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Axial Vibration of Threaded External Fixation Pins: Detection of Pin Loosening

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Abstract—The hypothesis of this study was that a nondestructive vibrational method could detect bone lysis at the external fixation pin–bone interface prior to current clinical and radiographic methods. *In vitro* models were used to simulate changes observed during pin loosening *in vivo*. Fixation pin axial natural frequency decreased with decreasing tensile modulus of the material into which it was implanted. In a live animal study the right tibia of 12 dogs was fractured and stabilized with a four-pin unilateral external fixation frame. The axial natural frequency of each pin was measured and radiographs were taken at 0, 2, 4, 6, 8, and 10 weeks after surgery. The natural frequency did not change when the first radiographic changes around the interface were observed. Pins were palpably stable at this point. As loosening progressed, the natural frequency did decrease. Frequency and quasistatic tests of dissected pin–bone structures revealed a good correlation between natural frequency and pin–bone interface stiffness. In addition, the measurement of natural frequency was more sensitive to bone structure changes at the pin–bone interface than low-load quasi-static stiffness. Therefore, a nondestructive vibration technique could be used instead of low-load quasistatic tests for assessing the pin–bone interface *ex vivo*. © *1998 Biomedical Engineering Society*. [S0090-6964(98)00803-0]

Keywords—Vibration, External Fixation Pin, Interface, Loosening, Nondestructive.

INTRODUCTION

External fixation is a fracture management technique that involves drilling percutaneous pins into the proximal and distal portions of a bone fracture. Each pin is clamped to a stiff connecting bar with the bone aligned. Many derivations of the basic frame configuration are available, allowing the external fixator to be used in a wide variety of fractures. Other advantages of external fixation include its resistance to most loading conditions, the ability of the patient to use the limb after fracture fixation, and its ability to be removed after bone union. External fixation has increased in popularity, both in human and veterinary surgery during the past two decades. However, the most common reason for failure continues to be loss of stability at the pin–bone interface.[2,4] Loose pins increase patient morbidity by contributing to pin-track infections, prolonged limb disuse, and delayed bone union or fracture nonunion.[4]

Pin loosening is suggested by the presence of exudate around the pin track and the detection of radiolucency. Instability is confirmed by palpation and assessing patient discomfort. The presence of radiolucency around the pin is considered the most sensitive measure of failure of the pin–bone interface.[1,13] Radiographic examination is helpful in determining whether there is a large amount of bone resorption around a pin but is subjective, and standard radiographs may not provide sufficient detail to identify the early stages of pin tract bone resorption. A method that is more sensitive to initial degenerative changes at the external fixation pin–bone interface is needed to enable early detection of pin loosening so that management practices can be implemented to halt the loosening process and facilitate fracture healing.

Vibrational response[5,10,16] and ultrasonic[6,15] techniques have been found to be sensitive to subtle changes in bone quality but there has been relatively little information reported on the use of vibration to assess the pin–bone interface of fixation pins. Previous research from our laboratory has shown that a driving point vibration technique is nondestructive to biological materials and has been used to estimate the elastic modulus of cancellous bone,[16] monitor fracture healing,[14] and estimate the pullout strength of porous coated cylindrical osseous implants.[8]

The objectives of this investigation were to answer the following questions: (1) Does a decrease in axial pin natural frequency correlate with a decrease in pin tract quality? (2) Can a driving point mechanical inertance technique be used to quantitate changes in bone structure at the pin–bone interface *in vivo* and is it more sensitive

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than current radiographic analyses? (3) Is the representation of *ex vivo* interface stiffness with pin natural frequency more sensitive to changes in bone structure that occurred *in vivo* than the measurement of stiffness using a materials testing machine? These questions were answered with investigations involving phantom models (*in vitro* tests), live animals (*in vivo* tests), and pin–bone structures dissected from the live animal experiment (*ex vivo* tests).

MATERIALS AND METHODS

Measurement of Interface Parameters

Vibration Analysis Technique. The axial natural frequency (w_o) was measured for pins in each investigation. For a lightly damped single degree of freedom system the resonant frequency and the natural frequency are approximately the same. The effect of significant damping is to reduce the resonant frequency while the natural frequency remains constant. The natural frequency of the pin system was measured here since we were interested in recording changes in interface stiffness (assuming the mass of the pin to be constant) without having to account for the effect of damping, which is unknown (see the Appendix).

Pin axial natural frequency was measured using a driving point inertance technique. A stationary electromagnetic shaker (B&K Minishaker type 4810, Bruel and Kjaer, Naerum, Denmark) was used during *in vitro* tests and a hand-held electromagnetic shaker was used during the *in vivo* and *ex vivo* tests (B&K Hand-held Exciter type 5961). Each shaker, driven by a broadband binary noise excitation signal originating from a dual-channel fast Fourier transform (FFT) analyzer (model 660B, Nicolet Scientific Corporation, Northvale, NJ), provided axial excitation to the pin while a piezoelectric impedance head transducer (type 8001, Bruel and Kjaer, Naerum, Denmark) sensed the excitation force and system acceleration at the driving point (Fig. 1). A 0.50 kg static load was applied to ensure contact between the pin and the impedance head during the test. Force and acceleration signals were amplified using charge amplifiers (types 2635 and 2628, Bruel and Kjaer, Naerum, Denmark). Data acquisition and spectral analyses were performed using the dual-channel FFT analyzer and the data were downloaded to a 80286 personal computer for storage.

The FFT analyzer computed the inertance of the system by taking the complex ratio of acceleration divided by force. It was assumed that the force-deformation response of the pin–bone system was linear for loads between 0.0 and 1.0 kg and that damping was viscous. Therefore, the frequency response of the system to binary noise could be assumed to be the same as its response to a variable frequency forcing function (principle of superposition). Results of preliminary quasistatic tests of the canine pin–bone interface validated the assumptions and all loads applied to pins during each testing condition ranged from 0.0 to 1.0 kg. The undamped natural frequency was determined from the inertance spectra as the frequency where the real component of the inertance passed through zero [Fig. 2(a)] and where a peak occurred in the imaginary component [Fig. 2(b)]. The mathematical theory of finding an undamped natural frequency and the basis of using it as a representation of stiffness is given in the Appendix.

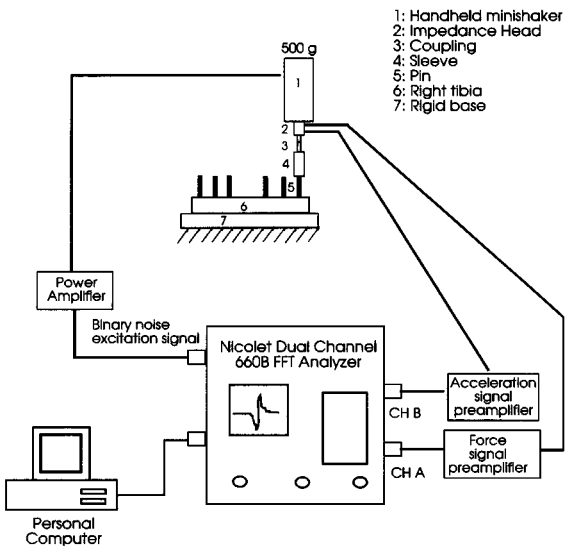

FIGURE 1. Schematic diagram of the vibrational test system.

Measurement of Quasistatic Stiffness. A materials testing machine (model 1122, Instron Corp., Canton, MA) was used to conduct a low-load quasistatic test to mea-

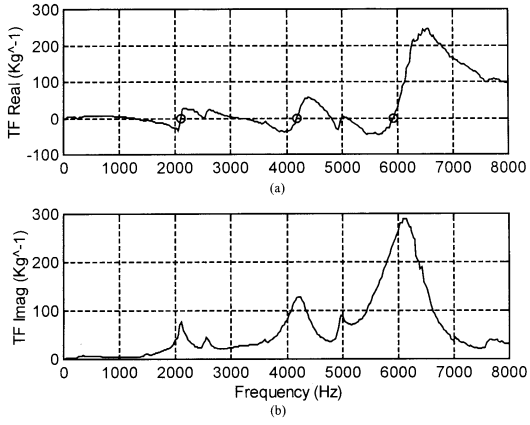

FIGURE 2. Real (a) and imaginary (b) components of inertance from an *in vivo* test showing the location of three undamped natural frequencies (denoted with an "○").

TABLE 1. Urethane sample data.

Harkness* Product ID	Density (g cm^{-3})	Tensile modulus (MPa)
MP400	1.28	0.913
MP600	1.26	3.447*
MP750	1.26	10.34*
MP850	1.26	17.24*
MP950	1.15	62.05*
MP175	1.18	350.0

*Harkness Industries, Inc., Cheshire, CT.

sure the stiffness of the pin–bone system during the *in vitro* and *ex vivo* studies. Pins were loaded at a constant deformation rate of 1 mm/min, and the test was halted when a load of 0.75 kg was achieved. Deformation was measured by crosshead displacement. A 80286 personal computer and dedicated software [Series IX (version 4), Instron Corp., Canton, MA] were used for data acquisition and analyses. The stiffness between 0.30 and 0.70 kg was calculated from the force-deformation curve by fitting a regression line between the load limits. The stiffness was computed between 0.30 and 0.70 kg to provide a better representation of the stiffness of the system in the 0.50 kg region, which was the static loading condition during the vibration tests.

Experimental Protocol

In Vitro Tests. Ten square specimens (30×30 ×3.2 mm) were cut from six different types of urethane (Harkness Industries, Inc., Cheshire, CT). Urethane tensile moduli ranged from 0.913 to 350.0 MPa (Table 1). This range was chosen to include the possible modulus of tissue at the pin–bone interface for loose pins as well as for pins that had begun the loosening process.[7] Holes, 3.20 mm in diameter, were punched in the middle of each specimen. Interface™ (Imex™, Longview, TX) Half-pins (3.17 mm shank diam, 3.97 mm thread diam, 11 threads/cm, 316LS stainless steel, 11 cm long) were inserted into the punched hole of each specimen. Pin–urethane specimens were held in a custom-made ring clamp that had holes of either 6.35 or 4.57 mm in diameter. This resulted in either 1.19 or 0.3 mm of clearance around the pin.

Two additional stages of pin loosening within each type of urethane were simulated with these two clearance hole diameters because the pin–urethane system is more compliant at larger clearance hole diameters. The axial natural frequency and quasistatic stiffness were measured for each pin–urethane specimen. Five specimens from each sample of urethane were tested using the 6.35 mm clearance hole and the other five specimens were tested using the 4.57 mm clearance hole.

In Vivo Tests. External fixation pin loosening was monitored in 12 mixed breed canines (six males). Ten were skeletally mature and two (males) showed evidence of an active tibial physis. Mean tibia length was approximately 17 cm.

Surgery was performed to apply a unilateral four-pin external fixation device. Animals were anesthetized, and the right hind limb was prepared for aseptic surgery. Six pilot holes (numbered 1–6, with 1 being most proximal and 6 most distal) were drilled in the tibia using a 3.1 mm diameter drill bit (Imex™, TX). A positioning template ensured a consistent spacing of 2 cm between pins in the proximal and distal portions and of 4 cm between pins 3 and 4. Half-pins (the same as those used during the *ex vivo* tests) were threaded into the pilot holes by hand until the threads fully engaged both cortices. A midshaft tibial osteotomy (osteotomy group) was performed between pins 3 and 4 in six dogs and a 1.5 cm tibial ostectomy (ostectomy group) was performed between pins 3 and 4 in the remaining six dogs. The bone was cut using a power surgical bone saw. The fibula was transected using bone cutters.

A unilateral medium external fixation frame incorporating pins 1, 3, 4, and 6 was used to stabilize the bone. A 4.75 mm diam connecting rod was used in all frames. Pins 2 and 5 were not incorporated into the fixation frame and were monitored as unloaded controls. The nominal fixation pin length, from cut edge to trocar tip, was 5.5 cm. Analgesia (oxymorphone, 0.1 mg/kg intravenously) was administered to each dog as needed to reduce discomfort during the first 24 h of recovery. Activity was limited to small runs (1.5 m long, 3 m wide), and animals were checked daily. Fixation frame stability, pin drainage, and sensitivity were graded weekly.

The axial natural frequency of each pin was measured immediately after surgery and at two-week intervals for ten weeks. Each dog was sedated with acepromazine (0.025–0.05 mg/kg) and oxymorphone IV (0.05–0.1 mg/kg), radiographed, and placed on a table before vibrational testing. The right hind limb was extended on a heavy pedestal test stand that provided a balanced and rigid base. The limb was positioned so that the fixation pins were perpendicular to the base (Fig. 1). The connecting bar and clamp was removed from the pins before testing. When testing pins 1, 3, 4, and 6, a connecting bar was attached to pins 2 and 5 prior to removing the primary connecting bar so that the fracture would not be disturbed. When testing pins 2 and 5, the connecting bar was replaced on pins 1, 3, 4, and 6. After each vibrational test, pin looseness was assessed by palpation. Drainage and swelling around the pin were also noted.

Medial–lateral and cranial–caudal radiographs were taken of the fractured tibia at the time each dog underwent vibrational testing. Each pin track was graded ac-

cording to the amount of radiolucency around the pin. A large amount of radiolucency indicated bone lysis and an unstable pin–bone interface. Grades were given to each pin at each time period by the same observer as follows (radiograph score): (1) solid, (2) slight progress towards loosening, (3) moderate progress towards loosening, (4) likely loose, and (5) obviously loose.

Ex Vivo Tests. Animals were sacrificed during the 11th week and the right tibia harvested. The fixation frame was left intact, and all soft tissues were removed. The tibiae were wrapped in gauze soaked with buffered saline (pH 7.4) and frozen ($-20.0\,^{\circ}C$). Prior to testing, each tibia was thawed in a refrigerator overnight and equilibrated to room temperature for 2 h. Tibiae were then cut into 2-cm segments, each containing one pin. Segments were clamped in a vice that was attached to a rigid base and the axial natural frequency of the pin–bone system was measured. The vice was then placed under the crosshead of the materials testing machine and the quasistatic stiffness of the pin–bone system was measured. Proximal–distal, lateral–medial, and cranial–caudal microradiographs were taken of each bone segment after vibrational and quasistatic testing. Each pin track was graded by one observer according to the amount of pin track lysis (microradiograph score): (1) no lysis, (2) lysis at either near or far cortex, but not both, (3) moderate lysis of the near cortex, and (4) lysis along the entire pin track.

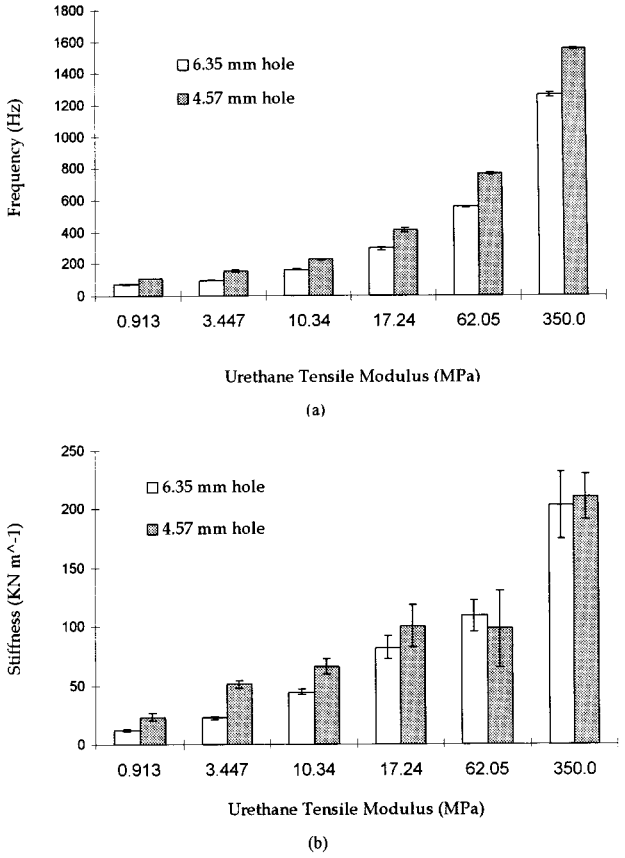

FIGURE 3. Means of natural frequency (a) and quasistatic stiffness (b) from *in vitro* tests ($n=5$ for each urethane–hole combination). Error bars indicate standard deviations.

RESULTS

In Vitro Tests

Natural frequency and quasistatic stiffness increased with an increase in the tensile modulus of the urethane ($p<0.0001$) (Fig. 3). In general, frequency and quasistatic stiffness both decreased ($p<0.05$) when the larger clearance hole diameter (6.35 mm) was used to clamp specimens. There was a significant interaction between tensile modulus and clearance hole diameter ($p<0.001$). The mean frequency for each combination of urethane and clearance hole was significantly different from every other combination ($p<0.05$) [Fig. 3(a)].

Changes in the stiffness of the pin–urethane interface were not always detected from quasistatic results. There was no significant difference between quasistatic stiffness, measured using the 4.57 mm clearance hole, when 17.24 or 62.05 MPa modulus urethane was used or when 3.447 or 10.34 MPa modulus urethane was used [Fig. 3(b)]. However, mean stiffness, measured using the 6.35 mm clearance hole, was significantly different ($p<0.05$) for each type of urethane used. Clearance hole diameter did not have a statistically significant effect upon the quasistatic stiffness when 17.24, 62.05, and 350.0 MPa moduli urethane materials were used.

In Vivo Tests

All dogs recovered from surgery without complications. Drainage, swelling, and discomfort were associated with a number of pins by the end of the first week. Although this was likely the result of local infection, symptoms mostly resolved without treatment. During the second week, it was found that two ostectomy dogs had deformed the connecting bar. These bars were replaced.

By week 10, 28 pins (13 from the osteotomy group and 15 from the ostectomy group) in 11 dogs showed radiographic evidence of pin track lysis and had received a radiograph score greater than 1. Seven pins from the ostectomy group and four from the osteotomy group were classified as either ''likely loose'' or ''obviously loose'' by week 10. Pin tract lysis was more likely to occur around the most proximal pins (Fig. 4). Lysis was observed around three of the unloaded pins but none of these pins were ''likely loose'' or ''obviously loose'' by week 10. Pin-track lysis was not observed in any pin for one dog.

Multiple resonant frequencies were present in the *in vivo* inertance signals where only one resonant frequency

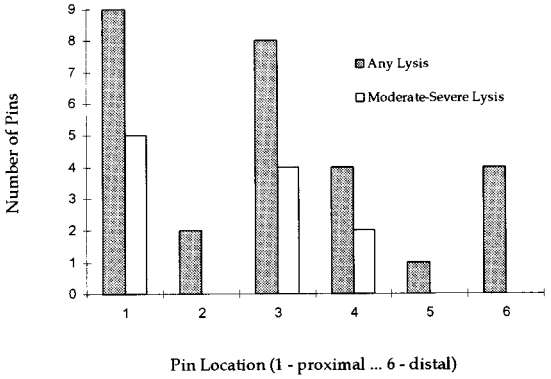

FIGURE 4. Standard radiographic evidence of pin-tract lysis for each pin location.

was found in *in vitro* and *ex vivo* signals. A typical *in vivo* inertance spectrum is shown in Fig. 2. The frequency at which the real component was zero and corresponded to the largest imaginary peak was considered the dominate undamped frequency of the system and was the frequency used for further analysis ($n=72$). The frequency used for analysis in Fig. 2 was 5900 Hz.

A temporal decrease in frequency was measured in 71% of the pins where radiographic evidence of pin-tract lysis was detected (20 out of 28). The decrease in frequency began before radiograph scores increased in 13 pins. In six, a decrease in frequency and an increase in radiograph grade began at the same time. In one pin a decrease in frequency occurred after an increase in radiographic grade. Frequencies for a pin that proceeded to gross loosening are shown in Fig. 5. A temporal decrease in frequency was not detected in 29% of the pins (8 out of 28) that showed radiographic evidence of pin-tract lysis. All of the pins in this grouping had week 10 radiograph scores of 2 or 3.

No radiographic evidence of pin-tract lysis was found in 44 pins. A decrease in frequency over time was present in 32% of these pins (14 out of 44), two of

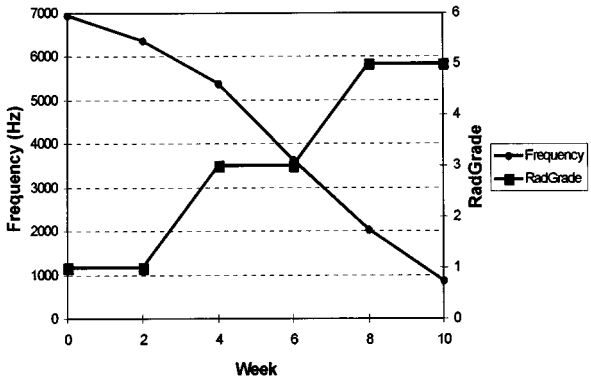

FIGURE 5. Natural frequency and standard radiograph grade versus time for a pin that proceeded to gross loosening.

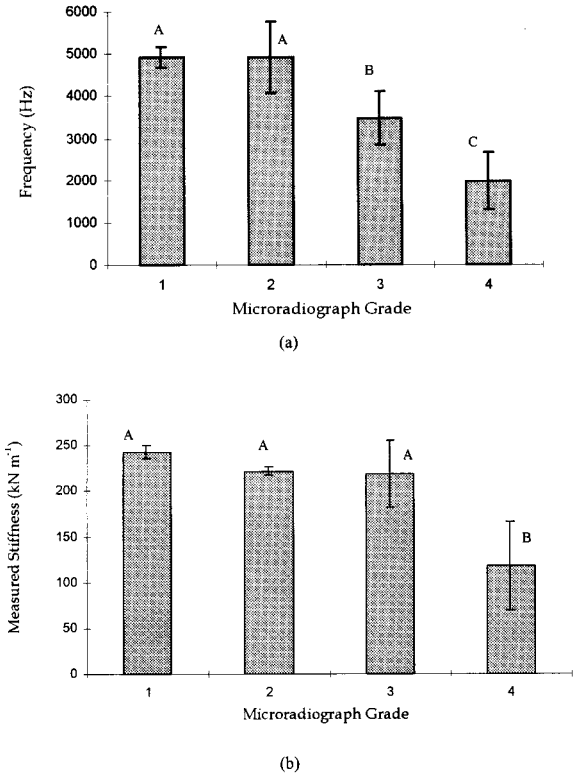

FIGURE 6. Mean natural frequency (a) and mean quasistatic stiffness (b) for pins classified according to microradiograph grade ($n=46$, 5, 7, and 9 for grades 1, 2, 3, and 4, respectively). Error bars indicate 95% confidence intervals. Different letters adjacent to columns indicate significant differences between grades.

which were control pins. Nineteen of the 30 pins for which a temporal decrease in frequency was not detected were control pins.

Ex Vivo Tests

The average frequency was approximately 5000 Hz for pins with microradiograph scores of 1 and 2 and significantly decreased ($p<0.05$) in pins with microradiographic scores of 3 and 4 [Fig. 6(a)]. No significant differences in quasistiffness were detected between pins with microradiograph grades of 1, 2, or 3 [Fig. 6(b)]. The stiffness of the load cell, fixtures, and loading frame of the materials testing machine was 250 kN m^{-1} between 0.30 and 0.70 kg of load. A quasistatic stiffness of 250 kN m^{-1} and slightly greater was measured for 25 pins that were still rigidly implanted into bone indicating that this method was inappropriate for those specimens. To examine the relationship between frequency and quasistatic stiffness, specimens with a measured stiffness greater than 250 kN m^{-1} were excluded from the data set and quasistatic stiffness values were adjusted for machine compliance (Fig. 7). A curve fit, assuming a power model relationship, resulted in an exponent of 0.4.

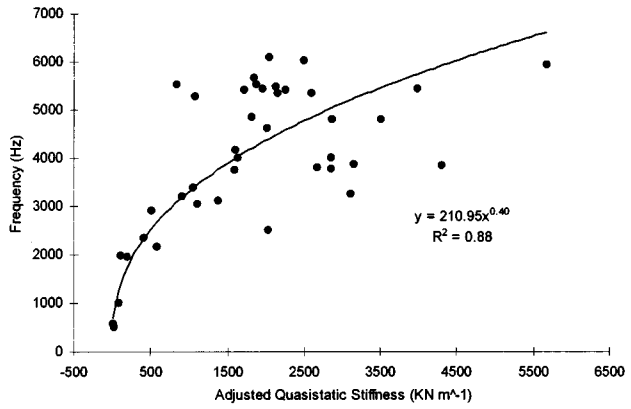

FIGURE 7. *Ex vivo* natural frequency versus *ex vivo* quasistatic stiffness adjusted for machine compliance.

DISCUSSION

The hypothesis, that a driving point mechanical vibration technique could detect bone lysis at the pin–bone interface, was supported by the results. The *in vitro* results have shown that a decrease in axial pin natural frequency correlates well with a decrease in pin-tract quality, which was simulated by varying the tensile modulus of the medium in which a fixation pin was implanted. The natural frequency of the pin–urethane system even responded to small changes at the pin–urethane interface that were induced by changing the clearance hole diameter. Similarly, it was found in the *in vivo* investigation and confirmed by *ex vivo* results that a decrease in bone thread quality results in a decrease in pin–bone natural frequency. *In vitro* results correlate well with those of Kaneko.[11]

A mechanical inertance technique can be used to quantitate changes in bone structure at the pin–bone interface *in vivo* but is not more sensitive than current radiographic analyses. In fact, it is less sensitive in some cases. Frequency did not decrease until after bone lysis was detected by the presence of radiolucency. A slight decrease in bone quality, which may be detected radiographically, may not be enough to decrease pin natural frequency. It is likely that the natural frequency of the pin will not decrease until the interface has broken down over its whole length.

The measurement of quasistatic stiffness was not sensitive enough to detect bone lysis at one cortex or even moderate bone lysis at the near cortex. The representation of *ex vivo* interface stiffness with pin natural frequency was more sensitive to changes in bone structure that occurred *in vivo*. Although the vibrational method is at present not suitable for the diagnosis of small changes in pin-track bone quality *in vivo*, it may be an attractive alternative to low-load quasistatic tests for the nondestructive assessment of the stiffness of the pin–bone interface *ex vivo*. The nondestructive nature of this vibration method enables it to be used on pin–bone specimens that will be subjected to histological examination.

A complicating factor in the analysis of quasistatic stiffness data was the relatively low machine stiffness ($294\ \mathrm{kN\ m^{-1}}$ for *in vitro* tests and $250\ \mathrm{kN\ m^{-1}}$ for *ex vivo* tests). This artifact had the greatest effect on the stiffer *in vitro* and *ex vivo* specimens and is likely to be the cause of quasistatic stiffness insensitivity for those specimens. Quasistatic stiffness tests for some pins with microradiograph grades of 1, 2, and 3 were effectively machine compliance tests. Correcting the data for machine compliance increased standard deviations but did not change statistical results. It should be noted that the stiffness values presented in this paper were measured stiffness values (except for stiffness data presented in Fig. 7) and are not actual stiffness values.

Applying a larger force to the pin during *ex vivo* quasistatic tests may have improved accuracy but would have damaged tissue that was to be evaluated later histologically. Low loads were also used since they are more representative of manual pin manipulation in a clinical setting. An extensometer could have been placed across the interface to eliminate the need for machine compliance correction. However, proper placement and mounting of an extensometer upon each specimen could have subjected the pin–bone interface to further deterioration before histological fixation. These complications with quasistatic testing make dynamic testing a much more attractive alternative.

Compensating for machine compliance and removing stiffness values above $250\ \mathrm{kN\ m^{-1}}$ from further analysis produced the expected relationship between frequency and stiffness (Fig. 7). The power model fit was consistent with the theoretical relationship between natural frequency (w_o) and stiffness (k) for a single degree of freedom mass (m) excited system: $w_o = \sqrt{k/m}$, and the exponent was approximately 0.5.

A potentially complicating factor in the measurement of resonant frequency in both the *in vivo* and *ex vivo* tests was the application of a 0.50 kg static load to the end of the pin. The load may have compressed any thin fibrous membrane that had formed between the pin and bone threads. As a result, if the fibrous membrane at the interface was thin, the natural frequency of the pin may not be significantly affected.

The finding of a greater incidence of loose pins in locations 1 and 3 (Fig. 4) is most likely explained by the greater amount of muscle and soft tissue located around the proximal tibia. Fixation pins that penetrate muscle have an increased probability of becoming loose.[3,9]

Our overall conclusion is that a change in pin stability may not be detected by axial vibration until the loosening process has progressed considerably. Li *et al.*[12] reported similar results when they investigated the loosen-

ing of an artificial hip femoral component using vibrational methods similar to those used in this study. Using an *in vitro* model that simulated the loosening of a femoral component, they found that late loosening can be reliably detected by a vibrational analysis, but this method has poor diagnostic sensitivity in early loosening when there is no obvious prosthetic instability. Data reported by Cornelissen *et al.*[5] also shows little change in transverse resonant frequencies of a fixation pin with large interface stiffness. A figure published in their paper shows mean torque at time of retrieval (MTR) versus an effective length ratio (ELR) measured by vibration of the fixation pin. There was little change in the ELR for MTR values above 20 N cm. The ultrasonic method described by Dickens *et al.*[6] seems to be the most sensitive vibrational method for measuring differences in interface parameters for fixation pins that show no obvious signs of instability. However, as the authors pointed out, the ultrasonic method described in their paper is difficult to apply in a clinical setting.

In summary, the vibrational technique used to monitor pin-track lysis in this investigation was not as sensitive as radiographs *in vivo* but was more sensitive than quasistatic tests *ex vivo*. It is recommended that axial pin natural frequency be measured to quantify the condition of the pin–bone interface *ex vivo* for specimens that could be used later for histological examination, instead of subjecting specimens to potentially destructive mechanical testing. This will limit the number of animals required for fixation pin investigations, allow quantification of the integrity of the pin–bone interface, and preserve the potential for histological evaluation.

ACKNOWLEDGMENT

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APPENDIX

The driving point inertance technique used throughout this investigation consisted of applying a sinusoidal forcing function to a system, measuring the force applied to the system, measuring the acceleration of the system as a result of that force, and taking the ratio of the two [Eq. (A1)]. This ratio is known as the mechanical inertance and is dependent upon the frequency of the driving force (w) and the mechanical properties of the system:

$$\text{Inertance} = \frac{\text{output}}{\text{input}} = \frac{\text{Acceleration}}{\text{Force}}\ (1/\text{kg}). \qquad \text{(A1)}$$

Each mode of vibration can be analyzed as a single degree of freedom oscillator. The governing differential equation of motion for a forced single degree of freedom system is

$$F(t) = M\ddot{x}(t) + c\dot{x}(t) + kx(t), \qquad \text{(A2)}$$

where $F(t)$=forcing function (N), t=time (s), M=mass (kg), $x(t)$=displacement (m), c=damping coefficient (kg/s), and k=stiffness (N/m).

A resonance occurs when very little input (force) produces large output (acceleration) with respect to input and output values for nonresonant frequencies. Usually, resonance occurs near the undamped natural frequency. If the undamped natural frequency of this system [Eq. (A2)] is to be found experimentally, force and acceleration data must be gathered for each forcing frequency in a range of frequencies that includes the resonant frequency of the system. Therefore, the frequency of the forcing function must be variable in order to ''scan'' for a resonance.

If the forcing function is sinusoidally changing with time, then

$$F(t) = F_0 \cos(wt + \theta), \qquad \text{(A3)}$$

where F_0=force amplitude (N), θ=phase angle (rad), and w=forcing frequency (rad/s). Using Euler's formula and allowing the forcing function to vary with discrete frequencies, Eq. (A3) can be written as

$$F_n(t) = F_n \cos(w_n t + \theta) = F_n e^{i(w_n t + \theta)}, \qquad \text{(A4)}$$

where the subscript n denotes discrete frequencies of excitation.

Equation (A4) is applied experimentally by exciting a system at discrete frequencies within a frequency range. To achieve this, binary noise was used to drive the shaker. Binary noise can be thought of as a signal made up of many discrete sinusoid waves of specific frequencies. If the relationship between force and displacement (k) for the vibrating system is linear and if damping (c) is viscous, then its frequency response to binary noise will be the same as its response to a variable frequency forcing function (principle of superposition).

In the steady-state response to a sinusoidal input, $F_n(t)$, the motion of the mass will be sinusoidal with a frequency equal to that of the excitation. The displacement, velocity, and acceleration of the mass take the following forms:

$$x(t) = X_n e^{i(w_n t + \theta)}, \qquad \text{(A5)}$$

$$\dot{x}(t) = i w_n X_n e^{i(w_n t + \theta)}, \qquad \text{(A6)}$$

$$\ddot{x}(t) = -w_n^2 X_n e^{i(w_n t + \theta)}, \tag{A7}$$

where X_n=displacement amplitude (m). Substituting Eqs. (A4)–(A7) into Eq. (A2) yields

$$F_n e^{i(w_n t + \theta)} = (-Mw_n^2 + icw_n + k) X_n e^{i(w_n t + \theta)}. \tag{A8}$$

The inertance frequency response function, $I(w_n)$, can be found by substituting Eqs. (A7) and (A8) into Eq. (A1):

$$I(w_n) = \frac{\ddot{x}(t)}{F(t)} = \frac{-w_n^2(k - Mw_n^2 - icw_n)}{(k - Mw_n^2)^2 + c^2 w_n^2}. \tag{A9}$$

The resonant frequency can be found by examining the inertance signal in the frequency domain and locating the frequency of maximum inertance.

The undamped natural frequency (w_o) is near system resonance for light damping and can be pinpointed using the following method. First, the inertance is decomposed into real and imaginary components:

$$\text{REAL}(I(w_n)) = \frac{-w_n^2(k - Mw_n^2)}{(k - Mw_n^2)^2 + c^2 w_n^2}, \tag{A10}$$

$$\text{IM}(I(w_n)) = \frac{w_n^3 c}{(k - Mw_n^2)^2 + c^2 w_n^2}. \tag{A11}$$

Second, the undamped natural frequency is determined by setting the real component of inertance equal to zero:

$$k - Mw_0^2 = 0, \tag{A12}$$

$$w_0 = \sqrt{\frac{k}{M}}. \tag{A13}$$

The maximum of the imaginary component [Eq. (A11)] is very close to w_o for light damping.

In summary, the undamped natural frequency is the frequency where the real component of inertance is zero and near the maximum value of the imaginary component. This relation can be used to find an undamped natural frequency experimentally from a display showing inertance as a function of frequency (Fig. 2). Finally, if the mass (M) of the system is known then the stiffness (k) can be computed directly from Eq. (A13). If the mass (i.e., the mass of an external fixation pin) is constant then natural frequency is proportional to the square root of stiffness.

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