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Vertical Heat and Salt Fluxes Induced by Inertia-Gravity Internal Waves on Sea Shelf

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Abstract—Free inertia-gravity internal waves are considered in a two-dimensional vertically nonuniform flow in the Boussinesq approximation. The equation for vertical velocity amplitude includes complex factors caused by the gradient of the flow velocity component transverse to the wave-propagation direction; therefore, the eigenfunction and wave frequency are complex. It is shown that the decrement of damping (imaginary correction to the frequency) of 15-min internal waves is two orders of magnitude smaller than the wave frequency; i.e., the waves weakly damp. Vertical wave fluxes of heat and salt are nonzero due to the phase shift between fluctuations of the vertical velocity and temperature (salinity) different from $\pi/2$. The vertical component of the Stokes drift speed is also nonzero and contributed into the vertical transport.

Keywords: inertia-gravity internal waves, Stokes drift, wave fluxes of heat and salt

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INTRODUCTION

Internal waves play an important role in dynamic processes on a sea shelf, due to ever-present energy sources that generate the waves, i.e., fluctuations of air pressure, wind stresses on the sea surface, interactions of flows and tidal waves with bottom inhomogeneities, and instability of flows.

The vertical exchange is an important part of the marine ecosystem. It is usually connected with turbulent diffusion. Causes of generation of a small-scale turbulence are very different. We should distinguish those that act in the stratified ocean depth, including hydrodynamic instability of flows and internal wave breaking. One important contribution into the vertical exchange is provided by the double diffusion [1–3], where the temperature and salinity increase with depth under stable stratification or decrease with depth. The "salt fingers" mode is possible in the last case; it is quite typical for the world ocean [1]. Only double diffusion explains the occurrence of step structures in the ocean, which form the fine vertical structure of hydrophysical fields [2]. The development of this approach resulted in the development of nonlinear mathematical models of ocean microstructure formation on the basis of amplitude equations of thermohaline convections [3]. The contribution of internal waves in the formation of the fine vertical structure due to the kinematic effect of high modes, internal wave breaking [4], nonlinear generation of mean flows by a wave packet [5–8], and vertically high-oscillating correction to the density field [5, 6] should be taken into account separately.

The effect of a small-scale turbulence on the internal waves was considered in several works [9–11]. It was shown that internal waves damp when considering turbulent viscosity and diffusion. Vertical wave fluxes of heat and salt are nonzero in this case [12]. It is interesting to find these fluxes in a vertically inhomogeneous flow for inertia-gravity internal waves (taking into account the Earth's rotation). It is noticeable that the wave fluxes are nonzero in this case even neglecting turbulent viscosity and diffusion. Below we show that the boundary problem for internal waves with consideration of the Earth's rotation has complex factors in the presence of a 2D flow, where the velocity component transverse to the wave propagation direction depends on the vertical coordinate; therefore, the wave frequency and eigenfunction are complex. This means weak damping of a wave and nonzero vertical wave fluxes of heat $\overline{Tw}$ and salt $\overline{Sw}$ (T, S, and w are the wave disturbance of the temperature, salinity, and vertical velocity). The vertical component of the Stokes drift velocity is also nonzero and contributes into the vertical transfer.

STATEMENT OF THE PROBLEM

Let us consider free internal waves in a baroclinic flow taking into account the Earth's rotation in an unbounded fixed-depth basin. Two components of the velocity of a horizontally stratified mean flow depend on the vertical coordinate. The dispersion relation is derived in a linear approximation. Vertical wave heat and salt fluxes and the Stokes drift velocity are found on the second order of amplitude. A set of hydrodynamic equations for wave disturbances in the Boussinesq approximation has the form

$$\frac{Du}{Dt} - fv + w\frac{dU_0}{dz} = -\frac{1}{\rho_0(0)}\frac{\partial P}{\partial x}, \quad (1)$$

$$\frac{Dv}{Dt} + fu + w\frac{dV_0}{dz} = -\frac{1}{\rho_0(0)}\frac{\partial P}{\partial y}, \quad (2)$$

$$\frac{Dw}{Dt} = -\frac{1}{\rho_0(0)}\frac{\partial P}{\partial z} - \frac{g\rho}{\rho_0(0)}, \quad (3)$$

$$\frac{D\rho}{Dt} = -w\frac{d\rho_0}{dz}, \quad (4)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (5)$$

where x, y, and z are two horizontal and a vertical coordinates—the z axis is upward directed; u, v, and w are two horizontal and a vertical components of the wave flow velocity; ρ and P are the wave disturbances of density and pressure; H is the sea depth; $\rho_0(z)$ is the mean density profile; f is the Coriolis parameter; $U_0(z)$ and $V_0(z)$ are the components of the mean flow velocity; g is the gravity acceleration; and $\frac{D}{Dt} = \frac{\partial}{\partial t} + (u + U_0)\frac{\partial}{\partial x} + (v + V_0)\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$.

The "solid top" condition is a boundary condition on the sea surface ($z = 0$); it filters internal waves from the surface waves [13]:

$$w(0) = 0. \quad (6)$$

The impermeability condition is a boundary condition on the bottom:

$$w(-H) = 0. \quad (7)$$

LINEAR APPROXIMATION

Let us find the linear approximation in the form

$$u_1 = u_{10}(z)Ae^{i\theta} + \text{c.c.}, \; v_1 = v_{10}(z)Ae^{i\theta} + \text{c.c.}, \; w_1 = w_{10}(z)Ae^{i\theta} + \text{c.c.}, \; P_1 = P_{10}(z)Ae^{i\theta} + \text{c.c.}, \; \rho_1 = \rho_{10}(z)Ae^{i\theta} + \text{c.c.}, \quad (8)$$

where c.c. means complex conjugated terms; A is the amplitude factor; θ is the wave phase; $\partial\theta/\partial x = k$, $\partial\theta/\partial t = -\omega$, k is the horizontal wavenumber, and ω is the wave frequency. The wave is assumed to propagate along the x axis.

Substitution of Eq. (8) in set (1)–(5) allows us to connect the amplitude functions $u_{10}, v_{10}, \rho_{10}, P_{10}$ with w_{10}:

$$u_{10} = \frac{i}{k}\frac{dw_{10}}{dz}, \; \Omega = \omega - kU_0, \quad (9)$$

$$\frac{P_{10}}{\rho_0(0)} = \frac{i}{k}\left[\frac{\Omega}{k}\frac{dw_{10}}{dz} + \frac{dU_0}{dz}w_{10} + \frac{f}{\Omega}\left(i\frac{dV_0}{dz}w_{10} - \frac{f}{k}\frac{dw_{10}}{dz}\right)\right], \quad (10)$$

$$\rho_{10} = -\frac{i}{\Omega}w_{10}\frac{d\rho_0}{dz}, \quad v_{10} = \frac{1}{\Omega}\left(\frac{f}{k}\frac{dw_{10}}{dz} - iw_{10}\frac{dV_0}{dz}\right). \quad (11)$$

The function w_{10} obeys the equation

$$\frac{d^2w_{10}}{dz^2} + k\left[\frac{if\frac{dV_0}{dz}}{\Omega^2 - f^2} - \frac{f^2\frac{dU_0}{dz}}{\Omega(\Omega^2 - f^2)}\right]\frac{dw_{10}}{dz} + kw_{10}\left[\frac{k(N^2 - \Omega^2) + \Omega\frac{d^2U_0}{dz^2} + if\frac{d^2V_0}{dz^2}}{\Omega^2 - f^2} + \frac{ifk\frac{dU_0}{dz}\frac{dV_0}{dz}}{\Omega(\Omega^2 - f^2)}\right] = 0, \quad (12)$$

where $N^2 = -\frac{g}{\rho_0(0)}\frac{d\rho_0}{dz}$ is the squared Brunt–Väisälä frequency.

Boundary conditions for w_{10} are

$$z = 0: \; w_{10} = 0, \quad (13)$$

$$z = -H: \; w_{10} = 0. \quad (14)$$

Equation (12) includes complex factors with small imaginary parts; therefore, we change to dimensionless variables (strokes mean dimensionless physical parameters):

$$z = Hz', \; t = t'/\omega_*, \; w_{10} = w'_{10}V_{0*}, \; V_0 = V'_{0*}V_{0*}, \; U_0 = U'_0V_{0*}, \; k = k'/H, \; f = f'\omega_*, \; \omega = \omega'\omega_*, \; N = N'\omega_*, \; \Omega = \Omega'\omega_*, \quad (15)$$

where ω_* is the characteristic wave frequency; V_{0*} is the characteristic flow velocity transverse to the wave propagation direction.

Then Eq. (12) takes the form

$$\frac{d^2w'_{10}}{dz'^2} + k'\left[\frac{i\varepsilon f'\frac{dV'_0}{dz}}{\Omega'^2 - f'^2} - \frac{\varepsilon f'^2\frac{dU'_0}{dz}}{\Omega'(\Omega'^2 - f'^2)}\right]\frac{dw'_{10}}{dz'}$$
$$+ k'w'_{10}\left[\frac{k'(N'^2 - \Omega'^2) + \varepsilon\Omega'\frac{d^2U'_0}{dz'^2} + i\varepsilon f'\frac{d^2V'_0}{dz^2}}{\Omega'^2 - f'^2} + \frac{i\varepsilon^2 f'k'\frac{dU'_0}{dz}\frac{dV'_0}{dz}}{\Omega'(\Omega'^2 - f'^2)}\right] = 0, \quad (16)$$

where $\varepsilon = V_{0*}/H\omega_*$ is a small parameter. The imaginary part of factors in Eq. (16) is on the order ε; therefore, the imaginary part of solution w_{10} is also proportional to ε; i.e., the solution of Eq. (16) can be represented as

$$w'_{10}(z') = w'_0(z') + \varepsilon i w'_1(z'), \quad (17)$$

where $w'_0(z')$ and $w'_1(z')$ are the real functions. The frequency can also be expanded in the parameter ε:

$$\omega' = \omega'_0 + \varepsilon\sigma'_1 + \ldots, \quad (18)$$

then $\Omega' = \Omega'_0 + \varepsilon\sigma'_1 + \ldots$. After the substitution of Eqs. (17) and (18) in Eq. (16), we determine the boundary problems for $w'_0(z')$ and $w'_1(z')$. The function $w'_0(z')$ obeys the equation (accurate to terms ~ε):

$$\frac{d^2w'_0}{dz'^2} - \varepsilon k'\frac{dw'_0}{dz'}\frac{dU_0}{dz'}\frac{f'^2}{\Omega'_0(\Omega'^2_0 - f'^2)}$$
$$+ \frac{k'w'_0}{(\Omega'^2_0 - f'^2)}\left[k'(N'^2 - \Omega'^2_0) + \varepsilon\Omega'_0\frac{d^2U'_0}{dz'^2}\right] = 0. \quad (19)$$

The boundary conditions for w'_0 are

$$w'_0(0) = 0, \ w'_0(-1) = 0. \quad (20)$$

The function $w_1(z')$ obeys the equation (accurate to terms ~ε):

$$\frac{d^2w'_1}{dz'^2} - \varepsilon k'\frac{dw'_1}{dz}\frac{dU'_0}{dz'}\frac{f'^2}{\Omega'_0(\Omega'^2_0 - f'^2)}$$
$$+ \frac{k'w'_1}{(\Omega'^2_0 - f'^2)}\left[k'(N'^2 - \Omega'^2_0) + \varepsilon\Omega'_0\frac{d^2U'_0}{dz'^2}\right] = F'(z'), \quad (21)$$

where

$$F'(z') = -k'\frac{dw'_0}{dz'}\frac{dV'_0}{dz'}\frac{f'}{(\Omega'^2_0 - f'^2)} + ik'\frac{dw'_0}{dz'}\frac{dU'_0}{dz'}\frac{\sigma'_1 f'^2(3\Omega'^2_0 - f'^2)}{\Omega'^2_0(\Omega'^2_0 - f'^2)^2}$$
$$- \frac{k'w'_0}{(\Omega'^2_0 - f'^2)}\left[k'\frac{2i\Omega'_0\sigma'_1(N'^2 - f'^2)}{(\Omega'^2_0 - f'^2)} + \varepsilon\frac{d^2U'_0}{dz'^2}\frac{i\sigma'_1(\Omega'^2_0 + f'^2)}{(\Omega'^2_0 - f'^2)} + f'\frac{d^2V'_0}{dz'^2} + \varepsilon\frac{f'k'}{\Omega'_0}\frac{dU'_0}{dz'}\frac{dV'_0}{dz'}\right].$$

The boundary conditions for w'_1 are

$$w'_1(0) = 0, \ w'_1(-1) = 0. \quad (22)$$

When changing to dimensional variables, Eq. (19) takes the form

$$\frac{d^2w_0}{dz^2} - k\frac{dw_0}{dz}\frac{dU_0}{dz}\frac{f^2}{\Omega_0(\Omega_0^2 - f^2)} + \frac{kw_0}{(\Omega_0^2 - f^2)} \times \left[k(N^2 - \Omega_0^2) + \Omega_0\frac{d^2U_0}{dz^2}\right] = 0, \quad (23)$$

where $\Omega_0 = \omega_0 - k\,U_0$ is the wave frequency with the Doppler shift.

Equation (23) should be supplemented by the boundary conditions

$$w_0(0) = 0, \ w_0(-H) = 0. \quad (24)$$

If there is no flow at $U_0 = 0$, boundary problems (23) and (24) have an enumerable set of eigenfunctions—a set of modes. A certain value of the frequency $\omega_0 < \max(N)$, answering a given mode, corresponds to each value of the wavenumber k. At $U_0 \neq 0$, there is a possibility that no discrete spectrum of real eigenfrequencies exists [14] because of singularities in Eq. (23) at $\Omega_0 = 0$ and $\Omega_0 = \pm f$ (hydrodynamically stable flows are considered). There is a critical layer where the wave phase velocity is equal to the flow velocity in the presence of singularity at $\Omega_0 = 0$. The singularity shifts to the level $\Omega_0 = f$ if considering the Earth's rotation. The effect of this singularity to the dispersion curves is shown by the calculations below.

Let $a(z) = -\frac{f^2 k}{\Omega_0(\Omega_0^2 - f^2)}\frac{dU_0}{dz}$ and $b(z) = \frac{k}{(\Omega_0^2 - f^2)} \times \left[k(N^2 - \Omega_0^2) + \Omega_0 \frac{d^2U_0}{dz^2}\right]$; then Eq. (23) is written as

$$\frac{d^2w_0}{dz^2} + a(z)\frac{dw_0}{dz} + b(z)w_0 = 0. \quad (25)$$

Let us reduce Eq. (25) to a self-conjugated form by multiplying both parts of the equation by $p(z) = \exp\left(\int a(z)dz\right)$:

$$\frac{d}{dz}\left(p(z)\frac{dw_0}{dz}\right) - q(z)w_0 = 0, \quad (26)$$

where $q(z) = -b(z)p(z)$.

After changing to dimensional variables, Eq. (21) is transformed to the form

$$\frac{d^2w_1}{dz^2} + a(z)\frac{dw_1}{dz} + b(z)w_1 = F(z), \quad (27)$$

where

$$F(z) = -k\frac{dw_0}{dz}\frac{dV_0}{dz}\frac{f}{(\Omega_0^2 - f^2)} + ik\frac{dw_0}{dz}\frac{dU_0}{dz}\frac{\sigma_1 f^2(3\Omega_0^2 - f^2)}{\Omega_0^2(\Omega_0^2 - f^2)^2}$$
$$- \frac{kw_0}{(\Omega_0^2 - f^2)}\left[k\frac{2i\Omega_0\sigma_1(N^2 - f^2)}{(\Omega_0^2 - f^2)} + i\frac{d^2U_0}{dz^2}\frac{\sigma_1(\Omega_0^2 + f^2)}{(\Omega_0^2 - f^2)} + f\frac{d^2V_0}{dz^2} + \frac{fk}{\Omega_0}\frac{dU_0}{dz}\frac{dV_0}{dz}\right].$$

The boundary conditions for w_1 are

$$w_1(0) = 0, \quad w_1(-H) = 0. \quad (28)$$

Multiplying both parts of linearly inhomogeneous Eq. (27) by the function $p(z)$, we find the self-conjugated operator in the left part, the same as in linearly homogeneous Eq. (26):

$$\frac{d}{dz}\left(p(z)\frac{dw_1}{dz}\right) - q(z)w_1 = F_1(z), \quad (29)$$

where $F_1(z) = p(z)F(z)$.

The solvability condition of boundary problem (28) and (29) [15] is

$$\int_{-H}^{0} F_1 w_0 dz = 0. \quad (30)$$

From this we derive σ_1:

$$\sigma_1 = \frac{a}{b},$$

where

$$a = ifk\int_{-H}^{0}\frac{pw_0}{(\Omega_0^2 - f^2)}\left(\frac{d}{dz}\left(w_0\frac{dV_0}{dz}\right) + w_0\frac{k}{\Omega_0}\frac{dU_0}{dz}\frac{dV_0}{dz}\right)dz,$$
$$b = \int_{-H}^{0}\frac{pkw_0}{(\Omega_0^2 - f^2)^2}\left[w_0\left(2k\Omega_0(N^2 - f^2) + \frac{d^2U_0}{dz^2}(\Omega_0^2 + f^2)\right) - f^2\frac{dw_0}{dz}\frac{dU_0}{dz}\frac{(3\Omega_0^2 - f^2)}{\Omega_0^2}\right]dz. \quad (31)$$

NONLINEAR EFFECTS

The velocity of Stokes drift of liquid particles is defined by the equation [16]

$$\mathbf{u}_s = \overline{\int_0^t \mathbf{u}d\tau\nabla\mathbf{u}}, \quad (32)$$

where $\mathbf{u}$ is the field of Euler wave velocities; the bar means averaging over the wave period.

The vertical component of the Stokes drift velocity is defined as

$$w_s = iA_1A_1^*\left(\frac{1}{\omega} - \frac{1}{\omega^*}\right)\frac{d}{dz}\left(w_{10}w_{10}^*\right), \quad (33)$$

where $A_1 = A\exp(\delta\omega \cdot t)$, $\delta\omega = \sigma_1/i$ is the wave damping decrement; σ_1 is purely imaginary.

In the presence of a mean flow, where the velocity component V_0, transverse to the wave propagation

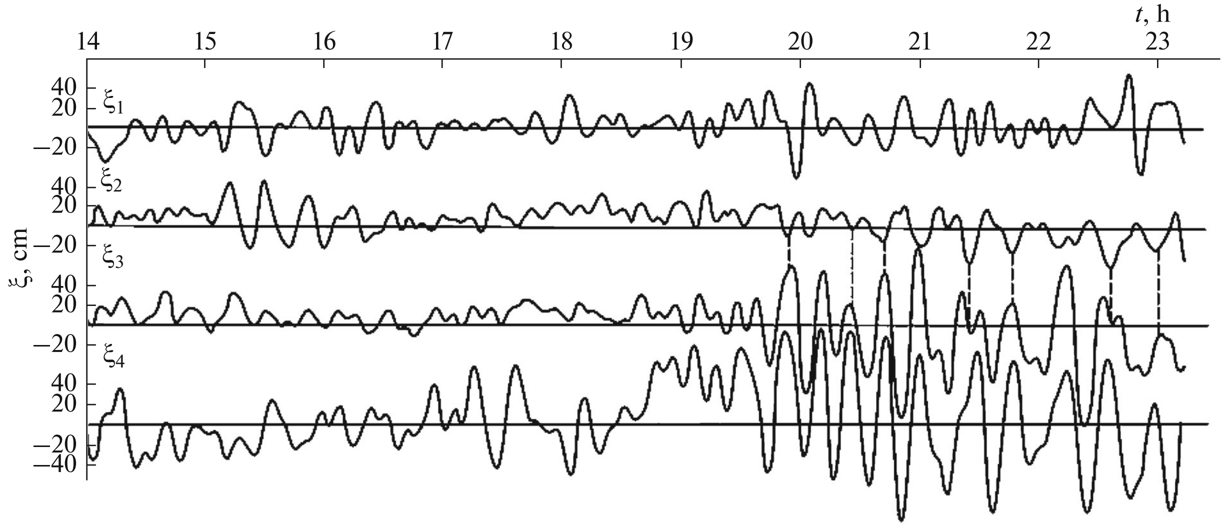

Fig. 1. Time variations of vertical shifts of temperature isolines.

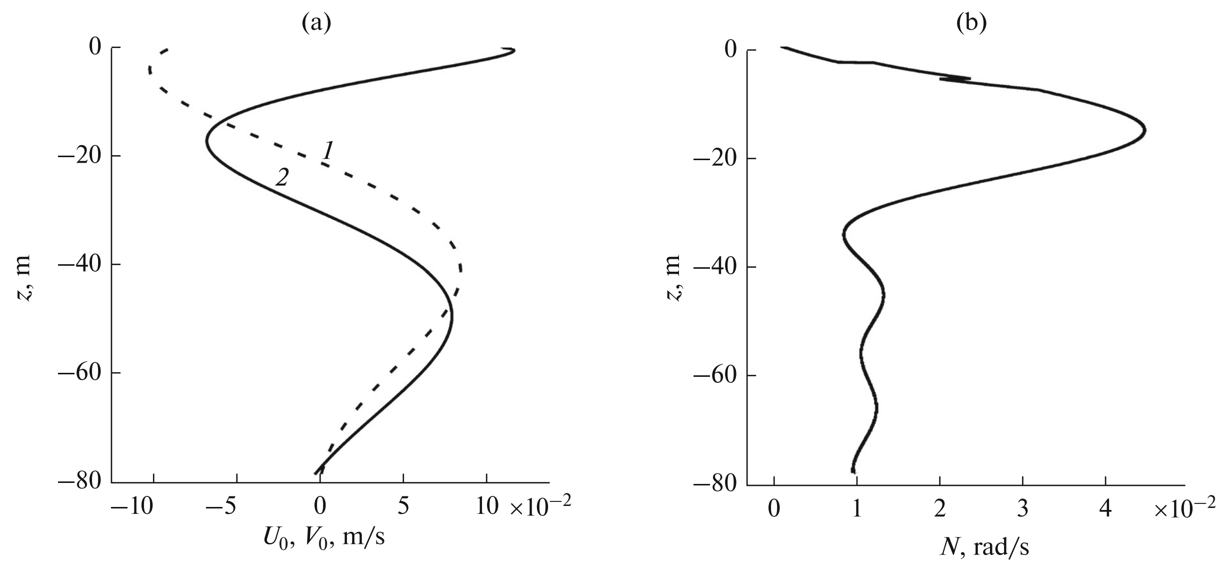

Fig. 2. Vertical profiles of (a) the flow velocity components V_0(*1*) and U_0 (*2*) and (b) the Brunt–Väisälä frequency.

direction, depends on the vertical coordinate, w_s is nonzero.

The vertical wave fluxes of heat and salt are defined by the equations

$$\overline{Tw}/|A_1|^2 = -w_{10}w_{10}^*\left(\frac{i}{\Omega} - \frac{i}{\Omega^*}\right)\frac{dT_0}{dz}, \quad (34)$$

$$\overline{Sw}/|A_1|^2 = -w_{10}w_{10}^*\left(\frac{i}{\Omega} - \frac{i}{\Omega^*}\right)\frac{dS_0}{dz}. \quad (35)$$

CALCULATION RESULTS

Let us calculate the wave fluxes of heat and salt for internal waves, which were observed in an in situ experiment during the third stage of the 44th voyage of *Mikhail Lomonosov* research vessel on the northwestern shelf of the Black Sea.

Figure 1 shows four temperature isolines recorded by gradient-distributed temperature sensors [17]. The first sensor is measured in the layer 5–15 m, the second one is measured in the layer 15–25 m, the third one is measured at depths 25–35 m, and the fourth sensor is measured in the layer 35–60 m. It is seen that high-amplitude oscillations with a period of 15 min in the 25–60 m layer are out of phase with the oscillations in the 15–25 m layer, which witnesses the second-mode oscillations. Vertical profiles of the Brunt–Väisälä frequency and two components of the flow velocity are shown in Figs. 2a and 2b. Boundary prob-

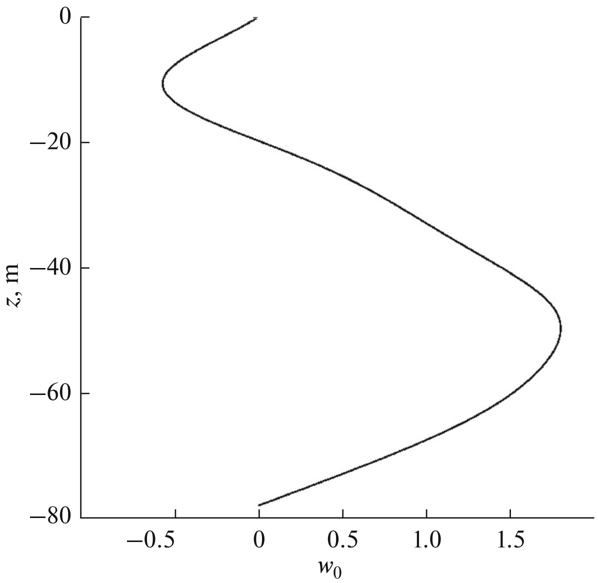

Fig. 3. Eigenfunction of the second mode of 15-min internal waves.

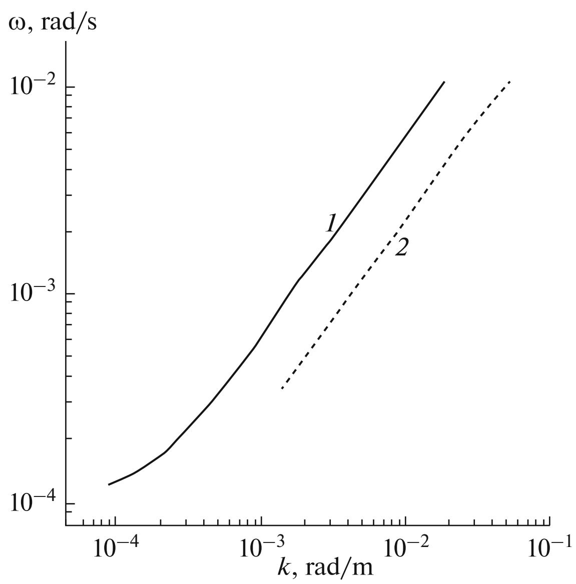

Fig. 4. Dispersion curves of the first (*1*) and second (*2*) modes.

lem (23) and (24) for internal waves is solved by the implicit Adams scheme on the third order of accuracy. The wavenumber is found by the shooting method from the requirements for the fulfillment of boundary conditions (24). The eigenfunction of 15-min second-mode internal waves is shown in Fig. 3.

The wavenumber is 0.032 rad/m. Let us found the normalizing factor A_1 from the known maximum of the amplitude of vertical shifts. For this, let us express the vertical shift ζ using the relation $\frac{d\zeta}{dt} = w$:

$$\zeta = \frac{iw_0}{\Omega_0} A_1 \exp(ikx - i\omega_0 t) + \text{c.c.}$$

From this,

$$A_1 = \frac{\max \zeta}{2 \max |w_0/\Omega_0|}.$$

Thus, the amplitude of vertical shifts is proportional to w_0. Extreme points of the function w_0 correspond to maximal vertical shifts according to the experimental data (Figs. 1, 3); i.e., the second mode was observed in the experiment. The wavelength of 15-min second-mode internal waves is 196 m. Dispersion curves of the first two modes are shown in Fig. 4. If neglecting the flow, then the dispersion curves in the low-frequency region would begin from a minimal frequency equal to the inertia frequency. If the flow is considered, the dispersion curves are cut in the low-frequency region due to the effect of the singularity at $\Omega_0 = f$. The minimal frequency of the first mode corresponds to 1.22×10^{-4} rad/s, and, that of the second mode, to 3.49×10^{-4} rad/s (let us note for the comparison that the Coriolis frequency is 1.04×10^{-4} rad/s).

Boundary problem (28) and (29) for the function w_1 is solved numerically by the implicit Adams scheme on the 3rd order of accuracy; the only solution orthogonal to w_0 is found, as well as the wave damping decrement $\delta\omega$. The damping decrement $\delta\omega = -1.12 \times 10^{-5}$ rad/s for 15-min second-mode internal waves. For the comparison, the damping decrement of the first mode $\delta\omega = -1.26 \times 10^{-5}$ rad/s. The damping decrement is two orders of magnitude lower than the wave frequency.

The vertical wave heat flux is composed by flux $\overline{Tw}$ (34) and a flux induced by the vertical component of the Stokes drift velocity $J_{qs} = T_0(z) w_s$, ($T_0(z)$ is the mean temperature profile, w_s is the vertical component of Stokes drift velocity (33), and T is the wave disturbance of the temperature). These two fluxes normalized to the squared wave amplitude are compared in Fig. 5a for the first mode and in Fig. 5b for the second mode. For the first mode, the flux J_{qs} prevails over $\overline{Tw}$ in the absolute magnitude out of a pyknocline located in the depth range 10–20 m. For the second mode, the flux induced by the vertical component of the Stokes drift velocity predominates. A similar comparison of the wave salt fluxes is shown in Fig. 6a for the first mode and in Fig. 6b for the second mode. The flux induced by the vertical component of the Stokes drift velocity prevails everywhere over the flux $\overline{Sw}$.

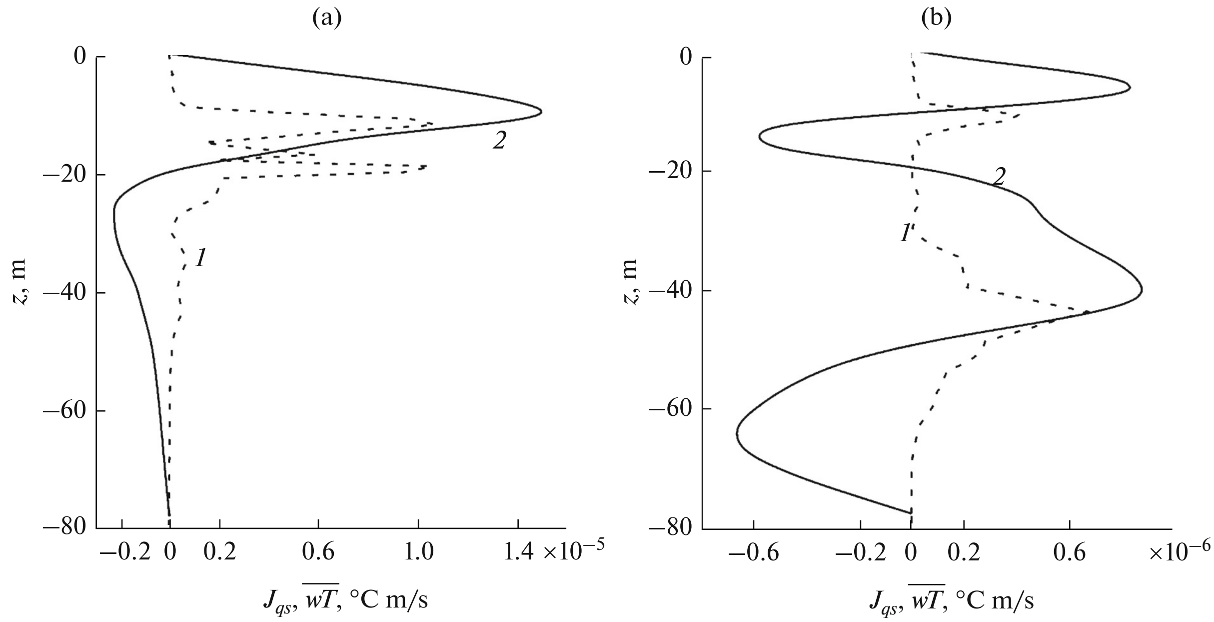

Fig. 5. Comparison of wave fluxes of heat $\overline{wT}$ (*1*) and J_{qs} (*2*) for (a) the first and (b) the second modes.

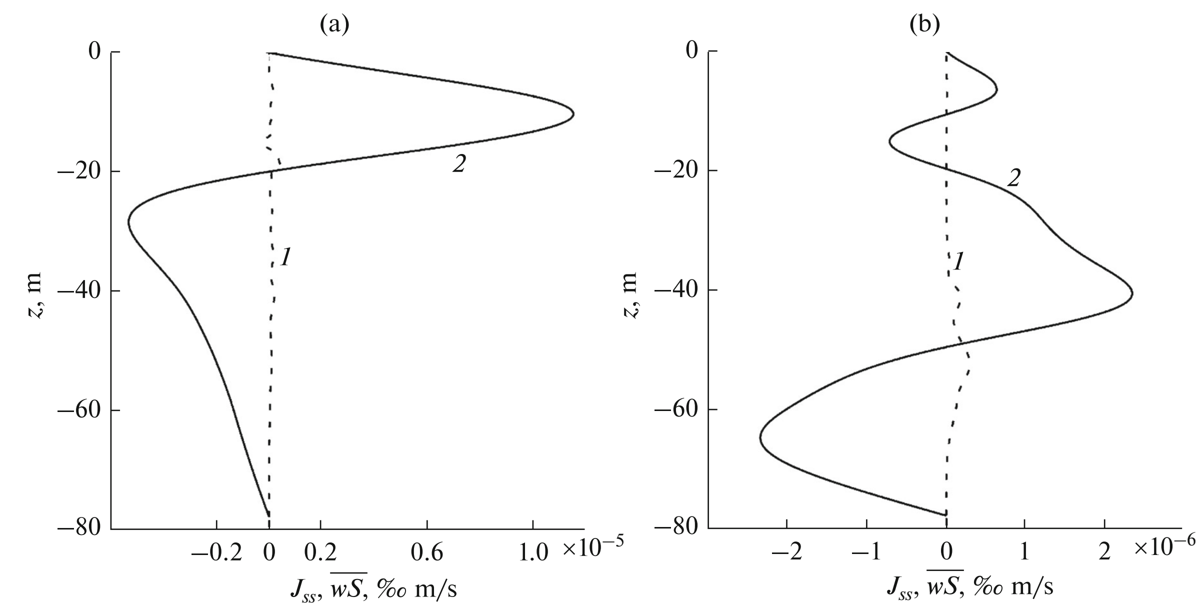

Fig. 6. Comparison of wave fluxes of salt $\overline{wS}$ (*1*) and J_{ss} (*2*) for (a) the first and (b) the second modes.

The total wave flux of heat is defined by the formula $J_q = \overline{Tw} + T_0(z)w_s$, and the turbulent flux is defined by the formula $\overline{T'w'} = -M_z \dfrac{dT_0}{dz}$. The coefficient of vertical eddy diffusion is estimated by the formula $M_z \cong 0.93 \times 10^{-4} N_c^{-1}$ m^2/s; N_c corresponds to the Brunt–Väisälä frequency in cycles per hour [18]. The vertical profiles of temperature and salinity are shown in Fig. 7. Vertical wave fluxes of heat normalized to the squared wave amplitude are compared with the turbulent flux in Fig. 8. The first mode predominates in the upper 20-m layer; these fluxes are comparable in value deeper. For the second mode, the wave flux is weaker than turbulent in the upper 40-m layer; the wave and turbulent fluxes are comparable in value

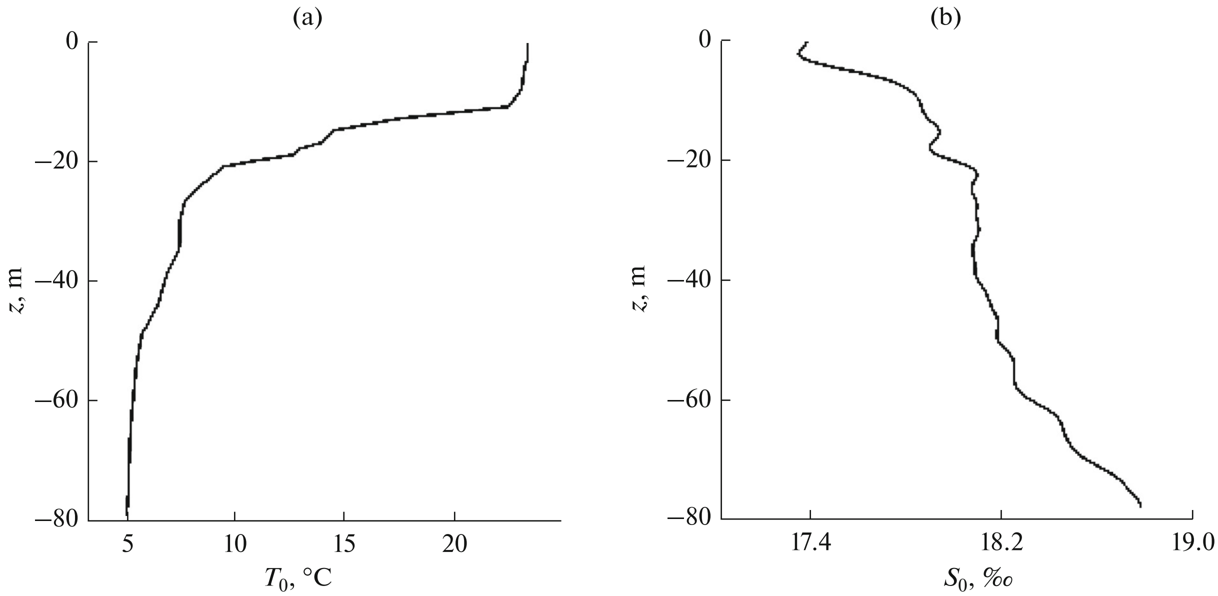

Fig. 7. Vertical profiles of (a) temperature and (b) salinity.

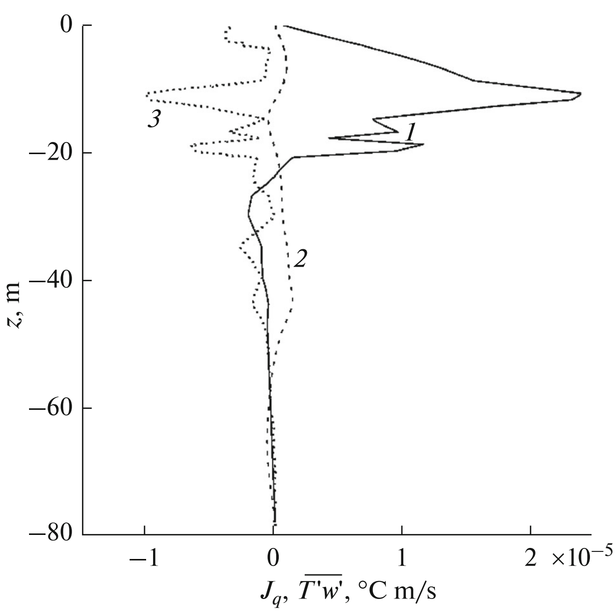

Fig. 8. Profiles of the vertical fluxes of heat for the first (*1*) and second (*2*) modes; turbulent flux (*3*).

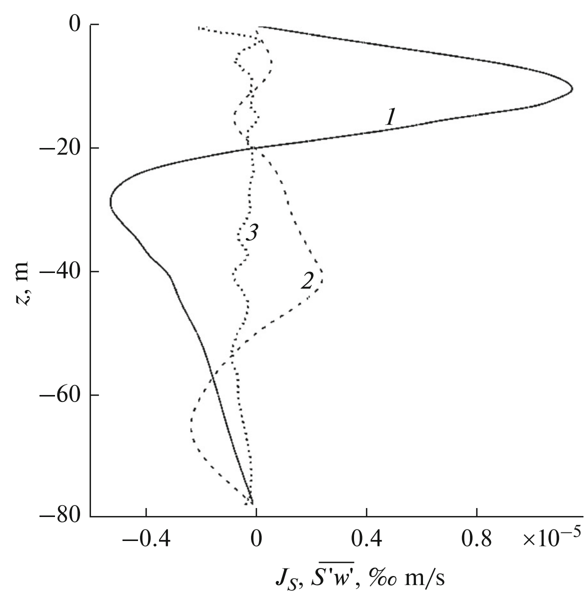

Fig. 9. Profiles of the vertical fluxes of salt for the first (*1*) and second (*2*) modes; turbulent flux (*3*).

deeper. The wave and turbulent fluxes of salt are defined similarly: $J_s = \overline{Sw} + S_0(z)w_s$, $\overline{S'w'} = -M_z \frac{dS_0}{dz}$. The wave flux of salt exceeds the turbulent flux for the first mode (Fig. 9). For the second mode, the wave flux is comparable with the turbulent one in the upper 20-m layer; the wave flux exceeds the turbulent flux deeper. The comparison between the total wave fluxes in Figs. 8 and 9 and fluxes induced by the vertical component of the Stokes drift velocity (Figs. 5, 6) shows a decisive contribution of the vertical component of the Stokes drift velocity into the wave transfer of salt. The flux $\overline{Tw}$ noticeably contributes into the vertical wave flux of heat, especially for the first mode.

CONCLUSIONS

1. The vertical component of the Stokes drift velocity of internal waves is nonzero and contributes decisively in the wave transfer of salt.

2. The first-mode wave flux of heat exceeds the turbulent flux in the subsurface 20-m layer; these fluxes are comparable deeper. For the second mode, the wave flux is lower than the turbulent one in the upper 40-m layer.

3. The vertical wave flux of salt exceeds the turbulent flux for the first mode; for the second mode, the wave flux exceeds the turbulent flux deeper than 20 m.

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REFERENCES

1. G. Veronis, "On finite amplitude instability in thermohaline convection," J. Mar. Res. **23** (1), 1–17 (1965).
2. J. S. Turner, "Double-diffusive phenomena," Ann. Rev. Fluid Mech. **6**, 37–56 (1974).
3. S. B. Kozitskii, "Amplitude equations for a system with thermohaline convection," J. Appl. Mech. Tech. Phys. **41** (3), 429–438 (2000).
4. N. A. Panteleev, I. N. Okhotnikov, and A. A. Slepyshev, *Small-Scale Structure and Dynamics of the Ocean* (Naukova Dumka, Kiev, 1993) [in Russian].
5. Yu. D. Borisenko, A. G. Voronovich, A. I. Leonov, and Yu. Z. Miropol'skii, "On the theory of nonstationary low-linear internal waves in a stratified fluid," Izv. Akad. Nauk SSSR: Fiz. Atmos. Okeana **12** (3), 293–301 (1976).
6. A. G. Voronovich, A. I. Leonov, and Yu. Z. Miropol'skii, "On the theory of the formation of a fine structure of hydrophysical fields in the ocean," Okeanologiya **11** (5), 490–497 (1976).
7. R. Grimshaw, "The modulation of an internal gravity wave packet and the resonance with the mean motion," Stud. Appl. Math. **56**, 241–266 (1977).
8. R. Grimshaw, "The effect of dissipative processes on mean flows induced by internal gravity-wave packets," J. Fluid Mech. **115**, 347–377 (1982).
9. P. Le Blond and L. Mysak, *Waves in the Ocean* (Elsevier, 1977; Mir, Moscow, 1981).
10. P. H. Le Blond, "On damping of internal gravity waves in a continuously stratified ocean," J. Fluid Mech. **25** (1), 121–142 (1966).
11. L. A. Ostrovskii and I. A. Soustova, "The upper mixing layer as a sink of internal wave energy," Okeanologiya **19** (6), 973–981 (1979).
12. A. A. Slepyshev, "Transport processes induced by weakly nonlinear internal waves in the presence of turbulence," Izv., Atmos. Ocean. Phys. **33** (4), 494–505 (1997).
13. Yu. Z. Miropol'skii, *Dynamics of Internal Gravity Waves in the Ocean* (Gidrometeoizdat, Leningrad, 1981) [in Russian].
14. W. H. Banks, P. G. Drazin, and M. B. Zaturska, "On the normal modes of parallel flow of inviscid stratified fluid," J. Fluid Mech. **75** (1), 149–171 (1976).
15. E. Kamke, *Differentialgleichungen: Lösungsmethoden und Lösungen* (Geest & Portig K.-G., Leipzig, 1959; Nauka, Moscow, 1976).
16. M. S. Longuet-Higgins, "On the transport of mass by time varying ocean current," Deep-Sea Res. **16** (5), 431–447 (1969).
17. *Report on Activities of the 44-th Route (3rd Stage) by the Mikhail Lomonosov Research Vessel on August 7–September 15, 1985* (MGI AN USSR, Sevastopol, 1985), vol. 1 [in Russian].
18. V. A. Ivanov, A. S. Samodurov, A. M. Chukharev, and A. V. Nosova, "Intensification of vertical turbulent Exchange in areas of pairing of the shelf and the continental slope in the Black Sea," Dop. Nats. Akad. Nauk Ukr. **6**, 108–112 (2008).

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