TECHNICAL ARTICLE

Assessment of the Displacement Field Along a Surface Crack in a Flat Plate Using Optical Techniques

J. Ribeiro[1], M. Vaz[2], H. Lopes[1], F.Q. de Melo[3], and J. Monteiro[4]

1 ESTG—Instituto Politécnico de Bragança, Campus de Sta Apolónia, Apt 1134, 5301-857 Bragança, Portugal
2 DEMec—Faculdade de Engenharia da Universidade do Porto, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal
3 UA—Universidade de Aveiro, Campo Universitário de Santiago, 3810-193 Aveiro, Portugal
4 Laboratory of Optics and Experimental Mechanics, Instituto de Engenharia Mecânica e Gestão Industrial, Rua do Barroco, 174, 4465-591, Portugal

Keywords
Digital Image Correlation, Fracture Mechanics, Interferometry, Moiré, Optical Methods, Speckle Interferometry

Correspondence
J. Ribeiro
ESTG—Instituto Politécnico de Bragança,
Campus de Sta Apolónia,
Apt. 134,
5301-857 Bragança, Portugal
Email: jribeiro@ipb.pt

Received: May 22, 2012; accepted: August 1, 2012

doi:10.1111/j.1747-1567.2012.00856.x

Abstract

Structural components may contain cracks, and for these components a crack tip represents a highly singular stress. The evaluation of the associated strain gradient is difficult to achieve with experimental discrete methods. An efficient alternative are the optical techniques which are non-contact, and so delivering continuous information of the displacement fields and associated derivatives for strain evaluation. This paper describes some experimental methods to fully characterize the displacement field near a crack tip existing in flat plates. Three optical field techniques based on image analysis were used in the present work; respectively, Electronic Speckle Pattern Interferometry, Moiré Interferometry, and Digital Image Correlation. These methods allow different resolutions which can be adjusted according to the expected strain gradient. While the first method depends on the laser wavelength and position of the illumination sources, the second depends on the grating pitch and the last on the surface texture or painted speckle. Algorithms to derivate and filtering the displacement field are developed to compute the strain field and will be used for further works.

Introduction

Many structural failures could be associated with the fracture of the materials used in the fabrication of that structural component. Such events occur as consequence of structural defects as cracks, where a high stress concentration is generated. The detection of such structural singularities is of primary importance, being essential to focus the attention on fracture mechanics parameters characterizing the eventually catastrophic consequences in case of components failure when designing modern structures. The study of fracture condition of structural components with cracks has been performed since last century[1–3] on using analytical approaches[4,5,6,7] combined with different experimental techniques[8,9,10] and numerical simulations.[11] The analytical approaches are based on assumptions which are simplifications of the real aspect of the problem; still reliable, but only for application on simple cases. The experimental techniques can effectively assess the displacement or strain fields close to a crack line and this can be achieved in discrete and continuous measurements. The discrete measurements are usually performed with electrical strain gages,[12,13] while continuous measurements are possible with optical techniques like Moiré Interferometry (MI),[14,15] Electronic Speckle Pattern Interferometry (ESPI),[16] and Digital Image Correlation (DIC).[17,18] The electrical strain gages measure a strain average correspondent of the sensor area. On the other hand, the previous referred optical methods (ESPI, Moiré, and DIC) are field techniques that allow measurements of in-plane and out-plane displacements without contact and with a high resolution. The assessment to the displacement field using holographic interferometric techniques has been developed since the 1970s[19] until the beginning of 1990s.[20,21] In these first works, the determination of the stress intensity factor (SIF) with experimental methods was achieved on measuring displacements in the area of crack apex using the holographic interferometry to record holograms by the scheme of opposing beams. With the advances in the design of digital cameras, another interferometric method started to be used, the ESPI.[22] This technique uses the surface roughness which a laser illumination to generate a speckle pattern to measure the in-plane and out-of-plane displacement field. The ESPI has been used to evaluate K_I, J integral, and other fracture mechanics parameters for different kinds of specimens.[16,23,24] All these techniques use light to extract object surface information prior and after application of the load system to enhance displacements or strain fields. ESPI and MI need coherent illumination to generate the fringe patterns which characterize the object surface deformation. DIC is an optical technique used frequently in fracture mechanics field.[25,26] This technique is also based on the comparison of two images acquired at different deformation states, one before and other after deformation. In this technique, the object is illuminated by a non-coherent light and the intensity patterns result due to the surface texture. Nowadays, an important tool used to study the influence of the stress field close to a crack in a cracked component is the numerical simulation.[27–29] Using this technique it is possible to predict with good accuracy the crack growth; however, the complex phenomena that occur in the crack tip are very difficult to simulate despite the improvement of numerical algorithms and computer calculation capacity. To overcome these limitations, hybrid methods are frequently used, allowing that some experimental measurements are inserted in the numerical simulations.[30,31]

Experimental Procedures

In this work, three different optical experimental techniques were used: ESPI, MI, and DIC. ESPI and MI are interferometric techniques and both setups have capability for in-plane displacement measurement; however, the ESPI setup can be adapted to simultaneously measure in-plane and out-of-plane displacements. A commercial system of DIC (ARAMIS®—GOM, Braunschweig, Germany) was also applied in the characterization of in-plane displacements and strains on the surface of a three-point bending test specimen (3PB) based on the setup described by ASTM E399-90 standard.[32]

Electronic speckle pattern interferometry

The applications of this method to fracture mechanics has been carried out in previous work where the SIF of a part-through crack was measured.[16] To achieve this step, the experimental setup was coupled with accurate image processing tools and designed to cancel rigid body motion only obtaining the abovementioned displacements and rotations along the crack line of the plate. The optical setups used to measure in-plane and out-of-plane displacement fields with ESPI are schematically represented in Fig. 1. In the case of in-plane measurements, the object surface is illuminated with two beams with an equal incident angle that defines a sensibility vector parallel to the object surface. The mirror mounted on a piezoelectric transducer is used to implement the phase stepping technique, which in combination with image processing algorithms, allows the calculation of the spatial phase distribution. For the out-of-plane measurements, the laser beam is splitted into two, one being the object beam and used to illuminate the object and the other one the reference beam; this combination generates a sensibility vector normal to the object surface. The light source used was a 2 W laser from COHERENT (Verdi) with a radiation wave length of 514 nm.

The specimen consisted of a polymeric (elastometer) plate which had a part-through cut which was made with the help of a paper cutter. The crack profile was approximately elliptical, having $l_0/h = 0.5$ ($h =$ 6 mm the plate thickness) and $h/a = 0.43$ (where $2a \cong 28$ mm is the crack length) as geometric parameters.

The plate is uniformly stressed by a pair of equal tensile load applied at both free ends of the plate. To materialize this force system, a gravity load was stretched with two identical pieces of string looped around precision sheaves fitted with needle bearings. Figure 2 shows the test specimen inserted in the out-of-plane displacement setup and part of the load system. The loading system was designed to avoid any rotation of the specimen and, in this way, prevent the rigid body motion. This purpose was achieved by posterior analysis of the fringes.

The interferometric fringe pattern was obtained by applying a low tensile load of about 10 N along the horizontal direction.

Moiré interferometry

Another optical method used in the assessment of the displacement field near a singularity is the MI. This technique relies on the superposition of two grids, one is the reference resulting from the interference

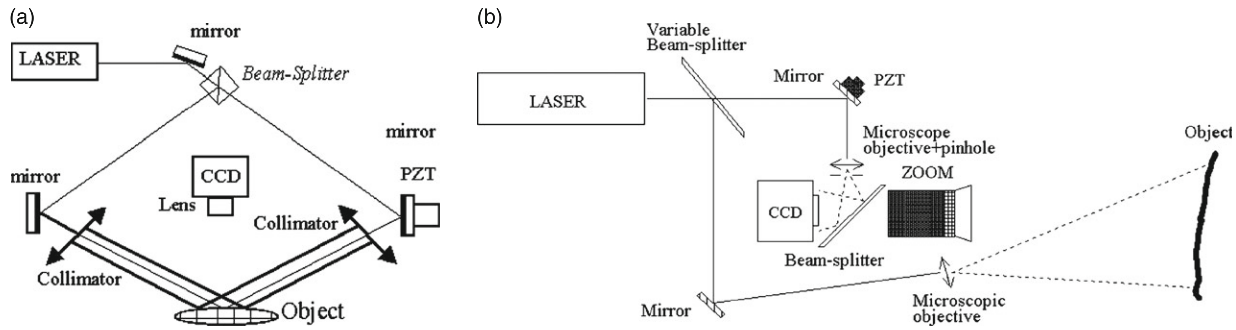

Figure 1 ESPI setup to measure displacement fields. (a) In-plane and (b) out-of-plane.[16]

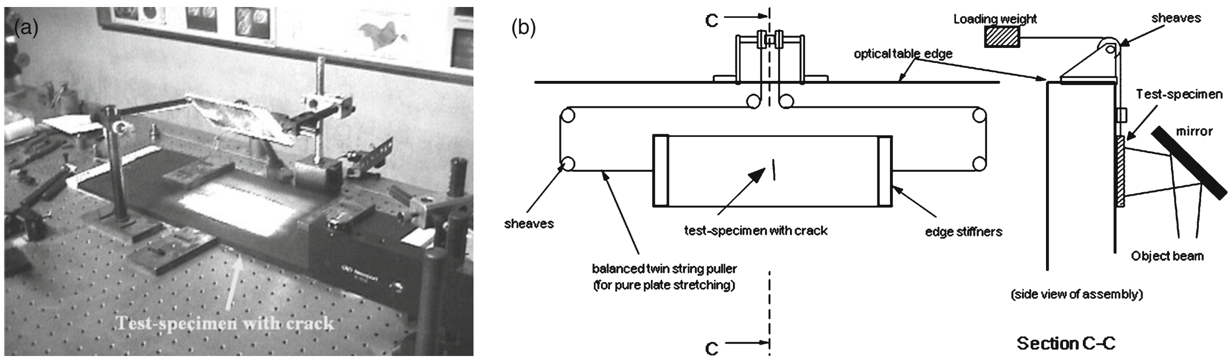

Figure 2 Specimen in an out-of-plane displacement setup (a). Top and side views of the experimental setup directly assembled on an optical table (b).[16]

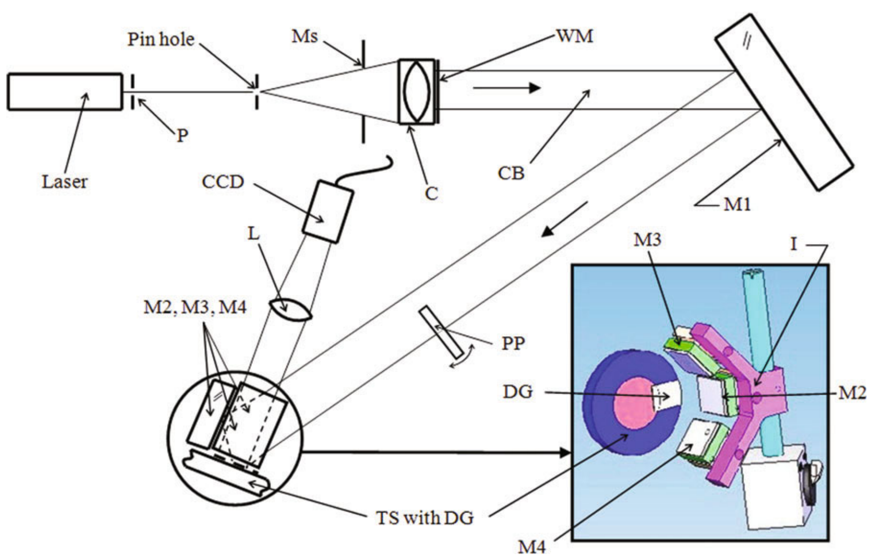

Figure 3 Schematic presentation of the moiré interferometry optical setup used.[34]

of two coherent laser collimated beams, which fall on the surface, and the other follows the deformation of the object surface. High-frequency grids (up to 1200 lines/mm or more) are recorded on the object surface. The resolution depends on the grid pitch and can be selected according to the expected displacement amplitude. This method is restricted to in-plane measurements, yet with accuracy to characterize structural singularities like a through crack.[33]

Image processing algorithms, involving filtering, phase calculation, unwrapping, and spatial differentiation are used as data post-processing tools to

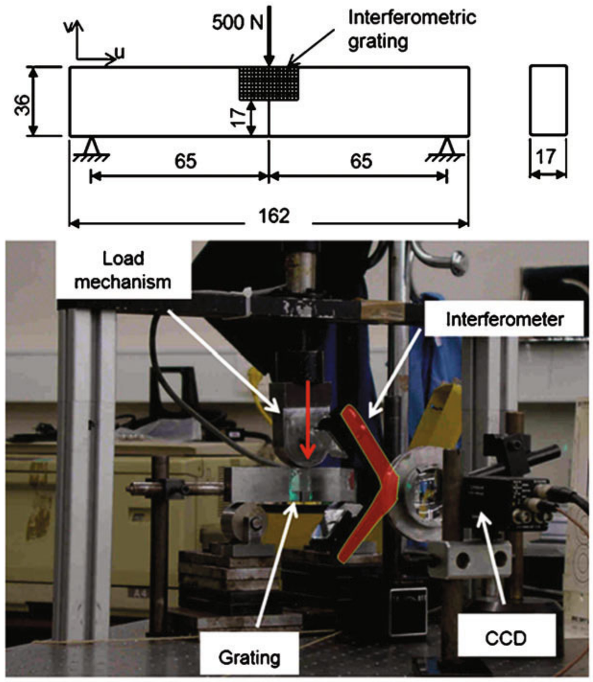

Figure 4 MI setup to obtain the 2D displacement on a 3PB test specimen.

transform surface displacement into stress fields. In Fig. 3 a schematic presentation of the MI optical setup used in this work can be seen. Here, Laser is the laser source, Ms is a mask, C is the collimator, WM is a window mask, CB is the collimated beam, M1, M2, M3, and M4 are plane mirrors, PP is a parallel plate glass, L is a lens, TS is the test specimen, DG is the diffraction grating, and I is the interferometer.

A four-beam optical system to produce displacement fringe patterns in x and y directions was used,[35] where x is parallel to horizontal direction and y to vertical direction. The collimated laser beam is partially obstructed by the window mask. This is constituted by three elements whose function is to illuminate simultaneously the grating and the mirrors: illuminating M2 and grating by opening the middle element allows the measurement of the displacement in the x direction, otherwise illuminating M3 and M4 by opening the upper and lower elements the measurement of displacement is done in the y direction.

The light source used in this setup was a 2 W laser from Coherent (Verdi, Santa Clara, CA). A CCD camera, Sony XC-8500CE, with a resolution of 782 × 582 pixels was used for image acquisition. A computer controlled PI piezoelectric actuator was used to perform the phase modulation needed to calculate the phase maps using the four-interferogram algorithm described by Creath.[36] A high-frequency grating was previously replicated on the specimen surface. The grating used was a 1200 lines/mm, obtained with aluminum vaporization on the top of an epoxy replication of a master grating from Photomechanics and has the dimensions of 19 mm × 26 mm. For this grating frequency, the displacement measurement sensitivity is 2.4 fringes/μm and the displacement resolution is 0.1 μm. The grating replication process described next is due to Post.[37] Also, the setup proposed by Post[38] was used in the virtual grating generation by laser interferometry. The reference images were recorded after superposition of the virtual grating over the object replication grating. A temporal phase modulation was used to increase the resolution of the displacement fields. A tiltable parallel glass plate was used to promote phase modulation to perform a four-image phase calculation algorithm.[36]

As seen in Fig. 4, the MI was used to characterize the displacements on the surface of a 3PB steel (P20) specimen. The measurements were performed according with the parameters defined in ASTM E399-90 standard.[32]

The first part of experimental test was to control the specimen grating and mirror positions to obtain a null field at room temperature. After the charge application of 500 N, four-phase maps are recorded for each orthogonal direction with phase shift step of $\pi/2$. The phase steps are created by tilting the PP. The obtained phase maps were used to measure the displacement field near the crack tip.

Digital image correlation

This new optical technique is being spread recently in the scientific community. Essentially, in the DIC the object is illuminated by a non-coherent light and the intensity patterns depend on the surface texture. These intensity patterns, which should have a random distribution, will be divided into smaller areas. Each subdivision of the picture is initially recorded and compared by correlation with the obtained images of different deformation states of the object. The determination of displacement and strain fields are obtained by the correlation between the random pattern of initial image (the reference one) and the final one (assigned to the deformed state).[39] The texture can be natural or prepared with special painting and influences the accuracy of the measurement. In this particular technique the software plays an important role as the displacements are calculated from image details. To reach a higher accuracy, correlations are based on squares of pixels, known as facets, rather than individual pixel tracing. The facets have an array of grayscale values which

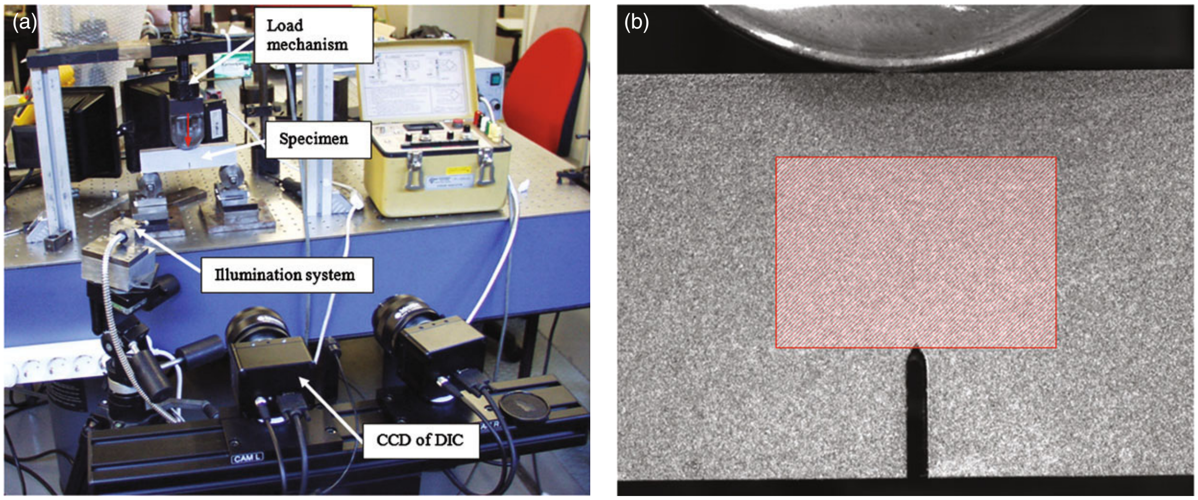

Figure 5 DIC setup to obtain the 2D displacement on a 3PB test specimen (a) and the measurement region on the specimen (b).

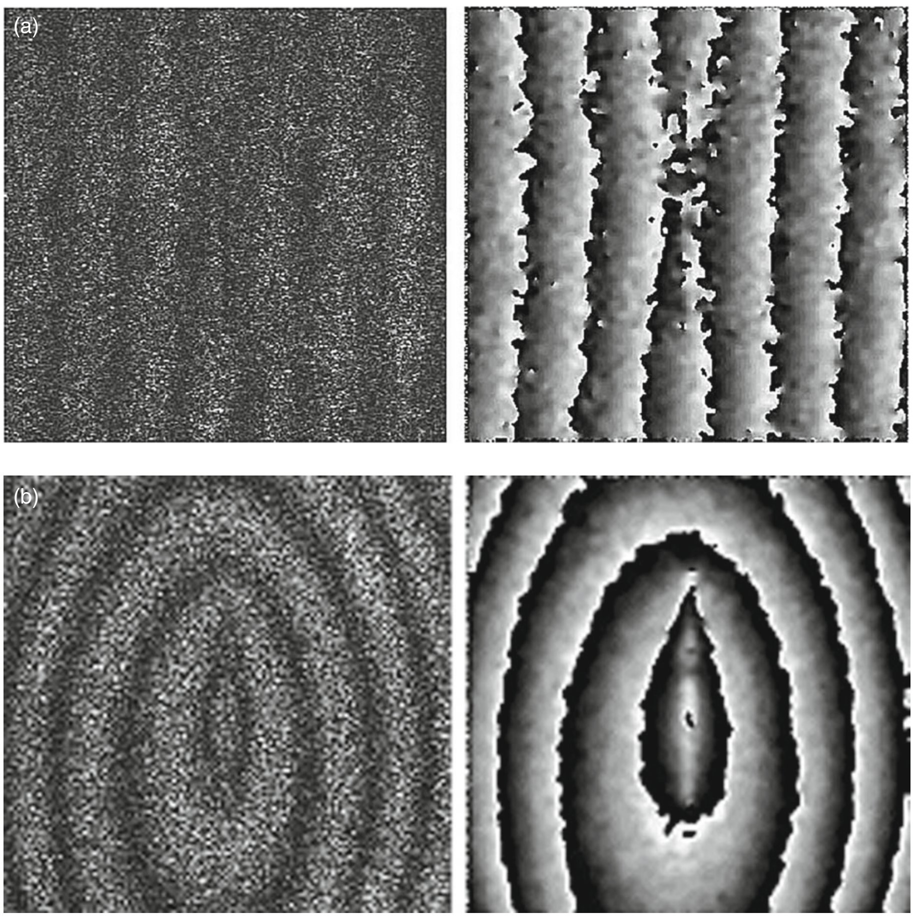

Figure 6 Fringe pattern and phase map obtained for in-plane (a) and out-of-plane (b) displacement measured with ESPI.[11]

correspond to the pattern texture, therefore allowing tracking through the subsequent image stages. This method could be used to measure 2D and 3D strain and displacement fields. Recently there are available integrated systems offering easy operation with reliable results.

Before measuring the displacement field, a surface preparation of the test specimen was carried out.

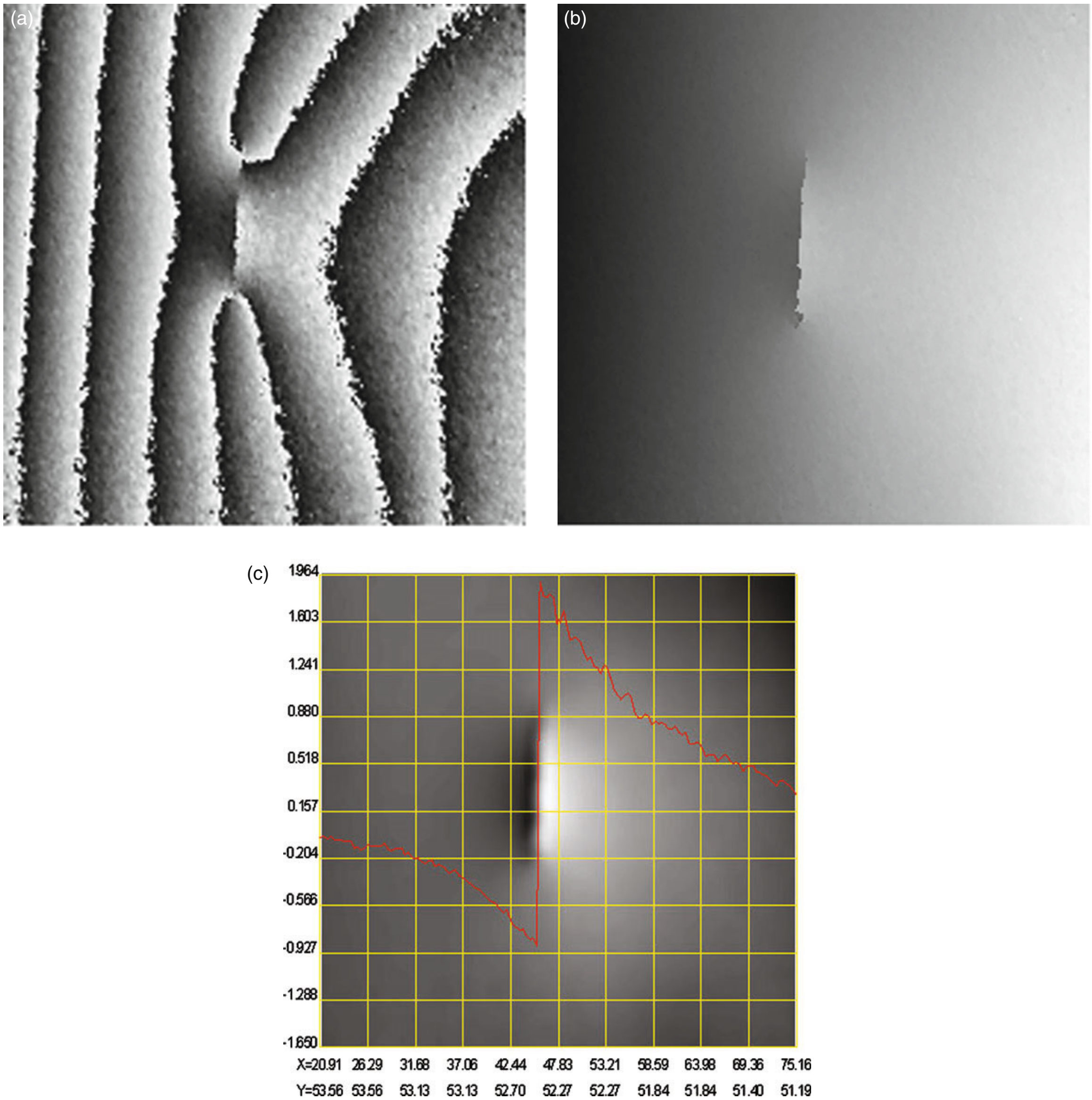

Figure 7 In-plane displacement: (a) phase map; (b) displacement field (unwrapping phase map), and (c) SIF.

For that, the surface to be examined was sprayed with a thin uniform coat of matte white paint for the background and a spray of matte black paint, so materializing a random pattern of black dots.

For this experimental work, the DIC was conducted using a CCD camera integrated in the GOM package ARAMIS 6.0.2. The CCD sensor has a resolution of 1280 × 1024 pixels which allowed the detection of facets with small physical size and consequently a good resolution and accuracy.

Similar to the previous experimental measurement with MI, the DIC was used to characterize the displacements on the surface of the same 3PB specimen. The measurements were performed according to the parameters defined in ASTM E399-90 standard.[32] The setup can be seen in Fig. 5, as well as the region of measurement in interest. For this study, the setup was prepared to measure in-plane displacements; hence only needing one camera placed in front of specimen surface to capture the reference and deformed images.

Before capturing and saving the reference image, it is essential to calibrate the setup. The first task consisted in defining two parallel lines at a precise distance. The software allows the selection of two points to define their distance; this value will be used as a scale for the next computations of displacement field.

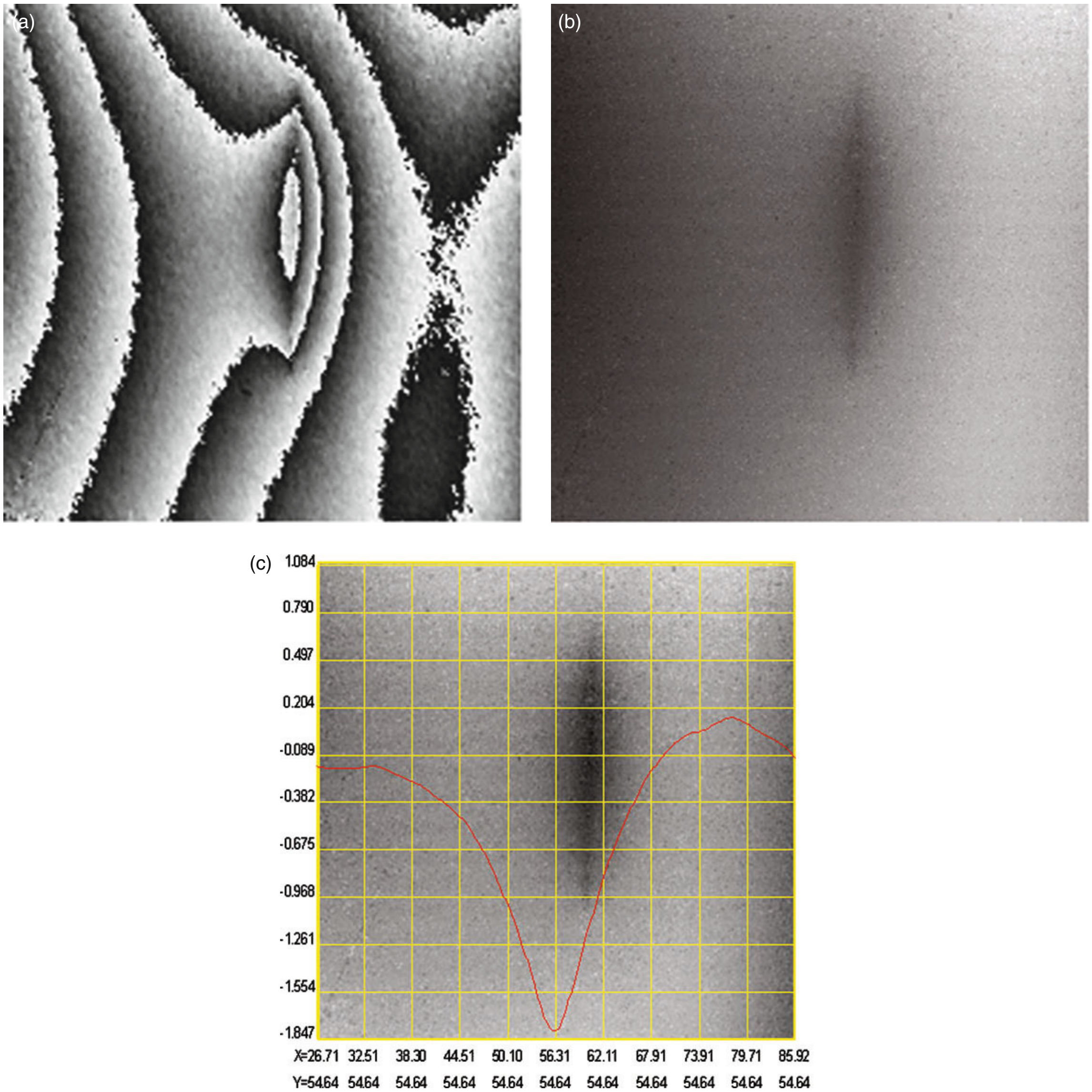

Figure 8 Out-of-plane displacement: (a) phase map; (b) displacement field (unwrapping phase map), and (c) SIF.

The first captured image is taken prior changing any test specimen state, so being the reference image saved in the computer hard disc. A second image is then captured after loading the test specimen with a 500 N force; also saving the image corresponding to this deformed state. The correlation algorithm developed for Aramis system allows the determination of displacement field on the region close to the crack tip.

Analysis of Experimental Results

Electronic speckle pattern interferometry

The illumination ESPI setup was the tool to measure the in-plane and out-of-plane displacements. This operation resulted of the interferometry pattern represented in Fig. 6(a and b), respectively. The fringe pattern corresponds to the phase map obtained by image processing after phase calculation with a phase shift technique. These phase maps have discontinuities in $-\pi$ and π. To overcome this limitation, an algorithm is used to unwrap the phase making possible to obtain a continuous displacement field.

Once the displacement is calculated, a hybrid methodology allows the determination of compliance factors for each slice element of the line spring model (side edge cracked elements) placed along the crack

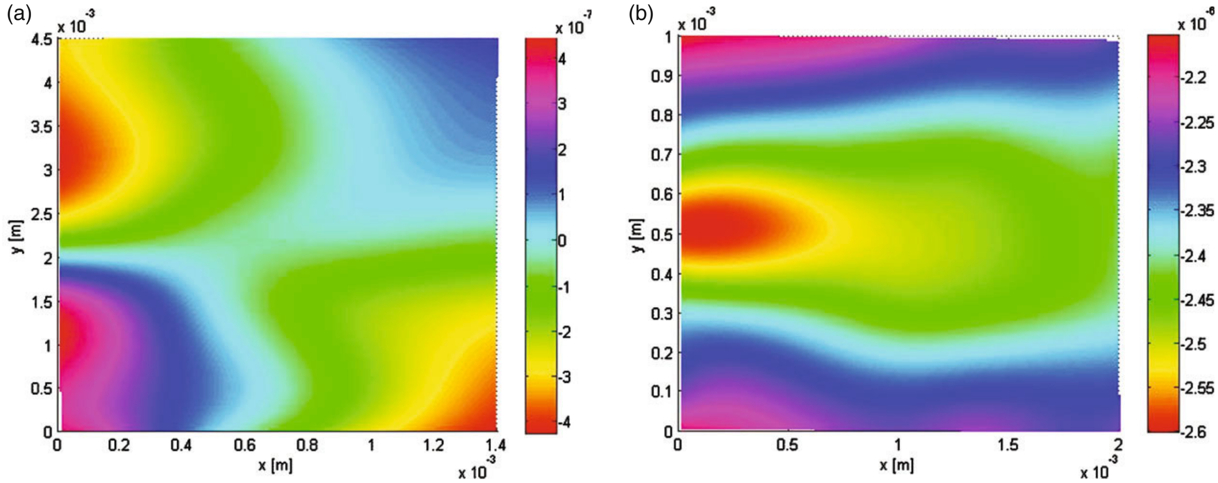

Figure 9 Displacement field, in *u* direction (a) and *v* direction (b), obtained with Moiré interferometry. Displacement units in meter.

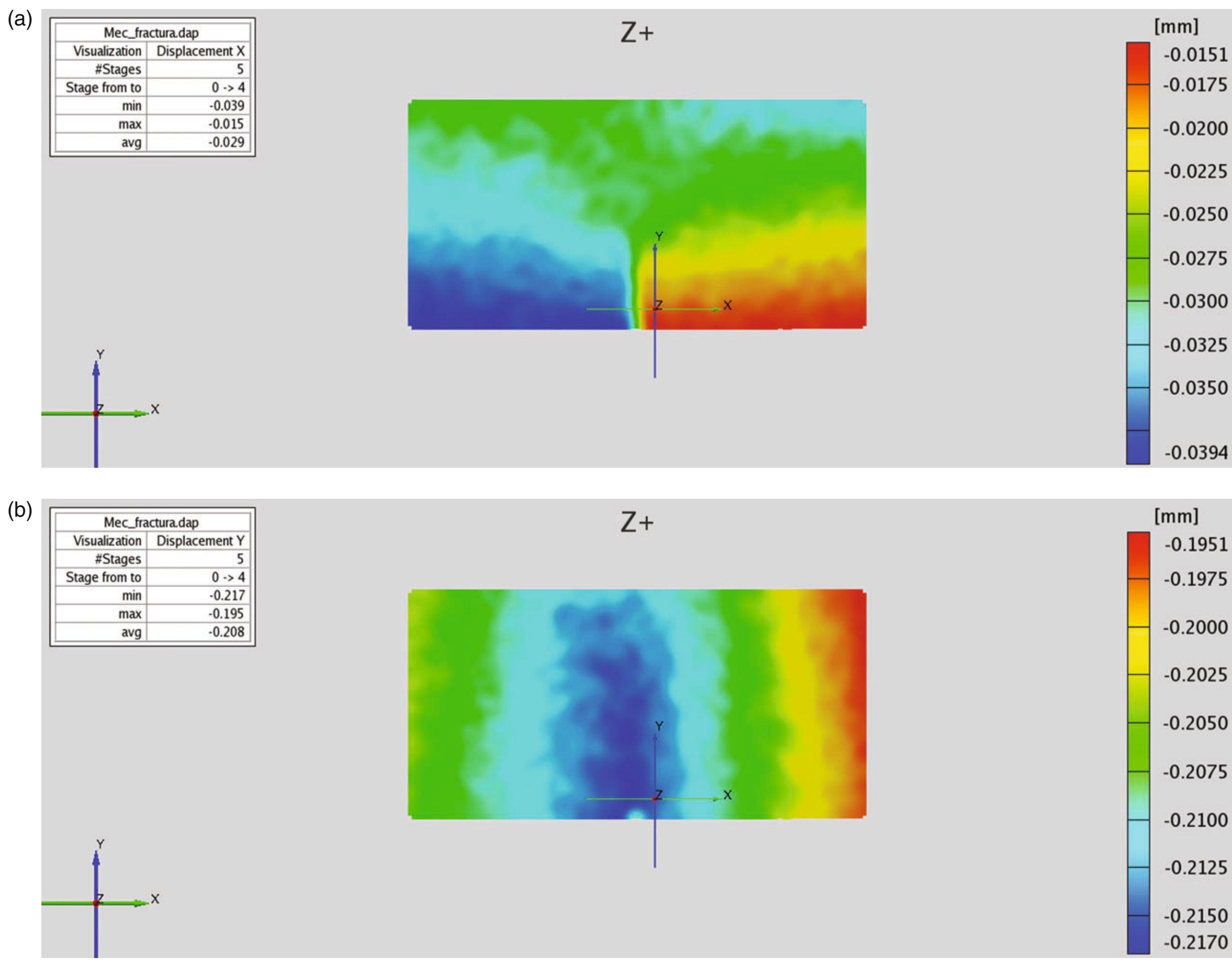

Figure 10 Displacement fields obtained with DIC in *x* direction (a) and in *y* direction (b).

plane. This allows the evaluation of the consequent stress field and further determination of the SIF.

The problem of a part-through crack is approached with precision using 3D Finite Element Techniques. However, this simulation is expensive, once it means a heavy computation effort near the singularity due to the highly refined necessary for precision of results. To overcome this drawback a line spring model was proposed by Rice and Levy.[40] The model consisted in replacing the residual ligament along the part trough crack surface by a set of side edge cracked plates were each crack depth reconstruct the real crack profile. Then the structural behavior of each side edge cracked plate of the model acts like a spring when an extensional displacement and crack edge rotations are coupled. Using optical techniques like ESPI it is possible to record with high precision the relative distortion of the conjugate crack surfaces along its length. After obtaining the mentioned displacements and rotations they are input in the line spring equations to compute the SIF as detailed in a previous work.[16]

The main steps to obtain the SIF using in-plane and out-of-plane displacements are shown in brief in Figs. 7 and 8.

Moiré interferometry

The displacement fields are obtained by filtering and unwrapping the phase maps. The displacement fields for u and v directions are shown in Fig. 9.

In spite of being an accurate tool for the experimental measurement of displacements, its application field is restricted to in-plane measurements which leave the line spring method incomplete for the edge crack rotations. However, the MI can be used to adjust a numerical solution and complement it with the surface rotation obtained with the FEM (Finite Element Method). Still the MI method is fully applicable to fracture mechanics problems dealing with through cracks or side edge crack. Also in this technique, image processing techniques could be involved to obtain the continuous field displacement with temporal phase shift.

Digital image correlation

The results of displacement measurements are shown in Fig. 10.

The obtained images had an important level of noise, so they were smoothed using a median filter.

Comparing the results of the measurements between MI and DIC, it is possible to verify that they are very similar in the behavior of the displacement fields. Even the values are nearest.

Conclusions

The procedures described appear as an interesting step in the progress of hybrid processes in the evaluation of the SIF in cracked plates. ESPI technique allows both in-plane and out-of-plane measurements; however, it is complex, expensive, and easily decorrelates; MI is less sensitive to rigid body motion and accurate enough for this application being the grid recording a difficult and long-standing task. Finally, DIC appears to be the less expensive technique, it also allows in-plane and out-of-plane measurements and the direct obtainment of strain fields, but is very demanding in the computation effort and for very high resolutions it is difficult to create the speckle small enough.

Another approach to solve these problems is to use the numerical simulation. This is a very powerful tool; however, to create an accurate model the input of experimental data is necessary. ESPI, MI, and DIC already prove to be able to deliver the necessary experimental data for the numerical simulation. Some results have been already obtained using DIC by Trummer et al.[41]

All the methods referred in this work can be applied to experimental fracture mechanics with good results not only in SIF assessment, but also in the adjustment of numerical models for the same goal.

References

1. Airy, G.B., *On the Strains in the Interior of Beams*, British Association Report, Cambridge (1862).
2. Airy, G.B., ''On the Strains in the Interior of Beams,'' *Philosophical Transactions of the Royal Society* **153**:49–80 (1863).
3. Inglis, C., ''Stresses in a Plate due to the Presence of Cracks and Sharp Corners,'' *Transactions of the Institute of Naval Architects* **55**:219–241 (1913).
4. Irwin, G.R., ''Analysis of Stresses and Strains Near the End of a Crack Traversing a Plate,'' *Journal of Applied Mechanics* **24**:361–364 (1957).
5. Muskhelishvili, N.I., *Some Basic Problems of the Mathematical Theory of Elasticity*, 3rd Edition, P. Noordhoff, Groningen (1953).
6. Williams, M.L., ''Stress Singularities Resulting from Various Boundary Conditions in Angular Corners of Plates in Extension,'' *Journal of Applied Mechanics* **19**:526–528 (1952).
7. Janssen, M., Zuidema, J., and Wanhill, R., *Fracture Mechanics*, 2nd Edition, Spon Press, New York (2004).
8. Parnas, L., Bilir, Ö., and Tezcan, E., ''Strain Gage Methods for Measurement of Opening Mode Stress Intensity Factor,'' *Engineering Fracture Mechanics*, **55**(3):485–492 (1996).

9. Moore, A.J., and Tyrer, J.R., ''Phase-Stepped ESPI and Moiré Interferometry for Measuring Stress-Intensity Factor and *J* Integral,'' *Experimental Mechanics* **35**(4):306–314 (1995).
10. Yoneyamaa, S., Ogawab, T., and Kobayashib, Y., ''Evaluating Mixed-Mode Stress Intensity Factors from Full-Field Displacement Fields Obtained by Optical Methods,'' *Engineering Fracture Mechanics* **74**(9):1399–1412 (2007).
11. Belytschko, T., Organ, D., and Krongauz, Y., ''A coupled finite element-element-free Galerkin method,'' *Computational Mechanics* **17**(3):186–195 (1995).
12. Kuang, J.H., and Chen, L.S., ''A Single Strain Gage Method for K_I Measurement,'' *Engineering Fracture Mechanics* **51**(5):871–878, 1995.
13. Dally, J.W., and Sanford, R.J., ''Strain-Gage Methods for Measuring the Opening-Mode Stress-Intensity Factor, K_I,'' *Experimental Mechanics* **27**(**4**):381–388 (1987).
14. Epsteina, J.S., Junga, H.Y., and Reuterb, W.G., ''Stress Intensity Factor Extraction Using Moiré Interferometry Based on a Two-Parameter Displacement Eigen Function: Validity Criteria and Comparison with ASTM E-399 K_{IC} Plane Strain Test Methods,'' *Optics and Lasers in Engineering* **13**(2):167–180 (1990).
15. Dhara, S., Clouda, G., and Paleebuta, S., ''Measurement of Three-Dimensional Mode-I Stress Intensity Factor Using Multiple Embedded Grid Moire Technique,'' *Theoretical and Applied Fracture Mechanics* **12**(2):141–147 (1989).
16. Monteiro, J.M., Vaz, M.A.P., Melo, F.Q., and Gomes, J.F.S., ''Use of Interferometric Techniques for Measuring the Displacement Field in the Plane of a Part-Through Crack Existing in a Plate,'' *International Journal of Pressure Vessels and Piping* **78**(4):253–259 (2001).
17. Yoneyama, S., Morimoto, Y., and Takashi, M., ''Automatic Evaluation of Mixed-Mode Stress Intensity Factors Utilizing Digital Image Correlation,'' *Strain* **42**(1):21–29 (2006).
18. Hamam, R., Hild, F., and Roux, S., ''Stress Intensity Factor Gauging by Digital Image Correlation: Application in Cyclic Fatigue,'' *Strain* **43**(3):181–192 (2007).
19. Dudderar, T.D., and Gorman, H.J., ''The Determination of Mode I Stress-Intensity Factors by Holographic Interferometry,'' *Experimental Mechanics* **13**(4):145–149 (1973).
20. Ovchinnikov, A.V., Safarov, Y.S., and Garlinskii, R.N., ''Determination of Stress Intensity Factors by a Method of Holographic Interferometry,'' *Materials Science* **19**(2):131–135 (1983).
21. Tyrin, V.P., ''Application of the Holographic Interferometry Method to Determine the Stress Intensity Factor,'' *Journal of Applied Mechanics and Technical Physics* **31**(1):142–145 (1990).
22. Burch, J.M., ''Interferometry with Scattered Light,'' Dickson, J. H., (ed.), *Optical Instruments and Techniques*, Oriel Press, Newcastle-Upon-Tyne, pp. 213–229 (1970).
23. Moore, A.J., and Tyrer, J.R., ''Evaluation of Fracture Mechanic's Parameters Using Electronic Speckle Pattern Interferometry,'' *The Journal of Strain Analysis for Engineering Design* **29**(4):257–262 (1994).
24. Miyata, H., Murakami, A., and Kato, M., ''Application of Laser Speckle Pattern Interferometry for Precise Measurements of Displacement Distribution in Porous Ceramics,'' *Journal of Solid Mechanics and Materials Engineering* **1**(11):1341–1351 (2007).
25. Mogadpalli, G.P., and Parameswaran, V., ''Determination of Stress Intensity Factor for Cracks in Orthotropic Composite Materials using Digital Image Correlation,'' *Strain* **44**(**6**):446–452 (2008).
26. Ju, S.H., ''Calculation of Notch H-Integrals Using Image Correlation Experiments,'' *Experimental Mechanics* **50**(4):517–525 (2010).
27. Sato, K., Saimoto, A., Hashida, T., and Imai, Y., ''Numerical Simulation of Crack Propagation and Coalescence in Randomly Distributed Crack System,'' *Strength, Fracture and Complexity* **1**(4):205–213 (2003).
28. Sekine, H., Yan, B., and Yasuho, T., ''Numerical Simulation Study of Fatigue Crack Growth Behavior of Cracked Aluminum Panels Repaired with a FRP Composite Patch Using Combined BEM/FEM,'' *Engineering Fracture Mechanics* **72**(16):2549–2563 (2005).
29. Sreeramulu, K., Sharma, P., Narasimhan, R., and Mishra, R., ''Numerical Simulations of Crack Tip Fields in Polycrystalline Plastic Solids,'' *Engineering Fracture Mechanics* **77**(8):1253–1274 (2010).
30. Nishioka, T., Murakami, R., and Takemoto, Y., ''The Use of the Dynamic J Integral (J′) in Finite-Element Simulation of Mode I and Mixed-Mode Dynamic Crack Propagation,'', *International Journal of Pressure Vessels and Piping* **44**(3):329–352 (1990).
31. Lei, J., Wang, Y., and Gross, D., ''Two Dimensional Numerical Simulation of Crack Kinking from An Interface Under Dynamic Loading by Time Domain Boundary Element Method,'' *International Journal of Solids and Structures* **44**(3): 996–1012 (2007).
32. ASTM E399-90, ''Standard Test Method for Plain-Strain Fracture of Metallic Materials,'' Annual book of ASTM Standards, 1992.
33. Fellows, L., and Nowell, D., ''Crack Closure Measurements Using Moiré Interferometry with

Photoresist Gratings,'' *International Journal of Fatigue* **26**:1075–1082 (2004).

34. Ribeiro, J., Monteiro, J., Vaz, M., Lopes, H., and Piloto, P., ''Measurement of Residual Stresses with Optical Techniques,'' *Journal of Strain: An International Journal for Experimental Mechanics* **45**: 123–130 (2009).
35. Ribeiro, J., Monteiro, J., Lopes, H., and Vaz, M., ''Moiré Interferometry Assessment of Residual Stress Variation in Depth on a Shot Peened Surface,'' *Journal of Strain: An International Journal for Experimental Mechanics* **47**:542–550 (2011).
36. Creath, K., and Schmit, J., ''N-Point Spatial Phase-Measurement Techniques for Non-Destructive Testing,'' *Optics and Lasers in Engineering* **24**:365–379 (1996).
37. Post, D., ''Moiré Interferometry at VPI & SU,'' *Experimental Mechanics* **23**:203–210 (1983).
38. Post, D., Han, B., and Ifju, P., *High Sensitivity Moiré: Experimental Analysis for Mechanics and Materials*, Springer Verlag, New York (1997).
39. Hu, T., Ranson, W., Sutton, M., and Peters, W., ''Application of Digital Image Correlation Techniques to Experimental Mechanics,'' *Experimental Mechanics* **25**:232–244 (1985).
40. Rice, J.R., and Levy, N., ''The Part-Through Surface Crack in an Elastic Plate,'' *Journal of Applied Mechanics, Transactions of the ASME* **39**:185–194 (1972).
41. Trummer, V.R., Moreira, P.M.G.P., Pastrama, S.D., Vaz, M.A.P., and Castro, P.M.S.T., ''Methodology for In Situ Stress Intensity Factor Determination on Cracked Structures by Digital Image Correlation,'' *International Journal of Structural Integrity* **1**(4):344–357 (2010).