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ELECTRONIC AND OPTICAL PROPERTIES OF SEMICONDUCTORS

The Conductivity Tensor and Frequency of Electron Momentum Relaxation in the Case of Scattering by Ionized Impurities in a Magnetic Field: The Density Matrix Method

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Abstract—A solution to the Liouville equation for a one-electron density matrix in relation to a homogeneous semiconductor in a magnetic field is obtained using perturbation theory. Expressions for the conductivity tensor and electron-momentum relaxation rate are obtained for the case of scattering by ionized impurities. These expressions provide a sufficiently accurate description of the concentration and magnetic field dependences of the longitudinal conductivity of a nondegenerate electron gas in the quantum limit that have been observed in some studies. No explanation for these dependences is found in the context of the current theory of magnetoresistivity. An explanation of the temperature dependences of the components of the conductivity tensor is suggested for a degenerate electron gas in magnetic fields that correspond to the quantum limit. *© 2005 Pleiades Publishing, Inc.*

1. INTRODUCTION

It is known that, in a magnetic field, the electron energy spectrum of semiconductors is quantized. However, for $\alpha = \hbar\omega/kT \ll 1$, where $\omega = qB/m$ is the cyclotron frequency, the effect of quantization appears, in most cases, to be small and electron transport can be described using the Boltzmann transport equation. It follows from this equation that a magnetic field substantially affects transverse (with respect to the field) transport phenomena if $\omega\tau = \mu B > 1$, where μ is the electron mobility. In the longitudinal direction, transport effects are independent of a magnetic field.

In a quantizing magnetic field, for $\alpha \gg 1$, the use of the transport equation for the electron distribution function cannot be applied to a description of transverse galvanomagnetic effects in semiconductors (for which the magnetic field vector $\mathbf{B}$ is perpendicular to the current-density vector $\mathbf{j}$). Moreover, this approach cannot, in principle, describe a number of specific effects related to quantization. The problem of transverse galvanomagnetic effects has been exactly solved by Adams and Holstein using the density matrix method [1]. According to [1–3], the character of conductivity in the presence of a magnetic field in the quantum limit does not substantially differ from the case of classical high fields. The only difference is that the density of states at the Fermi level and the relaxation times depend on the magnetic field [4, 5]. According to [1], the longitudinal conductivity of electron gas σ_{zz} ($\mathbf{B} \parallel \mathbf{j} \parallel z$) does not differ from the classical expression.

However, further studies have shown that in the case of scattering by ionized impurities, the expressions for σ_{xx} and σ_{zz} obtained in [1–3] are not quite exact in the quantum limit [4]. It was assumed that this circumstance is related to the fact that it is necessary to take into account the effects of screening and the quasi-one-dimensional character of electron motion in the field of a charged impurity. However, in the traditional approach, a consistent consideration of these effects [1] does not describe a number of features of magnetotransport. With regard to these effects, the longitudinal conductivity $\sigma_{zz}(B_z)$ for a nondegenerate electron gas increases with the magnetic field in the ultra-quantum limit when only the lowest Landau level is occupied, due to the suppression of small-angle scattering [4]; moreover, the greater the increase in conductivity, the smaller the electron concentration. In InSb, this pattern in the dependence of $\sigma_{zz}(B_z)$ was observed at a temperature of $T = 30$ K only for a relatively high electron concentration ($n \approx 10^{15}$ cm^{-3}). At lower electron concentrations, the opposite field dependence was observed [6]. For electron concentrations of $n \approx 10^{13}$ cm^{-3}, the disagreement between theory and experiment can be up to a factor of 40. In the context of the existing theories, this fact has no explanation. For a degenerate electron gas in doped semiconductors in a temperature range from 0.05 to 15 K in the ultra-quantum limit, a rather strong temperature dependence of the diagonal components of the conductivity was observed (see review [4] and the references therein). Longitudinal resistivity $\rho_{zz}(B_z)$ monotonically decreases with increasing temperature, whereas transverse conductivity $\sigma_{xx}(B_z)$ and resistivity $\rho_{xx}(B_z)$ ($\mathbf{j} \parallel x$, $\mathbf{B} \parallel z$) depend nonmonotonically on temperature. At the same time, the Hall conductivity $\sigma_{xy}(B_z)$ varies only very slightly with temper-

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ature. It is impossible to explain these temperature dependences in the context of the theory developed in [1].

Recently, it has been shown that, in some cases, a consistent consideration of quantization in a magnetic field is important when formulating an adequate description of galvanomagnetic effects [7, 8]. However, in [7, 8], the case of scattering by ionized impurities, which is very important from the experimental point of view, was not analyzed. In this study, the transport equation for a density matrix in arbitrary magnetic fields is solved for scattering by ionized impurities, assuming that the departure from equilibrium is small. Expressions for the momentum relaxation rate are also obtained and compared with the available experimental results.

2. THEORY

It is known that, in a magnetic field, the matrices of electron momentum components normal to the field have no diagonal elements. Due to this fact, the electron transport in a quantizing magnetic field cannot in principle be described using the Boltzmann transport equation. Generally, the most complete microscopic description of a state in a quantum system is a description using the statistical operator (the density matrix) R. In the Schrödinger representation, it obeys the Liouville equation

$$i\hbar\frac{\partial R}{\partial t} = [H, R]. \tag{1}$$

In what follows, we restrict the analysis to a one-electron approximation. We assume that the magnetic field B is directed along the z axis, $\mathbf{j} \parallel x$, and the electric field E_x is also directed along the x axis.

The properties of a system considered in the one-electron approximation are described by the Hamiltonian operator

$$H = H_0 + W + U = H_e + W + U, \tag{2}$$

where H_e is the Hamiltonian of an electron in a magnetic field, W is the operator of the interaction of electrons with phonons or impurities, and $U = -qE_x x$ is the potential-energy operator. Generally, the electric current density can be calculated using the relation

$$\mathbf{j} = \mathrm{Tr}(R\mathbf{J}), \tag{3}$$

where Tr(…) denotes the trace of an operator and $\mathbf{J}$ is the current-density operator in a magnetic field. To calculate the current, we must find the statistical operator using Eq. (1). To solve this equation, we treat the sum $W + U$ in expression (2) as a perturbation.

If we choose the vector potential of the magnetic field in the gage $\mathbf{A} = (0, Bx, 0)$, then the wave functions of the operator H_0 and of the electron energy for a homogeneous semiconductor are described by the well-known relations in the Landau representation [9]. In this representation, an electron state $|i\rangle$ is described by a set of quantum numbers (n, k_y, k_z, and s).

We solve Eq. (1) using the method of successive approximations. A justification of this method and the procedure for deriving an approximate equation are described in detail in [10]. For steady-state conditions, using the chosen basis of wave functions, we obtain from Eq. (1)

$$(E_1 - E_2)R_{12} + U_{13}R_{32} - R_{13}U_{32} + i\pi S_{12} = 0, \tag{4}$$

where

$$S_{12} = 2\sum_3 \{\delta(E_2 - E_3)[M_{14}^+(\delta_{43} - R_{43})M_{35}R_{52} - (\delta_{14} - R_{14})M_{43}R_{35}M_{52}^+] + \delta(E_1 - E_3) \times [R_{14}M_{45}^+(\delta_{53} - R_{53})M_{32} - M_{13}R_{34}M_{45}^+(\delta_{52} - R_{52})]\}. \tag{5}$$

Here, M_{ij} are the matrix elements of the interaction of electrons with charged impurities; the form of these elements depends on the method used to describe this interaction.

It is well known that, strictly speaking, Eq. (1) does not describe the irreversible behavior of an electron system. To obtain irreversible behavior, we must use additional arguments [1]. For this purpose, we may either modify the Hamiltonian or take advantage of some artificial mathematical method that permits us to describe the interaction of the system with the medium. The most common method is derived from the assumption on an initial approximation for the diagonal part of the density matrix, i.e., for the distribution function. We represent the density matrix as

$$R_{12} = F_1\delta_{12} + G_{12}\delta(k_{y1} - k_{y2}), \tag{6}$$

where $G_{12} = G_{n_1n_2}(k_{z1}, k_{z2})$. If the electric field does not affect the spatial uniformity of the electron system, the quantity $F_1 = F(E_1)$ is usually chosen in the form of the Fermi–Dirac distribution function, which depends on the energy and quasi–Fermi level E_F. The choice of F_1 is actually based on the principle of local equilibrium, widely used in the theory of semiconductors. At high temperatures, this approximation is quite satisfactory. However, at low temperatures, it is necessary to take into account corrections to the distribution function related to the combined effect of the electric field and relaxation processes. Such a correction was obtained in [1] for the first nonvanishing order in scattering,. However, it can only be applied to high fields. A general procedure for obtaining the corrections is described in [10]. It consists in expansion of the Gibbs statistical operator in the interaction potential treated as a perturbation. In the approximation linear in an electric field, the distribution function can be written as

$$F_1 + \frac{\partial F_1}{\partial E}U_{11}Z, \tag{7}$$

where U_{11} is the potential-energy matrix element and Z is the function representing the result of summation

over all the high-order scattering terms. Generally, this function depends on the magnetic field, relaxation parameters, and the distribution function and can be determined by solving Eq. (1) for the diagonal elements of the density matrix. Then, from (6), we obtain

$$R_{12} = \left(F_1 + \frac{\partial F_1}{\partial E}U_{11}Z\right)\delta_{12} + G_{12}\delta(k_{y1} - k_{y2}). \quad (8)$$

In this study, we do not analyze the exact form of the function Z. Instead, we restrict the analysis to first approximation in magnetic field. A standard solution to the Boltzmann transport equation for the distribution function shows that its nonequilibrium part can be written as [11]

$$\Delta f = \frac{\mathbf{f}_1\mathbf{k}}{k}.$$

In the case of a magnetic field, we find

$$\mathbf{f}_1 = -q\frac{\hbar k}{m}\frac{\partial F}{\partial E}\frac{\omega[\mathbf{BE}] + \nu B\mathbf{E}}{B(\nu^2 + \omega^2)},$$

where ν is the momentum relaxation frequency. We omit the term proportional to $\mathbf{E}$. The consideration of this term improves the accuracy of the calculation of the function P_{nm}, which is determined below. However, in this study, we disregard this problem. Then, we obtain

$$\Delta f = -\frac{\hbar q}{m}\frac{\partial F}{\partial E}\frac{\omega(\mathbf{k}[\mathbf{BE}])}{B(\nu^2 + \omega^2)}$$
$$= -qk_y\lambda^2 E_x\frac{\partial F}{\partial E}\frac{\omega^2}{\nu^2 + \omega^2} = U_{11}\frac{\partial F}{\partial E}\frac{\omega^2}{\nu^2 + \omega^2}. \quad (9)$$

Comparing this expression with (7), we obtain

$$Z = \frac{\omega^2}{\nu^2 + \omega^2}.$$

We see that $Z = 0$ for $B = 0$ and, in the high-field limit, $Z = 1$. In fact, in [1], the distribution function was written (although not explicitly) in form (9) with $Z = 1$. The divergence of the solutions in the low-field region [1] is mainly related to the fact that the field dependence of Z is disregarded.

We now search for a solution of Eq. (4) linear in G. We restrict further consideration of the interaction of electrons with a charged impurity to the case of a screened Coulomb potential. Then, we use (4), (5), and (8) to obtain

$$(E_n - E_m + i\hbar\nu_{nm}^{(0)})G_{nm} + U_{nm}(F_m - F_n)$$
$$- i\sqrt{2(n+1)}q\lambda E_x P_{nm} - i\hbar B_{nm} = 0, \quad (10)$$

where

$$\nu_{nm}^{(0)} = \nu_0\int_0^\infty dy\left\{\sum_{l=0}^{l_m}[(1 - F_m)K_{nl} + F_m K_{ml}]A_{ml}\right.$$
$$\left. + \sum_{l=0}^{l_n}[(1 - F_n)K_{ml} + F_n K_{nl}]A_{nl}\right\}, \quad (11)$$

$$P_{nm} = \frac{\hbar\nu_i}{\sqrt{n+1}}Z\int_0^\infty\sqrt{y}dy\left[(1 - F_n + F_m)\left(-\frac{\partial F_m}{\partial E_m}\right)\right.$$
$$\times\sum_{l=0}^{l_m}\left|C_{n,l}^{m,m-n+l}\right|A_{ml} + (1 + F_n - F_n)\left(-\frac{\partial F_n}{\partial E_n}\right) \quad (12)$$
$$\left.\times\sum_{l=0}^{l_n}\left|C_{m,l}^{n,n-m+l}\right|A_{ml}\right](\delta_{m,n+1} + \delta_{m,n-1}),$$

$$B_{nm} = \nu_i\int_0^\infty dy\left\{\sum_{l=0}^{l_m}[(1 - F_n)C_{nl}^{m,m-n+l}G_{l,m-n+l}\right.$$
$$+ F_m C_{ml}^{n,n-m+l}G_{n-m+l,l}]A_{ml}$$
$$+ \sum_{l=0}^{l_n}[(1 - F_m)C_{ml}^{n,n-m+l}G_{l,n-m+l}$$
$$\left. + F_n C_{nl}^{m,m-n+l}G_{m-n+l,l}]A_{nl}\right\},$$

$$A_{nl} = \frac{1}{2b_{nl}}\{[2y + a^2 + (x + b_{nl})^2]^{-2}$$
$$+ [2y + a^2 + (x - b_{nl})^2]^{-2}\},$$

$$x = k_z\lambda, \quad a = k_s\lambda, \quad l_n = n + \left[\frac{x^2}{2}\right],$$

$$b_{nl}^2 = x^2 + 2(n - l),$$

$$\nu_i = \frac{mq^4N^+\lambda^3}{2\pi^2\kappa^2\hbar^3},$$

N^+ is the concentration of ionized impurities, $k_s = 1/r_s$, r_s is the screening radius, and κ is the permittivity of the semiconductor. The functions $K_{nl}(y)$ and $C_{n,l}^{m,p}(y)$, as well as the method of integration applied to the matrix elements, are described in the Appendix. We see that the required function G appears in Eq. (10) as a sum. Therefore, in contrast to scattering by phonons [7, 8], it

is impossible to obtain a simple solution for G. However, the solution to Eq. (10) can be always written as

$$G_{nm} = \frac{U_{nm}(F_n - F_m) + i\sqrt{2(n+1)}qE_x\lambda P_{nm}}{E_n - E_m + i\hbar\nu_{nm}}. \quad (13)$$

The accuracy of the calculation of ν_{nm} depends on a number of parameters and on the chosen procedure for the solution to Eq. (10). Using the results obtained in the Appendix, we can show that $\nu_{nm}^{(0)} G_{nm} > B_{nm}$. Using this inequality, we can solve Eq. (10), e.g., by the method of successive approximations. In the first approximation, we may set $B_{nm} = 0$. These questions will be considered in more detail below. The result for the components of the conductivity tensor is

$$\sigma_{xx} = \frac{q^2\omega}{(2\pi)^2\hbar}\sum_{s,n}(n+1) \times \int dk_z \left[\frac{\nu_{n,n+1}(F_n - F_{n+1}) + \omega P_{n,n+1}}{\omega^2 + \nu_{n,n+1}^2}\right], \quad (14)$$

$$\sigma_{xy} = \frac{q^2\omega}{(2\pi)^2\hbar}\sum_{s,n}(n+1) \times \int dk_z \left[\frac{\omega(F_n - F_{n+1}) - \nu_{n,n+1}P_{n,n+1}}{\omega^2 + \nu_{n,n+1}^2}\right], \quad (15)$$

where $F_n \equiv F(E_n)$. We see that the structure of these equations coincides with that of the equations describing scattering by phonons [7, 8]. Therefore, to account for the combined effect of different relaxation mechanisms, we have to represent ν and P as sums over these mechanisms. In addition, the obtained expressions indicate that the function Z depends on the magnetic-level number n and can be written more correctly in the form

$$Z_{n,n+1} = \frac{\omega^2}{\omega^2 + \nu_{n,n+1}^2}.$$

Using these results, we can easily show that the longitudinal conductivity $\sigma_{zz}(B_z)$ is described by a standard expression [11].

For high magnetic fields ($\omega \gg \nu$), Eq. (13) formally transforms into the expression obtained in [1]. However, a substantial difference is that, in contrast to [1], in the low-field limit, this equation transforms into the well-known expressions for the conductivity tensor in a semiconductor [11]. Furthermore, in this study, we obtain a more exact expression than (11) for the frequency of ionized impurity scattering ν_{nm} in high magnetic fields (see below).

3. CALCULATION OF THE FREQUENCY OF ELECTRON SCATTERING

Equation (10) is Fredholm's equation of the second kind, which satisfies all the conditions for absolute convergence. Therefore, we use the method of successive approximations in the calculations. In first approximation, we set $B_{nm} = 0$. To obtain the second approximation, we take the fact that G_{nm} is a generalized (symbolic) function into account. In Eq. (10), we replace $\nu_{nm}^{(0)}$ by μ_{nm} and omit the term proportional to P_{nm}. This term corresponds to scattering effects that are quadratic in frequency. The consideration of these effects can be important in an analysis of magnetotransport in semiconductors with a low charge-carrier mobility. Taking into account the δ functions in the collision integral, we derive the following equality from the integrand in (10):

$$G_{l,m-n+l}(k_{z3}; k_{z4}) = G_{nm}\frac{\partial K_{z2}}{\partial k_{z3}} = G_{nm}\frac{k_{z3}}{k_{z2}}.$$

Using this equality in the collision integral, we obtain the following approximation for the scattering frequency:

$$\nu_{nm} = \frac{q^4\hbar\omega N^+}{16\pi\kappa^2\sqrt{2m}} \times \int_0^\infty dy\left[\sum_{l=0}^{l_m}\frac{K_{nl}}{b_{ml}(p_2)}D_{mnl}(p_2) + \sum_{l=0}^{l_n}\frac{K_{ml}}{b_{nl}(p_1)}D_{nml}(p_1)\right]. \quad (16)$$

Here,

$$b_{nm}^2(p_i) = \hbar\omega(n-m) + p_i^2, \quad i = 1, 2,$$

$$p_i^2 = \frac{\hbar^2 k_{zi}^2}{2m}, \quad l_n = n + \left[\frac{p_1^2}{\hbar\omega}\right], \quad E_s = \frac{\hbar^2 k_s^2}{2m},$$

$$D_{nml}(p_i) = \left[\frac{[p_i - b_{nl}(p_i)](1 + F_n - F_m)}{p_i[\hbar\omega y + (b_{nl} - p_i)^2 + E_s]^2} + \frac{[p_i + b_{nl}(p_i)](1 + F_n - F_m)}{p_i[\hbar\omega y + (b_{nl} + p_i)^2 + E_s]^2}\right].$$

For a nondegenerate electron gas, we replace the summation in (16) by integration and, in the limit of a zero magnetic field, obtain

$$\nu = \frac{q^4 N^+}{8\pi\kappa^2\sqrt{2m}p} \times \left[\frac{1}{(\sqrt{E} - p)^2 + E_s} - \frac{1}{(\sqrt{E} + p)^2 + E_s}\right], \quad (17)$$

where $p = p_1 = p_2$.

Now, we calculate the scattering frequency in the absence of a magnetic field directly from Eq. (5). In this

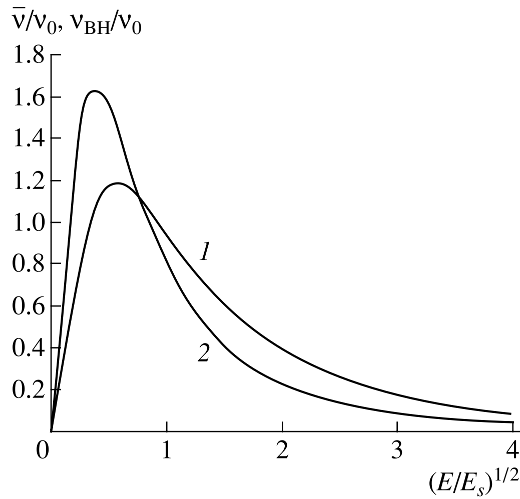

Fig. 1. The energy dependence of the rate of momentum relaxation induced by ionized impurities when (*1*) calculated using formula (18) and (*2*) calculated using the Brooks–Herring theory.

case, the electron state $|i\rangle$ is described by a set of quantum numbers (k_x, k_y, k_z, and s). An equation similar to Eq. (10) is obtained from Eq. (5). In the absence of a magnetic field, the frequency of the momentum relaxation depends only on the total electron energy. The iterative procedure of the solution converges well, and, in the second approximation, the well-known Brooks–Herring formula for the frequency of momentum relaxation ν_{BH} is obtained. We note here that further iterations only slightly improve the accuracy of the calculations.

We can see that frequency (16) depends on energy and k_z. However, the frequency ν_{BH} depends only on energy. This circumstance is due to the difference in the sets of quantum numbers used in the analysis in the absence and in the presence of a magnetic field. In order to obtain an expression for the scattering frequency from (16) that is equivalent to ν_{BH}, it is necessary to take into account that $p = \sqrt{E}\cos\Theta$ and average over the angle. The average frequency is given by

$$\bar{\nu} = \left(\int_0^\pi d\Theta \frac{\sin\Theta}{\nu(\Theta)}\right)^{-1}.$$

After simple calculations, we obtain the following expression from (16):

$$\bar{\nu} = \frac{60x}{15 + 40x^2 + 8x^4}\nu_0. \tag{18}$$

Here,

$$x^2 = \frac{E}{E_s}, \quad \nu_0 = \frac{q^4 N^+}{16\pi\kappa^2\sqrt{2m}E_s^{3/2}}.$$

The obtained expression differs from the Brooks–Herring formula. The disagreement is caused by a difference in the convergence of the iterative procedure for the calculations with different sets of quantum numbers. Figure 1 shows the results of the calculation of the relaxation frequencies $\bar{\nu}$ and ν_{BH}. We can see that the difference in the results is nevertheless not very large.

4. CALCULATION OF THE CONDUCTIVITY OF ELECTRON GAS AND COMPARISON WITH THE EXPERIMENTAL RESULTS

First, we consider the temperature dependence of the conductivity of a degenerate electron gas in the ultra-quantum limit. In this case, for the transverse conductivity, $\omega \gg \nu$ and (13) can be used to obtain

$$\sigma_{xx} = \frac{q^2}{(2\pi)^2\hbar}\int dk_z P_{01}. \tag{19}$$

Using this formula in the calculations, we must take into account that, for a degenerate electron gas in a semiconductor, the Fermi energy in high magnetic fields is low. For example, for InSb with an electron concentration of $n = 10^{16}$ cm^{-3} at $B = 5$ T, the Fermi energy is $E_F = 1.5$ meV and the ratio E_F/kT is small if the temperature is not too low. Therefore, the procedure in which the derivative of the distribution function is replaced by the δ function is not quite exact. Taking this circumstance into account, we use (12) to obtain the following approximate expression for the function $P_{n,n+1}$:

$$P_{01} = (1 + F_0)\left(-\frac{\partial F_0}{\partial E}\right)V_{01}(E_F).$$

Here,

$$V_{01} = \frac{\hbar\omega q^4 N^+}{8\pi^2\kappa^2\sqrt{2m}}\int_0^\infty \sqrt{y}C_{00}^{11}dy$$
$$\times\left[\frac{1}{(\hbar\omega\nu + 4E_F + E_s)^2} + \frac{1}{(\hbar\omega\nu + E_s)^2}\right].$$

Substituting this expression into (19) and then integrating, we find that the conductivity σ_{xx} in the ultra-quantum limit is

$$\sigma_{xx} = \frac{\sigma_{xx0}}{\left[1 + \exp\left(-\frac{E_F}{kT}\right)\right]^2},$$

where σ_{xx0} is the conductivity at a temperature of zero. Thus, for $n = 10^{16}$ cm^{-3}, σ_{xx} decreases by a factor of 1.4 as the temperature changes from 3 to 10 K. Since σ_{xy} is virtually independent of temperature and in the ultra-quantum limit, $\sigma_{xy} \gg \sigma_{xx}$, it follows that ρ_{xx} also decreases with temperature, in good agreement with the

experimental results [4]. Using (16), we can obtain a similar dependence of σ_{zz}.

From this analysis, we can see that, for a degenerate electron gas in a semiconductor, the temperature corrections for the diagonal components of the conductivity tensor are not large and do not modify the main form of the magnetic field dependence. At the same time, the obtained results allow us to interpret the experimentally observed dependences.

For nondegenerate semiconductors, the situation is different. As was noted in Section 1, the existing theory cannot explain the experimentally observed dependences of the longitudinal resistivity ρ_{zz} on the electron concentration n and magnetic field B_z. Therefore, we need to analyze this case in more detail. The desired dependences can be obtained using expression (16). However, to improve the accuracy of the calculations, we must take into account a number of factors that were not discussed above.

First, it is necessary to take into account the dependence of the concentration of activated impurities and, accordingly, the electron concentration on the magnetic field. Second, if not all the impurity atoms are ionized or the semiconductor is compensated, the electron concentration n_{eff}, which is used for the calculation of the screening radius, can differ from the concentration of free carriers $n = N^+$. In this case, screening can be also due to localized carriers moving between the centers [12]. For example, if, in an n-type semiconductor with the donor concentration N_D, there are some acceptors, whose concentration is $N_A \ll N_D$, then

$$n_{\text{eff}} = n + (N_D - N_A - n)(N_A + n)/N_D.$$

It follows from this relation that, in our case, when calculating the Debye radius r_s, we should replace n by n_{eff}. We should note that taking this fact into account does not strongly affect the calculated dependences. Third, we must take into account the relatively weak scattering by the deformation potential of acoustic phonons (DA), for which the frequency of momentum relaxation in a magnetic field was calculated in [7, 8].

Figure 2 shows the magnetic-field dependence of the longitudinal magnetoresistivity $\rho_{zz}(B_z)$ for doped InSb (for two different electron concentrations) calculated taking into account the above factors. This dependence is expected to be observed in the low-field limit. For the calculations, we used the following parameters from [5]: the Lande factor $g = -40$ and the momentum relaxation time $\tau_{DA}(300\text{ K}) = 40$ ps. The impurity ionization energy was taken to be $E_d = 5$ meV. The calculation shows that the shape of the curves for the relative longitudinal magnetoresistivity weakly depends on E_d in a fairly wide range of E_d. The calculated curves agree with the experimental results [13] at an accuracy of no worse than 20% in the entire range of field variation. We note here that no decrease in resistivity was observed in the experimental curve for $n = 10^{13}$ cm^{-3} in the region of low fields. Under the specified conditions, at an average thermal energy, the parameter $(E/E_s)^{1/2}$ in Fig. 1 is equal to 1.1 for the higher electron concentration and to 11 for the lower concentration. We can see from Fig. 1 that the scattering frequencies calculated using the two formulas are approximately equal. Therefore, the presence of a region of decreasing resistivity in the calculated curves may indicate that, for weak magnetic fields, it is necessary to take into account other scattering mechanisms.

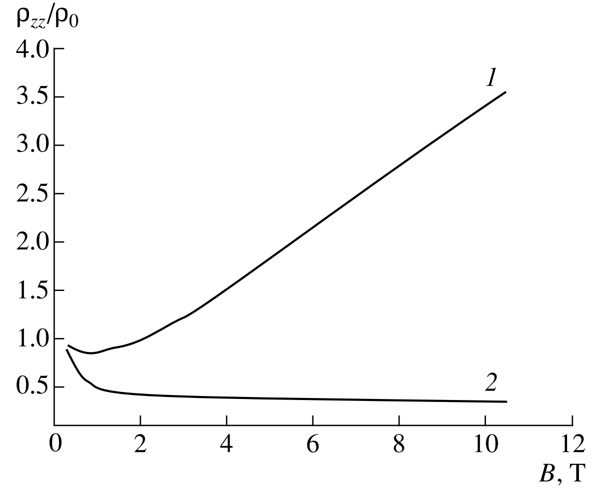

Fig. 2. The magnetic field dependence of the longitudinal resistivity at $T = 30$ K for semiconductors with the electron concentrations $n = (1)\ 10^{13}$ and $(2)\ 10^{15}$ cm^{-3}.

5. CONCLUSIONS

Thus, the results obtained in this study allow us to describe the frequency of electron-momentum relaxation by ionized impurities in arbitrary magnetic fields much more precisely. These results also make it possible to describe the temperature dependences of the diagonal components σ_{xx} and σ_{zz} of the conductivity tensor of a degenerate electron gas in the ultra-quantum limit, without taking localization effects into account. In a nondegenerate electron gas, the obtained expressions satisfactorily describe the dependence of the longitudinal conductivity σ_{zz} on the electron concentration and magnetic field over a wide range of variation in these parameters.

APPENDIX

Integration of Matrix Elements and Orthogonal Polynomials

Using the equality $k_{y1} = k_{y2}$, we can represent the screened Coulomb potential as follows:

$$\frac{\exp(-k_s r)}{4\pi r} = \frac{1}{(2\pi)^3}\int \frac{d^3 q}{q^2 + k_s^2}\exp(i\mathbf{qr}).$$

Then, the product of the matrix elements in (5) can be reduced to the integral

$$M_{14}^{+}M_{32} = 2\pi\left(\frac{q^2}{\kappa}\right)^2 N^{+}\int\frac{d^3q}{(q^2+k_s^2)^2}\delta(k_{y3}-k_{y1}+q_y)$$
$$\times\delta(k_{z3}-k_{z1}+q_z)\delta(k_{y2}-k_{y4}+k_{y3}-k_{y1})$$
$$\times\delta(k_{z2}-k_{z4}+k_{z3}-k_{z1})I_s,$$

where

$$I_s = \int dx_1 dx_2 f_n\left(x_2-\frac{q_y}{2}\lambda^2\right)f_p\left(x_2+\frac{q_y}{2}\lambda^2\right)$$
$$\times f_l\left(x_2+\frac{q_y}{2}\lambda^2\right)f_m\left(x_1-\frac{q_y}{2}\lambda^2\right)\exp[iq_x(x_1-x_2)].$$

If we introduce the definition

$$I_{nl} = \int_{-\infty}^{\infty} dy f_n\left(y-\frac{q_y}{2}\lambda^2\right)f_l\left(y+\frac{q_y}{2}\lambda^2\right)$$
$$\times\exp(iq_x y) = \exp\left(-\frac{x}{2}\right)z_{nl},$$

then we obtain

$$I_s = \exp(-x)z_{ml}z_{np}^{*},$$

where

$$q_r = \sqrt{q_x^2+q_y^2}, \quad x = (q_r\lambda)^2/2,$$
$$z_{nl} = \frac{1}{\sqrt{\pi n!l!2^{n+l}}}$$
$$\times\int_{-\infty}^{\infty} dy\exp(-y^2)H_n\left(y+\frac{z_{10}}{\sqrt{2}}\right)H_l\left(y+\frac{z_{01}}{\sqrt{2}}\right).$$

Using these relations, we can easily show that

$$\int_0^{\pi} d\varphi I_{lm}I_{pn} = 2\pi\delta_{p,n-m+l}C_{m,l}^{n,n-m+l},$$
$$C_{m,l}^{n,n-m+l} = \exp(-x)z_{ml}z_{n,n-m+l}^{*},$$
$$C_{m,l}^{m,l} = K_{ml} = K_{ln} = \exp(-x)z_{ml}z_{ml}^{*},$$
$$\int_0^{\infty} C_{ml}^{n,n-m+l}dx = \delta_{nm}.$$

REFERENCES

1. E. N. Adams and T. D. Holstein, J. Phys. Chem. **10**, 254 (1959); in *Problems in Quantum Theory of Irreversible Processes* (Inostrannaya Literatura, Moscow, 1961), p. 255.
2. R. Kubo, H. Hesegawa, and N. Hashitdume, J. Phys. Soc. Jpn. **14**, 56 (1959); in *Problems in Quantum Theory of Irreversible Processes* (Inostrannaya Literatura, Moscow, 1961), p. 89.
3. P. N. Agryes and L. M. Roth, Phys. Chem. Solids **12**, 89 (1959).
4. S. S. Murzin, Usp. Fiz. Nauk **170**, 387 (2000) [Phys. Usp. **43**, 349 (2000)].
5. V. F. Gantmakher and I. B. Levinson, *Scattering of Carriers in Metals and Semiconductors* (Nauka, Moscow, 1984; North-Holland, Amsterdam, 1987), Chap. 14.
6. S. S. Murzin and P. V. Popov, Pis'ma Zh. Éksp. Teor. Fiz. **58**, 280 (1993) [JETP Lett. **58**, 289 (1993)].
7. V. É. Kaminskiĭ, Fiz. Tekh. Poluprovodn. (St. Petersburg) **36**, 1360 (2002) [Semiconductors **36**, 1276 (2002)].
8. V. É. Kaminskiĭ, Phys. Rev. B **67**, 085201 (2003).
9. L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics*, Vol. 3: *Quantum Mechanics: Non-Relativistic Theory*, 4th ed. (Nauka, Moscow, 1989; Pergamon, Oxford, 1977), Chap. 15.
10. A. I. Akhiezer and S. V. Peletminskiĭ, *Methods of Statistical Physics* (Nauka, Moscow, 1977; Pergamon, Oxford, 1981), Chaps. 3–5.
11. A. I. Ansel'm, *Introduction to the Theory of Semiconductors* (Nauka, Moscow, 1977), Chap. 7.
12. L. M. Falicov and M. Quevas, Phys. Rev. **164**, 1025 (1967).

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