Effect of the configuration of a quantum wire on the electron–phonon interaction

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The dissipation of the electric current on acoustic phonons in a one-dimensional conductor is of an activational character. The activation energy of phonon scattering of an electron depends on the spatial configuration of the conductor. For this reason, electron scattering by phonons can be substantially weakened and the phonon contribution to the resistance thereby can be decreased by choosing an appropriate form of the conductor. © *1999 American Institute of Physics.* [S1063-7826(99)01810-4]

One of the high-priority problems in modern semiconductor physics is to develope structures with prescribed values of the electrical parameters, most importantly the conductivity. It is known that the conductivity of real structures is limited by scattering by lattice defects and by phonons. Modern technologies make it possible to eliminate defects of the crystal structure, while in order to decrease phonon scattering it is necessary to develop a method for controlling the electron–phonon interaction. Previous investigations[1,2] have shown that the matrix elements of the electron–phonon interaction can be substantially decreased and the phonon scattering mechanism can therefore be substantially suppressed by creating artificial crystal structures with prescribed parameters of the electron energy spectrum or by manipulating the electron energy spectrum in two- and three-dimensional systems by means of external perturbations (quantizing magnetic field, static deformation of the crystal, and others).

A qualitatively different (and substantially more effective) method of decreasing scattering by phonons can be implemented in one-dimensional conducting structures (quantum wires) with a nonlinear configuration. The physical factor responsible for the effect of the configuration of a quantum wire (QW) on the phonon scattering of electrons is that the potential of any elementary interaction in a solid (electron–electron, electron–photon, electron–phonon, and so on) is three-dimensional, while the translational motion of an electron in a QW is one-dimensional. By prescribing the electron trajectory in three-dimensional space by means of the configuration of the QW it is possible to change qualitatively the effective interaction potential. This was first noted in Ref. 3. Subsequent investigations[4,5] have shown that the configuration of the wire influences very critically the electron–electron and electron–photon interaction processes. In this sense, the electron–phonon interaction also should not be ruled out. As will be shown below, electron scattering by acoustic phonons can be greatly weakened by making an appropriate choice of the form of the QW.

Real QWs are conducting channels which are embedded in a three-dimensional crystal. For this reason, phonon scattering in QWs is due to the interaction with three-dimensional phonons and the electron–phonon interaction potential at the point $\mathbf{r}$, in general, has the form

$$U(\mathbf{r},\mathbf{q},t)=\tilde{U}(\mathbf{q})\exp(i\,\mathbf{q}\cdot\mathbf{r})\exp[-i\tilde{\varepsilon}(\mathbf{q})t/\hbar], \tag{1}$$

where $\mathbf{q}$ is the three-dimensional phonon wave vector, $\tilde{\varepsilon}(\mathbf{q})$ is the phonon energy, and the explicit form of the function $\tilde{U}(\mathbf{q})$ depends on the choice of the specific model of the electron–phonon interaction. The electron wave function in a one-dimensional rectilinear conductor is

$$\psi_k(s,t)=\sqrt{1/L}\,\exp(i\,k\,s)\exp[-i\varepsilon(k)t/\hbar], \tag{2}$$

where k is the wave number corresponding to the motion of an electron along the QW, s is the coordinate measured along the QW, L is the length of the QW, and $\varepsilon(k)=\hbar^2k^2/2m$ is the electron energy. In accordance with the standard quantum-mechanical relations, the lifetime $\tau(k)$ for an electron in the state k in the presence of the perturbation (1) is determined by the expression

$$\frac{1}{\tau(k)}=\sum_{\mathbf{q}}\frac{1}{\tau(k,\mathbf{q})}, \tag{3}$$

where

$$\begin{aligned}\frac{1}{\tau(k,\mathbf{q})}=\frac{2\pi}{\hbar}\sum_{k'}\{1-f[\varepsilon(k')]\}\\ \times\{|U^{(-)}_{k'k}(\mathbf{q})|^2\delta[\varepsilon(k')-\varepsilon(k)-\tilde{\varepsilon}(\mathbf{q})]\\ +|U^{(+)}_{k'k}(\mathbf{q})|^2\delta[\varepsilon(k')-\varepsilon(k)+\tilde{\varepsilon}(\mathbf{q})]\}\end{aligned} \tag{4}$$

determines the electron lifetime $\tau(k,\mathbf{q})$ in the state k in the presence of an interaction with the phonon $\mathbf{q}$. Here $f[\varepsilon(k)]$ is the Fermi–Dirac distribution function

$$\begin{aligned}U^{(\pm)}_{k'k}(\mathbf{q})=\frac{\tilde{U}^{(\pm)}(\mathbf{q})}{L}\int_{-L/2}^{L/2}\exp\{i[q_x x(s)+q_y y(s)\\ +q_z z(s)]\}\exp[i(k-k')s]ds\end{aligned} \tag{5}$$

is the matrix element of the electron–phonon interaction potential (1) for a transition of an electron from the state k into the state k', the signs $\pm$ in the matrix element correspond to phonon emission and absorption, and $x(s)$, $y(s)$, and $z(s)$ are the Cartesian coordinates of the point s on the one-dimensional conductor. We shall consider a QW embedded

in a three-dimensional isotropic elastic continuum, characterized by density ρ and longitudinal sound velocity v_l, such that the Fermi velocity of the electrons in the QW satisfies $v_F \gg v_l$. We shall use the deformation potential method to describe the interaction of electrons with acoustic phonons, so that the expression (1) assumes the form

$$U(\mathbf{r},\mathbf{q},t)=\Xi \operatorname{div}\mathbf{u}(\mathbf{q}), \tag{6}$$

where Ξ is the deformation potential constant, and $\mathbf{u}(\mathbf{q})$ is the deformation vector of the elastic medium for a phonon with wave vector $\mathbf{q}$. For the interaction potential (6) $\tilde{U}^{(-)}(\mathbf{q})$ is given by

$$\tilde{U}^{(-)}(\mathbf{q})=\left[\frac{\hbar n(q)q}{2V\rho v_l}\right]^{1/2}\Xi, \tag{7}$$

where $n(q)$ is the phonon occupation number, described by the Bose–Einstein function, and V is the volume of the three-dimensional elastic continuum. The quantity $\tilde{U}^{(+)}(\mathbf{q})$, corresponding to phonon emission, is obtained from Eq. (7) by formally replacing $n(q)$ with $n(q)+1$. With Eqs. (4)–(7) the expression (3) becomes

$$\frac{1}{\tau(k)}=\frac{\pi\Xi^2}{\rho v_l L^2 V}\sum_{\mathbf{q}}\sum_{k'} q\{1-f[\varepsilon(k')]\}|J(k,k',\mathbf{q}|^2 \times\{n(q)\delta[\varepsilon(k')-\varepsilon(k)-\tilde{\varepsilon}(\mathbf{q})] +[n(q)+1]\delta[\varepsilon(k')-\varepsilon(k)+\tilde{\varepsilon}(\mathbf{q})]\}, \tag{8}$$

where

$$J(k',k,\mathbf{q})=\int_{-L/2}^{L/2}\exp\{i[q_x x(s)+q_y y(s) +q_z z(s)]\}\exp\{i(k-k')s\}ds.$$

Since the Fermi surface in a QW consists of only two points, dissipation of the electric current in the QW occurs as a result of the scattering of electrons from one Fermi point into the other. It is obvious that in a rectilinear QW, such scattering processes are due to the interaction of electrons with phonons whose wave vector $q \geqslant 2k_F$. Since the energy of such phonons satisfies $\tilde{\varepsilon}(\mathbf{q}) \geqslant \tilde{\varepsilon}(2k_F)$ and is substantially different from zero for a prescribed value of the Fermi wave vector k_F, electron scattering by acoustic phonons in a QW is an inelastic process, in contrast to scattering by acoustic phonons in two- and three-dimensional systems. This results in a qualitatively different picture of current dissipation in a QW at low temperatures

$$T \ll \tilde{\varepsilon}(2k_{\mathrm{F}})/K_{\mathrm{B}}, \tag{9}$$

where $\tilde{\varepsilon}(2k_{\mathrm{F}})=2\hbar v_l k_{\mathrm{F}}$, and K_B is Boltzmann's constant. The scattering probability due to absorption of a phonon (the process 1 in Fig. 1) is exponentially small, since at low temperatures (9) there are few phonons with energy $\tilde{\varepsilon}(\mathbf{q}) \geqslant \tilde{\varepsilon}(2k_{\mathrm{F}})$. The scattering probability due to emission of a phonon (the process 2 in Fig. 1) is exponentially small because of the Pauli principle, since at temperatures corresponding to the inequality (9) the region of thermal broadening of the Fermi–Dirac distribution function near the Fermi level is much smaller than the minimum energy $\tilde{\varepsilon}(2k_{\mathrm{F}})$ of the emitted phonon. Therefore, when the condition (9) holds, the probability of phonon scattering of an electron satisfies

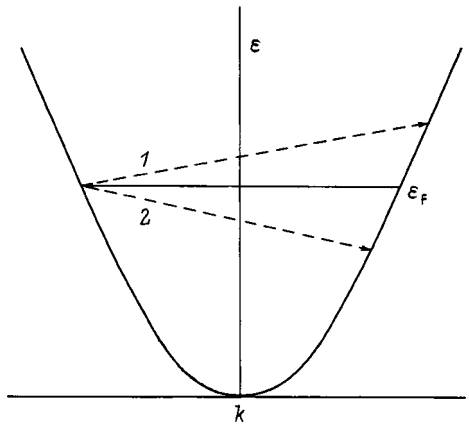

FIG. 1. Electron–phonon scattering processes near the Fermi energy ε_{F} in a rectilinear quantum wire: *1*—with phonon absorption, *2*—with phonon emission.

$$W \propto \exp\left(-\frac{2\hbar v_l k_{\mathrm{F}}}{K_{\mathrm{B}}T}\right). \tag{10}$$

It follows from the relation (10) and the preceding qualitative arguments that phonon scattering of electrons in a QW is of an activational character, and the activation energy of the phonon mechanism of scattering in a rectilinear QW is

$$\varepsilon_{a0}=2\hbar v_l k_{\mathrm{F}}. \tag{11}$$

Using Eq. (8) to calculate the electron lifetime $\tau(k_{\mathrm{F}})$ at the Fermi level in a rectilinear QW for the scattering processes shown in Fig. 1, we obtain the expression

$$\frac{1}{\tau(k_{\mathrm{F}})}=\frac{4\Xi^2 m K_{\mathrm{B}}T k_{\mathrm{F}}}{\pi\rho v_l^2\hbar^3}\exp\left(-\frac{\varepsilon_{a0}}{K_{\mathrm{B}}T}\right), \tag{12}$$

if the condition (9) is satisfied. This expression is in complete agreement with the qualitative analysis. The activation energy of phonon scattering can be substantially increased by changing the configuration of the QW. This will expand the temperature range in which the phonon contribution to the resistance is exponentially small. As a specific example, we shall consider a QW in the form of a helix. Since the curvature of such a QW is the same at all points, the wave vector of the electron along the helical QW is a conserved quantity just as the wave vector of an electron along a rectilinear QW. Under this circumstance, the relation (8), written for a rectilinear QW, remains in force for a helical QW, where

$$x(s)=R\cos(2\pi s/l), \quad y(s)=R\sin(2\pi s/l), \quad z(s)=sh/l,$$

R is the radius of the helix, h is the pitch of the helix, and $l=\sqrt{4\pi^2R^2+h^2}$ is the length of the helix. Calculating for a helical QW the electron lifetime at the Fermi level using Eq. (8), we find for $K_{\mathrm{B}}T \ll \varepsilon_a$ and $K_{\mathrm{B}}T \ll h v_l/R$

$$\frac{1}{\tau(k_{\mathrm{F}})}=\frac{4\Xi^2 m K_{\mathrm{B}}T k_{\mathrm{F}}}{\pi\rho v_l^2\hbar^3}\left(\frac{l}{h}\right)^3\exp\left(-\frac{\varepsilon_a}{K_{\mathrm{B}}T}\right), \tag{13}$$

where the activation energy of phonon scattering of an electron in a helical QW (the energy of a phonon which transfers an electron from one Fermi point into another on the helical QW) is

$$\varepsilon_a = \frac{2\hbar v_l k_F l}{h}. \tag{14}$$

Comparing the activation energies (11) and (14), we obtain

$$\frac{\varepsilon_a}{\varepsilon_{a0}} = \frac{l}{h}. \tag{15}$$

It follows from the relations (13)–(15) that to increase the lifetime $\tau(k_F)$ at the Fermi level and to obtain the associated increase in conductivity the values of the parameters h/l and R of the helical QW must be small. Specifically, at liquid-helium temperatures and $k_F \simeq 10^6$ cm^{-1}, $v_l \simeq 10^6$ cm/s, $R \simeq 10^{-6}$ cm, and $h/l \simeq 10^{-2}$ it follows from Eqs. (12) and (13) that the phonon-scattering-induced transport electron relaxation time at the Fermi level in a helical QW is tens of orders of magnitude greater than the relaxation time in such rectilinear QW.

The single-electron approximation was used in the derivation of the basic relations obtained in this paper. This approximation is valid if the electronic system can be treated as a Fermi gas. At the same time, it is known that the formation of an electron liquid in a one-dimensional conductor (Tomonaga–Luttinger liquid) leads to an energy spectrum of elementary excitations of the electronic system that is qualitatively different from that of a Fermi gas (see, for example, the review in Ref. 6). Thus, it is necessary to formulate a criterion of applicability of the single-electron approximation in the problem solved in the present paper. For the helical QW considered here, the single-electron approximation certainly holds for the electron–phonon processes shown in Fig. 1, if the energy (14) of such phonons is much higher than the characteristic electron–electron interaction energy $e^2/\epsilon r$, where ϵ is the dielectric constant of the medium, and $r = \pi/k_F$ is the distance between the electrons. This criterion can be written explicitly as

$$\frac{e^2}{2\pi\hbar v_l \epsilon l}\frac{h}{l} \ll 1,$$

and it holds, in particular, for the parameter values used to obtain the previous estimate of $\tau(k_F)$ in a helical QW. The conclusion that the phonon dissipation of current in a helical QW is strongly suppressed therefore remains valid when the electron–electron interaction is taken into account.

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