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MARINE PHYSICS

Extreme Statistics of Storm Surges in the Baltic Sea

E. A. Kulikov[a], * and I. P. Medvedev[a, b]
[a] *Shirshov Institute of Oceanology of the Russian Academy of Sciences, Moscow, Russia*
[b] *Fedorov Institute of Applied Geophysics, Moscow, Russia*
**e-mail: KulikovE@ocean.ru*
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Abstract—Statistical analysis of the extreme values of the Baltic Sea level has been performed for a series of observations for 15–125 years at 13 tide gauge stations. It is shown that the empirical relation between value of extreme sea level rises or ebbs (caused by storm events) and its return period in the Baltic Sea can be well approximated by the Gumbel probability distribution. The maximum values of extreme floods/ebbs of the 100-year recurrence were observed in the Gulf of Finland and the Gulf of Riga. The two longest data series, observed in Stockholm and Vyborg over 125 years, have shown a significant deviation from the Gumbel distribution for the rarest events. Statistical analysis of the hourly sea level data series reveals some asymmetry in the variability of the Baltic Sea level. The probability of rises proved higher than that of ebbs. As for the magnitude of the 100-year recurrence surge, it considerably exceeded the magnitude of ebbs almost everywhere. This asymmetry effect can be attributed to the influence of low atmospheric pressure during storms. A statistical study of extreme values has also been applied to sea level series for Narva over the period of 1994–2000, which were simulated by the ROMS numerical model. Comparisons of the "simulated" and "observed" extreme sea level distributions show that the model reproduces quite satisfactorily extreme floods of "moderate" magnitude; however, it underestimates sea level changes for the most powerful storm surges.

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1. INTRODUCTION

Investigation of sea level variations is of particular importance for understanding the nature of the formation of such dangerous phenomena as floods and ebbs in coastal areas. It is well known that the frequency (or return period) of floods of various strength is considered when constructing coastal objects, calculating insurance risks, estimating possible environmental damage, etc. Ebb phenomena are no less dangerous, because they cause shoaling of the offshore zone, which implies difficulties for navigation. Extreme negative surge can also dry out water intakes required to cool nuclear power plant reactors. The occurrence of such situations could be a high hazard.

Storm surges and ebbs in the Baltic Sea result from meteorological effects (tangential wind stress and variable atmospheric pressure) on the sea surface. It is considered that extreme sea level rises in the Gulf of Finland are caused by cyclones over the water area of the Baltic Sea with a prevalence of westerly winds, which form surges [14]. The water rises increase because of shallow water and narrowing of the Gulf of Finland toward the mouth of the Neva River. Storm surges in St. Petersburg manifest themselves as sea level rise in the mouth of the Neva and in Neva Bay. During severe storm surges, a considerable part of the historical city center was flooded. According to the official catalog of the Northwest Department on Hydrometeorology and Environmental Monitoring, in the city's history, 309 floods have occurred, three of which are considered catastrophic: the sea level rise exceeded 3 m above the zero point of the Baltic System of Heights (BSH). The maximum sea level rise was observed on November 7(19), 1824, when the level attained a height of 421 cm above the zero point of the BSH. This flood caused huge economic damage in the city; hundreds of people died [13].

Baltic sea-level variability is mainly of a random character. Their regular components, e.g., tides or seasonal components, make a considerable but not dominant contribution to the formation of extreme flood–ebb phenomena [11, 12]. It is known that the probability of flood–ebb phenomena increases in winter due to intensified cyclonic activity in the atmosphere. If we consider sea level variations as a realization of a stochastic process, we can use the mathematical statistics apparatus widely applied in probabilistic analysis of flash floods, extreme wind loading on building structures, life expectancy, etc. The classical approach is set forth in E. Gumbel's book *Statistics of Extreme Values* [4]. The probabilistic analysis of extreme events is based on the concept of the return period. The return period T of an extreme event with a given magnitude is the mean time interval during which this event is supposed to occur or be exceeded only one time. This value can be written as $\langle T(x)\rangle = \frac{1}{p} = \frac{1}{1-F(x)} > 1$, where the function $F(x)$ is the probability distribution. Thus, if an event can occur over a one-year period with probability p, then for this event to occur one time, the observation series should have an average duration of $1/p$ years.

The use of the apparatus of extreme value statistics to describe sea level variations—wind waves or storm surges—has become usual to oceanographers. Among numerous papers devoted to this problem, we can mention [2, 10, 21, 27]. A detailed review of the extreme value statistics' applications to studying the sea level variations, including those in the Baltic Sea, is presented in [1]. In particular, the curves of the probability distribution of annual maximum sea level heights for different areas of the Baltic Sea are presented in this paper. As for contemporary works, one can find detailed investigations of the statistics of extreme Baltic sea-level values (ebbs and floods) in [16, 29]. In [29], based on data of 31 tide gauges over the period from 1960 to 2010, the spatial distribution of extreme (positive/negative surges) sea level values of the 100-year recurrence was calculated and corresponding graphs were plotted. It was shown that the largest sea level extrema are observed in the Gulf of Riga, the Gulf of Finland, in the head of the Gulf of Bothnia, and in the southwestern Baltic Sea.

It is obvious that adequate estimation of the return periods for extreme storm surges phenomena in the Baltic Sea requires long-term observation series. There are a few tens tide gauges on the Baltic seacoast where the duration of observations exceeds 100 years. Correspondingly, such observation series make it possible to calculate statistically confident estimates for return periods up to a few decades. For longer return periods, it is necessary to extrapolate with a probabilistic model corresponding to the statistical properties of an observation series. In this work, with several Baltic tide gauge stations as examples, we have attempted to estimate the return periods for extreme floods/ebbs, construct probabilistic models for positive and negative surges, and determine the physical interpretation of the character of extreme sea level value distributions. Particular attention is paid to the statistics of extreme sea level variations at the head of the Gulf of Finland, where the most intense floods in the Baltic Sea are observed.

2. DATA AND STATISTICS

In this work, we used the long-term hourly observations of sea level variations carried out at 13 tide gauges situated along the Baltic Sea coast and a long-term series of monthly sea level maxima and minima at the Vyborg station from 1889 to 2014, which were collected from the European Marine Observation and Data Network (EMODnet) (http://emodnet.eu/), the University of Hawaii Sea Level Center (UHSLC, http://uhslc.soest.hawaii.edu/), and the Unified State System of Information on the World Ocean of Russian Federation (ESIMO, http://portal.esimo.ru/) (Fig. 1, Table 1). For the analysis, stations with qualitative observation series in different parts of the Baltic Sea were selected. Thus, the Hornbæk, Gedser, Klagshamn, Warnemünde, and Sassnitz tide gauges are located in the southwestern Baltic Sea, in the Danish straits. In the main gulfs, the Ratan (the Gulf of Bothnia), Pärnu (the Gulf of Riga), Tallinn, Vyborg, Narva, Kronstadt, and Gorny Institute (the Gulf of Finland) stations (Fig. 1) were used. For the deep-water areas of the Baltic Sea, hourly sea level observations in Stockholm were taken. The duration of observations at the above-mentioned stations varied considerably (Table 1), from 15 (Kronstadt) to 125 years (Stockholm). To study the statistics of floods in St. Petersburg, we used the archival and present-day data on floods from 1703 to the present from the official catalog of the Northwest Department on Hydrometeorology and Environmental Monitoring (http://www.meteo.nw.ru/).

One of the important conditions necessary for statistical analysis of extreme values is the stationarity of the long-term series of observations used. As is known, taking into account the global sea level rise in the World Ocean, the Baltic Sea region undergoes considerable postglacial land uplifts/downlifts [25], which create pronounced unidirectional trends in the long-term series of observations of seal level variations [17]. The most considerable land uplift is observed on the Scandinavian Peninsula, to the north of the Gulf of Bothnia [17], due to which interannual sea level variations at the Ratan station (Fig. 1) show a negative trend (a decrease in the seal level) at a rate of up to 1 cm/year, i.e., up to 100 cm per 100 years. Therefore, for the statistical analysis data, we used sea level deviation from the linear average long-term trend. The centered series of values obtained after subtraction of the trend obviated the necessity of reducing the sea level data to a height reference system.

The deviation of the Baltic sea-level from the mean value is to a large extent a random value. Nevertheless, in different frequency ranges of sea level variations, it is possible to detect the regular components. Thus, in the mesoscale range of Baltic sea-level variability the astronomical tides occurred to be the background variation of sea level. The amplitude of tides in the Baltic Sea attains maximum values up to 23 cm at the head of the Gulf of Finland and in the southwestern part of the Baltic Sea, near the Danish [20]. It is typical of the long-term sea level variations to have a regular seasonal variability expressed by annual and semi-annual components. The amplitude of seasonal Baltic Sea level variations varies considerably [3, 11]. The maximum average long-term seasonal sea level variations (up to 12–13 cm) are observed at the head of the

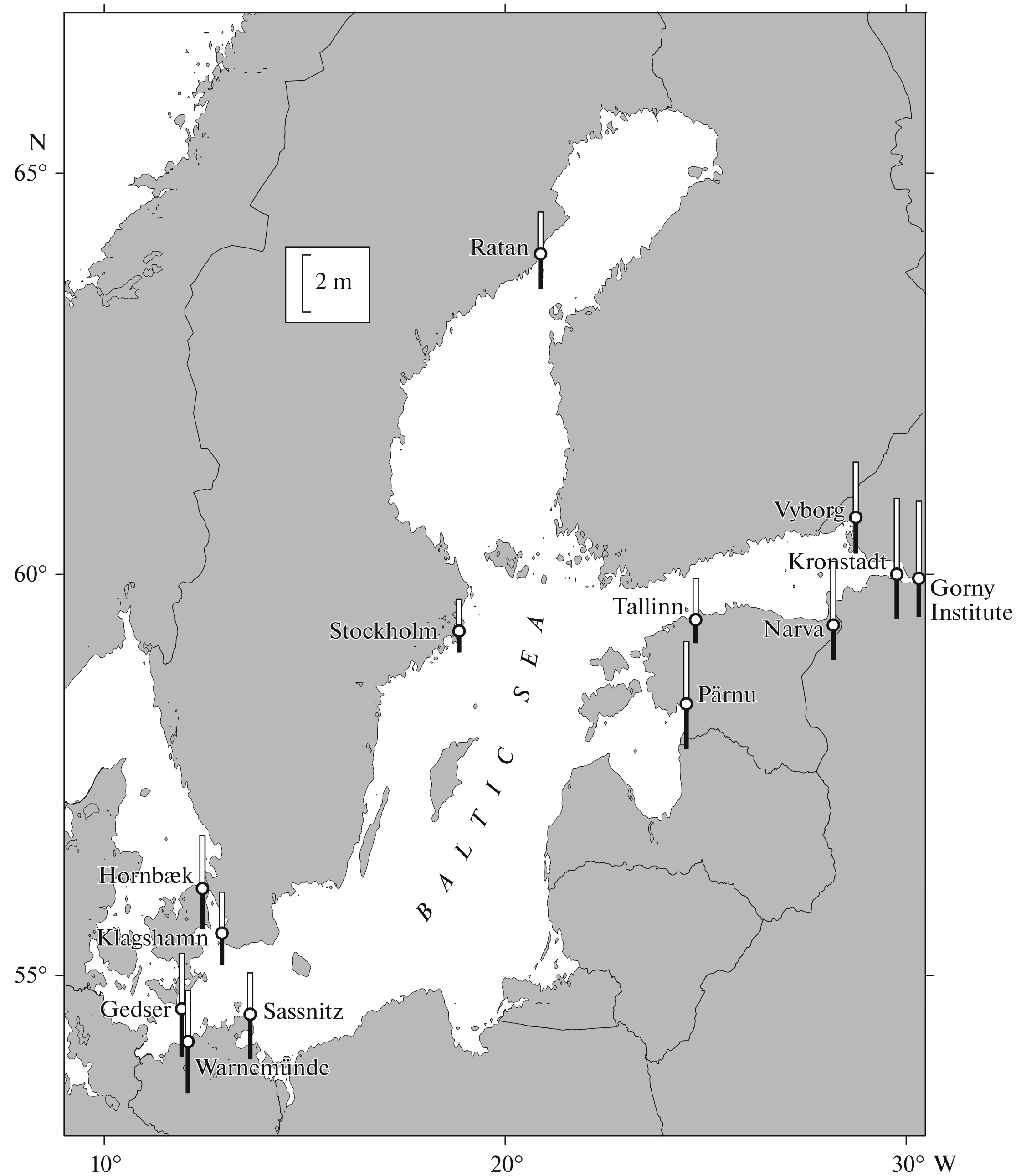

Fig. 1. Map of Baltic Sea. Circles refer to positions of 13 tide gauges whose long-term observation data were used. White sticks show values of extreme sea level rises (storm surges) with 100-year return period for each station. Black sticks refer to values of extreme drops in sea level (ebbs) with 100-year return period.

Gulf of Bothnia and Gulf of Finland [11]. Their amplitude varies with time, attaining 30–32 cm in certain years [11].

Usually the amplitude of sea level variations is described by the standard deviation $\sigma = \sqrt{\langle h^2 \rangle}$, where $\langle\ \rangle$ is the sign of averaging. It is seen from Table 1 that the maximum value of σ is attained at the head of the Gulf of Finland (Gorny Institute, Kronstadt, and Vyborg). In the Gulf of Riga (Pärnu), sea level variations also show a considerable amplitude. The minimum standard deviation is observed in Stockholm, situated in the central part of the Baltic Sea and in the neighborhood of the Danish (Sassnitz and Klagshamn). It should be noted that the spatial distribution of eigenmode amplitude of sea level oscillations (the dominant mode with a period of about 26–29 h) is such that the amplitude of variations is maximum at the head of the Gulf of Finland and the nodal line is located somewhere in the vicinity of Stockholm [6].

Table 1. Data of sea level observations in Baltic Sea at 13 tide gauges and statistical properties of sea level variations: standard deviation (σ), asymmetry coefficient (γ), extreme sea level rises (h_{max}^{100}) and ebbs (h_{min}^{100}) with 100-year return period

Station	Observation period	σ, cm	γ	h_{min}^{100}, cm	h_{max}^{100}, cm
Stockholm	1889–2013	19.1	0.177	–69 (–71)*	116
Ratan	1891–2013	23.9	0.268	–122	142
Gedser	1891–2005	23.2	–0.075	–163	184
Vyborg (month)	1889–2014	–	–	–118 (–120)	175 (185)
Vyborg	1992–2007	28.6	0.552	–	–
Hornbæk	1891–2005	23	0.411	–134	178
Klagshamn	1929–2013	18.3	0.148	–102	134
Sassnitz	1954–2006	19.9	0.246	–139	133
Warnemünde	1956–2006	21.6	0.244	–169	157
Narva	1977–2009	27.8	0.609	–107	185
Pärnu	1978–2009	29.3	0.661	–115	271
Gorny Institute	1977–2007	31.2	0.724	–123	242
Tallinn	1978–1995	25.2	0.4	–74	124
Kronstadt	1992–2006	29.3	0.565	–124	215

* Estimates of extreme sea level maximum and minimum values are indicated in parentheses.

Correspondingly, the increase in the amplitude of variations towards the head of the Gulf of Finland and its minimum values near Stockholm can be well explained. However, small sea level deviations in the southwestern Baltic Sea are not well understood. It is possible that this phenomenon can be explained by the particularities of atmospheric processes in the Baltic region. Predominantly westerly winds and a typical eastward propagation of storms form such an "asymmetrical" sea level variation.

The asymmetry in sea level variations manifests itself as asymmetry in the probability of sea level extremes (positive and negative surges). The asymmetry parameter of the sea level probability density $\gamma = \left\langle h^3 \right\rangle / \sigma^3$ was calculated for the hourly series of observations (Table 1). The asymmetry parameter is positive if the right tail of the probability distribution is "heavier" than the left and is negative otherwise. In other words, the positive value of the asymmetry parameter we observe for most of the stations (see Table 1) means that sea level rises occur with a higher probability than for an ebb of the same magnitude. The maximum positive asymmetry is observed at the head of the Gulf of Finland and reaches a value of 0.724. At the Gedser stations, the coefficient γ is about zero and has a negative value –0.075. It is worth mentioning that the values of the asymmetry parameters calculated in this work for different Baltic Sea stations agree with the estimates for γ obtained more than 50 years ago [8]. Figure 2 shows the histograms of hourly sea level values at the Gorny Institute station (a) over the period from 1977 to 2007 and in Stockholm (b) over the period from 1889 to 2013. The dashed line refers to histogram approximated by the normal distribution. For the Stockholm station, the distribution is close to normal (symmetrical), while for the Gorny Institute station, positive deviations are more probable than negative ones.

The above-mentioned asymmetry between the floods and ebbs can be explained, on the one hand, by the anisotropic wind rose over the Baltic Sea. The west winds dominate and cause the most powerful storm surges in the eastern part of the Baltic region, in the Gulf of Finland and in the Gulf of Riga. However, it is seen from Table 1 that such asymmetry is also typical of the western part of the Baltic Sea coast (except for the Gedser tide gauge). We can suggest a simple explanation of this difference in the formation of floods and ebbs. The extreme sea level deviations are formed, as a rule, under the storm conditions, i.e., during deep cyclones accompanied by lowering atmospheric pressure. So, according to the inverse barometer effect, a rise in sea level is observed. Therefore, irrespective of the wind direction, storm surges increase because of the drop of the air pressure in a cyclone, whereas decreases in sea level are partly "compensated" by a positive barometric sea level response.

3. ANALYSIS OF THE RECURRENCE OF EXTREME SEA LEVEL VALUES

The extreme value distributions are defined as distributions for the minimum or the maximum of a very

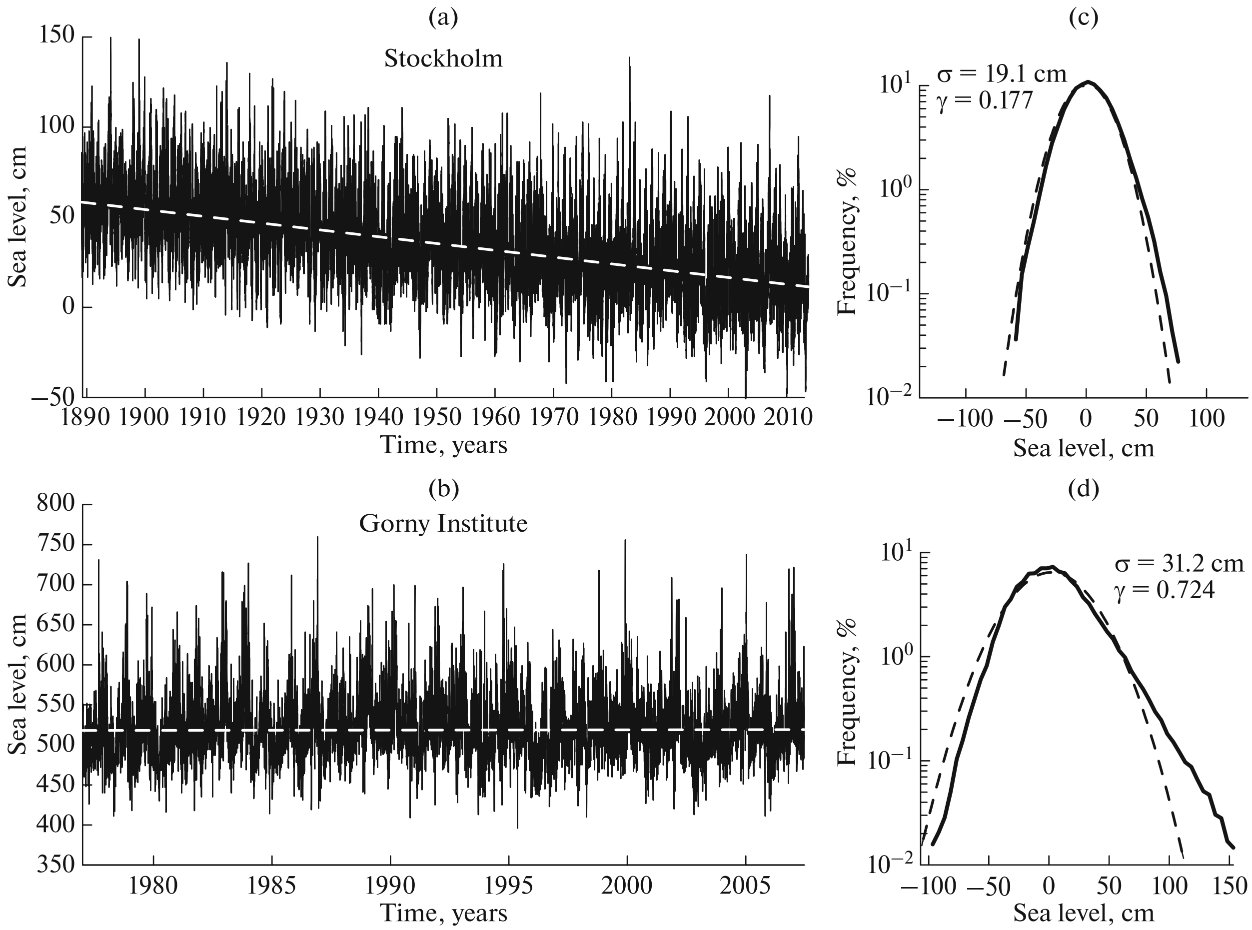

Fig. 2. Hourly records of sea level (a) in Stockholm and (b) at Gorny Institute station. White dashed line shows linear trend. (c, d) Histograms of hourly sea level values (frequency per 5 cm interval) in (c) Stockholm over period of 1889–2013 and (d) Gorny Institute station over period of 1977–2007. Dashed line shows approximations of histograms by normal distribution.

large collection of random observations from the same arbitrary distribution. Usually, the following three types of distributions are attributed to the class of extreme value distributions [5]:

the Gumbel distribution:

$$F(x) = \exp\left(-e^{-(x-\xi)/\theta}\right); \qquad (1)$$

the Fréchet distribution:

$$F(x) = \begin{cases} 0, & x < \xi \\ \exp\left(-\left(\dfrac{(x-\xi)}{\theta}\right)^{-k}\right), & x \ge \xi \end{cases}; \qquad (2)$$

the Weilbull distribution:

$$F(x) = \begin{cases} \exp\left(-\left(\dfrac{(\xi-x)}{\theta}\right)^{k}\right), & x < \xi \\ 1, & x \ge \xi \end{cases}. \qquad (3)$$

The Gumbel distribution (1) is unlimited; i.e., theoretically, the extreme values can attain any magnitudes. The last two distributions are limited. The Fréchet distribution (2) is limited from below, while the Weilbull distribution (3) is limited from above. Note that distributions (2) and (3) can be derived from one another by changing the sign of a random value.

As a rule, limitation in a probabilistic distribution is governed by physical reasons. Thus, in when a solid fracture (e.g., explosion), the size of a fragment is considered a random value, but limited from above by the size of the breaking sample. The extreme values of floods and ebbs are governed by limitations in the duration and force of a storm wind over the water area. According to [19], winds with an extreme speed of up to 30 m/s are observed over the Baltic water area in November–December.

The existence of the absolute maximum or minimum of a random physical value is quite natural from the viewpoint of common sense. Therefore, when using unbounded distribution (1) to approximate empirical data, a researcher in fact acknowledges the insufficient knowledge about the stochastic process. However, in the case of relatively short (of short duration) data sampling, these limits may not "reveal" themselves in the empirical distribution function. Additionally, the Gumbel type 1 distribution is more "convenient" from the viewpoint of fitting to the observation data, as it has only two parameters, whereas distributions (2) and (3) belong to the type of three-parameter distributions. Apparently, the approximation of an empirical distribution function by the dependence with three unknown parameters often proves very problematic due to the instability of the procedure for fitting their values. Distribution (1) can be easily reduced to a linear dependence whose parameters can easily be estimated for empirical values through the slope of the approximating line and its *y*-intercept.

Fitting the parameters of theoretical models (1)–(3) to describe the return period distributions for the extreme values is perhaps one of the most complicated stages of the analysis [28]. Unfortunately, it is impossible to exclude a subjective approach in this procedure. The point is that the approximation procedure is carried out only for the tail of the distribution, which corresponds to the rarest extreme events. The choice of the extreme value of the return period, from which it is possible to use the extreme value distribution model (1)–(3), is quite arbitrary and largely depends on the formulated problem and researcher's scientific intuition. In [29], the maximum-likelihood method was used to fit the parameters of models; in [15] the distribution parameters were calculated with the Hydrognomon software package (http://hydrognomon.org/). In this study, we suggest a graphical approach in which fitting (e.g., by the least squares method) is done for a linear dependence reflecting the asymptotic approximation of models (1)–(3) with periods $T \to \infty$. In order to represent the asymptotic dependence on the graph as a straight line for each of the models (1)–(3), the points are plotted in specially transformed $X(T)$ and $Y(h)$ coordinates.

Let us consider the procedure for estimating the return periods for the extreme sea level values and the methods for approximating the plot of the return period by the model dependences using data on the monthly level deviation maxima in Vyborg from 1889 to 2014.

To calculate the return period for an extreme event (in this case, it when the sea level attains or exceeds a particular value h), it is necessary, first, to sort all recorded events in ascending order, designating them with index j from the minimum $h_1 = -30$ cm to the maximum $h_N = 175$ cm, where $N = 1402$ is the number of events. Thus, the duration of observations is $T_0 = 116.8$ years ($\Delta t = 1$ month).

For a given value j, the number of events with $h \geq h_j$ is $N - j + 1$, and, correspondingly, the return period can be estimated as

$$T_j = \frac{N+1}{N-j+1}\Delta t. \qquad (4)$$

Note that, according to the recommendation suggested in [4], the numerator in this expression is $N + 1$ and not N. For example, for $h = 145$ cm, index $j = 1387$, the number exceedances this sea level value is obtained or exceeded is $N - j + 1 = 16$, and, correspondingly, the return period is 87.7 months (7.3 years).

Next, the probability distribution $F(x)$ of the level h_j can be written as

$$F(h_j) = 1 - \frac{\Delta t}{T_j}. \qquad (5)$$

The probability of the event when the sea level attains $h = 145$ cm within one month of observations is $p_j = \Delta t / T_j = 1.14\%$.

The standard error in estimating probability p_j is evaluated as

$$\sigma^2(p_j) = \frac{j(N - j + 1)}{(N+2)(N+1)^2}. \qquad (6)$$

Thus, when calculating the return period for 145 cm, the corresponding spread of values lies within the interval from 5.9 to 9.7 years.

Figure 3a shows the graph of the return period for monthly sea level maxima in Vyborg. These estimates were obtained from the data of long-term observations from 1889 to 2014. It is seen that in the graph, plotted in the logarithmic scale along the recurrence period axis $X(T) = \log T$, the distribution of extrema points corresponding to the return period values less than 20 years approximates quite well with the straight line. The scale parameter θ of an extreme value x in (1) is ≈50 cm and $\xi \approx 104$ cm; the rarity of events is characterized by $\xi/\theta \approx 2$. This means that an increment of the dimensionless value of an extremum of x/θ by 2 (the sea level value at 100 cm) increases its return period by one order of magnitude. However, for rarer events, the estimates of return periods deviate considerably from the asymptotic of the Gumbel distribution. For large values of the return period, the Weibull distribution (3) proves to be the most appropriate asymptotic. To estimate the parameters of this approximation, it is not sufficient to use the method of fitting the distribution of points on the graph by a linear dependence, as is done for the asymptotic of the Gumbel distribution (1), when $\log T$ is plotted along the x axis and the value of the extreme event h, along the y axis. In the case of distribution (3), it is necessary to use two asymptotic approximations (for $T \to \infty$):

(1) Points are potted on the graph (Fig. 3b) on axes $X(T) = \log T$ and $Y(h) = \log(h_{\max} - h)$; the asymptotic of the Weibull distribution at $T \to \infty$ is a straight line:

$$\log(h_{\max} - h) \sim \frac{1}{k}\log T,$$

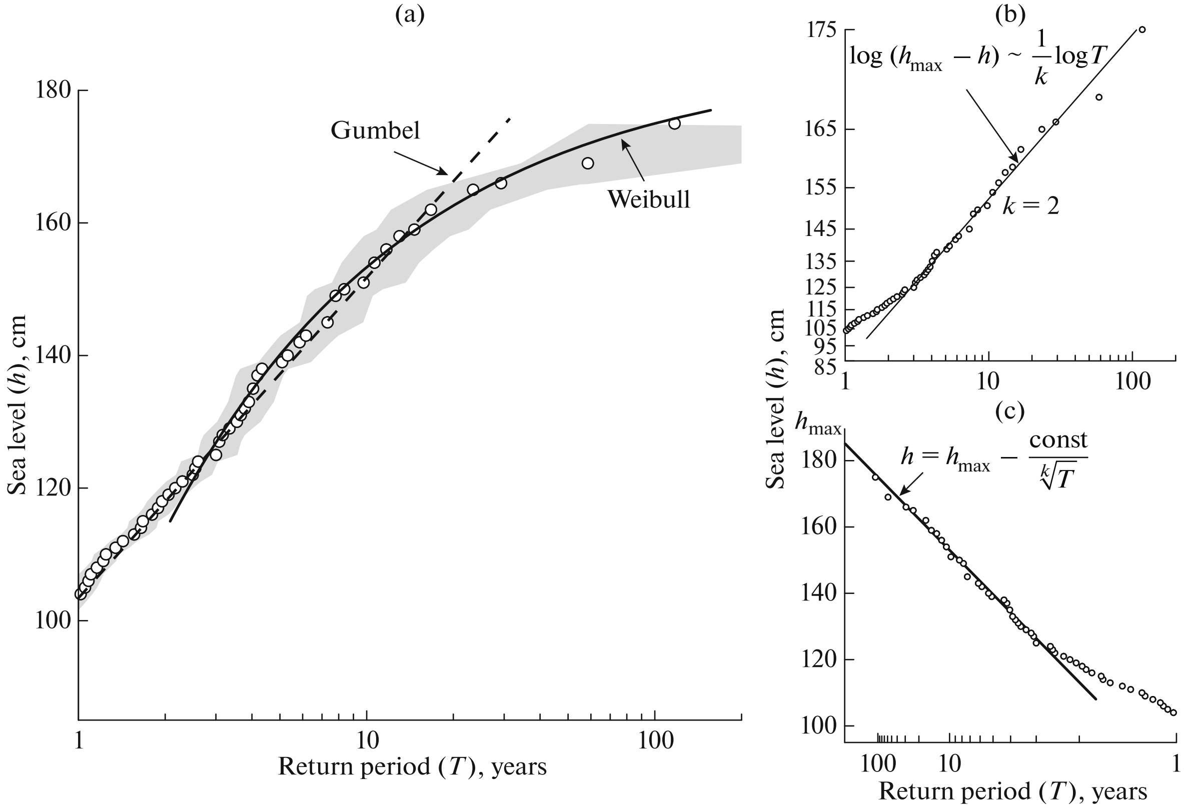

Fig. 3. Distribution function for monthly sea level maxima in Vyborg (1889–2014). Dashed line refers to approximation corresponding to Gumbel distribution (1); solid line refers to Weibull distribution (3). Gray indicates spread of intervals of return period estimates calculated by formula (6). (b, c) Fitting procedure for observed return periods with two asymptotics of Weibull distribution for $T \to \infty$.

and, fitting the slope of the line approximating the distribution, it is easy to estimate the exponential parameter $k \approx 2$ in dependence (3).

(2) Points on the graph (Fig. 3c) are plotted in axes $X(T) = 1/\sqrt[k]{T}$ and $Y(h) = h$. Fitting the approximating line, we can precise the extreme value of $h_{\max}$ (the value ξ in (3)). In this case, at $T \to \infty$, the asymptotic of the Weibull distribution "sets" against the value $h_{\max} \approx 185$ cm at the ordinate axis.

Figure 4 shows the results of analysis of the return period distribution for the extreme sea level values in Stockholm. We were unable to reveal the limit value for such a long-term series of observations (125 years) in the extreme flood distribution (Fig. 4a). At the same time, the character of the distribution of the extreme decrease in sea level in Stockholm (Fig. 4b) shows a trend toward convergence of the sequence of minimum values to an asymptotic limit $h_{\min} \approx -71$ cm (the Fréchet distribution (2)).

For all 13 series of sea level observations, the calculations of empirical distributions for the extreme sea level deviations (positive/negative surges) were carried out and their approximations were fitted. For most stations (except for Stockholm and Vyborg), approximation by the Gumbel distribution proved quite satisfactory. In accordance with the obtained approximations, the extreme sea level rises and ebbs with a 100-year return period in the Baltic Sea were estimated and plotted on the chart (Fig. 1) as sticks. The same values are shown in Table 1. For Stockholm and Vyborg, the extreme sea level values calculated with the Weibull distribution model (Vyborg) and the Fréchet model (Stockholm) are given in parentheses. On the whole, the character of the distribution for the extreme storm surges and ebbs agrees with the estimates made in [29].

When dealing with single events of maximum magnitude, sometimes it is difficult to determine whether a given flood corresponds to a supposed theoretical distribution. On January 9, 2005, in Pärnu (the Gulf of Riga), a storm surge was observed that attained a height of 775 cm in the height system corresponding to the zero BSH + 500 cm, which is 271 cm counted from the mean sea level value. In this case, the next extremum, smaller in magnitude, is 684 (from the zero BSH + 500 cm) or 178 cm from the mean sea level value, respectively. This jump of almost 1 m seems to be an outlier of the flood statistics in Pärnu. In fact, this event is related to the unique storm conditions that

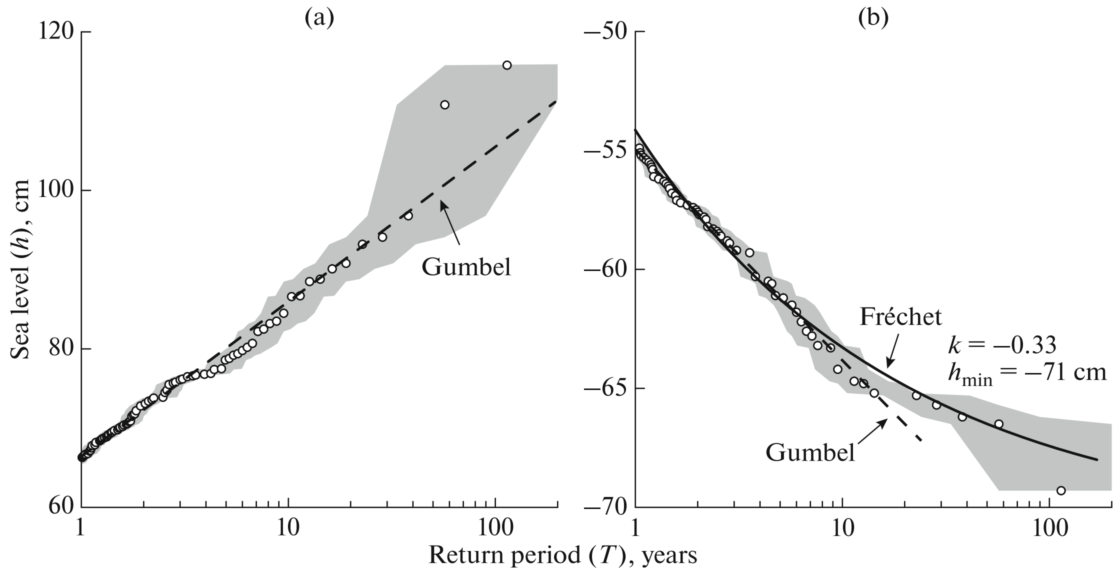

Fig. 4. Recurrence period distribution function for daily sea level (a) maxima and (b) minima in Stockholm (1889–2013). Dashed line refers to approximation corresponding to Gumbel distribution; solid line, to Fréchet distribution (for extreme drop in sea level).

occurred in the Baltic region due to the cyclone Ervin. A hurricane-force wind attained a speed of 34 m/s and wind gusts reached 46 m/s [26]. The storm was one of the most severe in the Baltic region in 50 years: 17 people died; littoral forest areas, power supply, and the environment were damaged. Extrapolating the empirical distribution of flood maxima in Pärnu from 1978 to 2009 by the Gumbel distribution (Fig. 5), we can estimate the probability of a flood with a height equal to or exceeding the storm surge that occurred on January 9, 2005, as 10^{-3} year^{-1}. In fact, it means that over 30 years of observations, an event with an average latency period of 1000 years was recorded. At first glance, this seems impossible. However, the estimate of the probability that such an event occurs within an observation period of 30 years is about 3%, which cannot be considered a vanishing value.

One more example is the analysis of the distribution of Neva flood heights as a function of their return period calculated from historical data recorded since 1703. Today, it is thought that floods are sea level rises with a height of above 160 cm with respect to the zero BSH or above 150 cm with respect to the ordinary level near the Gorny Institute station. Floods are classified as follows [13]: floods with a sea rise of up to 210 cm are dangerous, those from 211 to 299 cm are extremely dangerous, and those above 300 cm are catastrophic. According to the official statistics of sea level observations, 309 floods were recorded in St. Petersburg from 1703 to the present. Among them, 58 floods are considered extremely dangerous. Catastrophic floods were recorded three times: on September 21, 1777; November 19, 1824; and September 23, 1924. So, on average, floods occur once a year; extremely dangerous ones have a return period of about 5–6 years; catastrophic floods occur approximately once per 100 years. Figure 6 shows the empirical dependence of the flood height on its return period. It is seen that for a recurrence period from 1 to 80 years, this dependence is approximated quite satisfactorily by the Gumbel distribution (1); it is shown by a straight line on the graph. However, for $T > 100$ years, the graph is "broken": three events attributed as catastrophic considerably

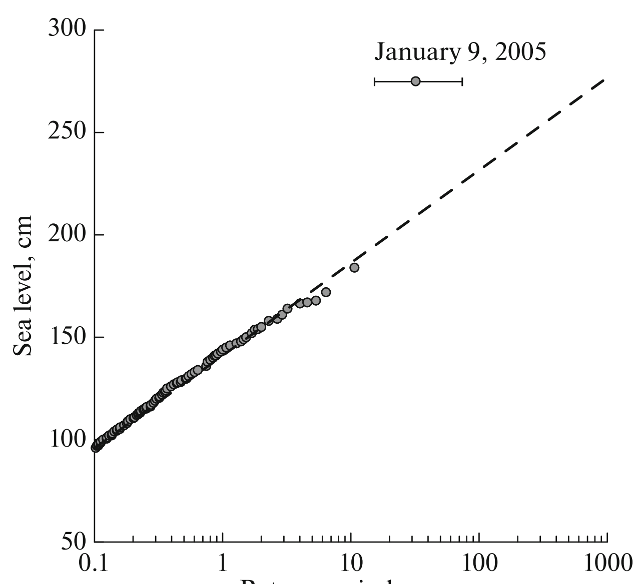

Fig. 5. Distribution function for return period of daily sea level maxima in Pärnu from 1978 to 2009. For estimation of return period of extreme on January 9, 2005, interval corresponding to standard deviation is indicated. Dashed line refers to approximation by Gumbel distribution.

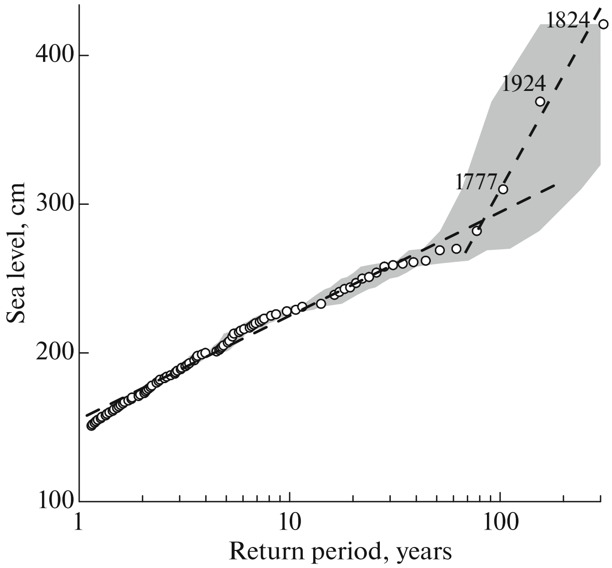

Fig. 6. Distribution function for recurrence periods of floods in St. Petersburg based on historical data from 1703 to present. Sea level values with respect to zero BSH are plotted along y axis. Three catastrophic events that occurred in 1777, 1824, and 1924 are indicated. Dashed lines refer to approximations according to Gumbel distribution for two intervals of recurrence periods: 1–80 and 80–300 years.

deviate from the straight line approximating the distribution of shorter recurrence periods. In this case, the probability of such a "triple" deviation is vanishingly small. In fact, we see in the recurrence period distribution for St. Petersburg a violation of the similarity hypothesis ($h \sim \log T$) implied by the Gumbel distribution (1), which works quite satisfactorily for a return period ranges from 1 to 80 years, but is not valid for the entire observation period (from 1 to 300 years). Catastrophic floods exceeding a level of 300 cm form differently compared with those that are simply dangerous (below 300 cm). One possible explanation for this exceptional peculiarity of catastrophic floods is that the most powerful events can also be related to the influence of the mouth of the Neva. It is known that in some cases, a surge wave propagates a few tens of kilometers upstream. So, it is natural that when the action of the west wind suddenly stops, a flash flood effect can occur, substantially increasing the flood level in St. Petersburg. This additional physical effect, which does not correspond to the formation mechanism (in terms of the similarity hypothesis) of a usual storm surge, should manifest itself as a change in the character of the return period distribution of the most severe floods.

4. NUMERICAL SIMULATION OF SEA LEVEL VARIATIONS

A version of the well-known ROMS (Regional Ocean Modeling System) model [23] adapted to reproduce anemobaric fluctuations in the Baltic Sea level was presented in [7]. The driving force was given in the form of shear stress exerted by the wind $\boldsymbol{\tau} = (\tau_x, \tau_y) = \rho_a C_D |\vec{\mathbf{U}}_\mathbf{W}| (U_W, V_W)$, where $\vec{\mathbf{U}}_\mathbf{W}$ is the wind speed in m/s; ρ_a = 1.25 kg/m^3 is the air density; $C_D = 1.3 \times 10^{-3}$ is the wind drag coefficient.

The force associated with live atmospheric pressure was taken into account in the equations of motion through the field pressure gradients $\vec{F} = \frac{1}{\rho g}\nabla P_a$, where P_a is atmospheric pressure, ρ is sea water density, and g is gravitational acceleration.

GEBCO data on the Baltic Sea floor topography with one-minute resolution were used in the model. The Øresund strait, the Great Belt, and the Little Belt were artificially closed, which made it possible to exclude the influence of the North Sea level variability (external oscillations). The calculations were carried out on a grid with constant latitude and longitude steps and with resolutions of Δx = 2' and Δy = 2', respectively.

It is shown in [18] that the external oscillations in the Kattegat strait enter the Baltic region considerably weakened: for a period of 10 days, they show a tenfold reduction in amplitude. This means that the model for the closed Baltic Basin, in which water exchange through the Danish straits is not considered, quite adequately reproduces internal oscillations with periods shorter than 10–20 days. In fact, the point is reconstruction of the background sea level oscillations, including the eigenmodes of sea level oscillations in the basin, as well as cases of storm surges.

Specification of driving forces is a very important part of the model. To calculate the meteorological effect (of the shear stress exerted by wind and air pressure) on the sea level, NCEP/CFSR reanalysis data [22] were used; atmospheric pressure was given with a step of 0.5°, and the wind, with a step of 0.3125°.

The main characteristic of the developed diagnostic model is its orientation toward adequate reconstruction of the statistic properties of Baltic sea level variability. It concerns the mean oscillation energy (variance), its distribution over the water area, the spectral structure of time registration of the sea level, etc.

Simulation of 7-year series of hourly observations in Narva over 1994–2000 showed that the RMS deviation of sea level variations σ calculated with the model series is 13 cm, which nearly coincides with the estimate σ = 12 cm for the observation data. The asymmetry parameters γ are also close to each other, 0.596 and 0.554, respectively. To carry out an adequate comparison, the series of hourly sea level observations were subjected to high-frequency filtering; the filter width with the Hamming window was 10 days. Thus, we were able eliminate "external" oscillations caused by variations in the level of the North Sea.

Figure 7 shows the return period distributions for the exceedance of the daily extreme level in Narva obtained using observation data (1994–2000) and the model calculations for the same period. One can see that the dependence of the daily level maxima on their return periods shows good agreement between the model series and the observed values in a height range up to 70 cm. The model considerably understates the values for larger values of the extreme sea level rise. This understating for the two most intensive storm surges that occurred on November 29, 1999, and on October 12, 1994, is about 25%.

This shortcoming of the model is a consequence of its "universality." The model parameters were selected so as to approximate sea level fluctuations simultaneously over the entire Baltic Sea Basin [7]. However, as experience in flood simulations shows, the most precise reconstruction of an extreme event requires adjustment of the model for a particular water area, e.g., the Gulf of Finland.

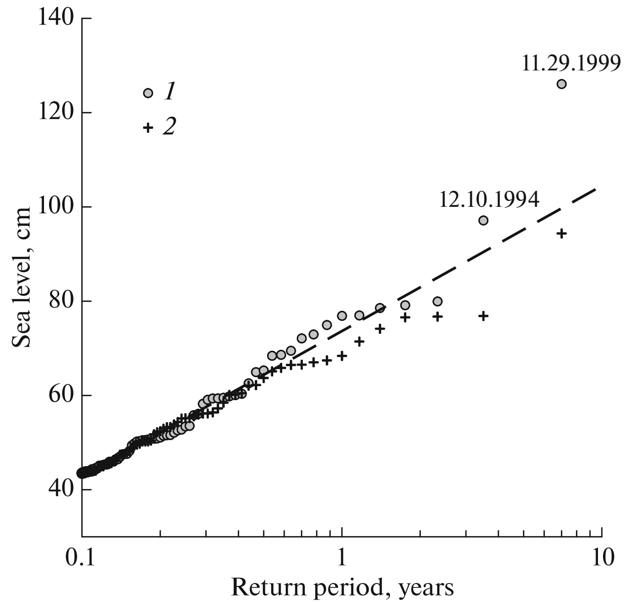

Fig. 7. Distribution function of recurrence periods for daily sea level maxima in Narva (1994–2000) based on observations (*1*) and numerical simulation data (*2*). Dashed line refers to approximation corresponding to Gumbel distribution.

5. DISCUSSION AND CONCLUSIONS

Estimation of the extreme sea level return period is a necessary stage of survey designed works for construction planning in the coastal area. Depending on the type of building (an industrial project, transport terminals, etc.), the requirements on such estimates can be very different. The standard approach is as follows: at the survey stage, when the service life of the construction project is determined, e.g., 50 years, an extreme sea level rise is estimated with a nonexceedance probability of 95 or 99% during this period. The strictest requirements are objects of the nuclear power industry. When constructing nuclear power plants, all natural impacts with a frequency of occurrence above 10^{-4} per year are subjects for consideration [9]. The consideration of extreme sea level ebbs is also of great importance, since they can cause problems for navigation (shallowing of navigating channels), as well as drying of water intakes.

To estimate the frequency of a rare extreme event with a return period much greater than the duration of observations, it is already insufficient to base calculations only on empirical estimates, since the point is extrapolation of the dependence of the magnitude of an extreme event on its return period. For example, if the duration of sea level observations is 100 years, then events with a frequency of 10^{-3} year^{-1} or less are beyond the capacities of "feasible" empirical extrapolation. Therefore, extrapolation of the estimate of a recurrence period is usually based on a probabilistic model of extreme value statistics. Proceeding from the physical similarity hypothesis of the formation processes of extreme events with different scales of the return period, an extreme value distribution model is chosen (e.g., the Gumbel model, etc.) and its parameters are calculated by fitting them to the empirical dependence. Thus, when choosing the Gumbel model, the physical self-similar (scaling) phenomena implies that the relation between the extrema and their return periods is defined as $h \sim \log T$ in a wide range of recurrence periods. As shown in this study, the empirical distributions for the return periods of extreme sea level values for the majority of the 13 stations in the Baltic regions can be satisfactorily approximated by the dependence given by the formula for the double exponential Gumbel distribution (1). It is only for the two longest series of observation (125 years) in Stockholm and Vyborg that the deviation from distribution (1) becomes significant for large return periods. In this case, the analytical Fréchet (2) and Weibull (3) probability distribution functions should be used.

Discussing the problem of extrapolating empirical functions to the domain of low exceedance probabilities, V.Kh. German and S.P. Levikov wrote [2], "It should be taken into account that even a very well approximated empirical maximum distribution function does not ensure the reliability of extrapolated values." Every researcher extrapolates an empirical distribution at his/her own accord, assuming that events of low probability are physically similar to those recorded. Such an example of "unexpected" deviation of an empirical extremum distribution from the theoretical distribution is revealed for the Neva floods. It is seen in Fig. 6 that approximation (1) works satisfactorily on time scales that differ from each other by more than two orders of magnitude. There are only three catastrophic events, which occurred in 1777, 1824, and

1924, that fall out of the statistics determined based on an assumption of physical self-similarity $h \sim \log T$. It is evident that the most severe floods in St. Petersburg form under the action of another physical mechanism that does not manifest itself in "moderate" events.

It should be noted that in the more general case, extreme sea level deviations can form also as a result of coincidence of two or even several events of different nature. For instance, the high water of spring tide can coincide with the maximum sea level rise caused by a storm surge. Consequently, it is necessary to calculate the joint probability for coinciding extreme events [27]. For such a distribution, it is impossible to use the similarity hypothesis for the entire time scale range. Graphs of the return period distribution for the extreme sea level values for the Okhotsk Sea are presented in [24] taking into account the coincidence probability of three events: a high tide, storm surge, and tsunami. The character of the distribution for the greatest sea level deviations are determined by the recurrence period of the rarest tsunami events whose height considerably exceeds both that of a possible storm surge and tide.

In conclusion, we should note a significant result of the statistical analysis of the Baltic Sea level variability, namely, the asymmetry in the distribution of sea level deviations from the mean sea level. For the rarest extreme events, the probability of seal level rises (storm surges) is considerably higher than that of a ebbs (negative surges). Such an asymmetry is associated with the effect of decreasing atmospheric pressure during a cyclone, which always causes a rise in sea level independently of wind direction.

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