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MARINE PHYSICS

Spectrum of Mesoscale Sea Level Oscillations in the Northern Black Sea: Tides, Seiches, and Inertial Oscillations

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Abstract—Long-term data from 23 tide gauges were used to analyze the spectrum of mesoscale sea level variability of the Black Sea. The tides have sharp spectral peaks, and they are detected at diurnal and semidiurnal frequencies for all stations. A local wide spectral peak associated with inertial oscillations is located between the diurnal and semidiurnal tidal peaks. This peak is well known in the spectra of the current velocity variations of the Black Sea, but in the sea level spectrum it has been identified for the first time. At frequencies of >3 cpd, sea level spectra of the Black Sea have (1) wide maxima in the continuous spectrum, which correspond to the main eigenmodes of the sea with periods of 5.6, 4.8, 4.1, and 3.1 h, and (2) sharp peaks of radiational harmonics S_3, S_4, S_5, and S_6. The periods of seiches calculated in this study are close to the periods of eigenmodes of the Black Sea, obtained by the numerical modeling of other authors. The main factors influencing the formation of radiational tides in the Black Sea are presumably breezes and runoff from large rivers. The significant predominance of a harmonic with frequency of 5 cpd (S_5) over other radiational harmonics is caused by the influence of an eigenmode, with a frequency of about 5 cpd. The proximity of the periods of these oscillations leads to resonant amplification and to a corresponding increase in amplitude of the radiational harmonic S_5.

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INTRODUCTION

The sea surface always changes under the influence of various astronomical, meteorological, hydrological, and geological factors. These factors form complex spectrum of sea-level oscillations. In the open ocean, the greatest contribution (up to 85–90%) to the total variance of sea level oscillations is made by tides [22], and the contribution of meteorological sea level oscillations does not exceed 10%, while the energy balance of sea level oscillations in the marginal seas is different.

The Black Sea is one of the most isolated seas of the World Ocean; its isolation leads to the formation of a unique regime of mesoscale sea level oscillations in this sea. For example, the tide, which is formed within the sea is independent: it is a response of water mass to the direct effect of tidal forces [12]. Anemobaric oscillations in the Black Sea become of an induced stationary character, and these oscillations are generated by atmospheric processes with natural synoptic period [5].

The oscillations in the level of the Black Sea have been studied for many centuries (see, for example, monographs [14, 6, 8]). However, only a very small part of these studies has been devoted to the research of spectrum of the mesoscale sea level oscillations of the Black Sea [3, 4, 10, 23]. Some peculiarities of periodic and aperiodic sea level oscillations are still poorly understood. The authors of [4, 5], for example, have noticed a spectral peak with a period of about 5 h, which origin remains unknown.

The present study of the long-term series of observations from 23 tide gauges is based on a spectral analysis of sea level oscillations in the Black Sea. As will be shown below, spectral analysis has allowed us to reveal some interesting effects: in particular, the existence of radiational tides in some parts of the sea and the presence of inertial sea level oscillations in the northeastern Black Sea.

MATERIALS AND METHODS

In our paper, we used a long-term observation series from 23 coastal tide gauges (stations) located along the northern coast of the Black Sea (Fig. 1, table); the data were adjusted to the same time count (GMT). The sampling frequency was 1 h; spikes and errors in the series were removed; short gaps (of few days long) were interpolated. Series containing long gaps were excluded from the analysis. The spectra were calculated by procedure of fast Fourier transform with the use of a half-overlapping Kaiser–Bessel window.

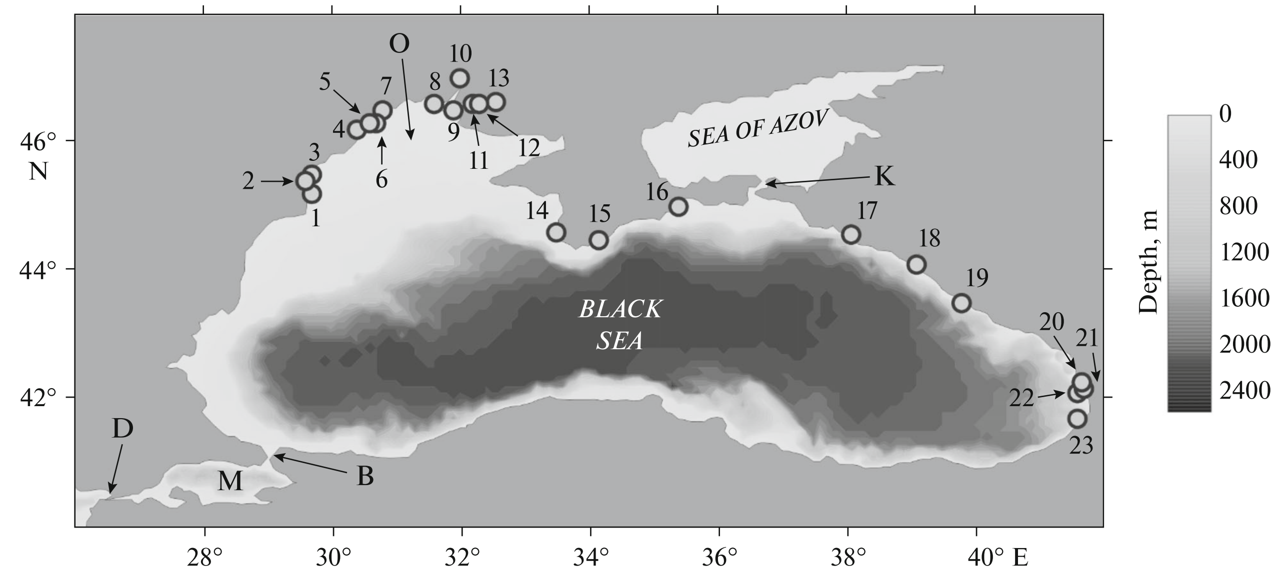

Fig. 1. Location scheme for tidal gauges whose data were used in the present study. Numbers (1–23) in the figure correspond to the numbers of tide gauges in table. Letter notations: M, Sea of Marmara; B, Bosphorus; D, Dardanelles; O, Gulf of Odessa; K, Kerch Strait.

RESULTS

Figure 2 presents the spectra of sea level oscillations for four stations located in different parts of the Black Sea: Prorva (northwestern coast, near the mouth of the Danube River), Odessa (Gulf of Odessa, northern Black Sea), Yalta (southern coast of Crimea), and Batumi (eastern coast of the Black Sea). The length of the time window varied depending on the total length of the record. For the series of Odessa, Yalta, and Batumi, the window length was 8192 measurements (spectral resolution $\Delta\omega \approx 0.00293$ cpd, or cycles per day), while for the series of Prorva it was 4096 measurements ($\Delta\omega \approx 0.00586$ cpd). Depending on the series length, numbers

Tide gauges whose observation series were used in present study

Ord. nos.	Station name	Latitude, deg. N	Longitude, deg. E	Time span, years
1	Bol’shoe	45.2	29.7	1977–1984
2	Vilkovo	45.4	29.6	1977–1984
3	Prorva	45.5	29.7	1977–1984
4	Belgorod-Dnestrovsky	46.2	30.4	1977–1995
5	Paromnaya Pereprava	46.3	30.6	1980–1995
6	Illichivsk	46.3	30.7	1977–1995
7	Odessa	46.5	30.8	1977–1995
8	Ochakov	46.6	31.6	1977–1995
9	Geroiskoe	46.5	31.9	1985–1995
10	Nikolaev	47.0	32.0	1977–1995
11	Stanislav	46.6	32.2	1989–1991
12	Kasperovka	46.6	32.3	1977–1995
13	Kherson	46.6	32.6	1977–1995
14	Sevastopol	44.6	33.5	1977–1995
15	Yalta	44.5	34.2	1977–1995
16	Feodosiya	45.0	35.4	1977–1995
17	Gelendzhik	44.6	38.1	1977–1992
18	Tuapse	44.1	39.1	1977–2014
19	Sochi	43.5	39.8	1977–2014
20	Kulevi	42.3	41.7	1977–1979
21	Poti (Rioni)	42.2	41.7	1977–1979
22	Poti	42.1	41.6	1977–1991
23	Batumi	41.7	41.6	1977–1991

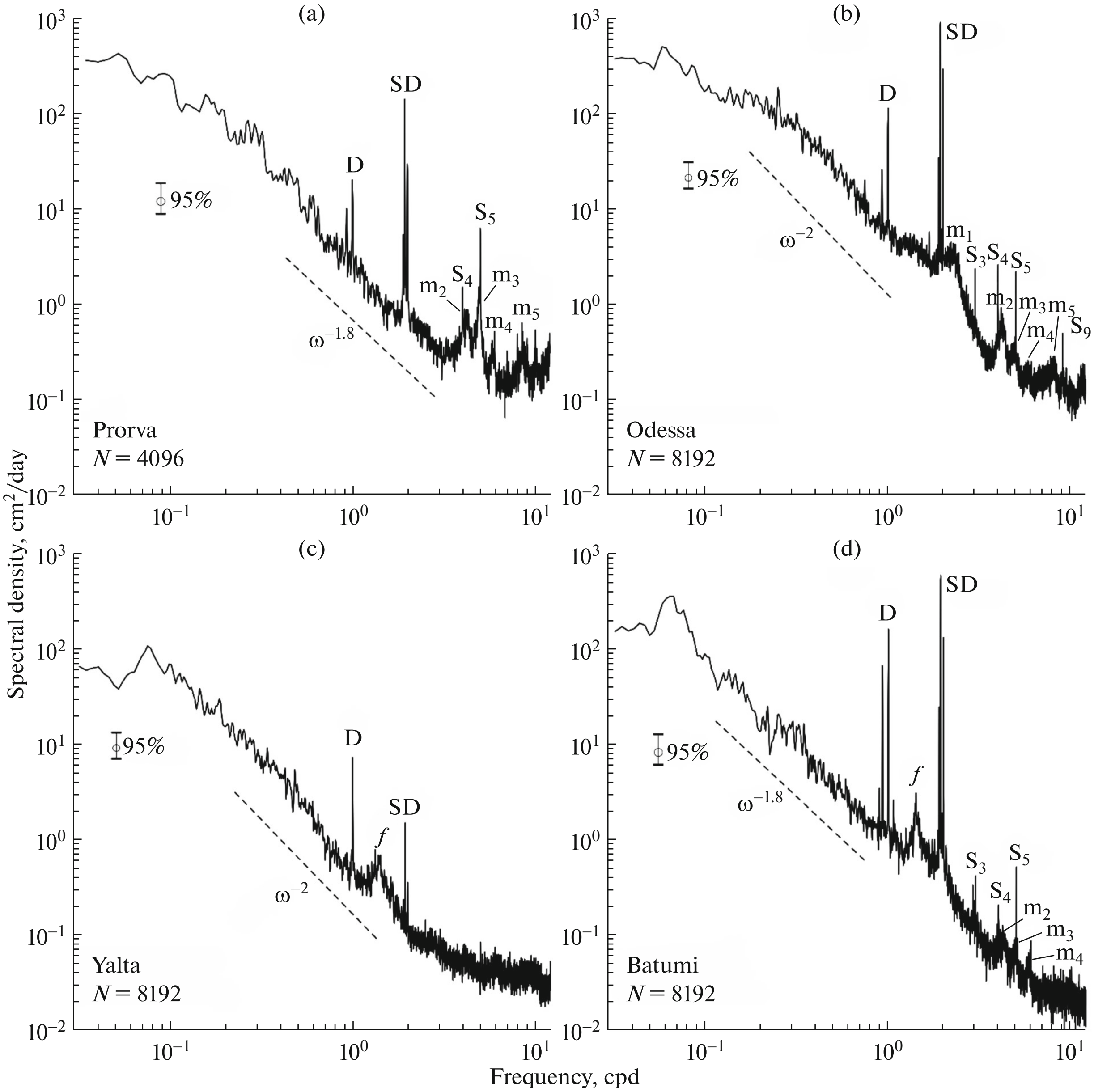

Fig. 2. Sea level spectra of the Black Sea at the stations of Prorva, Odessa, Yalta, and Batumi. D and SD are spectral peaks corresponding to diurnal and semidiurnal tides, respectively; *f* is peak of frequency of inertial oscillations; m_1, m_2, m_3, m_4, and m_5 are modes of the Black Sea level eigen oscillations; S_3, S_4, S_5, and S_9 are radiational peaks; *N* is the length of spectral window.

of degrees of freedom (ν) were different for different stations (60 for Prorva and Batumi and 70 for Odessa and Yalta); hence, confidence intervals were also different (see Fig. 2).

The main amount of energy in the considered spectra is concentrated at low frequencies, and energy rapidly decreases towards higher frequencies. In the frequency range of 0.1–2.5 cpd, the general power law of spectra decay is close to ω^{-2} law, where ω is the spectral frequency (indicated with dashed line in Fig. 2). This law is typical for the long-wave spectra in the open ocean [7]. Depending on the nature of sea-level oscillations, the spectrum can be *continuous* in character, with a continuous energy distribution on the frequency (this is typical of "noise" turbulent processes), or have a character of *discrete* spectrum, in the form of sharp delta-like peaks corresponding to regular harmonic components with fixed frequencies. Oscillations of the sea level caused by variations of atmospheric pressure are generally similar to a random noise pattern and their spectrum is in the form of continuous function of frequency. Tidal sea level oscillations occur in the spectrum as delta-like peaks with the frequencies of main tidal harmonics (K_1, O_1, M_2, S_2, and others). In the spectra of the oscillations of the Black Sea level (Fig. 2), both types of spectrum can be noticed.

Despite their small amplitudes, of a few centimeters, tides are clearly evident in the spectra of sea level oscillations (Fig. 2): these are narrow and sharp peaks corresponding to the frequencies of main tidal harmonics: diurnal K_1 (periods of 23.93 h) and O_1 (25.82 h), and semidiurnal M_2 (12.42 h) and S_2 (12.00 h). In the western and eastern parts of the Black Sea (Prorva, Odessa, and Batumi; see Fig. 2), the mixed, mainly semidiurnal character of tidal oscillations is observed [14], and respective semidiurnal peaks noticeably dominate over the diurnal ones in the spectra. The semidiurnal amphidromic point, at which the amplitude of oscillations is close to zero, is located in the central part of the Black Sea, and this is why semidiurnal tides on Crimean coast (see Yalta in Fig. 2) are weaker than diurnal ones.

In the high-frequency part of the spectrum (frequencies of >2.5 cpd), spectral peaks are observed at many stations in the Black Sea (for example, at the stations Prorva, Odessa, and Batumi; Fig. 2) at frequencies of 3, 4, 5, 6, 7, and 8 cpd, which are multiple of the *solar day*. The long-wave spectra at frequencies higher than tidal are rather monotonic, which is distorted at frequencies of approximately of 3, 4, 5, and 6 cycles per lunar day. These spectral peaks are associated with high-frequency tidal harmonics, which are formed in the shallow waters as a result of the nonlinear interaction of major gravitational tidal constituents associated mainly with movements of the Moon. The tides in the Black Sea are weak, and there are no conditions to generate nonlinear shallow-water harmonics. In the considered case, a precise diurnal cycle suggests that these oscillations are associated not with the Moon but with the Sun and are caused not by gravitational effects but by the solar radiational effects on the sea level. Movements directly or indirectly related to radiation of the Sun are called "*radiational tides*" [19]. Radiational tides are formed by the combined effect of various periodic factors: (1) air temperature variations and induced variations in sea surface temperature, (2) atmospheric tides, and (3) sea-breeze winds [11]. The prevalence of one of these factors depends on the specific physical and geographical conditions at the observation area. It was noted in [14, 15] that sea-breeze winds play a certain role in amplifying the diurnal sea level oscillations of the Black Sea.

At frequencies of >2.5 cpd, the sea level spectrum of the Black Sea also reveals local increases in the *continuous part of the spectrum* (continuum), which form under the effect of eigen sea level oscillations (m_{1-5} in Fig. 2). The sea level spectrum at the Odessa station (Fig. 2b) demonstrates five local peaks, corresponding to seiche modes, with central periods of 10.72 (m_1), 5.62 (m_2), 4.96 (m_3), 4.12 (m_4), and 3.06 h (m_5). At Prorva station (Fig. 2a), local peaks have central periods of 5.64 (m_2), 4.84 (m_3), 4.12 (m_4), and 3.06 h (m_5). At Batumi station, three local peaks, with periods of 5.76 (m_2), 4.8 (m_3), and 4.1 h (m_4), are clearly pronounced. The high-frequency part of spectrum of sea level oscillations at Yalta station contains no visible peaks in the continuous spectrum that could be interpreted as seiches.

An interesting feature of some spectra presented in Fig. 2 (Yalta and Batumi) is the presence of a local wide peak between the semidiurnal and diurnal sharp peaks. The central period of this peak is ~17.2 h, which is close to the inertial period of the Black Sea basin. The period of inertial oscillations is defined by the formula $T_f = 12$ h/sinφ, where φ is the geographic latitude. For example, the Black Sea is characterized by $T_f = 16.4$–18.1 h. The similar peak is observed in spectra of all stations off the northern (Crimean) and eastern (Caucasian) coasts of the Black Sea. There are many research publications devoted to inertial oscillations in the Black Sea (see, for example, [3, 8]), but these inertial oscillations are usually present in the records of variations in current velocity or water density as a result of baroclinic processes. The response in sea level oscillations at periods of inertial oscillations is likely formed by variations in the density field due to the meandering of the coastal currents and the passing of mesoscale vortices.

DISCUSSION

Spectral differences between oscillations of different natures allow us to distinguish and consider the particular components of the Black Sea level oscillations: tides, seiches, and inertial waves.

Periods of the seiches identified in the sea level spectra in the present study are in good agreement with the results of numerical modeling of eigen oscillations of the level of the Black Sea from [1, 2, 9, 10, 16–18, 21]. One-node seiches mode with a period of ~10.9 h was obtained in [9]. The authors of [1, 10] did not take into account the Earth's rotation, and therefore the period of the lower mode was slightly underestimated (9.5–9.7 h). The authors of [2, 17] did not take into consideration the northwestern shallow-water part of Black Sea, so the obtained lower modes were about 5–6 h in period. The two-node seiche with a period of ~5.6 h (this seiche was revealed in the spectra from Odessa, Batumi, and Prorva stations; m_2 in Fig. 2) was successfully reproduced in [1, 9, 10, 16, 17]; however, the maximum amplitude of these seiches is attained in the shallow water of the Gulf of Odessa. As is seen in Fig. 2, it is the sea level spectrum from the Odessa station where this mode is the best expressed. The seiche mode m_3 with a period of ~4.8–4.9 h is seen the best in the sea level spectrum from the Prorva station—this also agrees with the peaks in spatial distribution of amplitudes of eigen oscillations with this period, as obtained in [1, 9, 10, 16, 18]. A seiche with a period of ~4.1 h (m_4 in Fig. 2) was reproduced in [1, 16]. A seiche with a period of ~3.1 h manifests primarily in the northwestern shallow water part of the Black Sea (see Prorva and Odessa stations in Fig. 2) [1, 9, 10, 16]. A good coincidence between the periods of the distinguished local peaks in the continuous part of the spectrum and the results of numerical modeling of the Black Sea's eigen oscillations [1, 9, 10, 16–18] gives us reason to interpret these components as Black Sea seiches.

In [3, 8, 9], in addition to the above-mentioned seiche of the Black Sea, seiches with periods of 24 and 12 h were identified. It was supposed in [9] that seiches with a period of about 12 h are caused by the combined effect of atmospheric fluctuations and tidal forces. One of the grounds for such an interpretation was probably the wide spectral peaks [3, 4, 10] obtained by the spectral analysis of short-term (a few months to a year) series. In long-term series, tidal peaks in the spectra are sharp delta-like ones; i.e., they have the pattern differing from that of seiches, with wide peaks in the continuum (Fig. 3). Thus, spectral peaks of diurnal and semidiurnal periodicities are of a purely *tidal* nature (astronomical and radiational), and the suggestion that seiches with period of ~12.5 h are caused by atmospheric disturbances is erroneous.

The long-term series of observations from tide gauges make it possible to provide the detailed tidal spectroscopy and reveal the fine structure of tidal peaks. Figures 3c and 3d present high-resolution spectra for diurnal and semidiurnal tidal ranges, enabling more detailed examination of individual tidal harmonics. For the analysis a long-term (18 years) series from the Ochakov station was used. The length of the spectral window was $N = 65536$ h, the spectral resolution was $\Delta\omega \approx 0.000366$ cpd, the number of degrees of freedom $\nu = 6$. Spectral peaks corresponding to the main tidal harmonics O_1, P_1, S_1, K_1, N_2, M_2, S_2, and K_2 are clearly distinguishable from noise, noticeably exceeding the 95% confidence interval of the spectrum.

In the diurnal tidal range, we separated adjacent harmonics from one group (P_1–S_1–K_1). The amplitude of radiational harmonic S_1 exceeds those of harmonics O_1, P_1, and K_1. If the ratio between amplitudes of harmonics O_1 and K_1 is close to the theoretically predicted one (decomposition of tidal potential; see [20]), then the ratio between amplitudes of harmonics P_1 and O_1 greatly differs from the theoretical value (in tidal potential, the ratio between amplitudes of these harmonics is $H_{O_1}/H_{P_1} \approx 2.15$). The harmonics K_1 and P_1 probably have mixed origin—gravitational and radiational, due to the division of the main diurnal harmonic S_1 from seasonal modulation. In contrast to gravity harmonics, radiational harmonics are not strictly determined: their frequencies are stable, but amplitudes change under the effect of various hydrometeorological factors, varying from season to season. A similar formation mechanism for radiational tide (and, in particular, radiational harmonics P_1 and K_1) is observed in the CuronianLagoon, Baltic Sea [13].

Figures 3a and 3b demonstrate the mesoscale spectra of sea level oscillations for the Vilkovo and Ochakov stations. In the low-frequency part of mesoscale sea level spectrum, peaks corresponding to diurnal and semidiurnal tidal harmonics (O_1, K_1, M_2, S_2, and others) sharply differ from the continuous spectrum. In contrast, the high-frequency part demonstrates a more complicated pattern: sharp discrete peaks S_4, S_5, and S_6 corresponding to radiational tides are located near the peaks of main seiche modes m_2, m_3, m_4. It has been shown in [11] that radiational tides in the Baltic Sea occur in the spectra of sea level oscillations at some locations (Narva and Daugava) in the form of acute spectral peaks at frequencies of 3, 4, 5, 6, 7, and 8 cpd, which are multiple to the solar day cycle. Radiational tidal peaks in the Baltic Sea have close spectral density values (these values, however, slightly decrease with the growth of frequency). In the Black Sea, radiational tidal peaks manifest at particular stations, e.g., at the Vilkovo station (Fig. 3a), the peak S_5 with a frequency of 5 cpd (4.8 h in period) is clearly expressed, whereas the peaks S_4 and S_6 are almost absent. The spectrum of sea level oscillations from the Ochakov station (Fig. 3b) contains well-expressed harmonics S_3, S_4, S_5, and S_6, but the radiational peak with frequency of 5 cpd (S_5) is also considerably stronger than the others.

A peak with a frequency of about 5 cpd was revealed for the first time by German [4] in the sea level spectrum near Vilkovo (Odessa oblast, Ukraine). He proposed two hypotheses for the origin of this wave: (1) the peak corresponds to the longitudinal one-node seiche of the Black Sea with period of 4.5 h; (2) the peak corresponds to the tidal shallow water component. German supposed that, due to proximity of the Danube delta margin and Vilkovo (~19 km), nonlinear shallow water tidal components can form in this area [4]. However, he utilized relatively short observation series (up to three months long) in his work [4], so he could not define the frequency and Q-factor of the peak with a frequency of 5 cpd more precisely.

The results of the present analysis show that the revealed peak has a strict frequency of 5.00 cpd, and it is multiple to the solar day cycle. The shallow water gravitational harmonics formed by nonlinear interaction between the main diurnal and semidiurnal components do not contain a component with this frequency. Thus, such a component is not of a gravitational tidal origin but a radiational one. Moreover, the peak with this frequency manifests in the sea level spectra at Vilkovo station, on the one hand, and at Prorva, Ochakov, Odessa, Batumi, and other stations, on the other hand (Figs. 2 and 3). The proximity of large river mouths—the Danube for Prorva and Vilkovo; the Dniester for Odessa and Illichivsk; the

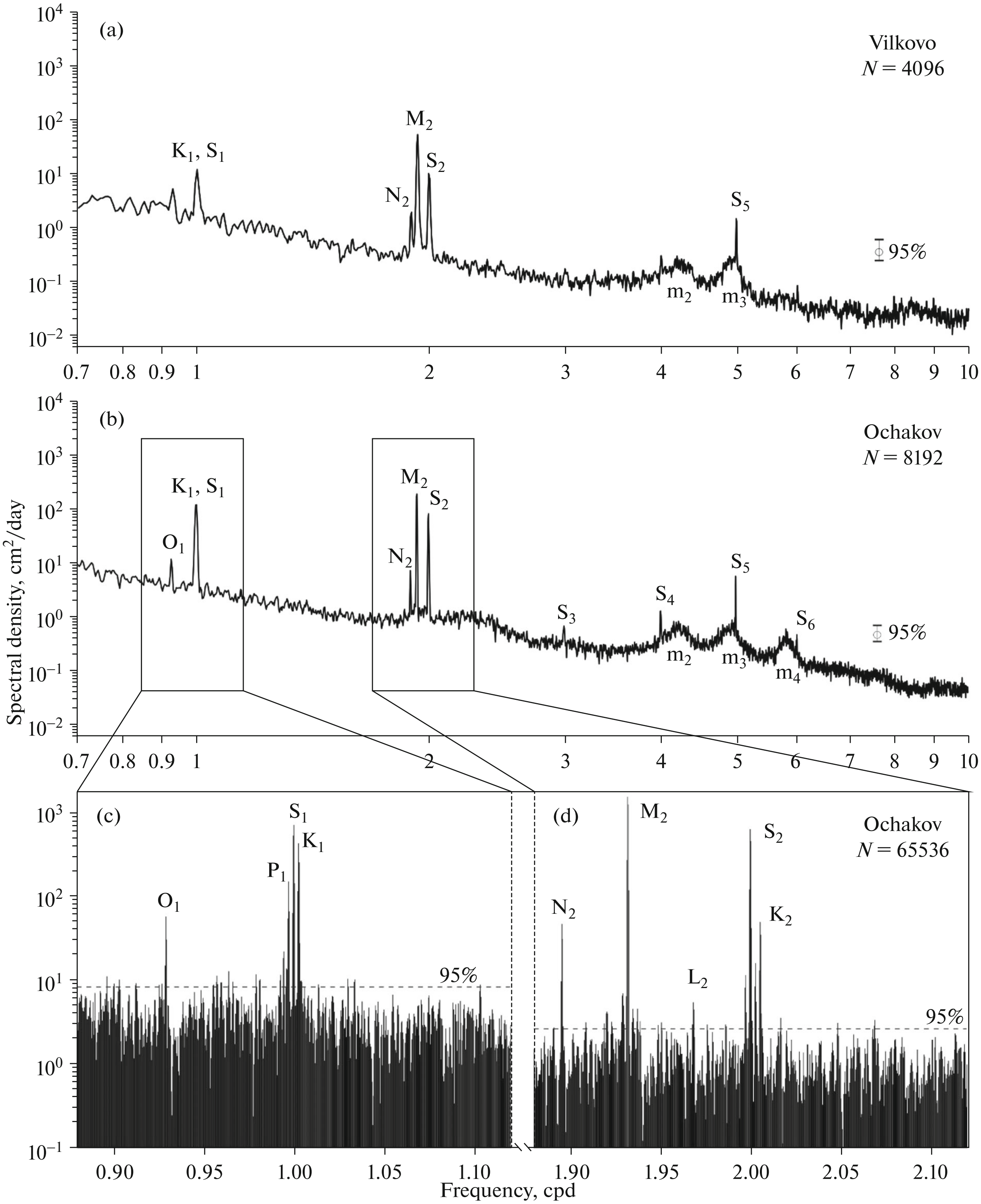

Fig. 3. Sea level spectra at the Vilkovo (a) and Ochakov (b) stations. Indicated are the main tidal peaks, high-frequency radiational harmonics (S_3, S_4, S_5, and S_6), and modes of eigen oscillations (m_2, m_3, and m_4). High-resolution spectra of sea level oscillations for the diurnal (c) and semidiurnal (d) frequency bands at the Ochakov station. Dashed line indicates 95% confidence interval for diurnal and semidiurnal harmonics, as calculated in accord with χ^2 distribution for each band.

South Bug and Dnieper for Ochakov, Nikolaev, and Kherson—seems to be an important factor in forming radiational tides in the Black Sea. But it is the influence of the seiche mode, with a period of 4.8–4.9 cpd that probably amplifies exactly the peak S_5, compared to the other radiational components (S_3, S_4, S_6). The proximity of periods of this seiche and radiational harmonics S_5 leads to resonance and an increase in amplitude of this harmonics. The periods of other radiational components (S_3, S_4, and S_6) considerably differ from periods of the main seiche modes, and hence these harmonics are not amplified. Figure 3b illustrates how resonance forms at frequency of 5 cpd and the absence of resonance at other frequencies are multiple for the solar day cycle.

CONCLUSIONS

Based on long-term observation series, a mesoscale sea level spectrum of the Black Sea has been considered. The continuous spectrum formed under the effect of atmospheric processes decreases by frequency according to the law ω^{-2}. Tides forming in the Black Sea manifest in the spectra in the form of sharp delta-like peaks at diurnal and semidiurnal tidal frequencies. A wide local peak at periods of inertial oscillations is observed between these spectral peaks. This peak is well known in the spectra of variations in current velocity in the Black Sea (see [3, 8]), but in the sea level spectra they have been revealed for the first time.

At frequencies of more than 3 cpd, the characteristic features of the spectrum of oscillations of the level of the Black Sea are wide peaks in the continuum (these peaks correspond to the main seiche modes of the Black Sea, with periods of 5.6, 4.8, 4.1, and 3.1 h) and sharp delta-like peaks of radiational harmonics S_3, S_4, S_5, and S_6. The periods of seiches in the calculated spectra are close to the periods of eigen oscillations of the Black Sea level obtained by numerical modeling in studies of different authors (see, for example, [1, 9, 10]).

The main factors affecting the formation of radiational tides in the Black Sea are supposedly breezes and runoff from large rivers. The noticeable predominance of the oscillation component with a frequency of 5 cpd (S_5) over other radiational harmonics is caused by the existence of seiche with close frequency: it leads to resonance and to the increase in the amplitude of radiational harmonic S_5.

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