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A Quasi-Classical Description of the Conductivity Oscillations in Layered Crystals Under the Condition of Charge-Carrier Scattering by Acoustic Phonons

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Abstract—The oscillating part of the longitudinal conductivity of layered crystals is considered within the quasi-classical approximation, where both the electric field and quantizing magnetic field are perpendicular to the layers. Our approach differs from the conventional one by taking into account both the nonparabolicity of a narrow conduction miniband and the dependence of the Fermi surface size in the direction of the magnetic field on charge-carrier concentration. This approach makes it possible to consider not only the standard case, with open Fermi surfaces, but also the case of closed Fermi surfaces. It is shown that, for closed Fermi surfaces, the existence of frequencies that do not correspond to extreme cross sections of the Fermi surfaces cut by planes normal to the magnetic field can serve as a criterion for the narrowness of the conduction miniband, which determines the translational motion of charge carriers across the layers. *© 2005 Pleiades Publishing, Inc.*

1. INTRODUCTION

If it is assumed that there are many Landau levels in the allowed band of a crystal, the Lifshitz–Kosevich theory of magnetic susceptibility [1] and the Kosevich–Andreev theory of transport coefficients [2, 3] are valid in the quasi-classical approximation. Furthermore, interband transitions are not forbidden, and charge-carrier scattering probabilities are either the same as they would be in the absence of a magnetic field or oscillate with the variations in a magnetic field [4]. The Kosevich–Andreev theory was developed for a case in which electric and magnetic fields are mutually perpendicular. However, experimental studies of the oscillations of transport coefficients, in particular, of thermopower, are quite often performed for a situation in which an electric field (or temperature gradient) and a magnetic field are parallel. Moreover, neither the Lifshitz–Kosevich theory nor the Kosevich–Andreev theory take into account the explicit form of the nonparabolicity of a conduction band and the dependence of the size of the Fermi surface along the direction of a magnetic field on charge-carrier concentration. However, there exist numerous highly anisotropic crystals with layered structures where the motion of the charge carriers in the layer plane is described using the effective mass approximation and the motion in the perpendicular direction by the tight-binding approximation or by some nonparabolic dispersion relation [5]. Examples of such crystals include transition metal dichalcogenides [6], intercalated graphite compounds (synthetic metals) [7], multinary semiconductor compounds with superlattices (in particular, II–VI–VII compounds) [8], quasi-two-dimensional organic conductors [9], etc. The aim of this study is to describe, within the quasi-classical approximation, the oscillations of electrical conductivity and to establish the conditions for the applicability of this description to crystals in which the electric and magnetic fields are parallel to each other (longitudinal electrical conductivity) and are perpendicular to the layers.

It should be noted that layered conductors are usually considered as being quasi-two-dimensional, i.e., conductors for which the Fermi energy is much greater than the width of the narrow conduction miniband, which determines charge-carrier motion in a direction normal to the layers. The theory relating to the Shubnikov–de Haas effect (in a magnetic field perpendicular to the current) for such crystals has already undergone a sufficiently detailed development in the quasi-classical approximation [9]. However, there are also layered crystals that, though they are described by a model of the band-spectrum characteristic of quasi-two-dimensional crystals, are not quasi-two-dimensional in the above sense. The Fermi energy in these crystals is smaller than the width of the narrow miniband that determines the motion of electrons across the layers; however, these quantities are comparable, which means that the usual effective mass approximation is not valid [7, 8]. Nevertheless, these crystals can be transformed into a quasi-two-dimensional form by doping. In this study, we derive expressions that describe the oscillating part of the longitudinal electrical conductivity in crystals of both types.

2. RESULTS AND DISCUSSION

The most general expression for the energy levels of charge carriers in a layered crystal in a quantizing magnetic field perpendicular to the layers is

$$\varepsilon(n, k_z) = \mu^* H(2n+1) + W(x), \tag{1}$$

where $\mu^* = \mu_B(m_0/m^*)$; μ_B is the Bohr magneton; n is the number of the Landau level; m^* is the effective mass of an electron in the layer plane, which is, for simplicity, assumed to be isotropic; k_z is the component of the quasi-momentum in the direction perpendicular to the layers; H is the quantizing magnetic field; $W(x)$ is the charge-carrier dispersion relation, which is not parabolic and describes the carrier motion in the direction normal to the layers; $x = ak_z$; and a is the distance between the translationally equivalent layers.

When describing the Shubnikov–de Haas effect in the quasi-crystal approximation for charge-carrier scattering by acoustic phonons, we assume, for simplicity, that the relaxation time of the longitudinal quasi-momentum does not depend on the carrier energy and that the temperature dependence of this time obeys the Bloch–Grüneisen law [10]. Then, the relaxation time is given by

$$\tau = \tau_0(\Theta_D/T^5), \tag{2}$$

where, for the crystal under consideration, τ_0 is a constant that has the dimensions of time and describes the scattering intensity, and Θ_D is the Debye temperature of the crystal.

The conductivity is obtained from the Kubo formula [11] by summation over the Landau levels, which can be precisely performed for spectrum (1) using the longitudinal quasi-momentum relaxation time given by (2) for any form of the function $W(x)$. In the approximation $\zeta/kT \gg 1$ and $\Delta/kT \gg 1$, we obtain the following expression for the magnetic-field-independent part of the conductivity:

$$\sigma_0 = \frac{32\pi\tau_0 e^2 m^* a\Theta_D^5}{h^4T^5}\int\limits_{W(x)\le\zeta}(W'(x))^2dx, \tag{3}$$

and the oscillating part of the conductivity assumes the form

$$\sigma_{osc} = \frac{32\pi\tau_0 e^2 m^* a\Theta_D^5}{h^4T^5} \times\sum_{l=1}^{\infty}(-1)^{l-1}f_l^{\sigma}\int\limits_{W(x)\le\zeta}(W'(x))^2\cos\left[\pi l\frac{(\zeta-W(x))}{\mu^*H}\right]. \tag{4}$$

In (4), f_l^{σ} denotes the temperature-related damping factor of the oscillations,

$$f_l^{\sigma} = \frac{\pi^2lkT/\mu^*H}{\sinh(\pi^2lkT/\mu^*H)}. \tag{5}$$

In (3)–(5), ζ is the Fermi energy measured from the bottom of the narrow conduction miniband and $W'(x)$ is the derivative. The integration in (3) and (4) is performed only with respect to positive values of x.

Using formulas (3) and (4), we can now calculate the conductivity for the dispersion relations $W(x)$.

The simplest tight-binding dispersion relation, which is used to describe a warped Fermi surface of a layered crystal, is written as [5]

$$W(x) = \Delta(1-\cos x), \tag{6}$$

where Δ is the half-width of the narrow miniband in the direction normal to the layers. For this dispersion law, as well as for nonquasi-two-dimensional crystals, i.e., for $\zeta \le 2\Delta$, the magnetic-field-independent part of the conductivity assumes the form

$$\sigma_0 = \frac{16\pi\tau_0 e^2 m^* a\Theta_D^5\Delta^2}{h^4T^5}(C_0 - C_2), \tag{7}$$

and the oscillating part is

$$\begin{aligned}\sigma_{osc} = {}& \frac{16\pi\tau_0 e^2 m^* a\Theta_D^5\Delta^2}{h^4T^5} \\ &\times\sum_{l=1}^{\infty}(-1)^{l-1}f_l^{\sigma}\left\{\cos\left[\frac{\pi l(\zeta-\Delta)}{\mu^*H}\right]\left[(C_0-C_2)J_0\left(\frac{\pi l\Delta}{\mu^*H}\right)\right.\right. \\ &\left.\times\sum_{r=1}^{\infty}(-1)^r(2C_{2r}-C_{2r+2}-C_{2r-2})J_{2r}\left(\frac{\pi l\Delta}{\mu^*H}\right)\right] \\ &-\sin\left[\frac{\pi l(\zeta-\Delta)}{\mu^*H}\right]\sum_{r=1}^{\infty}(-1)^r(2C_{2r+1}-C_{2r+3}-C_{|2r-1|}) \\ &\left.\times J_{2r+1}\left(\frac{\pi l\Delta}{\mu^*H}\right)\right\},\end{aligned} \tag{8}$$

where $J_m(y)$ are the Bessel functions of the real argument y, and C_m are the modulating coefficients defined by the relations

$$C_0 = \kappa_\zeta = \arccos\left(1-\frac{\zeta}{\Delta}\right), \tag{9}$$

$$C_m = \frac{\sin m\kappa_\zeta}{m}. \tag{10}$$

When deriving formula (8), we expanded the oscillating part of the integrand in (4) in the Bessel functions of integer index using dispersion relation (6) [12]. In expansion (8), there are many Bessel functions, since, for $\zeta \le 2\Delta$, the Fermi surface of a layered crystal is closed and occupies the region $[-\kappa_\zeta; \kappa_\zeta]$ within the one-dimensional Brillouin zone. For $\zeta > 2\Delta$, the Fermi surface is open, and the integration in (4) should be performed over the entire Brillouin zone; therefore expression (4) can be written in the more compact form

$$\sigma_{osc} = \frac{16\pi^2\tau_0 e^2 m^* a\Theta_D^5\Delta^2}{h^4T^5} \times \sum_{l=1}^{\infty}(-1)^{l-1}f_l^{\sigma}\left[J_0\left(\frac{\pi l\Delta}{\mu^*H}\right)+J_2\left(\frac{\pi l\Delta}{\mu^*H}\right)\right] \times \cos\left[\pi l\left(\frac{\zeta-\Delta}{\mu^*H}\right)\right]. \quad (11)$$

Let us analyze these results in more detail. First, we should note that, initially, the magnetic-field-independent part of the longitudinal electrical conductivity increases monotonically with κ_ζ, i.e., with the charge-carrier concentration. It then attains a maximum, and finally becomes independent of the carrier concentration as the Fermi level crosses the top of the narrow miniband. This behavior occurs because any restriction imposed on the free motion of the charge carriers reduces the conductivity of the crystal.

We now consider the oscillating component of the electrical conductivity. General formula (4) for the conductivity differs from the conventional expression in the explicit allowance it makes for the dependence of the Fermi surface size along the direction of the magnetic field on the carrier concentration as a result of a restriction of the region of integration with respect to x [13, 14]. This restriction is quite justified, since the "disappearance" of the Fermi surface implies that the oscillating component of the conductivity vanishes. Therefore, it follows from (8) that not only the frequencies but also the amplitudes of the conductivity oscillations depend on the charge-carrier concentration via the concentration dependence of the Fermi energy. For the considered specific case of a layered crystal with a superlattice, this dependence is determined by the modulating coefficients of the Bessel functions defined by expressions (9) and (10). The series in r and l in expression (8) converge quite rapidly. However, for the specific case in which $\zeta = \Delta$, a more compact formula without trigonometric factors can be obtained from (8):

$$\sigma_{osc} = \frac{8\pi^2\tau_0 e^2 m^* a\Theta_D^5\Delta^2}{h^4T^5} \times \sum_{l=1}^{\infty}(-1)^{l-1}f_l^{\sigma}\left[J_0\left(\frac{\pi l\Delta}{\mu^*H}\right)+J_2\left(\frac{\pi l\Delta}{\mu^*H}\right)\right]. \quad (12)$$

This circumstance is a direct consequence of dispersion relation (6), which is representative of the small width of the conduction miniband and is characteristic of crystals with superlattices. However, when the magnetic field is so weak that there are a large number Landau levels in the narrow conduction miniband, we may use the traditional quasi-classical approximation in (4), which is not based on any model assumptions about the form of the function $W(x)$, i.e., about the character of the warping of a cylinder (which is actually the Fermi surface of the layered crystal). To apply this approximation to (4), we must retain the first nonvanishing terms in the expansions of $W(x)$ and $W'(x)$ in x near the extreme cross sections of the Fermi surface cut by the planes normal to the direction of the magnetic field. Furthermore, we must evaluate the integrals obtained using the method of steepest descent and the subsequent differentiation with respect to the parameter. The result for the oscillating component of the longitudinal conductivity is

$$\sigma_{osc}^{cc} = \mp\frac{4\tau_0 e^2 m^* a\Theta_D^5(\mu^*H)^{3/2}|W''_{ex}|^{1/2}}{h^4T^5} \times \sum_{l=1}^{\infty}(-1)^{l-1}f_l^{\sigma}l^{-3/2}\sin\left(\pi l\frac{\zeta-W_{ex}}{\mu^*H}\pm\frac{\pi}{4}\right). \quad (13)$$

In this formula, W_{ex} and $|W''_{ex}|$ are values of the function $W(x)$ and the modulus of its second derivative at the extremum points (if there are several extrema, then the sum in (13) must be taken over all the extrema belonging to the Fermi surface). The plus sign at the initial phase and the minus sign at the amplitude correspond to the minimal cross section, and the opposite signs correspond to the maximal cross section of the Fermi surface. In contrast to the traditional formula [2, 3, 9], in expression (13), the magnetic-field dependence of the oscillation amplitude and the curvature of the Fermi surface near the extreme cross sections is different, and cosines are replaced by sines. These differences are exclusively due to the fact that, in this study, we consider the longitudinal electrical conductivity whereas, in the traditional approach, the transverse conductivity is considered.

If, in formulas (8) and (11), we use an asymptotic limit in the form $\Delta/\mu^*H \gg 1$, then, retaining only the leading terms in the asymptotic expansions of the Bessel function [12], we see that the oscillating component of the conductivity vanishes in an identical manner. By including the subsequent terms in these expansions, we obtain two quasi-classical formulas of type (13): the first for a Fermi surface with one extreme cross section (cut by the $k_z = 0$ plane) when $0 < \zeta < 2\Delta$ and the second for a Fermi surface with three extreme cross sections (cut by the $k_z = 0$ plane and the $k_z = \pm\pi/a$ planes) when $\zeta > 2\Delta$. We then combine these formulas into the single expression

$$\sigma_{osc}^{cc} = \frac{16\sqrt{2}\tau_0 e^2 m^* a\Theta_D^5\Delta^{1/2}(\mu^*H)^{3/2}}{h^4T^5} \times \sum_{l=1}^{\infty}(-1)^{l-1}l^{-3/2}f_l^{\sigma}\left\{\sin\left(\frac{\pi l\zeta}{\mu^*H}-\frac{\pi}{4}\right) - \theta(\zeta-2\Delta)\sin\left(\frac{\pi l(\zeta-2\Delta)}{\mu^*H}+\frac{\pi}{4}\right)\right\}. \quad (14)$$

In this expression, $\theta(y)$ is the θ-pulse step function. In this context, we should note that, in the traditional approach, we always obtain a formula for the Fermi surface with three extreme cross sections. At the same time, from an analysis of the geometry of the Fermi surface, it follows that, for $0 < \zeta \le 2\Delta$, this surface has only one extremal cross section; however, this case was not considered in [9]. It is also can be seen in (14) that, in the traditional quasi-classical approach, in which the dependence of the size of the Fermi surface along the direction of the magnetic field on the charge-carrier concentration is disregarded, the oscillating component of the conductivity changes abruptly with variations in ζ. At the same time, formulas (8) and (11) predict that the conductivity dependence on ζ is continuous. Such a contradiction can be explained by the fact that, in the model under consideration, the Fermi surfaces are closed for $0 < \zeta < 2\Delta$ and open for $\zeta > 2\Delta$. It can also be explained in a purely mathematical form: the formulas for the series summation over *r*, which appear after passing to the asymptotic representations of the Bessel function [11] in (8), are incorrect for $\kappa_\zeta = 0$ and $\kappa_\zeta = \pi$. Furthermore, it is clear from the expression itself that expression (14) is valid only if the Fermi level is not too close to the bottom or top of the narrow conduction miniband, whereas the general formulas (8) and (11) are valid for any relation between ζ and Δ. In addition, the quasi-classical condition, for which formula (2) is valid, must be satisfied; i.e., a large number Landau levels must lie below the Fermi level in the narrow conduction miniband.

This contradiction can be resolved if we note that, even for $\zeta < 2\Delta$, the oscillating component of the conductivity of a layered crystal contains two, rather than one, sets of oscillation frequencies. These frequencies can be defined by the formulas

$$h_l^H = \frac{l\zeta}{2\mu^*}, \tag{15}$$

$$h_l^{\prime H} = \frac{l|\zeta - 2\Delta|}{2\mu^*}. \tag{16}$$

The first of these sets is always associated with the maximal cross section of the Fermi surface cut by the $k_z = 0$ plane. The second set is not associated with any cross section of the Fermi surface cut by the plane perpendicular to the direction of the magnetic field if $0 < \zeta < \Delta$. However, it is associated with two nonextreme cross sections of the Fermi surface cut by the planes

$$k_z = \pm\frac{\arccos(3 - 2\zeta/\Delta)}{a}$$

if $\Delta < \zeta < 2\Delta$ and with two minimal cross sections of the Fermi surface cut by the $k_z = \pm\pi/a$ planes if $\zeta > 2\Delta$. The contribution of the harmonics with frequencies (16) increases if the ratio ζ/Δ increases and the ratio Δ/μ^*H decreases, i.e., if the Fermi level lies closer to the top of the miniband and the miniband is narrower, leading to larger anisotropy of the layered crystal. This behavior is caused by the fact that a decrease in the ratio Δ/μ^*H leads to a decrease in the dephasing of the oscillations related to the nonextremal cross sections of the Fermi surface. A similar decrease in dephasing can be also caused by an increase in the ratio ζ/Δ. Such behavior may be accounted for by a slower change, as the Fermi level approaches the top of the miniband, in the areas of the cross sections of the Fermi surface regarded as functions of the longitudinal quasi-momentum. In the quasi-classical approximation, at $\zeta < 2\Delta$, the contribution of frequencies (16) is only a small correction, on the order of μ^*H/Δ, to formula (14); i.e., this contribution gives rise to a fine structure when $\zeta < \Delta$ or beats when $\zeta < \Delta$. However, if $\zeta = \Delta$, frequencies (15) and (16) are indistinguishable.

We can illustrate the appearance of non-quasi-classical frequencies of the conductivity oscillations by expanding the integrand in (4) (taking into account (6)) in the Bessel functions of half-integer index [12], which are elementary functions expressed in terms of the products of sines and cosines by polynomials. A similar procedure was used in [14] for magnetic susceptibility. Then, the amplitudes of the oscillations with different frequencies depend continuously on the charge-carrier concentration, i.e., on ζ, thus ensuring the continuity of the change in the oscillating component of the conductivity dependence on ζ. However, the statement about the presence of non-quasi-classical oscillation frequencies is not true for all dispersion relations. For example, if we calculate the conductivity with formulas (3) and (4) using a purely quadratic function $W(x)$, we obtain the following formulas for the monotonic and oscillating components:

$$\sigma_0^{sq} = \frac{16\tau_0 e^2 m^* \Theta_D^5 \sqrt{8\zeta^3}}{3h^3 T^5 \sqrt{m_l^*}}, \tag{17}$$

$$\sigma_0^{sq} = \frac{16\tau_0 e^2 m^* \Theta_D^5 \sqrt{(\mu^* H)^3}}{\pi h^3 T^5 \sqrt{m_l^*}} \times \sum_{l=1}^{\infty} (-1)^{l-1} f_l^\sigma l^{-3/2} \left[\sin\left(\frac{\pi l\zeta}{\mu^* H}\right) \mathrm{Ci}\left(\sqrt{\frac{2l\zeta}{\mu^* H}}\right) - \cos\left(\frac{\pi l\zeta}{\mu^* H}\right) \mathrm{Si}\left(\sqrt{\frac{2l\zeta}{\mu^* H}}\right)\right]. \tag{18}$$

Here, m_l^* is the longitudinal electron effective mass, and $\mathrm{Ci}(y)$ and $\mathrm{Si}(y)$ are the cosine and sine Fresnel integrals, respectively (the other notation is specified above). Formula (18), as well as formulas (8) and (11) for a crystal with superlattice, take into account the effect of the charge-carrier concentration dependence of the Fermi surface size along the direction of the magnetic field on the oscillations of the longitudinal conductivity. When passing to the asymptotic limit $\zeta/\mu^*H \gg 1$ in (18) and retaining only the leading terms in the expansions of the Fresnel integrals, we obtain a formula of type (14) for the Fermi surface with a unique stationary cross section cut by the $k_z = 0$ plane. Thus, we find that, for $\zeta < 2\Delta$ in the traditional quasi-classical approximation, dispersion relation (6) is virtually parabolic. In the same way, without using the method of steepest descent, we can also obtain formula (13) for a general case. Using the expansions of the Fresnel integrals in the Bessel functions of half-integer index [12], we can show that, for a parabolic dispersion relation, having a finite size of the Fermi surface along the direction of the magnetic field does not result in the appearance of non-quasi-classical oscillation frequencies. Moreover, this conclusion is valid not only for a quadratic dispersion relation but also for a linear relation of the form

$$W(x) = \Delta_0|x|, \tag{19}$$

which is used, for example, to describe the band structure of graphite and synthetic metals based on graphite intercalation compounds (Δ_0 is a parameter of the model and has the dimensions of energy) [6]. For this dispersion relation, we obtain the following formulas for the components of the longitudinal conductivity of the crystal:

$$\sigma_0^{\rm lin} = \frac{32\pi\tau_0 e^2 m^* a\Theta_{\rm D}^5\Delta_0\zeta}{h^4T^5}, \tag{20}$$

$$\sigma_{\rm osc}^{\rm lin} = \frac{32\tau_0 e^2 m^* a\Theta_{\rm D}^5\Delta_0\mu^* H}{h^4T^5} \times \sum_{l=1}^{\infty}(-1)^{l-1}f_l^{\sigma}l^{-1}\sin\left(\frac{\pi l\zeta}{\mu^* H}\right). \tag{21}$$

Formula (21) describes the oscillations of the longitudinal conductivity related to the only stationary (maximal) cross section of the Fermi surface cut by the $k_z = 0$ plane; this circumstance is quite understandable if we take into account that, for model (19), the Fermi surface of the crystal consists of two cones with contacting bases. If we try to apply formula (13) to model (19), we obtain an obviously incorrect result, $\sigma_{\rm osc}^{\rm lin} = 0$, since, in this model, $W''(x) \equiv 0$. Thus, we arrive at the conclusion that the presence of harmonics with non-quasi-classical frequencies in the oscillating component of the longitudinal conductivity and the deviation of the field dependence of the oscillation amplitudes from the "$H^{3/2}$ law" under the conditions of applicability for the quasi-classical approximation can serve as a measure of the nonparabolicity of the conduction band. Furthermore, it can be seen from (13), (14), and (21) that, due to the quasi-classical condition, in each of the cases considered, the oscillating component of the conductivity is small compared to the magnetic-field-independent component. Comparing these results with those of [1, 2], we see that, under the quasi-classical conditions, the longitudinal magnetooscillation effects are much less pronounced than the transverse effects. However, in layered crystals, the magnitude of the former effects may be larger because of the pronounced anisotropy of the electronic spectrum.

Using an elementary model of the band spectrum of a layered crystal as an example, we now analyze the limits of the applicability of the obtained results. For this purpose, we should take into account that the phonon energy is approximately kT at low temperatures and the scattering-induced change in the energy of the longitudinal motion of an electron cannot exceed the width of the narrow miniband 2Δ if we disregard the Umklapp processes. Therefore, the scattering-induced change in the number of the Landau subband is estimated to be

$$|\delta n| = \frac{kT}{2\mu^* H} + \frac{\Delta}{\mu^* H}. \tag{22}$$

The second term on the right-hand side is approximately equal to 100 for $m^* = m_0$, $\Delta = 0.01$ eV, and magnetic fields of approximately 1 T; therefore, the quasi-classical condition is satisfied, intersubband transitions are not suppressed, representation (2) for the relaxation time is valid, and the results obtained above are correct. However, under conditions in which the Shubnikov–de Haas effect is clearly pronounced, it is not always possible to disregard the suppression of intersubband transitions. Indeed, at low temperatures, where $q_z a \ll 1$ (q_z is the longitudinal component of the phonon wave vector), the absolute value of the scattering-induced change in the energy of the longitudinal motion of an electron does not exceed $q_z a\Delta$. Therefore, taking into account that $q_z = 2\pi kT/hs$, where s is the velocity of sound in the crystal, we use the condition $\delta n \leq 1$ to estimate the freeze-out temperature for intersubband transitions:

$$T_f = \frac{2\mu^* Hhs}{k(2\pi a\Delta + hs)}. \tag{23}$$

This temperature is, of course, lower than $2\mu^* H/k$. For $m^* = m_0$, $\Delta = 0.01$ eV, $s = 5 \times 10^3$ m/s, and $a = 10$ nm, we obtain $T_f = 0.074$ K in magnetic fields of approximately 1 T. At first sight, this condition for the freeze-out of intersubband transitions seems to be quite restrictive, especially if we take into account that, generally, magnetoinsulating experiments are performed at much higher temperatures [13]. However, if, taking into account the magnitude of the factor of the thermal smearing of the oscillations determined by (5), we write the condition necessary for the Shubnikov–de Haas effect to be clearly pronounced in the form

$$\frac{\mu^* H}{kT} \geq \pi^2, \tag{24}$$

then, for the same parameters values, we obtain $T \leq 0.068$ K; i.e., the temperature must be lower than T_f.

Expression (23) has a quite clear physical meaning. If we set $\Delta = 0$ in this expression, we will transform the system of Landau subbands into a system of discrete levels for which $T_f = 2\mu^* H/k$. This formula allows us to understand why, in typical metals, e.g., alkali metals, even in a situation where the Shubnikov–de Haas effect is well pronounced, the intersubband transitions do not freeze out and the quasi-classical approximation of the above sense is valid. For the purpose of estimation, if we set $\Delta = 5$ eV, $a = 0.5$ nm, and $s = 5 \times 10^3$ m/s in (23), we find that $2\pi a\Delta/hs \approx 850$; this value has the same order of magnitude as the ratio $2\mu^* H/kT_f$. Thus we see that, even if the thermal smearing of the oscillations is negligible, intersubband transitions in typical metals are important and, therefore, the quasi-classical approximation is valid. A different situation is realized in semimetals, e.g., in bismuth. If a magnetic field of about 1.25 T is applied to a Bi crystal along the long bisector ellipsoid axis of the constant-energy surface for a conduction band with the effective mass $m^*/m_0 = 8.2 \times 10^{-3}$, the distance between the Landau levels is $2\mu^* H/k \approx 204$ K [13]. If we also take into account the data [13] on the value of the Fermi energy for electrons and, for the purposes of estimation, set $2\Delta = 0.03$ eV = 348 K, then the ratio $\Delta/\mu^* H = 1.71$; moreover, according to (22), we may assume that, in the observation of the Shubnikov–de Haas effect, the intersubband transitions are frozen out. The freeze-out temperature in this case can be determined from inequality (24), which gives a temperature value equal to 10.3 K, in satisfactory agreement with experiment [15]. A similar situation is encountered in highly anisotropic layered crystals. However, in this situation we must take into account the effect of the magnetic field on charge-carrier scattering and, since the quasi-classical approximation is no longer valid, use the formulas from [16] rather than the formulas of this study. In addition, we should note that the authors of [4] consider oscillations of the scattering probability of charge carriers as the main cause of the Shubnikov–de Haas effect. This is equivalent to the assumption that the relaxation time is inversely proportional to the density of states in the magnetic field, with an allowance made for the effect of all the lower lying Landau subbands on the density of states. However, the resulting correction to the oscillating component of the conductivity does not affect the oscillation frequencies.

3. CONCLUSIONS

Thus, we have shown that, under the conditions of the applicability of the quasi-classical approximation, the presence of frequencies in the spectrum of the oscillations of the longitudinal conductivity that are not associated with the stationary cross sections of the Fermi surface cut by planes normal to the field, as well as the deviation of the field dependence of the corresponding amplitudes from the linear law or "the 3/2 law," can serve as a measure of the deviation of the dispersion relation, which describes the charge carrier motion, from linear or parabolic, respectively. Furthermore, it is shown that, in crystals with narrow conduction minibands or with small values of the Fermi energy and small transverse charge-carrier effective masses, the intersubband transitions in the Shubnikov–de Haas effect can freeze out. This freeze out is, at least, in qualitative agreement with the experiment and with the theoretical results obtained by previous authors without using the assumption of the nonparabolicity of the conduction band. On the basis of the results obtained during the preparation of this publication, it should be noted that, in order to study fine details of the topology of the Fermi surface of conducting materials (especially unconventional ones), it is necessary to perform the experiments in strong quantizing magnetic fields, i.e., in the region of large deviations from the traditional quasi-classical approximation. However, for this purpose, the topology the Fermi surface must be first parametrized, e.g., on the basis of rough calculations of the band structure, and the optimum set of orientations of the magnetic field for which these deviations are pronounced most clearly must be determined.

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