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CONTROL SCIENCES

Aeromagnetic Gradiometry and Its Application to Navigation

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Abstract—Modern methods of airborne magnetic field measurements are described. A stochastic algorithm to compensate the deviations between the indications of an aeromagnetometer and an aeromagnetic gradiometer is considered. An integration algorithm for the inertial and correlation-extremal navigation systems is briefly described. An advantage of using magnetic field gradient measurements as navigational data is justified. The performance of the integration algorithm is illustrated by numerical simulation.

Keywords: correlation-extremal navigation system, aeromagnetic gradiometry, magnetic compensation, integrated navigation system

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1. INTRODUCTION

Nowadays, several research groups are working on autonomous navigation tools using the physical fields of the Earth—magnetic, gravitational, topographic, optical, thermal, and others [1–4]. Navigation in such systems is based on comparing the information acquired by an airborne measurement system of field parameters with the information stored in an airborne computer (a field map or a reference). Generally, comparison procedures calculate a certain functional (a correlation function) and find its extremum. The navigation systems that employ geophysical fields are accordingly called correlation-extremal navigation systems (CENS) [5, 6]. In Russia, the farther of this research direction are Academician A.A. Krassovskii [7] and Professor V.P. Tarasenko [8], who suggested correlation-extremal navigation methods using geophysical fields and radar imaging of the Earth surface in the early 1960s. Later on, correlation-extremal navigation algorithms were created with an active application of linear and nonlinear filtering methods [6, 9–12]. The state-of-the-art theory involves multi-alternative filtering [13] and sequential Monte Carlo simulation [9, 14].

A continuous development of navigation methods allows to introduce new approaches that improve the accuracy and performance of the existing systems as well as to design new airborne complexes.

In the sense of navigation, it is particularly interesting to consider measuring systems for the gradient of a certain geophysical field. Such measurements have enhanced sensitivity and noise immunity; in contrast to the surface radiation fields (optical, infrared, gamma radiation), the former fields are independent of insolation, meteorological conditions or seasons, which gives a certain advantage in navigation.

At the modern stage of development, among airborne gradiometric systems one should consider vector and tensor magnetic gradiometers as well as tensor gravity gradiometers. Note that only the magnetic gradiometers yield airborne measurements comparable with terrestrial measurements in terms of accuracy and spatial resolution [15]. For this reason, we will analyze magnetic gradiometers as most promising [16]. Other gradiometric systems can be studied using the same approaches.

2. DEVICES AND METHODS FOR AIRBORNE MAGNETIC FIELD MEASUREMENTS

Quantitative characteristics of the magnetic field are the magnetic induction vector B, which is measured in teslas (T), and the magnetic field vector H, which is measured in amperes per meter (A/m). Here and in the sequel, we will write the three-dimensional vectors as column matrices composed of three elements. In the SI system, these characteristics have the relationship $B = \mu\mu_0 H$, see [17], where μ denotes the relative permeability of a medium and $\mu_0 = 4\pi \times 10^{-7}$ H/m is the magnetic constant. Generally speaking, for anisotropic mediums the parameter μ represents a tensor. But, in this paper, we consider aeromagnetic measurements and this parameter can be treated as a constant scalar equal to the relative permeability of air, $\mu = 1.00000037$.

Therefore, the magnetic induction and magnetic field vectors in the systems under study differ merely in the scale factor and units of measure. Whenever no confusion occurs, we will use the term "magnetic vector."

As a rule, the components of the magnetic vector are measured on board using fluxgate magnetometers [18]. The existing devices of this type have an approximate sensitivity of 0.1 nT and a measurement error of 10 nT.

The absolute value of the magnetic induction vector, $|B| = \sqrt{B^T B}$, is often measured using quantum sensors of different modifications [18]. The modern airborne quantum sensors have an approximate sensitivity of 0.001 nT and a measurement error up to 0.1 nT.

A vector magnetic gradiometer is a structure that includes several quantum magnetometers working in the differential mode under a fixed known spacing of the sensors, which lies within a range of 1–10 m. Its measurement sensitivity depends on the rigidity of the basic line of the sensors and its length; for the existing magnetic gradiometry systems, this sensitivity is about 1 pT/m in the 1-Hz bandwidth [19]. The measurement error of the gradient components depends on calibration conditions. Due to their design, such systems are difficult to calibrate in a lab because even after a separate calibration of each sensor the effect of the whole gradiometer's structure remains neglected. The field of all constructive elements of this gradiometric system is estimated during a special calibration flight at a large altitude. As a rule, the external field gradient in such an experiment reaches 10 pT/m, which is comparable with the error of a quantum magnetometer related to the length of the basic line.

For obtaining the tensor components of the magnetic field gradient ∇B^T, one has to differentiate all components of the field vector. Obviously, a tensor gradiometer based on fluxgate magnetometers has a several orders greater error than its vector counterpart based on quantum sensors. However, since the recent time some researchers are endeavoring to design a tensor gradiometer using component quantum magnetometers—superconducting quantum interference devices (SQUIDs), see [20].

Note that the components of the magnetic induction vector (like the components of the magnetic field vector and the tensor components of the magnetic field gradient) are measured in the axes associated with a measuring device. To recalculate their values into another coordinate system, one needs to consider the orientation of this device during flight.

3. VARIABILITY ANALYSIS FOR GEOMAGNETIC FIELD PARAMETER BY REAL DATA

In order to estimate the stability of geomagnetic field parameters with the course of time, we used the results of aerogeophysical surveys of the same surface segment conducted in 1999 and 2011, see [21]. The approximate dimensions of this segment are 7×5 km. The maps for the absolute

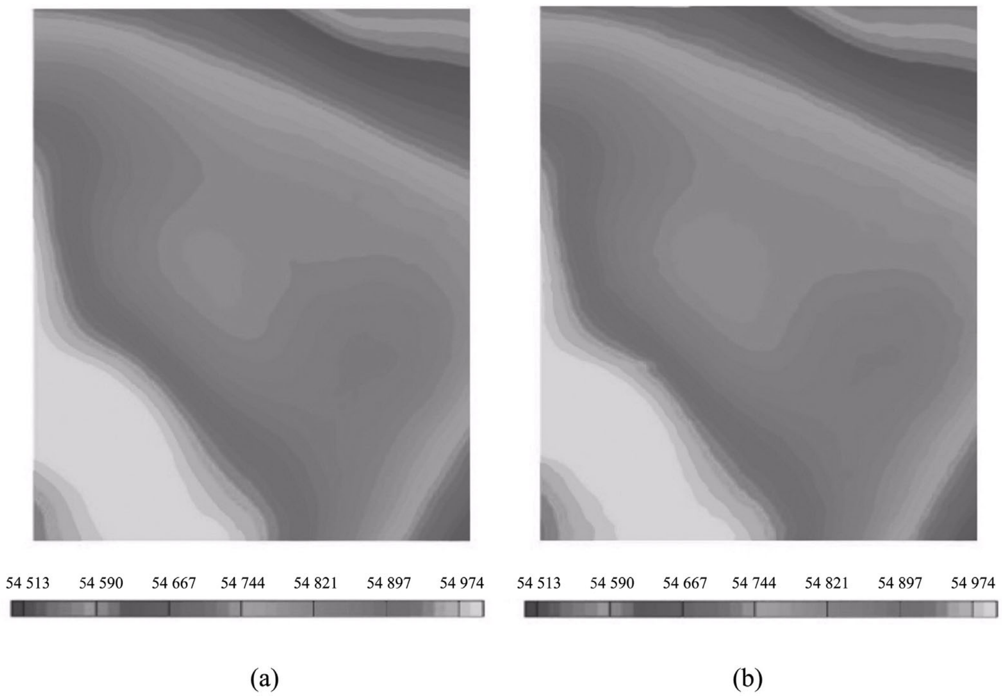

Fig. 1. Absolute values of magnetic field induction vector B, nT, in (a) 2011 and (b) 1999.

values of the magnetic field induction vector that correspond to these surveys are presented in Fig. 1 (flight altitude 70–80 m). Here the horizontal axis (not shown in the figure) is associated with the measurement points in the Gauss–Krüger coordinate system, the measured values are indicated by color in a given scale; for the available measurement data at separate points, the continuous maps were obtained using bilinear interpolation. The magnetic field induction in this segment varies within the range 54.5–55 μT.

The difference between these measurements is illustrated in Fig. 2. Taking into account the accuracy of modern quantum magnetic field sensors, we observe a considerable variation of the magnetic field over the whole segment. This restricts the applicability of the absolute values of the magnetic field induction vector to high-precision correlation-extremal navigation, because their temporal variability is much higher than the measurement errors of the quantum sensors.

At the same time, the use of the magnetic field gradient has a series of advantages. First of all, the anomalies of the gradient are mostly caused by local singularities of a medium, being less subjected to global changes of the geomagnetic field. Next, gradient measurements on a sufficiently small base ($\sim$ 1 m) allow to neglect the variational component of the magnetic field, which cannot be done for the measurements of the field itself. Finally, the anomalous field of the gradient has a smaller correlation radius: a major impact is exerted by near-surface objects, which produce more contrasting anomalies.

The horizontal gradient of the absolute values of the magnetic field induction vector is defined by

$$\nabla_H |B| = \sqrt{\left(\frac{\partial |B|}{\partial y_1}\right)^2 + \left(\frac{\partial |B|}{\partial y_2}\right)^2},$$

where the axes Oy_1 and Oy_2 are horizontal. The maps for the horizontal gradient of the absolute values of the magnetic field induction vector that were calculated by the data in Fig. 1 are shown Fig. 3. Clearly, there exist recurring fragments in Figs. 2 and 3 (the anomaly contours in the

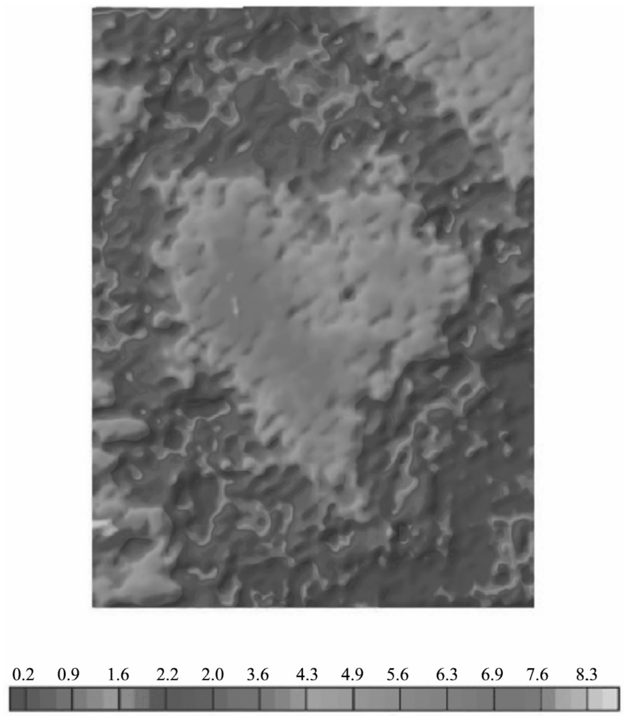

Fig. 2. Difference between absolute values of magnetic field induction vector measured in 2011 and 1999, $|B|_{2011} - |B|_{1999}$, nT.

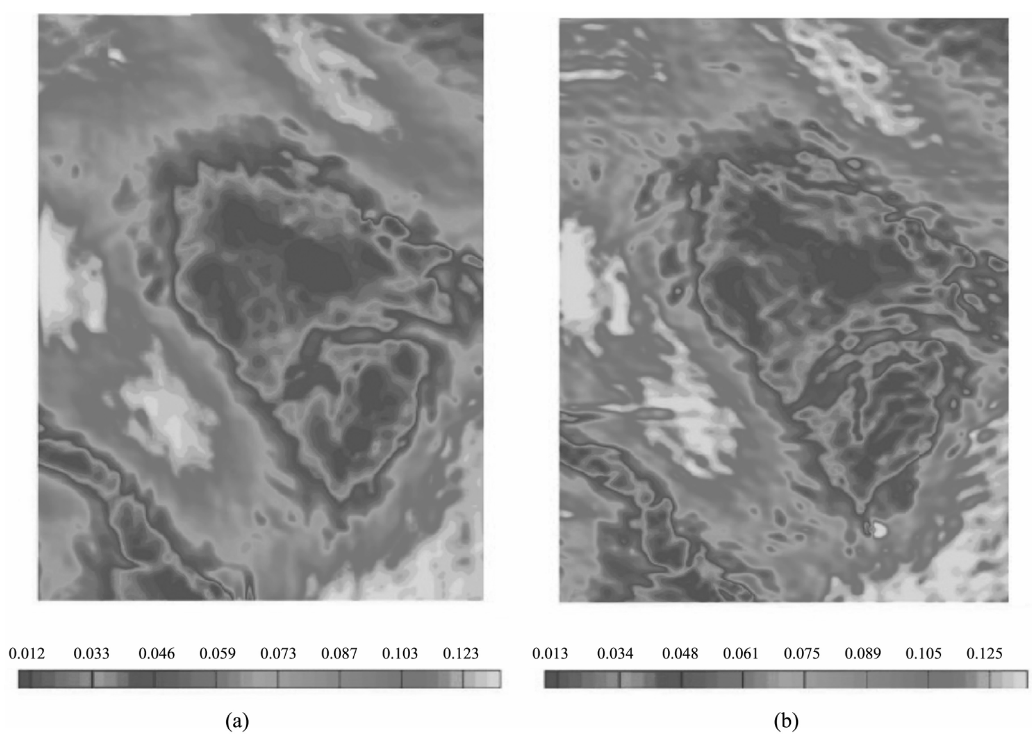

Fig. 3. Horizontal gradient of absolute values of magnetic field induction vector, $\nabla_H|B|$, nT/m, in (a) 2011 and (b) 1999.

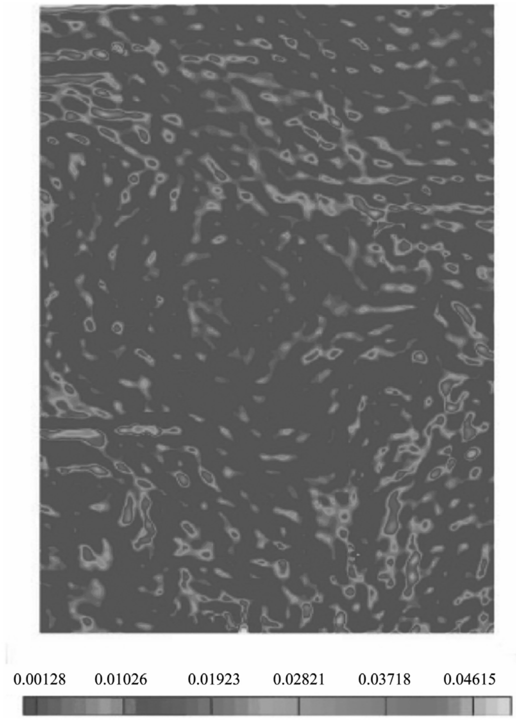

Fig. 4. Difference between gradients of absolute values of magnetic field induction vector measured in 2011 and 1999, $\nabla_H |B|_{2011} - \nabla_H |B|_{1999}$, nT/m.

central part of Fig. 2 match those in the central part of Figs. 3a and 3b), which are associated with the geological singularities of the given surface segment. This means that temporal variations affect both the normal and anomalous magnetic fields (the idea is to use the anomalous field for navigation). Therefore, in the absence of actual magnetic data, CENSs should consider only the anomalies of large size and amplitude with small temporal variations. Consequently, the CENSs based on magnetic field measurements can be designed using modern fluxgate sensors.

The difference between the gradient measurements of 2011 and 1999 is demonstrated in Fig. 4. The maximal deviation reaches 0.01 nT/m if we eliminate the differences induced by technogeneous anomalies and measuring equipment failures. This result well agrees with the error of the gradiometers based quantum optical pumping sensors. Thus, on the one hand, quantum vector magnetic gradiometric systems can be used for navigation even at the current stage of technological development; on the other, the gradient field has a sufficient stability for relying on the magnetic field gradient maps for decades.

4. COMPENSATION OF AIRCRAFT'S SELF-MAGNETIC FIELD IN MAGNETIC GRADIOMETRY

Except for the geomagnetic field, an airborne measuring system is affected by the self-magnetic field of a carrier aircraft, due to its magnetization and Foucault currents in its body. Obviously this field induces noises and reduces the accuracy of airborne measurements. So, it is required to compensate the impact of the aircraft's self-magnetic field on an airborne measuring device of the magnetic field (the so-called deviation compensation problem).

The compensation procedure for the impact of the aircraft's self-magnetic field needs a calibration flight to find the model parameters of these noises. During a calibration flight, an aircraft ascends to a maximal altitude $h \sim 1000$ m in order to minimize the influence of the anomalous magnetic field of the Earth. At this altitude in the straight flight mode, a series of evolutions with angles 5° in different channels (yaw, roll, pitch) are performed with four essentially different courses for reorienting the magnetic field vector with respect to the carrier.

To acquire measurements in the deviation compensation problem, it is necessary to use a vector magnetometer, i.e., a fluxgate magnetometer, to find the direction of the magnetic field vector B_F (here F denotes the field according to the vector sensor data). In the case of a tensor magnetic gradiometer, the source of information is any sensor of the gradiometer.

For a successful compensation, in addition to the measurement results of the gradient and magnetic field vector, one needs information about the coordinates and velocity of the carrier at the calibration stage. This information comes from a receiver of a satellite navigation system. Assume the self-magnetic field of the carrier is described by a linear model with several constant parameters. Then the compensation problem can be solved through parameter calculation of this model.

Let us reduce the compensation problem to a standard stochastic estimation problem. So it is required to pose a closed estimation problem, i.e., to obtain equations for the anomalous field, equations for the deviation parameters and equations for the measurements.

The deviation ΔB_m induced by magnetic masses has the form

$$\Delta B_m = K + LB_0,$$

which was originally established by Poisson in 1824; also, see [22–24]. Here B_0 denotes the geomagnetic field vector; K is the vector of a constant or "rigid" deviation component induced by the hard-magnetic materials of the aircraft; L gives a matrix of dimensions 3×3 that is associated with the inductive or "soft" component induced by the field of the soft-magnetic materials of the aircraft.

The deviation ΔB_i created by the induction currents under variations in the magnetic field, nonuniform movements of the aircraft or its motion in the field with large horizontal or vertical gradients [25] is calculated as

$$\Delta B_i = M\frac{dB_0}{dt},$$

where M indicates the current influence matrix of dimensions 3×3.

The measurement model in the estimation problem is constructed using the model of the complete deviation $\Delta B_{sum} = \Delta B_m + \Delta B_i$ jointly for the magnetic gradiometer and fluxgate sensor.

In the closed estimation model, the measurement model is supplemented by the difference equations that describe the components of the anomalous magnetic field and deviation parameters. Our setup proceeds from constant deviation parameters. The field has a stochastic model with parameters chosen by real data analysis.

After a parameter normalization subject to the typical values of the deviation parameters, we may write the expression

$$B = B_0 + \Delta B_m + \Delta B_i$$

for the measurable magnetic field vector in the following way [26], taking into account the magnetic noises and an infinitesimal order of corresponding terms:

$$b = b_0 + \varepsilon^3 K + \varepsilon^3 L^T b_0 + \varepsilon^4 b_0 M^T \frac{de_0}{d\tau}. \tag{1}$$

Here b denotes a dimensionless measurement vector that is related to the vector B by $B = B^* b$, $B^* = 50\,000$ nT; b_0 means the dimensionless vector of the geomagnetic field; all elements of the matrices K, L and M are values of infinitesimal order 1; $\varepsilon = 0.1$; $e_0 = b_0/|b_0|$ is the directional cosine vector; finally, τ indicates dimensionless time [27]. The accuracy of formula (1) varies from $O(\varepsilon^6)$ to $O(\varepsilon^8)$, depending on the altitude of aerial survey [26]. In the tensor notation, formula (1) can be represented as

$$b_i = b_{0i} + \varepsilon^3 K_i + \varepsilon^3 L_{ij} b^j + \varepsilon^4 b_s M_{ij} \frac{de_0^j}{d\tau}. \tag{2}$$

Here and in the sequel, all indices vary from 1 to 3, summation runs over recurrent indices and $b_s = \sqrt{b_{0i} b_0^i}$.

Now, using formula (2) and the notation $\Gamma_{ij} = \nabla_i b_j$ ($\Gamma_{0ij} = \nabla_i b_{0j}$) for the gradient tensor, the deviation model of the tensor magnetic gradiometer indications can be expressed as

$$\Gamma_{ij} = \varepsilon^3 \Gamma_{0ij} + K_{ij} + L_{ijk} b_0^k + \varepsilon b_s M_{ijk} \frac{de_0^k}{d\tau}, \tag{3}$$

where the values K_{ij}, L_{ikj}, and M_{ikj} have order 1. For high-precision SQUIDs, it is necessary to introduce the gradient model at the initial calibration stage.

The properties of the tensor of the magnetic field gradient lead to the following properties of the magnetization tensors:

$$K_{ij} = K_{ji}, \quad L_{ijk} = L_{jik}, \quad M_{ijk} = M_{jik}, \quad K_i^i = 0, \quad L_{ik}^i = 0, \quad M_{ik}^i = 0.$$

Hence, there are 35 parameters to be found.

Within the stochastic approach, the magnetic field gradient at measurement points is considered as a discrete Gauss–Markov process of the second order (also see the book [28]). This process obeys the difference equation

$$\begin{gathered} x_{j+1} = x_j + v_j u_j \Delta t, \\ u_{j+1} = u_j + q_j \Delta t, \\ x_0, \ u_0 = 0, \quad q_j \in N(0, \sigma^2), \quad E[q_i q_j] = \sigma^2 \delta_{ij}, \end{gathered} \tag{4}$$

where x_j is the jth measurement for any component of the tensor (or vector) of the magnetic field gradient; u_j denotes the derivative of the gradient component along the flight direction at the time instant j; q_j gives the noise vector at the time instant j; v_j is the aircraft velocity at the time instant j; Δt indicates the time interval between successive measurements; $N(\cdot)$ stands for the normal distribution; finally, $E[\cdot]$ means the expectation operator. The parameter σ^2 is adjusted using a statistical analysis of real magnetic field data.

Note that model (4) includes the derivative of a component of the gradient along the flight direction. Therefore, prior to using this model in the optimal estimation setup, we have to reproject the magnetic field values from the ground coordinate system (in which one of the axes coincides with the flight direction) into the coordinate system associated with the measuring device. Let the axis Oy_1 be coinciding with the flight direction; denote by O the transition matrix from the ground coordinate system to the one associated with the measuring device.

For the optimal estimation problem of the magnetic noise parameters to be well-posed without cumbersome calculations, we will introduce several auxiliary notations. For arbitrary matrices X, $X_k \in \mathrm{R}^{3\times 3}$, where $\mathrm{R}^{k\times m}$ is the set of real matrices of dimensions $k \times m$, write

$$X_{\{ij\}} = (X_{11}, X_{12}, X_{13}, X_{22}, X_{23}),$$
$$X_{\{ij\}k} = (X_{11k}, X_{12k}, X_{13k}, X_{22k}, X_{23k})$$

and

$$\hat{x}_0 = (\nabla_1 B_1,\, \nabla_1\nabla_1 B_1,\, \nabla_1 B_2,\, \nabla_1\nabla_1 B_2,\, \nabla_1 B_3,\, \nabla_1\nabla_1 B_3,\, \nabla_2 B_2,\; \nabla_1\nabla_2 B_2,\, \nabla_2 B_3,\, \nabla_1\nabla_2 B_3)\,.$$

Consider the matrices

$$A_j = \begin{pmatrix} 1 & V_j\Delta t \\ 0 & 1 \end{pmatrix}, \quad Q_j = \begin{pmatrix} 0 & 0 \\ 0 & V_j^2\Delta t^2\sigma^2 \end{pmatrix},$$
$$\hat{A}_j = \{A_j,\, A_j,\, A_j,\, A_j,\, A_j,\, I_{35}\}\,,$$

and

$$\hat{Q}_j = \{Q_j,\, Q_j,\, Q_j,\, Q_j,\, Q_j,\, \Theta_{35\times 35}\}\,,$$

which are block-diagonal matrices, where index j corresponds to the number of a current measurement, V_j gives the aircraft velocity at the measurement time instant, while I_n and $\Theta_{n\times k}$ mean the identity and zero matrices of compatible dimensions. For the jth measurement, denote

$$\hat{H} = H_1,\; \Theta_{5\times1},\; H_2,\; \Theta_{5\times1},\; H_3,\; \Theta_{5\times1},\; H_4,\; \Theta_{5\times1},\; H_5,\; \Theta_{5\times1},$$
$$I_5,\; I_5e_1,\; I_5e_2,\; I_5e_3,\; I_5e_1',\; I_5e_2',\; I_5e_3',$$

where index j is omitted for the sake of compactness and e' defines the finite difference

$$e_j' = \frac{(e_j - e_{j-1})}{\Delta t}, \quad j > 1,$$
$$e_1' = \frac{(e_2 - e_1)}{\Delta t}, \quad j = 1,$$

where e_j is the directional cosine vector $e_j = b_j/|b_j|$ at the time instant j, the matrices $H_i \in R^5$ have the form

$$(H_1,\; H_2,\; H_3,\; H_4,\; H_5)$$
$$= \begin{pmatrix} O_{11}^2 - O_{13}^2 & O_{21}^2 - O_{23}^2 & O_{11}O_{21} - O_{13}O_{23} & O_{11}O_{31} - O_{13}O_{33} & O_{21}O_{31} - O_{23}O_{33} \\ O_{12}^2 - O_{13}^2 & O_{22}^2 - O_{23}^2 & O_{12}O_{22} - O_{13}O_{23} & O_{12}O_{32} - O_{13}O_{33} & O_{22}O_{32} - O_{23}O_{33} \\ 2O_{11}O_{12} & 2O_{21}O_{22} & O_{12}O_{21} + O_{11}O_{22} & O_{12}O_{31} + O_{11}O_{32} & O_{22}O_{31} + O_{21}O_{32} \\ 2O_{11}O_{13} & 2O_{21}O_{23} & O_{13}O_{21} + O_{11}O_{23} & O_{13}O_{31} + O_{11}O_{33} & O_{23}O_{31} + O_{21}O_{33} \\ 2O_{12}O_{13} & 2O_{22}O_{23} & O_{13}O_{22} + O_{12}O_{23} & O_{13}O_{32} + O_{12}O_{33} & O_{23}O_{32} + O_{22}O_{33} \end{pmatrix}$$

(by analogy, R^k is the set of real vectors of dimension k).

Construct the state vector $x \in \mathrm{R}^{45}$,

$$x = \left(\hat{x}_0, K_{\{ij\}}, L_{\{ij\}1}, L_{\{ij\}2}, L_{\{ij\}3}, M_{\{ij\}1}, M_{\{ij\}2}, M_{\{ij\}3}\right)^T,$$

and the measurement vector $z \in \mathrm{R}^5$ $z^T = \Gamma_{\{ij\}}$, which satisfy the relationships

$$x_{j+1} = \hat{A}_j x_j + q_j, \quad q_j \in N(0, \hat{Q}), \quad E\left[q_i q_j^T\right] = \hat{Q}_j\delta_{ij}, \quad P_j = E\left[x_j x_j^T\right],$$
$$z_j = \hat{H}_j x + r_j, \quad r_j \in N(0, \hat{R}), \quad E\left[r_i r_j^T\right] = \hat{R}\delta_{ij},$$

on the strength of (3) and (4); here $r_j \in \mathrm{R}^5$ is the noise vector of the tensor measuring device with the diagonal covariance matrix $\hat{R}$. As is well-known, the optimal estimate of the state vector in this problem can be obtained using a discrete-time Kalman filter, with the initial values $x_0 = \Theta_{45\times 1}$ and P_0 adjusted through experimental data analysis (also see the book [27]). For a convergence analysis of these estimates, we may employ the so-called stochastic measures of estimability [29].

In the case of measuring the gradient of the absolute values of the magnetic induction vector, the compensation problem has another statement. More specifically, each component of the gradient represents the difference between the measurements of scalar magnetometers. For each of them, the deviation model [27] takes the form

$$b = b_0 + \varepsilon^3 e_F^T K + 0.5\varepsilon^3 e_F^T L_S b_F + \varepsilon^4 b_F^T M \frac{de_F}{d\tau} + o(\varepsilon^6).$$

Here $L_S = L + L^T$ is a symmetric matrix and $e_F = b_F/|b_F|$ denotes the directional cosine vector by the vector magnetometer data. Clearly, the measurements of the scalar sensors are not enough for solving the compensation problem. This model contains 16 observable parameters, namely,

$$K_1, K_2, K_3, \Delta L_{11}, \Delta L_{22}, L_{12}, L_{13}, L_{23}, \Delta M_{11}, M_{12}, M_{13}, M_{21}, \Delta M_{22}, M_{23}, M_{31}, M_{32};$$
$$\Delta L_{ii} = L_{ii} - L_{33}, \quad \Delta M_{ii} = M_{ii} - M_{33}.$$

By analogy with tensor measurements of the magnetic field gradient, it is possible to calculate the deviation of the gradient components $g_i = \nabla_i b$, i.e.,

$$g_i = \varepsilon^3 g_{0i} + K_{ij} e_F^j + L_{ijk} e_F^j b_F^k + \varepsilon M_{ijk} \frac{de_F^j}{d\tau} b_F^k + o\left(\varepsilon^3\right), \tag{5}$$

where the magnetization tensors $(K_{ij}, L_{ijk}, M_{ijk})$ generally differ from their counterparts figuring in expression (3). Hence, these values do not possess the same symmetry as for the tensor gradiometer. Problem (5) is decomposed into three estimation subproblems, each yielding 16 parameters of the tensors K, L, and M (the total number of parameters is 48).

Besides, note that the sensitivity of the scalar quantum magnetometers suffices for performing reliable gradient measurements at the calibration flight altitude. So, the complete model of the estimation problem also needs the model of the field gradient.

For posing the estimation problem of the magnetic deviation parameters, we again introduce some auxiliary notations. Let

$$\tilde{A}_j = \{A_j,\, A_j,\, A_j,\, I_{48}\},$$
$$\tilde{Q}_j = \{Q_j,\, Q_j,\, Q_j,\, \Theta_{48\times 48}\}$$

and, for the jth measurement,

$$\tilde{H} = \Big(O_1,\, \Theta_{3\times 1},\, O_2,\, \Theta_{3\times 1},\, O_3,\, \Theta_{3\times 1},\, I_3 e_1,\, I_3 e_2,\, I_3 e_3,\, I_3 e_1^2,\, I_3 e_2^2,\, I_3 e_2 e_3,$$
$$I_3 e_1 e_1',\, I_3 e_2 e_2',\, I_3 e_1 e_2',\, I_3 e_1 e_3',\, I_3 e_2 e_1',\, I_3 e_2 e_3',\, I_3 e_3 e_1',\, I_3 e_2 e'\Big),$$

where index j is omitted for brevity and O_i gives the ith column of the turn matrix O defined above. In addition, denote

$$\tilde{x}_0 = \Big(\nabla_1 |B|,\ \nabla_1\nabla_1 |B|,\ \nabla_2 |B|,\ \nabla_1\nabla_2 |B|,\ \nabla_3 |B|,\ \nabla_1\nabla_3 |B|\Big),$$
$$\tilde{x}_i = \Big(K_{1i}, K_{2i}, K_{3i}, L_{11i}, L_{22i}, L_{12i}, L_{13i}, L_{23i},$$
$$M_{11i}, M_{22i}, M_{12i}, M_{13i}, M_{21i}, M_{23i}, M_{31i}, M_{32i}\Big).$$

Construct the state vector $x \in \mathrm{R}^{54}$ $x = (\tilde{x}_0, \tilde{x}_1, \tilde{x}_2, \tilde{x}_3)^T$, and the measurement vector $z \in \mathrm{R}^3$, $z = (g_1, g_2, g_3)^T$, which satisfy the relationships

$$x_{j+1} = \tilde{A}_j x_j + q_j, \quad q_j \in N(0, \tilde{Q}_j), \quad E\left[q_i q_j^T\right] = \tilde{Q}_j \delta_{ij}, \quad P_j = E\left[x_j x_j^T\right],$$
$$z_j = \tilde{H}_j x + r_j, \quad r_j \in N(0, \tilde{R}), \quad E\left[r_i r_j\right] = \tilde{R}\delta_{ij},$$

due to expressions (4) and (5). Here $r_j \in \mathrm{R}^3$ is the noise vector of the vector measuring device with the diagonal covariance matrix $\tilde{R}$. Like in the previous case, the optimal estimate of the state vector can be obtained using a discrete-time Kalman filter, with the initial values $x_0 = \Theta_{54\times1}$ and P_0 adjusted through experimental data analysis.

First of all, once again note that the magnetometric system needs measuring devices for orientation angles: the field model is defined in the geographic coordinate system while the measurements are performed in the coordinate system associated with the aircraft. Next, in order to coordinate the measurements in different courses, it is necessary to introduce a coincidence condition for the gradient vectors, which yields an additional correcting measurement.

After the calibration procedure, the existing deviation can be compensated using the following formulas:

for the tensor gradiometer,

$$\Gamma_{0\{ij\}} = \Gamma_{\{ij\}} - \hat{H}\left(\Theta_{10\times1},\ K_{\{ij\}},\ L_{\{ij\}1},\ L_{\{ij\}2},\ L_{\{ij\}3},\ M_{\{ij\}1},\ M_{\{ij\}2},\ M_{\{ij\}3}\right)^T;$$

for the vector gradiometer,

$$g_{0i} = g_i - \tilde{H}\left(\Theta_{1\times6},\ \tilde{x}_1,\ \tilde{x}_2,\ \tilde{x}_3\right)^T.$$

All values correspond to the same measurement.

5. INTEGRATION ALGORITHM OF MAGNETOMETRIC AND INERTIAL NAVIGATION SYSTEMS

Consider an integration algorithm of the inertial and correlation-extremal navigation systems (INS-CENS) in the following setup. A two-component platform inertial system with a horizontal platform and a relatively free azimuthal orientation is chosen as the INS. A baroaltimeter is used for altitude measurements. Correcting information consists in the values of the magnetic field gradient $g = (g_1, g_2, g_3)^T$. For implementing the integration algorithm, the airborne computer reads navigational data from inertial sensors and also solves the difference equations for the error vector and its covariance matrix; this solution will be used at the correction stage. The covariance matrix must be calculated taking into account the error equations.

The premeasured values of the magnetic field gradient are defined as a set of measurements performed at several points in a certain three-dimensional bounded domain. Using some interpolation algorithm (in this paper, cubic interpolation), the measurements are transformed into continuous functions that relate the magnetic field gradient and its derivatives with respect to y_1 and y_2 to the coordinates of the measurement points $M(y_1, y_2, y_3)$. Denote the functions by $g = \Phi(M)$ and $\nabla_1 g = \nabla_1\Phi(M)$, $\nabla_2 g = \nabla_2\Phi(M)$. Assume the components of these vector-valued functions are written in the geographic coordinate system.

The integration algorithm includes two stages as follows [21]. The first stage is intended to roughly estimate the positioning errors by the measured physical field parameters. This stage starts when the aircraft flies along a reference segment in accordance with the indications of the continuously operating INS. For the first stage, it is necessary to analyze the statistical parameters of the reference map and to define the correlation radius of the current flight altitude. Next, the admissible value domain of the INS error vector is divided into a certain number of equal segments, each being smaller than the correlation radius. For all segments, a trajectory in the error vector space is initiated, which is calculated using the error equations with the initial conditions that match the center of the admissible domain. For each time instant, the field value can be found from the reference map for the current coordinates yielded by the inertial navigation system. For all trajectories, it is necessary to calculate an integral that describes the likelihood degree of a corresponding hypothesis. The hypotheses that do not satisfy the likelihood conditions are gradually eliminated by comparing this degree with a given threshold. The first stage ends as soon as a unique hypothesis is obtained.

At the second stage, the INS correction problem by the measured values of the magnetic field gradient is reduced to a stochastic optimal estimation problem. To this end, let us introduce some notations. For the jth measurement, define

$$A^{in} = \begin{pmatrix} I_2 & I_2\Delta t & \Theta_{2\times3} & \Theta_{2\times2} & \hat{\hat{y}} \\ -\omega_0^2 I_2\Delta t & I_2 + 2u_3 T\Delta t & \Theta_{2\times3} & \Theta_{2\times2} & \hat{\hat{w}} \\ \Theta_{3\times2} & \Theta_{3\times2} & I_3 + \hat{\omega}\Delta t & \Theta_{3\times2} & I_3\Delta t \\ \Theta_{2\times2} & \Theta_{2\times2} & \Theta_{2\times3} & I_2 & \Theta_{2\times3} \\ \Theta_{3\times2} & \Theta_{3\times2} & \Theta_{3\times3} & \Theta_{3\times2} & I_3 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

$$H^{in} = \left(-\Delta_1,\ \Theta_{3\times2}, -O_T\hat{\hat{\Phi}}_y(M) + \Delta_2\hat{y},\ \Theta_{3\times5}\right), \quad \Phi_y(M) = O_R\Phi(M),$$

$$\Delta_1 = O_T\left(\ \nabla_1\Phi_y(M)\ \ \nabla_2\Phi_y(M)\ \right), \quad \Delta_2 = \left(\ \Delta_1\ \ \Theta_{3\times1}\ \right),$$

where M is a model point with model coordinates $y \in \mathrm{R}^3$; $w \in \mathrm{R}^3$ means the model relative velocity; $u \in \mathrm{R}^3$ gives the earth angular velocity projected into the axes of the model trihedron; $\omega \in \mathrm{R}^3$ is the absolute angular velocity of the model trihedron; ω_0 denotes the Schuler frequency; O_R specifies the turn matrix from the geographic trihedron to the model trihedron; O_T indicates the turn matrix from the instrument trihedron to the trihedron associated with the measuring device (this matrix is assumed to be defined without errors); note that index j is omitted. In addition, for an arbitrary vector $f \in \mathrm{R}^3$, let

$$\hat{f} = \begin{pmatrix} 0 & f_3 & -f_2 \\ -f_3 & 0 & f_1 \\ f_2 & -f_1 & 0 \end{pmatrix}$$

and

$$\hat{\hat{f}} = \begin{pmatrix} 0 & f_3 & -f_2 \\ -f_3 & 0 & f_1 \end{pmatrix}.$$

For the jth measurement, write the state vector $x \in \mathrm{R}^{12}$, $x = \left(\delta y^T, \delta w^T, \beta^T, \Delta f^T, v^T\right)^T$, and also the measurement vector $z \in \mathrm{R}^3$, $z = g - O_T\Phi_y(M)$, where $\delta y \in R^2$ are the dynamic errors of the model coordinates; $\delta w \in \mathrm{R}^2$ mean the dynamic errors of the model velocities; $\beta \in \mathrm{R}^3$ denotes a small turn angle from the model trihedron to the instrument trihedron; $\Delta f \in \mathrm{R}^2$ is the instrument error vector of newton meters; $v \in \mathrm{R}^3$ gives the drift of the gyro platform [30]; finally, $g \in \mathrm{R}^3$ is the measured vector of the magnetic field gradient. The vector g can be written as $g = O_T(I_3 + \hat{\beta})\Phi(M')$, where the point M' has the coordinates $y' = y - \delta y - \hat{\beta}y$, $\delta y_3 = 0$. Then the measurement vector allows the representation $z = H^{in}x + r$ (up to the linear terms of the

Taylor expansion), where $r \in \mathrm{R}^3$ is the sensor noise vector. Hence, the above vectors satisfy the relationships

$$x_{j+1} = A_j^{in} x_j + q_j, \quad q_j \in N\left(0, Q^{in}\right), \quad E\left[q_i q_j^T\right] = Q^{in}\delta_{ij},$$
$$z_j = H_j^{in} x_j + r_j, \quad r_j \in N\left(0, R^{in}\right), \quad E\left[r_i r_j\right] = R^{in}\delta_{ij},$$

where the covariance matrices Q^{in} and R^{in} are defined by the characteristics of the inertial and magnetic sensors. The discrete Kalman filter gives the optimal estimate of the state vector components; this estimate is used at each step of the algorithm for INS correction.

6. NUMERICAL SIMULATION

To illustrate the efficiency of the suggested integration algorithm for the data supplied by the platform inertial navigation system and magnetic gradiometers, we performed a numerical simulation of this algorithm using the real data of the magnetic field from a test flight on a segment. During simulation, it was assumed that the aircraft has a straight flight with a constant velocity and a slowly changing orientation. The model of the inertial system errors was adjusted for a correct description of medium accuracy sensors. As a result, we obtained a relationship between the positioning error of the aircraft and the distance travelled S.

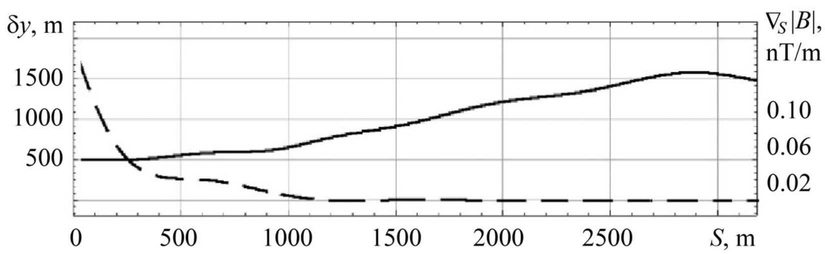

Fig. 5. Numerical simulation results for integration algorithm.

The simulation results are shown in Fig. 5. Here solid line corresponds to the magnetic field gradient projected into the flight direction, $\nabla_S|B|$; dashed line, to the horizontal component of the positioning error

$$\delta y = \sqrt{(y_1' - y_1)^2 + (y_2' - y_2)^2}$$

by the data obtained using the integration algorithm. Clearly, the error rapidly decreases below 20 m. This testifies to a high efficiency of the suggested integration solution for the data from magnetic gradiometers and the INS.

7. CONCLUSIONS

This paper has presented an analytical survey of the existing solutions for airborne measurements of the magnetic field gradient. It has been demonstrated that the key attributes of the magnetic field gradient (such as temporal instability and a small correlation radius) make it promising to use correlation-extremal navigation systems based on magnetic gradiometry. Much attention has been paid to the compensation problem for the indications of magnetic gradiometers. For solving this problem, we have developed an approach with a stochastic model of the anomalous field and designed a Kalman filter to estimate the compensation coefficients. A numerical simulation by real data has illustrated an efficient decrease of the inertial navigation system errors in case of integration with the magnetometric correlation-extremal navigation system.

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