_d + \omega \times I\omega.$$

As a result, $\dot{\omega} \equiv \rho_d$.

(5) The moment M_d gives us 3 linear equations on the controls U_i. Having defined the fourth control in any way (for example, by setting the total thrust of the four propellers), we can completely calculate the control U_i.

The scalar product in step 1 is necessary for the orthogonality condition of τ_d and q in $\mathbb{R}^4$ to be satisfied, which in turn is a consequence of the condition $\|q\| \equiv 1$. It turns out that it is enough that, with the quaternion multiplication in step 2, the scalar part of the resulting quaternion would turn to 0 and ω_d would be a vector.

Note that the only parameters of the controller are the coefficients k_p and k_d, which do not depend on the physical parameters of the system. That is, once found, the coefficients will be able to stabilize any quadrotor (provided that there are no errors in determining the physical parameters of the system).

3.2. Altitude Stabilization

The moment of forces M_d defines 3 equations on U_i. One option to fully define controls is to add an altitude controller. Let the target altitude H_d be given, which the quadrotor needs to withstand. Apply the PD-controller, setting target acceleration:

$$\Upsilon_d = A_p\,(H_d - H) - A_d\dot{H}, \tag{4}$$

where A_p and A_d are positive. From Eq. (3) we obtain the fourth equation for the control actions:

$$\sum_{i=1}^{4} U_i = \frac{m}{K}\frac{(\Upsilon_d + g)}{k_{mod}}. \tag{5}$$

3.3. General Control Law

Let us introduce the G matrix connecting the target moments and accelerations with the control actions U_i:

$$G = \begin{pmatrix} 0 & KL & 0 & -KL \\ -KL & 0 & KL & 0 \\ K\xi & -K\xi & K\xi & -K\xi \\ \dfrac{K}{m} & \dfrac{K}{m} & \dfrac{K}{m} & \dfrac{K}{m} \end{pmatrix}^{-1} = \begin{pmatrix} 0 & -\dfrac{1}{2KL} & \dfrac{1}{4K\xi} & \dfrac{m}{4K} \\ \dfrac{1}{2KL} & 0 & -\dfrac{1}{4K\xi} & \dfrac{m}{4K} \\ 0 & \dfrac{1}{2KL} & \dfrac{1}{4K\xi} & \dfrac{m}{4K} \\ -\dfrac{1}{2KL} & 0 & -\dfrac{1}{4K\xi} & \dfrac{m}{4K} \end{pmatrix}.$$

And then the controls are calculated simply from the matrix multiplication:

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix} = \begin{pmatrix} 0 & -\dfrac{1}{2KL} & \dfrac{1}{4K\xi} & \dfrac{m}{4K} \\ \dfrac{1}{2KL} & 0 & -\dfrac{1}{4K\xi} & \dfrac{m}{4K} \\ 0 & \dfrac{1}{2KL} & \dfrac{1}{4K\xi} & \dfrac{m}{4K} \\ -\dfrac{1}{2KL} & 0 & -\dfrac{1}{4K\xi} & \dfrac{m}{4K} \end{pmatrix} \begin{pmatrix} M_{dx} \\ M_{dy} \\ M_{dz} \\ \dfrac{\Upsilon_d + g}{k_{mod}} \end{pmatrix}. \tag{6}$$

3.4. Stability of Control Law

The stability of the altitude when using the controller (4) is obvious. Let us now prove that Algorithm provides the goal of altitude control for almost all initial states. Let $q_d = (1,0,0,0)$. This assumption is non-limiting, because otherwise we can introduce a modified quaternion of rotation $q' = q_d^* * q$ as a new state. With this choice of q_d, the equations of the closed-loop system for orientation are reduced to

$$\begin{aligned} \dot{q} &= \frac{1}{2} q * \omega, \\ \dot{\omega} &= -2k_p k_d q_v - k_d \omega. \end{aligned} \tag{7}$$

Theorem 1. *Let* $k_p, k_d > 0$. *Then the point* $L_1 = [(1,0,0,0),(0,0,0)]$ *is an asymptotically stable equilibrium point of the system* (7), $L_2 = [(-1,0,0,0),(0,0,0)]$ *is an unstable equilibrium point, and the region of attraction of the point* L_1 *is the domain* $\mathbb{S}^4 \times \mathbb{R}^3 \backslash L_2$.

Proof. As a candidate for the Lyapunov function, consider the function

$$V(q,\omega) = h_{11} \Big\| q_v{}^2 + 2h_{12}\langle q_v, \omega\rangle + h_{22} \Big\| \omega^2 + 2\nu(1 - q_w). \tag{8}$$

Obviously, $V(L_1) = 0$ and $V(L_2) = 4\nu$. It is necessary to find the coefficients h_{ij} and ν so that the derivative V with respect to the system will be negative everywhere, except for the points L_1 and L_2, and V itself will be positive definite. For convenience, we denote $k_q = 2k_p k_d$.

The derivative of the function V with respect to the system can be written as

$$\dot{V} = g_{11}(q_w)\|q_v\|^2 + 2g_{12}(q_w)\langle q_v, \omega\rangle + g_{22}(q_w)\|\omega\|^2, \tag{9}$$

where the coefficients g_{ij} satisfy following equality

$$\begin{aligned} G(q_w) = \begin{pmatrix} g_{11}(q_w) & g_{12}(q_w) \\ g_{12}(q_w) & g_{22}(q_w) \end{pmatrix} = \begin{pmatrix} 0 & \frac{\nu}{2} \\ \frac{\nu}{2} & 0 \end{pmatrix} \\ + \begin{pmatrix} h_{11} & h_{12} \\ h_{12} & h_{22} \end{pmatrix} \begin{pmatrix} 0 & \frac{q_w}{2} \\ -k_q & -k_d \end{pmatrix} + \begin{pmatrix} 0 & -k_q \\ \frac{q_w}{2} & -k_d \end{pmatrix} \begin{pmatrix} h_{11} & h_{12} \\ h_{12} & h_{22} \end{pmatrix}. \end{aligned} \tag{10}$$

Another way to write Eq. (9)

$$\dot{V} = \begin{pmatrix} q_v^\top & \omega^\top \end{pmatrix} [G(q_w) \otimes I_3] \begin{pmatrix} q_v \\ \omega \end{pmatrix},$$

shows that the negative definiteness of $\dot{V}$ is equivalent to the negative definiteness of the matrix G. It is also obvious that $(q_v, \omega) \equiv (0,0)$ only at the points L_1 and L_2.

Now let's show that there are coefficients h_{ij} and ν such that the matrix G will be negative definite. The problem is complicated by the fact that, according to Eq. (10), the coefficients of the matrix, in general, depend on q_w. To solve this problem, we use the KYP-lemma (see [6, 12]).

Let

$$A = \begin{pmatrix} 0 & 0 \\ -k_q & -k_d \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad H = \begin{pmatrix} h_{11} & h_{12} \\ h_{12} & h_{22} \end{pmatrix}, \quad N = \begin{pmatrix} 0 & \frac{\nu}{2} \\ \frac{\nu}{2} & 0 \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

Introduce a linear dynamic system

$$\dot{x} = Ax + Bu.$$

The condition of negative definiteness of the matrix G is then written as

$$\begin{pmatrix} x_1 & x_2 & \frac{q_w}{2}x_2 \end{pmatrix} \left[\begin{pmatrix} HA + A^\top H & HB \\ B^\top H & 0 \end{pmatrix} + \begin{pmatrix} N & 0 \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} x_1 \\ x_2 \\ \frac{q_w}{2}x_2 \end{pmatrix} < 0, \tag{11}$$

$$|x_1| + |x_2| \neq 0.$$

Let

$$u = \frac{q_w}{2}x_2, \qquad |q_w| \leqslant 1 \ \Rightarrow \ (x_2 + 2u)(x_2 - 2u) \geqslant 0$$

$$\Rightarrow \begin{pmatrix} x_1 & x_2 & u \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ u \end{pmatrix} \geqslant 0.$$

Define

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{pmatrix} \quad \text{and} \quad S = M + \begin{pmatrix} N & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \frac{\nu}{2} & 0 \\ \frac{\nu}{2} & 1 & 0 \\ 0 & 0 & -4 \end{pmatrix}.$$

Then condition (11) will follow from the matrix inequality

$$S + \begin{pmatrix} HA + A^\top H & HB \\ B^\top H & 0 \end{pmatrix} < 0. \tag{12}$$

The KYP-lemma states that the existence of the matrix $H = H^\top$ such that inequality (12) is satisfied is equivalent to the fulfillment of the inequality

$$\begin{pmatrix} x^* & u^* \end{pmatrix} S \begin{pmatrix} x \\ u \end{pmatrix} < 0 \tag{13}$$

for all $(x, u) \in \mathcal{M}_{i\omega}$ for each $\omega \in \mathbb{R} \cup \{\infty\}$, where the sets $\mathcal{M}_\lambda$ for $\lambda \in \mathbb{C}$ is defined as

$$\mathcal{M}_\lambda = \begin{cases} (x, u) \mid x \in \mathbb{C}^2 \backslash \{0\}, \ u \in \mathbb{C}, \ \lambda x = Ax + Bu, \ \text{for } |\lambda| < \infty \\ (0, u) \mid u \in \mathbb{C}^1 \backslash \{0\}, \ \text{for } |\lambda| = \infty. \end{cases}$$

In the case when $i\omega$ is not an eigenvalue of A, condition (13) can be written in terms of the frequency response:

$$\begin{pmatrix} (i\omega I - A)^{-1} B \\ 1 \end{pmatrix}^* S \begin{pmatrix} (i\omega I - A)^{-1} B \\ 1 \end{pmatrix} < 0. \tag{14}$$

Expressing frequency response

$$(i\omega I - A)^{-1} B = \begin{pmatrix} i\omega & 0 \\ k_q & i\omega + k_d \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{-\omega^2 + i\omega k_d} \begin{pmatrix} i\omega + k_d \\ -k_q \end{pmatrix} = \begin{pmatrix} -\dfrac{i}{\omega} \\ \dfrac{k_q(\omega + ik_d)}{\omega(\omega^2 + k_d^2)} \end{pmatrix}$$

and substituting it in (14), we obtain the condition

$$\begin{pmatrix} \dfrac{i}{\omega} & \dfrac{k_q(\omega - ik_d)}{\omega(\omega^2 + k_d^2)} & 1 \end{pmatrix} \begin{pmatrix} 0 & \dfrac{\nu}{2} & 0 \\ \dfrac{\nu}{2} & 1 & 0 \\ 0 & 0 & -4 \end{pmatrix} \begin{pmatrix} -\dfrac{i}{\omega} \\ \dfrac{k_q(\omega + ik_d)}{\omega(\omega^2 + k_d^2)} \\ 1 \end{pmatrix} = \frac{k_q^2 - \nu k_q k_d}{\omega^2(\omega^2 + k_d^2)} - 4 \quad < 0,$$

for which fulfilment it is enough to put

$$\nu > \frac{k_q}{k_d} > 0. \tag{15}$$

If $i\omega$ is an eigenvalue of A, then condition (13) should be checked for all vectors of the form $(x_\omega + x_s, u)$ such that

$$x_\omega \in \ker(i\omega I - A), \quad x_s \perp \ker(i\omega I - A), \quad Ax_s - i\omega x_s + Bu = 0.$$

In our case, $\omega = 0$, and for it

$$x_\omega = \begin{pmatrix} k_d & -k_q \end{pmatrix}^\top.$$

Moreover,

$$x_s = \begin{pmatrix} 0 & 1 \end{pmatrix}^\top \perp B,$$

which means $(x_s, u) = (0, 0)$. Then the frequency condition reduces to the inequality

$$\begin{pmatrix} k_d & -k_q \end{pmatrix} \begin{pmatrix} 0 & \dfrac{\nu}{2} \\ \dfrac{\nu}{2} & 1 \end{pmatrix} \begin{pmatrix} k_d \\ -k_q \end{pmatrix} < 0,$$

which in turn is equivalent to (15). Thus, the existence of the matrix H and the number ν such that the derivative of V is negative definite with respect to the system, is proven.

It remains to show that the function V is positive definite. From the negative definiteness of the matrix G and equality (10), choosing $q_w = 1$, we get:

$$-2k_q h_{12} = g_{11} < 0,$$
$$-2k_d h_{22} + h_{12} = g_{22} < 0,$$

whence immediately follows the positivity of the coefficient h_{22}. Therefore, since $\|q_v\| \leqslant 1$ and $|q_w| \leqslant 1$, function $V \to +\infty$ with $\|\omega\| \to +\infty$. Therefore, there exists $R > 0$ such that for any $q \in \mathbb{S}^4$ from $\|\omega\| > R$ it follows $V > 1$. Consider the set $E = \mathbb{S}^4 \times B^3(R)$ containing all pairs of quaternions of rotation q and angular velocities w such that $\|w\| \leqslant R$. The set E is compact, so there is a point L_3 in which

$$V(L_3) = \min_{x \in E} V(x).$$

Since $V(L_1) = 0$ and $L_1 \in E$, the value of $V(L_3) \leqslant 0$. Thus, L_3 cannot be on the boundary of the set E, which means that there is an open set $U \subset E$ containing L_3. Denote by $\psi_{L_3}(t)$ the solution of system (7), emitted from the point L_3. Then there will be $\Delta t > 0$ such that $\psi_{L_3}(\tau) \in U$ for all $\tau \in [0, \Delta t]$. Now suppose that $\dot{V}(L_3) < 0$. Choosing Δt sufficiently small, we get that $V(\psi_{L_3}(\tau))$ decreases, which contradicts the minimality of $V(L_3)$. So $\dot{V}(L_3)$ cannot be less than zero. However, there are only two such points: L_1 and L_2. $V(L_1) < V(L_2)$, and then $L_3 \equiv L_1$. That is, the function V does not take negative values, and zero is reached only at the point L_1. Hence, the function V is positive definite.

So, we have found a positive definite function V, the derivative of which with respect to the system is negative definite everywhere, except for two equilibrium points, L_1 and L_2. By Lyapunov's theorem on asymptotic stability, point L_1 is asymptotically stable, while its domain of attraction is the entire space, except for point L_2.

4. ADAPTATION SYSTEM

In a real machine, unfortunately, due to small errors in the manufacture of propellers or engines, as well as due to possible damage, the coefficients K for each propeller will differ slightly. And this difference has a significant effect, first of all, on the angular stabilization, since the robot begins to bank one side, and the condition for quaternions $q \to q_d$ is not satisfied. Further, we consider the case when each engine has its own thrust coefficient K_i, as well as its own estimate of this coefficient $\hat{K}_i$. Consider a closed system, which is obtained in this case.

The equations of the model can be written as:

$$\begin{pmatrix} I\dot{\omega} + \omega \times I\omega \\ \dfrac{\ddot{H} + g}{k_{mod}} \end{pmatrix} = \begin{pmatrix} 0 & LK_2 & 0 & -LK_4 \\ -LK_1 & 0 & LK_3 & 0 \\ \xi K_1 & -\xi K_2 & \xi K_3 & -\xi K_4 \\ \dfrac{K_1}{m} & \dfrac{K_2}{m} & \dfrac{K_3}{m} & \dfrac{K_4}{m} \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix}, \tag{16}$$

and the controller Eqs. (6) are as follows:

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix} = \begin{pmatrix} 0 & -\dfrac{1}{2\hat{K}_1 L} & \dfrac{1}{4\hat{K}_1 \xi} & \dfrac{m}{4\hat{K}_1} \\ \dfrac{1}{2\hat{K}_2 L} & 0 & -\dfrac{1}{\hat{4}K_2 \xi} & \dfrac{m}{4\hat{K}_2} \\ 0 & \dfrac{1}{2\hat{K}_3 L} & \dfrac{1}{4\hat{K}_3 \xi} & \dfrac{m}{4\hat{K}_3} \\ -\dfrac{1}{2\hat{K}_4 L} & 0 & -\dfrac{1}{4\hat{K}_4 \xi} & \dfrac{m}{4\hat{K}_4} \end{pmatrix} \begin{pmatrix} M_{dx} \\ M_{dy} \\ M_{dz} \\ \dfrac{\Upsilon_d + g}{k_{mod}} \end{pmatrix}. \tag{17}$$

Denote $\frac{1}{\hat{K}_i}$ by Ψ_i. Substituting (17) into (16) and expressing M_d according to Algorithm, we get

$$\begin{pmatrix} I\dot{\omega} + \omega \times I\omega \\ \dfrac{\ddot{H} + g}{k_{mod}} \end{pmatrix} = B \begin{pmatrix} -Ik_d\omega - Ik_q q_v + \omega \times I\omega \\ \dfrac{\Upsilon_d + g}{k_{mod}} \end{pmatrix}, \tag{18}$$

where the coefficients of the matrix B are as follows:

$$B[1:2,\bullet] = \begin{pmatrix} \frac{1}{2}(K_2\Psi_2 + K_4\Psi_4) & 0 \\ 0 & \frac{1}{2}(K_1\Psi_1 + K_3\Psi_3) \\ -\frac{\xi}{2L}(K_2\Psi_2 - K_4\Psi_4) & -\frac{\xi}{2L}(K_1\Psi_1 - K_3\Psi_3) \\ \frac{1}{2Lm}(K_2\Psi_2 - K_4\Psi_4) & -\frac{1}{2Lm}(K_1\Psi_1 - K_3\Psi_3) \end{pmatrix},$$

$$B[3,\bullet] = \begin{pmatrix} -\frac{L}{4\xi}(K_2\Psi_2 - K_4\Psi_4) \\ -\frac{L}{4\xi}(K_1\Psi_1 - K_3\Psi_3) \\ \frac{1}{4}(K_1\Psi_1 + K_2\Psi_2 + K_3\Psi_3 + K_4\Psi_4) \\ \frac{1}{4\xi m}(K_1\Psi_1 - K_2\Psi_2 + K_3\Psi_3 - K_4\Psi_4) \end{pmatrix},$$

$$B[4,\bullet] = \begin{pmatrix} \frac{Lm}{4}(K_2\Psi_2 - K_4\Psi_4) \\ -\frac{Lm}{4}(K_1\Psi_1 - K_3\Psi_3) \\ \frac{\xi m}{4}(K_1\Psi_1 - K_2\Psi_2 + K_3\Psi_3 - K_4\Psi_4) \\ \frac{1}{4}(K_1\Psi_1 + K_2\Psi_2 + K_3\Psi_3 + K_4\Psi_4) \end{pmatrix}.$$

As might be expected, in the case of exact coincidence of the estimates of the coefficients with their real values, i.e., in the case of $K_i\Psi_i \equiv 1$, the matrix B coincides with the identity one.

In Eq. (18) let's proceed to the linear system

$$\begin{pmatrix} I_x\dot{\omega}_x \\ I_y\dot{\omega}_y \\ I_z\dot{\omega}_z \\ \ddot{H} + g \end{pmatrix} = B \begin{pmatrix} -I_x k_d \omega_x - I_x k_q q_x \\ -I_y k_d \omega_y - I_y k_q q_y \\ -I_z k_d \omega_z - I_z k_q q_z \\ A_p(H_d - H) - A_d\dot{H} + g \end{pmatrix}. \tag{19}$$

Now let's introduce new notation to go to the full matrix formulation of the problem. Define the new state vector $X = \left(X_1^\top, X_2^\top\right)^\top$, where

$$X_1 = \left(2I_x q_x,\ 2I_y q_y,\ 2I_z q_z,\ H - H_d\right)^\top,$$
$$X_2 = \left(I_x\omega_x,\ I_y\omega_y,\ I_z\omega_z,\ \dot{H}\right)^\top,$$

and denote by the vector $\tilde{g}$ the total effect of gravity on the system:

$$\tilde{g} = (0,\ 0,\ 0,\ g)^\top.$$

Then system (19) can be rewritten as

$$\begin{aligned} \dot{X}_1 &= X_2, \\ \dot{X}_2 &= B(-T_1X_1 - T_2X_2 + \tilde{g}) - \tilde{g}, \end{aligned} \tag{20}$$

where the matrices T_1 and T_2 are the matrices of controller coefficients:

$$T_1 = \operatorname{diag}\left(\frac{k_q}{2}, \frac{k_q}{2}, \frac{k_q}{2}, A_p\right), \qquad T_2 = \operatorname{diag}\left(k_d, k_d, k_d, A_d\right).$$

In fact, they are diagonal, but only their symmetry, positive definiteness and, for the sake of convenience of computations, the commutability between themselves is assumed below.

Now, to estimate Ψ_i, we can apply the speed-gradient method in differential form (see [5]). This method allows one to synthesize a control that ensures that the given objective function $Q(X)$ tends to zero. For this, the function only has to satisfy four conditions:

1) condition of regularity: $Q(X)$ and $\dot{Q}(X, \Psi)$ are continuous;

2) growth condition: $Q(X) \geqslant 0$ and $Q(X) \to +\infty$ at $|X| \to +\infty$;

3) the condition of convexity of the derivative with respect to the system:

$$\dot{Q}(X, \Psi) - \dot{Q}(X, \Psi') \geqslant (\Psi - \Psi')^\top \nabla_\Psi \dot{Q}(X, \Psi');$$

4) goal reachability condition: $\exists \Psi^* \in \mathbb{R}^4$ and $\rho > 0 : \dot{Q}(X, \Psi^*) < -\rho Q(X) \quad \forall X$.

As an objective function for the speed-gradient method, let's consider the function

$$Q(X) = \|T_1X_1 + T_2X_2\|^2 + X_1^\top T_1^2 X_1 = \begin{pmatrix} X_1^\top & X_2^\top \end{pmatrix} \begin{pmatrix} 2T_1^2 & T_1T_2 \\ T_1T_2 & T_2^2 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}. \tag{21}$$

Due to the positive definiteness of the matrices T_1 and T_2, $Q(X)$ is also a positive definite quadratic form, which means that its tendency to zero ensures that X also tends to zero, moreover, the growth and regularity conditions are obviously satisfied.

Take the derivative with respect to the system for the function $Q(X)$:

$$\dot{Q}(X, B) = 2\left(X_2^\top - X_1^\top T_1 B^\top - X_2^\top T_2 B^\top + g^\top (B^\top - I)\right) \begin{pmatrix} 2T_1^2 & T_1T_2 \\ T_1T_2 & T_2^2 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}. \tag{22}$$

The convexity condition of the function $\dot{Q}$ is satisfied, since it is linear in the B matrix, which in turn is linear in the estimates of Ψ_i.

Let us introduce an auxiliary function $Z(X, J)$ such that $Z(X, J) = -\nabla_{\Psi_i} \dot{Q}(X, B)$, where $J = \frac{\partial B}{\partial \Psi_i}$:

$$Z(X, J) = \begin{pmatrix} X_1^\top & X_2^\top & \tilde{g}^\top \end{pmatrix} \begin{pmatrix} 2T_1T_2JT_1 & T_1T_2JT_2 + T_1J^\top T_2^2 & -J^\top T_1T_2 \\ T_2J^\top T_1T_2 + T_2^2JT_1 & 2T_2^2JT_2 & -J^\top T_2^2 \\ -T_1T_2J & -T_2^2J & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ \tilde{g} \end{pmatrix}.$$

Theorem 2. *Let $T_2^2 > T_1$. Then the algorithm*

$$\dot{\hat{K}}_i = -\gamma \hat{K}_i^2 Z\left(X, \frac{\partial B}{\partial \Psi_i}\right) \tag{23}$$

ensures the stability of the linearized system and the tendency of $\hat{K}_i \to K_i$.

Proof. We have already shown that for the function $Q(X)$ the conditions of regularity, growth and convexity of the speed-gradient method are satisfied. It remains to check the last condition—the condition of the reachability of the goal. To do this, we substitute $B \equiv I$ in function (22), which corresponds to ideally found estimates of Ψ_i. We shall obtain

$$\dot{Q}_I(X) = -\begin{pmatrix} X_1^\top & X_2^\top \end{pmatrix} \begin{pmatrix} 2T_1^2T_2 & 2T_1T_2^2 - 2T_1^2 \\ 2T_1T_2^2 - 2T_1^2 & 2T_2^3 - 2T_1T_2 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = -X^\top RX.$$

It suffices to show that the matrix of this quadratic form R is positive definite. Let us try to find a non-singular coordinate transformation that turns this matrix into a block-diagonal one. One such transformation may be

$$R = \begin{pmatrix} 2T_1^2T_2 & 2T_1T_2^2 - 2T_1^2 \\ 2T_1T_2^2 - 2T_1^2 & 2T_2^3 - 2T_1T_2 \end{pmatrix}$$
$$= \begin{pmatrix} T_1 & T_1T_2^{-1} \\ 0 & I \end{pmatrix} \begin{pmatrix} 2T_1T_2^{-1} & 0 \\ 0 & 2T_2^3 - 2T_1T_2 \end{pmatrix} \begin{pmatrix} T_1 & 0 \\ T_1T_2^{-1} & I \end{pmatrix},$$

from where the criterion of positive definiteness of a matrix of R follows:

$$T_1T_2^{-1} > 0,$$
$$T_2^3 - T_1T_2 > 0,$$

which, taking into account the positivity of the matrices T_1 and T_2, is transformed into the condition

$$T_2^2 > T_1. \tag{24}$$

So, if condition (24) is fulfilled for the coefficient matrices, then the adaptation of the coefficients Ψ_i according to the speed-gradient method will lead to the limit $Q(X, B) \to 0$ and, therefore, $X \to 0$, which proves statement about the stability of the linearized system. Does this mean that the estimates of Ψ_i converge to their true values $\frac{1}{K_i}$? To prove this fact, consider the system (20). Since the point $X = 0$ is a stable equilibrium point for an adaptive system, in the limit the following equality should be satisfied

$$\tilde{g} = B\tilde{g},$$

which means that the matrix B in the limit has an eigenvector $(0, 0, 0, 1)^\top$ with an eigenvalue 1. Expanding this equality as a system of equations, we get

$$K_2\Psi_2 - K_4\Psi_4 = 0,$$
$$K_1\Psi_1 - K_3\Psi_3 = 0,$$
$$K_1\Psi_1 - K_2\Psi_2 + K_3\Psi_3 - K_4\Psi_4 = 0,$$
$$K_1\Psi_1 + K_2\Psi_2 + K_3\Psi_3 + K_4\Psi_4 = 4,$$

whence a set of equalities follows immediately: $K_i\Psi_i = 1$ and, accordingly, $\Psi_i = \frac{1}{K_i}$ for all $i \in \{1, \dots, 4\}$.

The adaptation by the speed-gradient method can be written as:

$$\dot{\Psi}_i = -\frac{\dot{\hat{K}}_i}{\hat{K}_i^2} = \gamma Z\left(X, \frac{\partial B}{\partial \Psi_i}\right).$$

Unknown, but constant values of the real thrust coefficients K_i enter the corresponding partial derivative $\frac{\partial B}{\partial \Psi_i}$ linearly, and therefore can be taken out of the function Z into the factor γ. Expressing from here $\dot{\hat{K}}_i$, we obtain (23).

For the completeness of the algorithm, we present the concrete values of the used derivatives $\frac{\partial B}{\partial \Psi_i}$, while counting the coefficients K_i taken as a factor of γ:

$$\frac{\partial B}{\partial \Psi_1} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{L}{4\xi} & -\frac{Lm}{4} \\ 0 & -\frac{\xi}{2L} & \frac{1}{4} & \frac{\xi m}{4} \\ 0 & -\frac{1}{2Lm} & \frac{1}{4\xi m} & \frac{1}{4} \end{pmatrix}, \quad \frac{\partial B}{\partial \Psi_2} = \begin{pmatrix} \frac{1}{2} & 0 & -\frac{L}{4\xi} & \frac{Lm}{4} \\ 0 & 0 & 0 & 0 \\ -\frac{\xi}{2L} & 0 & \frac{1}{4} & -\frac{\xi m}{4} \\ \frac{1}{2Lm} & 0 & -\frac{1}{4\xi m} & \frac{1}{4} \end{pmatrix},$$
$$\frac{\partial B}{\partial \Psi_3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{L}{4\xi} & \frac{Lm}{4} \\ 0 & \frac{\xi}{2L} & \frac{1}{4} & \frac{\xi m}{4} \\ 0 & \frac{1}{2Lm} & \frac{1}{4\xi m} & \frac{1}{4} \end{pmatrix}, \quad \frac{\partial B}{\partial \Psi_4} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{L}{4\xi} & -\frac{Lm}{4} \\ 0 & 0 & 0 & 0 \\ \frac{\xi}{2L} & 0 & \frac{1}{4} & -\frac{\xi m}{4} \\ -\frac{1}{2Lm} & 0 & -\frac{1}{4\xi m} & \frac{1}{4} \end{pmatrix}. \tag{25}$$

5. MODELING

The verification of the algorithm presented in the previous section was carried out using the simulation of a quadrotor flight in the MATLAB R2016a system. For this, a quadrotor model, a control system and an adaptation system were implemented. The physical parameters for the simulation were approximately measured in one of the real manufactured quadrotor (robot FlyMaple), and they are listed in the table.

It also shows the applied coefficients of the controllers, which satisfy condition (24), and the "true" thrust coefficients chosen for the simulation, which are unknown for the control algorithm. In the initial position, the robot is assumed to be at rest on a horizontal surface, its coordinate in altitude is taken as 0.

To test the algorithm, 3 experiments were carried out, in the first of them there was no adaptation ($\gamma = 0$), in the second—$\gamma = 0.0005$, in the third—$\gamma = 0.005$. The initial estimates of the coefficients $\hat{K}_i$ are 8.6 N, the simulation time is 10 s.

Parameter values in the simulation

Physical parameters	Parameters of controllers	Thrust coefficients
$m = 1.3$ kg $L = 0.22$ m $\xi = 0.017$ m $I_x = 0.12$ kg×m^2 $I_y = 0.12$ kg×m^2 $I_z = 0.22$ kg×m^2 $g = 9.8$ m/s^2	$k_p = 2.6$ $k_d = 10.0$ $A_p = 2.0$ $A_d = 5.0$ $H_d = 3$ m	$K_1 = 12.6$ N $K_2 = 18.6$ N $K_3 = 4.6$ N $K_4 = 7.0$ N

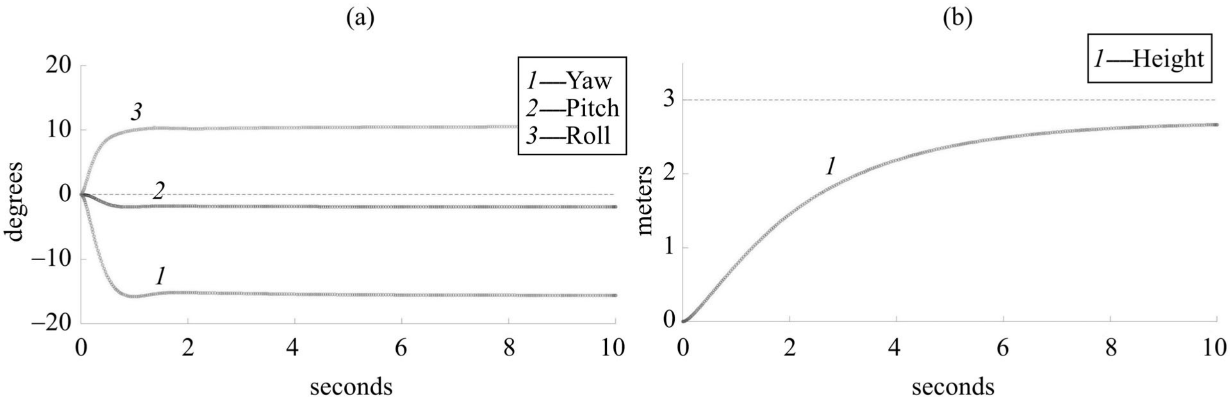


Fig. 3. Lack of adaptation, $\gamma = 0$: (a) orientation, Euler angles; (b) altitude H.

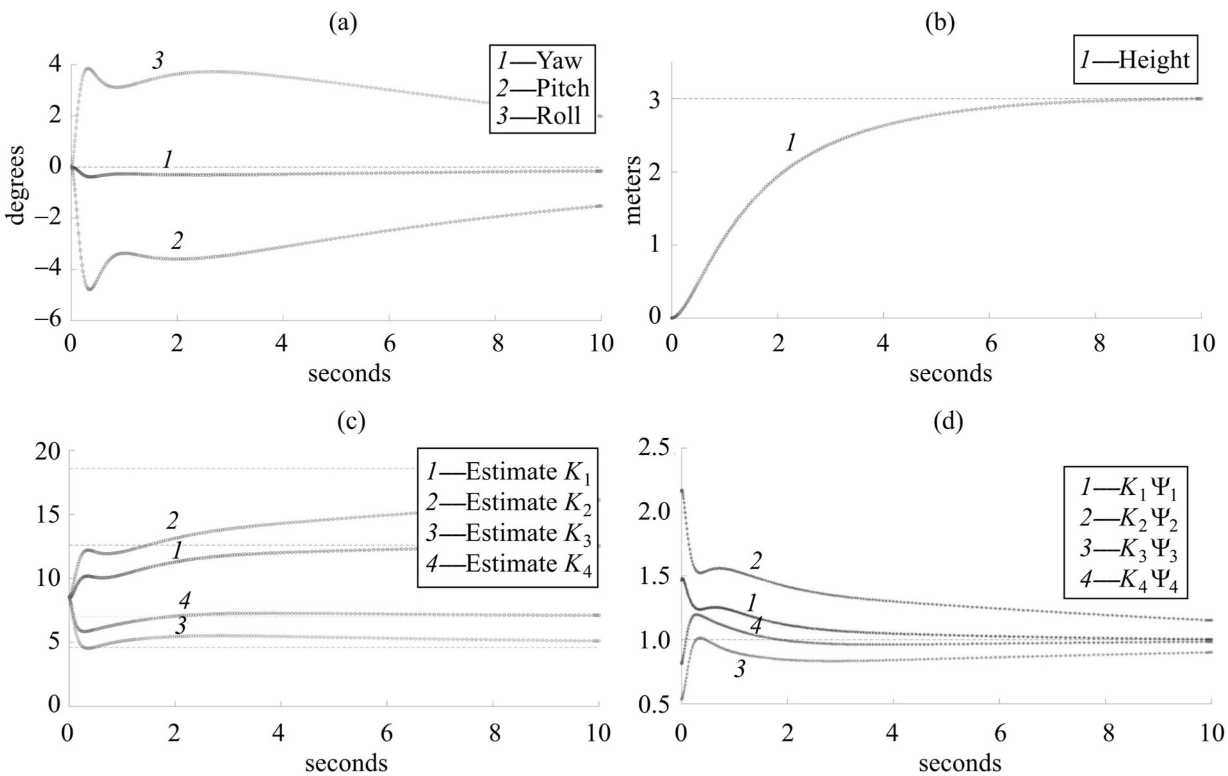


Fig. 4. Slow adaptation, $\gamma = 0.0005$: (a) orientation, Euler angles; (b) altitude H; (c) estimates of the coefficients $\hat{K}_i$; (d) values $K_i\Psi_i$.

In the first case (Fig. 3), in the absence of adaptation, the robot enters a stable movement, but does not achieve the control goal, in particular, it has a nonzero roll and pitch values, which in a real system would result in acceleration in the XY plane.

In the case of slow adaptation (Fig. 4), the state of the robot asymptotically reaches its target values, as well as the thrust coefficients. It can also be noted that in the absence of adaptation, the error in roll and pitch was 10–15 degrees, whereas in its presence it did not exceed 5, and after 10 seconds it was already 2 degrees. Errors of this order may already be acceptable in a real system.

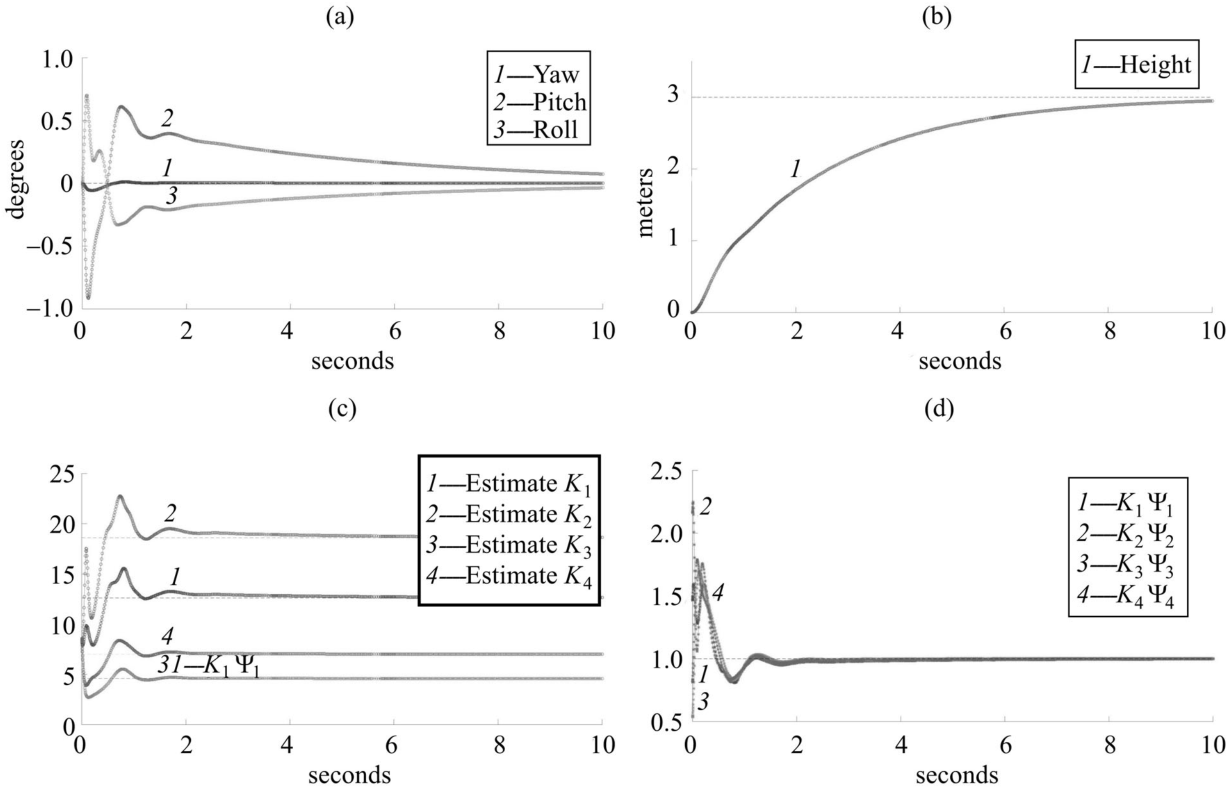


Fig. 5. Fast adaptation, $\gamma = 0.005$: (a) orientation, Euler angles; (b) altitude H; (c) estimates of the coefficients $\hat{K}_i$; (d) values $K_i \Psi_i$.

With fast adaptation (Fig. 5), the coefficients converge to their true values much faster, and the roll and pitch error does not exceed 1 degree. However, it can be noted that in the initial period the coefficients, and with them the orientation, experience strong fluctuations. A further increase in the coefficient γ causes the calculations to become unstable.

6. CONCLUSIONS

In this paper, a quaternion-based parametric controller for stabilizing a quadrotor, as well as a system for identifying thrust coefficients on each propeller, was described. A numerical simulation was also performed, confirming the efficiency of the proposed algorithms.

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