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Improved ratio-type estimators using stratified double–ranked set sampling

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ABSTRACT

In this article we propose improved ratio-type estimators for estimating the finite population mean $(\bar{Y})$ under stratified double-ranked set sampling (*S_tDRSS*) using the auxiliary information. The biases and mean squared errors (*MSE*) of the proposed ratio-type estimators are derived up to first order of approximation. The proposed estimators are compared with some competitor estimators both theoretically and numerically. It is identified through numerical and simulation studies that the proposed ratio-type estimators based on *S_tDRSS* are more efficient than the corresponding estimators in stratified ranked set sampling (*S_tRSS*) given by Mandowara and Mehta.

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1. Introduction

Ranked set sampling (*RSS*) technique was first introduced by Mclntyre (1952), and stratified ranked set sampling (*S_tRSS*) was suggested by Samawi and Muttlak (1996) to obtain a more efficient estimator for a population mean. Samawi (1996) proposed an efficient estimator in stratified ranked set sampling. Using *S_tRSS*, the performances of the combined and separate ratio estimates were obtained by Samawi and Siam (2003). Al-Saleh and Al-Kaddiri (2000) introduced the concept of double-ranked set sampling (*DRSS*) and showed that the *DRSS* estimator is more efficient than the usual *RSS* estimator in estimating the finite population mean. Al-Omari (2012) suggested ratio estimation of the population mean using auxiliary information in simple random sampling (*SRS*) and median ranked set sampling (*MRSS*). Al-Omari and Khalifa (2010) developed an estimator for the population mean using double extreme ranked set sampling (*DERSS*). Syam et al. (2014) introduced the concept of stratified double quartile ranked set sampling. Khan and Shabbir (2016a) suggested a class of Hartely–Ross type unbiased estimators in *RSS*. Khan and Shabbir (2016b) also proposed Hartely–Ross type estimators in RSS and (*S_tRSS*). Khan et al. (2016) proposed unbiased ratio estimators of the finite population mean in *S_tRSS*. Following Kadilar and Cingi (2003), Mandowara and Mehta (2014) used the idea of *S_tRSS* instead of stratified simple random sampling (*S_tSRS*) and obtained more efficient ratio-type estimators. In this article we use the idea of (*S_tDRSS*) instead of *S_tRSS* to improve the precision of ratio-type estimators given by Mandowara and Mehta (2014).

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2. Stratified ranked set sampling

In stratified ranked set sampling, for the hth stratum of the population, first choose m_h independent random samples each of size m_h (h = 1,2,. . .,L). Rank the observations in each sample and use the RSS procedure to get L independent RSS samples each of size m_h, to get $m_1 + m_2 + \ldots + m_L = m$ observations. This completes one cycle of S_tRSS. The whole process is repeated r times to get the desired sample size $n = mr$.

To obtain bias and MSE of the estimators, we define:

$$\bar{y}_{[S_tRSS]} = \bar{Y}(1 + e_0), \quad \bar{x}_{(S_tRSS)} = \bar{X}(1 + e_1)$$

such that

$$E(e_i) = 0, \quad (i = 0, 1), \quad E(e_0^2) = \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(C_{yh}^2 - \frac{1}{m_h}\sum_{i=1}^{m_h} W_{yh[i:mh]}^2\right),$$

$$E(e_1^2) = \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(C_{xh}^2 - \frac{1}{m_h}\sum_{i=1}^{m_h} W_{xh(i:mh)}^2\right),$$

$$E(e_0 e_1) = \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(\rho_{yxh} C_{yh} C_{xh} - \frac{1}{m_h}\sum_{i=1}^{m_h} W_{yh[i:mh]} W_{xh(i:mh)}\right),$$

where

$$W_{yh[i:mh]} = \frac{\tau_{yh[i:mh]}}{\bar{Y}}, \qquad W_{xh(i:mh)} = \frac{\tau_{xh(i:mh)}}{\bar{X}},$$

$$\tau_{xh(i:mh)} = (\mu_{xh(i:mh)} - \bar{X}_h), \quad \tau_{yh[i:mh]} = (\mu_{yh([i:mh]} - \bar{Y}_h).$$

Here C_{xh} and C_{yh} are the coefficients of variations of X and Y, respectively; $\mu_{yh[i:mh]} = E[y_{h[i:mh]}]$ and $\mu_{xh(i:mh)} = E[x_{h(i:mh)}]$; $\bar{Y}$ and $\bar{X}$ are the population means; $\bar{Y}_h$ and $\bar{X}_h$ are the population means of the hth stratum of the variables Y and X, respectively; and ρ_{yxh} is the population correlation coefficient in the hth stratum.

Using S_tRSS, the combined ratio estimator of population mean $(\bar{Y})$ given by Samawi and Siam (2003) is defined as

$$\bar{y}_{R(S_tRSS)SS} = \bar{y}_{[S_tRSS]}\left(\frac{\bar{X}}{\bar{x}_{(S_tRSS)}}\right), \tag{1}$$

where $\bar{y}_{[S_tRSS]} = \sum_{h=1}^{L} W_h \bar{y}_{h[RSS]}$ and $\bar{x}_{(S_tRSS)} = \sum_{h=1}^{L} W_h \bar{x}_{h(RSS)}$ are the unbiased estimators of population means $\bar{Y}$ and $\bar{X}$, respectively; $W_h = \frac{N_h}{N}$ is the known stratum weight; N_h is the hth stratum size; N is the total population size; and L is the total number of strata $(h = 1, 2, \ldots, L)$.

The bias and MSE of $\bar{y}_{R(S_tRSS)SS}$, up to the first order of approximation are respectively given by

$$Bias(\bar{y}_{R(S_tRSS)SS}) \cong \bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(C_{xh}^2 - \rho_{yxh}C_{xh}C_{yh}\right) - \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\left(\sum_{i=1}^{mh}W_{xh(i:mh)}^2 - \sum_{i=1}^{mh}W_{xh(i:mh)}W_{yh[i:mh]}\right)\right] \tag{2}$$

and

$$MSE(\bar{y}_{R(S_tRSS)SS}) \cong \bar{Y}^2\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(C_{yh}^2 + C_{xh}^2 - 2\rho_{yxh}C_{xh}C_{yh}\right) - \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\sum_{i=1}^{mh}\left(W_{yh[i:mh]} - W_{xh(i:mh)}\right)^2\right]. \tag{3}$$

Following Sisodia and Dwivedi (1981), Mandowara and Mehta (2014) suggested a modified ratio-type estimator for population mean $(\bar{Y})$ using S_tRSS, when the population coefficient of variation of the auxiliary variable for the hth stratum (C_{xh}) is known as

$$\bar{y}_{R(S_tRSS)SD} = \bar{y}_{[S_tRSS]}\frac{\sum_{h=1}^{L}W_h(\bar{X}_h + C_{xh})}{\sum_{h=1}^{L}W_h(\bar{x}_{h(RSS)} + C_{xh})}. \tag{4}$$

The bias and MSE of $\bar{y}_{R(S_tRSS)SD}$, up to first degree of approximation, are respectively given by

$$Bias(\bar{y}_{R(S_tRSS)SD}) \cong \bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(\lambda_1^2 C_{xh}^2 - \lambda_1\rho_{yxh}C_{xh}C_{yh}\right) - \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\left(\lambda_1^2\sum_{i=1}^{mh}W_{xh(i:mh)}^2 - \lambda_1\sum_{i=1}^{mh}W_{xh(i:mh)}W_{yh[i:mh]}\right)\right] \tag{5}$$

and

$$MSE(\bar{y}_{R(S_tRSS)SD}) \cong \bar{Y}^2\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(C_{yh}^2 + \lambda_1^2 C_{xh}^2 - 2\lambda_1\rho_{yxh}C_{xh}C_{yh}\right) - \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\sum_{i=1}^{mh}\left(W_{yh[i:mh]} - \lambda_1 W_{xh(i:mh)}\right)^2\right], \tag{6}$$

where

$$\lambda_1 = \frac{\sum_{h=1}^{L}W_h\bar{X}_h}{\sum_{h=1}^{L}W_h(\bar{X}_h + C_{xh})}.$$

Following Kadilar and Cingi (2003), Mandowara and Mehta (2014) proposed another ratio-type estimator for $\bar{Y}$ using stratified ranked set sampling as follows:

$$\bar{y}_{R(S_tRSS)KC} = \bar{y}_{[S_tRSS]}\frac{\sum_{h=1}^{L}W_h(\bar{X}_h + \beta_{2(xh)})}{\sum_{h=1}^{L}W_h(\bar{x}_{h(RSS)} + \beta_{2(xh)})}. \tag{7}$$

Here, $\beta_{2(xh)}$ is the coefficient of kurtosis of the auxiliary variable X in the hth stratum. The bias and MSE of $\bar{y}_{R(S_tRSS)KC}$, up to first degree of approximation, are respectively given by

$$Bias(\bar{y}_{R(S_tRSS)KC}) \cong \bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(\lambda_2^2 C_{xh}^2 - \lambda_2\rho_{yxh}C_{xh}C_{yh}\right) - \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\left(\lambda_2^2\sum_{i=1}^{mh}W_{xh(i:mh)}^2 - \lambda_2\sum_{i=1}^{mh}W_{xh(i:mh)}W_{yh[i:mh]}\right)\right] \quad (8)$$

and

$$MSE(\bar{y}_{R(S_tRSS)KC}) \cong \bar{Y}^2\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(C_{yh}^2 + \lambda_2^2 C_{xh}^2 - 2\lambda_2\rho_{yxh}C_{xh}C_{yh}\right) - \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\sum_{i=1}^{mh}\left(W_{yh[i:mh]} - \lambda_2 W_{xh(i:mh)}\right)^2\right], \quad (9)$$

where

$$\lambda_2 = \frac{\sum_{h=1}^{L}W_h\bar{X}_h}{\sum_{h=1}^{L}W_h(\bar{X}_h + \beta_{2(xh)})}.$$

Based on Upadhyaya and Singh (1999), Mandowara and Mehta (2014) proposed two more ratio-type estimators, using both coefficient of variation and coefficient of kurtosis of the auxiliary variable in S_tRSS, which are given as

$$\bar{y}_{R(S_tRSS)US1} = \bar{y}_{[S_tRSS]}\frac{\sum_{h=1}^{L}W_h(\bar{X}_h\beta_{2(xh)} + C_{xh})}{\sum_{h=1}^{L}W_h(\bar{x}_{h(RSS)}\beta_{2(xh)} + C_{xh})} \quad (10)$$

and

$$\bar{y}_{R(S_tRSS)US2} = \bar{y}_{[S_tRSS]}\frac{\sum_{h=1}^{L}W_h(\bar{X}_h C_{xh} + \beta_{2(xh)})}{\sum_{h=1}^{L}W_h(\bar{x}_{h(RSS)}C_{xh} + \beta_{2(xh)})}. \quad (11)$$

The bias and MSE of $\bar{y}_{R(S_tRSS)US1}$ and $\bar{y}_{R(S_tRSS)US2}$, up to first order of approximation, are respectively given by

$$Bias(\bar{y}_{R(S_tRSS)US1}) \cong \bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(\lambda_3^2 C_{xh}^2 - \lambda_3\rho_{yxh}C_{xh}C_{yh}\right) - \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\left(\lambda_3^2\sum_{i=1}^{mh}W_{xh(i:mh)}^2 - \lambda_3\sum_{i=1}^{mh}W_{xh(i:mh)}W_{yh[i:mh]}\right)\right], \quad (12)$$

$$Bias(\bar{y}_{R(S_tRSS)US2}) \cong \bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(\lambda_4^2 C_{xh}^2 - \lambda_4\rho_{yxh}C_{xh}C_{yh}\right) - \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\left(\lambda_4^2\sum_{i=1}^{mh}W_{xh(i:mh)}^2 - \lambda_4\sum_{i=1}^{mh}W_{xh(i:mh)}W_{yh[i:mh]}\right)\right], \quad (13)$$

$$MSE(\bar{y}_{R(S_tRSS)US1}) \cong \bar{Y}^2\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(C_{yh}^2+\lambda_3^2C_{xh}^2-2\lambda_3\rho_{yxh}C_{xh}C_{yh}\right)\right.$$
$$\left.-\sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\sum_{i=1}^{mh}\left(W_{yh[i:mh]}-\lambda_3W_{xh(i:mh)}\right)^2\right] \tag{14}$$

and

$$MSE(\bar{y}_{R(S_tRSS)US2}) \cong \bar{Y}^2\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(C_{yh}^2+\lambda_4^2C_{xh}^2-2\lambda_4\rho_{yxh}C_{xh}C_{yh}\right)\right.$$
$$\left.-\sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\sum_{i=1}^{mh}\left(W_{yh[i:mh]}-\lambda_4W_{xh(i:mh)}\right)^2\right], \tag{15}$$

where $\lambda_3 = \frac{\sum_{h=1}^{L} W_h\bar{X}_h\beta_{2(xh)}}{\sum_{h=1}^{L} W_h(\bar{X}_h\beta_{2(xh)}+C_{xh})}$ and $\lambda_4 = \frac{\sum_{h=1}^{L} W_h\bar{X}_hC_{xh}}{\sum_{h=1}^{L} W_h(\bar{X}_hC_{xh}+\beta_{2(xh)})}$.

3. Stratified double-ranked set sampling procedure

In stratified double-ranked set sampling, for the hth stratum of the population, first choose m_h^3 independent random samples ($h = 1, 2, \ldots, L$). Arrange these selected units randomly into m_h sets, each of size m_h^2. The procedure of *RSS* is then applied on each of the sets to obtain the m_h sets of ranked set samples each of size m_h. These ranked set samples are collected together to form m_h sets of observations each of size m_h. The *RSS* procedure is then applied again on this set to obtain L independent *DRSS* samples each of size m_h, to get $m_1 + m_2 + \cdots + m_L = m$ observations. This completes one cycle of S_tDRSS. The whole process is repeated r times to get the desired sample size $n = mr$.

To estimate population mean $(\bar{Y})$ in S_tDRSS using ratio estimator, the procedure can be summarized as follows:

- Step 1: Select m_h^3 bivariate sample units randomly from the hth stratum of the population.
- Step 2: Arrange these selected units randomly into m_h sets, each of size m_h^2.
- Step 3: The procedure of *RSS* is then applied on each of the sets to obtain the m_h sets of ranked set samples each of size m_h. Here, ranking is done with respect to the auxiliary variable X. These ranked set samples are collected together to form m_h sets each of size m_h units.
- Step 4: The *RSS* procedure is then applied again on this set to obtain L independent *DRSS* samples, each of size m_h, to get $m_1 + m_2 + \cdots + m_L = m$ observations.
- Step 5: Repeat the preceding steps r times to get the desired sample size $n = mr$.

We use the following notations for the S_tDRSS when ranking is done with respect to the auxiliary variable X. For the jth cycle and the hth stratum, the S_tDRSS is denoted by $\left(Y_{h[1]j}^{[1]}, X_{h(1)j}^{(1)}\right), \left(Y_{h[2]j}^{[2]}, X_{h(2)j}^{(2)}\right), \ldots, \left(Y_{h[mh]j}^{[mh]}, X_{h(mh)j}^{(mh)}\right)$ $j = 1,2,\ldots,r$, and $h = 1, 2, \ldots, L$. Here

$Y^{[i]}_{h[i]j}$ is the ith rank unit in the ith ranked set sample at the jth cycle of the hth stratum. To find the bias and MSE, we define the following terms:

$$\delta_0 = \frac{\bar{y}_{[S_tDRSS]} - \bar{Y}}{\bar{Y}} \quad \text{and} \quad \delta_1 = \frac{\bar{x}_{(S_tDRSS)} - \bar{X}}{\bar{X}},$$

such that

$$E(\delta_i) = 0, \quad (i = 0, 1), \quad E(\delta_0^2) = \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(C_{yh}^2 - \frac{1}{m_h}\sum_{i=1}^{m_h} W^{2[i:mh]}_{yh[i:mh]}\right),$$

$$E(\delta_1^2) = \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(C_{xh}^2 - \frac{1}{m_h}\sum_{i=1}^{m_h} W^{2(i:mh)}_{xh(i:mh)}\right),$$

$$E(\delta_0\delta_1) = \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(\rho_{yxh} C_{yh} C_{xh} - \frac{1}{m_h}\sum_{i=1}^{m_h} W^{[i:mh]}_{yh[i:mh]} W^{(i:mh)}_{xh(i:mh)}\right),$$

where

$$W^{[i:mh]}_{yh[i:mh]} = \frac{\tau^{[i:mh]}_{yh[i:mh]}}{\bar{Y}}, \quad W^{(i:mh)}_{xh(i:mh)} = \frac{\tau^{(i:mh)}_{xh(i:mh)}}{\bar{X}},$$

$$\tau^{(i:mh)}_{xh(i:mh)} = (\mu^{(i:mh)}_{xh(i:mh)} - \bar{X}_h), \quad \tau^{[i:mh]}_{yh[i:mh]} = (\mu^{[i:mh]}_{yh([i:mh]} - \bar{Y}_h)$$

$$\mu^{[i:mh]}_{yh[i:mh]} = E\left[y^{[i:mh]}_{h[i:mh]}\right] \quad \text{and} \quad \mu^{(i:mh)}_{xh(i:mh)} = E\left[x^{(i:mh)}_{h(i:mh)}\right].$$

4. Proposed estimators in S_tDRSS

Following Samawi and Siam (2003), we propose combined ratio-type estimator of population mean $(\bar{Y})$ using S_tDRSS and is defined as

$$\bar{y}_{R(S_tDRSS)SS} = \bar{y}_{[S_tDRSS]}\left(\frac{\bar{X}}{\bar{x}_{(S_tDRSS)}}\right), \tag{16}$$

where $\bar{y}_{[S_tDRSS]} = \sum_{h=1}^{L} W_h \bar{y}_{h[DRSS]}$ and $\bar{x}_{(S_tDRSS)} = \sum_{h=1}^{L} W_h \bar{x}_{h(DRSS)}$.

In terms of $\delta' s$, we have

$$\bar{y}_{R(S_tDRSS)SS} = \bar{Y}(1 + \delta_0)(1 + \delta_1)^{-1}$$

$$(\bar{y}_{R(S_tDRSS)SS} - \bar{Y}) \cong \bar{Y}(\delta_0 - \delta_1 + \delta_1^2 - \delta_0\delta_1 + \ldots).$$

Taking expectations, we get

$$Bias(\bar{y}_{R(S_tDRSS)SS}) \cong \bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(C_{xh}^2 - \rho_{yxh}C_{xh}C_{yh}\right) - \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\left(\sum_{i=1}^{mh} W_{xh(i:mh)}^{2(i:mh)} - \sum_{i=1}^{mh} W_{xh(i:mh)}^{(i:mh)} W_{yh[i:mh]}^{(i:mh)}\right)\right]. \quad (17)$$

Squaring and then taking expectation, the MSE of $\bar{y}_{R(S_tDRSS)}$ is given by

$$MSE(\bar{y}_{R(S_tDRSS)SS}) \cong \bar{Y}^2\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(C_{yh}^2 + C_{xh}^2 - 2\rho_{yxh}C_{xh}C_{yh}\right) - \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\sum_{i=1}^{mh}\left(W_{yh[i:mh]}^{[i:mh]} - W_{xh(i:mh)}^{(i:mh)}\right)^2\right]. \quad (18)$$

Following Mandowara and Mehta (2014), we propose a ratio-type estimator for $\bar{Y}$ using S_tDRSS, when the population coefficient of variation of the auxiliary variable for the hth stratum (C_{xh}) is known as

$$\bar{y}_{R(S_tDRSS)SD} = \bar{y}_{[S_tDRSS]}\frac{\sum_{h=1}^{L} W_h(\bar{X}_h + C_{xh})}{\sum_{h=1}^{L} W_h(\bar{x}_{h(DRSS)} + C_{xh})}. \quad (19)$$

In terms of $\delta's$, we have

$$\bar{y}_{R(S_tDRSS)SD} = \bar{Y}(1+\delta_0)(1+\lambda_1\delta_1)^{-1},$$

$$(\bar{y}_{R(S_tDRSS)SD} - \bar{Y}) \cong \bar{Y}(\delta_0 - \lambda_1\delta_1 + \lambda_1^2\delta_1^2 - \lambda_1\delta_0\delta_1 + \ldots).$$

Taking expectations, we get

$$Bias(\bar{y}_{R(S_tDRSS)SD}) \cong \bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(\lambda_1^2 C_{xh}^2 - \lambda_1\rho_{yxh}C_{xh}C_{yh}\right) - \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\left(\lambda_1^2\sum_{i=1}^{mh} W_{xh(i:mh)}^{2(i:mh)} - \lambda_1\sum_{i=1}^{mh} W_{xh(i:mh)}^{(i:mh)} W_{yh[i:mh]}^{(i:mh)}\right)\right]. \quad (20)$$

Squaring and then taking expectation, the MSE of $\bar{y}_{R(S_tDRSS)SD}$ is given by

$$MSE(\bar{y}_{R(S_tDRSS)SD}) \cong \bar{Y}^2\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(C_{yh}^2 + \lambda_1^2 C_{xh}^2 - 2\lambda_1\rho_{yxh}C_{xh}C_{yh}\right) - \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\sum_{i=1}^{mh}\left(W_{yh[i:mh]}^{[i:mh]} - \lambda_1 W_{xh(i:mh)}^{(i:mh)}\right)^2\right]. \quad (21)$$

Following Mandowara and Mehta (2014), we propose another ratio-type estimator for $\bar{Y}$ using S_tDRSS as follows:

$$\bar{y}_{R(S_tDRSS)KC} = \bar{y}_{[S_tDRSS]}\frac{\sum_{h=1}^{L} W_h(\bar{X}_h + \beta_{2(xh)})}{\sum_{h=1}^{L} W_h(\bar{x}_{h(DRSS)} + \beta_{2(xh)})}. \quad (22)$$

The bias and MSE of $\bar{y}_{R(S_tDRSS)KC}$ are given by

$$Bias(\bar{y}_{R(S_tDRSS)KC}) \cong \bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(\lambda_2^2 C_{xh}^2 - \lambda_2\rho_{yxh}C_{xh}C_{yh}\right) - \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\left(\lambda_2^2\sum_{i=1}^{mh}W_{xh(i:mh)}^{2(i:mh)} - \lambda_2\sum_{i=1}^{mh}W_{xh(i:mh)}^{(i:mh)}W_{yh[i:mh]}^{(i:mh)}\right)\right], \tag{23}$$

and

$$MSE(\bar{y}_{R(S_tDRSS)KC}) \cong \bar{Y}^2\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(C_{yh}^2 + \lambda_2^2 C_{xh}^2 - 2\lambda_2\rho_{yxh}C_{xh}C_{yh}\right) - \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\sum_{i=1}^{mh}\left(W_{yh[i:mh]}^{[i:mh]} - \lambda_2 W_{xh(i:mh)}^{(i:mh)}\right)^2\right]. \tag{24}$$

Based on Mandowara and Mehta (2014), we propose two more ratio-type estimators, using both the coefficient of variation and coefficient of kurtosis in S_tDRSS as follows:

$$\bar{y}_{R(S_tDRSS)US1} = \bar{y}_{[S_tDRSS]}\frac{\sum_{h=1}^{L} W_h(\bar{X}_h\beta_{2(xh)} + C_{xh})}{\sum_{h=1}^{L} W_h(\bar{x}_{h(DRSS)}\beta_{2(xh)} + C_{xh})} \tag{25}$$

and

$$\bar{y}_{R(S_tDRSS)US2} = \bar{y}_{[S_tDRSS]}\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h C_{xh} + \beta_{2(xh)}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_{h(DRSS)}C_{xh}\right) + \beta_{2(xh)}}. \tag{26}$$

The bias and MSE of $\bar{y}_{R(S_tDRSS)US1}$ and $\bar{y}_{R(S_tDRSS)US2}$, up to first order of approximation, are respectively given by

$$Bias\left(\bar{y}_{R(S_tDRSS)US1}\right) \cong \bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(\lambda_3^2 C_{xh}^2 - \lambda_3\rho_{yxh}C_{xh}C_{yh}\right) - \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\left(\lambda_3^2\sum_{i=1}^{mh}W_{xh(i:mh)}^{2(i:mh)} - \lambda_3\sum_{i=1}^{mh}W_{xh(i:mh)}^{(i:mh)}W_{yh[i:mh]}^{(i:mh)}\right)\right], \tag{27}$$

$$Bias\left(\bar{y}_{R(S_tDRSS)US2}\right) \cong \bar{Y}\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(\lambda_4^2 C_{xh}^2 - \lambda_4\rho_{yxh}C_{xh}C_{yh}\right) - \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\left(\lambda_4^2\sum_{i=1}^{mh}W_{xh(i:mh)}^{2(i:mh)} - \lambda_4\sum_{i=1}^{mh}W_{xh(i:mh)}^{(i:mh)}W_{yh[i:mh]}^{(i:mh)}\right)\right], \tag{28}$$

$$MSE\left(\bar{y}_{R(S_tDRSS)US1}\right) \cong \bar{Y}^2\left[\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(C_{yh}^2 + \lambda_3^2 C_{xh}^2 - 2\lambda_3\rho_{yxh}C_{xh}C_{yh}\right) - \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r}\sum_{i=1}^{mh}\left(W_{yh[i:mh]}^{[i:mh]} - \lambda_3 W_{xh(i:mh)}^{(i:mh)}\right)^2\right], \tag{29}$$

and

$$MSE(\bar{y}_{R(S_tD\,RSS)US2}) \cong \bar{Y}^2 \left[\sum_{h=1}^{L} \frac{W_h^2}{m_h r} \left(C_{yh}^2 + \lambda_4^2 C_{xh}^2 - 2\lambda_4 \rho_{yxh} C_{xh} C_{yh} \right) - \sum_{h=1}^{L} \frac{W_h^2}{m_h^2 r} \sum_{i=1}^{mh} \left(W_{yh[i:mh]}^{[i:mh]} - \lambda_4 W_{xh(i:mh)}^{(i:mh)} \right)^2 \right]. \quad (30)$$

5. Efficiency comparison

We obtain the conditions under which the proposed ratio-type estimators under S_tDRSS are more efficient than the corresponding ratio estimators in S_tRSS.

(1) Comparison: By Eqs. (3) and (18),

$$MSE(\bar{y}_{R(S_tDRSS)}) < MSE(\bar{y}_{R(S_tRSS)}),$$

$$\text{if} \quad \sum_{h=1}^{L} \frac{W_h^2}{m_h^2 r} \left[\sum_{i=1}^{mh} \left(W_{yh[i:mh]}^{[i:mh]} - W_{xh(i:mh)}^{(i:mh)} \right)^2 - \sum_{i=1}^{mh} \left(W_{yh[i:mh]} - W_{xh(i:mh)} \right)^2 \right] > 0.$$

(2) Comparison: By Eqs. (6) and (21),

$$MSE(\bar{y}_{R(S_tDRSS)SD}) < MSE(\bar{y}_{R(S_tRSS)SD}),$$

$$\text{if} \quad \sum_{h=1}^{L} \frac{W_h^2}{m_h^2 r} \left[\sum_{i=1}^{mh} \left(W_{yh[i:mh]}^{[i:mh]} - \lambda_1 W_{xh(i:mh)}^{(i:mh)} \right)^2 - \sum_{i=1}^{mh} \left(W_{yh[i:mh]} - \lambda_1 W_{xh(i:mh)} \right)^2 \right] > 0.$$

(3) Comparison: By Eqs. (9) and (24),

$$MSE(\bar{y}_{R(S_tDRSS)KC}) < MSE(\bar{y}_{R(S_tRSS)KC}),$$

$$\text{if} \quad \sum_{h=1}^{L} \frac{W_h^2}{m_h^2 r} \left[\sum_{i=1}^{mh} \left(W_{yh[i:mh]}^{[i:mh]} - \lambda_2 W_{xh(i:mh)}^{(i:mh)} \right)^2 - \sum_{i=1}^{mh} \left(W_{yh[i:mh]} - \lambda_2 W_{xh(i:mh)} \right)^2 \right] > 0.$$

(4) Comparison: By Eqs. (14) and (29),

$$MSE(\bar{y}_{R(S_tDRSS)US1}) < MSE(\bar{y}_{R(S_tRSS)US1}),$$

$$\text{if} \quad \sum_{h=1}^{L} \frac{W_h^2}{m_h^2 r} \left[\sum_{i=1}^{mh} \left(W_{yh[i:mh]}^{[i:mh]} - \lambda_3 W_{xh(i:mh)}^{(i:mh)} \right)^2 - \sum_{i=1}^{mh} \left(W_{yh[i:mh]} - \lambda_3 W_{xh(i:mh)} \right)^2 \right] > 0.$$

(5) Comparison: By Eqs. (15) and (30),

$$MSE(\bar{y}_{R(S_tDRSS)US2}) < MSE(\bar{y}_{R(S_tRSS)US2}),$$

$$\text{if} \quad \sum_{h=1}^{L} \frac{W_h^2}{m_h^2 r} \left[\sum_{i=1}^{mh} \left(W_{yh[i:mh]}^{[i:mh]} - \lambda_4 W_{xh(i:mh)}^{(i:mh)} \right)^2 - \sum_{i=1}^{mh} \left(W_{yh[i:mh]} - \lambda_4 W_{xh(i:mh)} \right)^2 \right] > 0.$$

Note: When the preceding conditions 1–5 are satisfied, the proposed ratio-type estimators in S_tDRSS are more efficient than in the corresponding ratio-type estimators in S_tRSS. All conditional values are positive, given in Table 1.

Table 1. Conditional values.

Comparison (1)	Comparison (2)	Comparison (3)	Comparison (4)	Comparison (5)
0.029238	0.024378	0.024384	0.024377	0.024374
0.007699	0.007700	0.007700	0.007701	0.007700
0.036153	0.036148	0.036136	0.036153	0.036143
0.005643	0.005643	0.005641	0.005643	0.005630
0.019705	0.019705	0.019702	0.019705	0.019704

6. Simulation study

To compare the performances of the estimators, a simulation study is conducted where ranking is performed on the auxiliary variable X. Bivariate random observations $\left(X_{(i)h}, Y_{[i]h}\right)$, $i = 1, 2, \ldots, m_h$, and $h = 1, 2, \ldots, L$, are generated from a bivariate normal population having parameters $\left(\mu_{xh}, \mu_{yh}, \sigma_{xh}, \sigma_{yh}, \rho_{yxh}\right)$ and from a bivariate gamma population with correlation parameter ρ_{yxh}. Using 10,000 simulations, estimates of $MSEs$ for ratio-type estimators are computed under S_tRSS and S_tDRSS. Estimators are compared in terms of percent relative efficiencies (PRE_s) and percentage relative bias (PRB). The simulation results are presented in Tables 2, 3, and 4, respectively. We used the following expressions to obtain the simulated MSE_s, PRE_s and PRB.

$$MSE\left(\bar{y}_{R(S_tDRSS)p}\right) = \frac{1}{10000}\sum_{i=1}^{10000}\left(\bar{y}_{R(S_tDRSS)pi} - \bar{Y}\right)^2, \quad p = SS, SD, KC, US1, US2.$$

$$MSE\left(\bar{y}_{R(S_tRSS)p}\right) = \frac{1}{10000}\sum_{i=1}^{10000}\left(\bar{y}_{R(S_tRSS)pi} - \bar{Y}\right)^2,$$

Table 2. PRE_s and (PRB) of various estimators obtained through bivariate normal distribution when m_h = (3, 4, 5).

L = 3; W_h = (.30, .30, .40)	μ_{xh} = (2, 3, 4); m_h = (3, 4, 5)	μ_{yh} = (3, 4, 6); r = 3	σ_{yh} = (1, 1, 1); n_h = (9, 12, 15)	σ_{xh} = (1, 1, 1)	
ρyxh	*PRE* (*SS*)	*PRE* (*SD*)	*PRE* (*KC*)	*PRE* (*US*1)	*PRE* (*US*2)
0.99, 0.99, 0.99	118.2746 (1.22)	121.2747 (1.21)	126.2752 (1.19)	121.7503 (1.03)	130.2754 (−0.97)
0.90, 0.90, 0.90	116.9044 (1.25)	118.9045 (−1.22)	121.9049 (1.20)	118.9589 (1.05)	126.7650 (−0.99)
0.70, 0.70, 0.70	109.0934 (1.28)	110.0907 (1.24)	118.0804 (1.22)	110.5179 (1.08)	122.9386 (−1.01)
0.50, 0.50, .50	105.0934 (−1.29)	105.0907 (1.27)	104.0804 (1.23)	105.5179 (1.11)	109.9386 (−1.02)
0.30, 0.30, 0.30	102.4654 (1.32)	102.0476 (1.29)	101.5508 (1.26)	102.9776 (1.13)	102.3118 (−1.09)
0.99, 0.90, 0.70	108.0087 (1.05)	108.2894 (1.03)	110.9562 (1.03)	108.7988 (1.01)	113.6345 (−0.94)
0.90, 0.70, 0.50	107.9294 (1.09)	107.6328 (−1.04)	107.9408 (1.06)	108.6698 (1.03)	112.1773 (−1.00)
0.70, 0.50, 0.30	101.2879 (1.17)	101.5856 (1.14)	103.2235 (−1.10)	102.2010 (1.03)	105.1147 (−1.02)

Table 3. *PREs* and (*PRB*) of various estimators obtained through bivariate normal distribution when m_h = (6, 7, 8).

L = 3; W_h = (.30,.30, .40)	μ_{xh} = (2, 3, 4); m_h = (6, 7, 8)	μ_{yh} = (3, 4, 6); r = 5	σ_{yh} = (1, 1, 1); n_h = (30, 35, 40)	σ_{xh} = (1, 1, 1)	
ρyxh	*PRE* (*SS*)	*PRE* (*SD*)	*PRE* (*KC*)	*PRE* (*US*1)	*PRE* (*US*2)
0.99, 0.99, 0.99	127.1027 (1.12)	130.2247 (−1.11)	134.2752 (1.09)	131.7500 (0.93)	140.2951 (−0.91)
0.90, 0.90, 0.90	125.1344 (1.15)	128.4511 (1.12)	131.9049 (1.10)	127.0189 (0.95)	136.7015 (−0.93)
0.70, 0.70, 0.70	117.0434 (1.18)	119.6909 (1.14)	129.0804 (1.12)	120.5107 (0.98)	131.2938 (−0.94)
0.50, 0.50, .50	114.0966 (1.21)	116.1807 (−1.17)	118.1080 (1.14)	116.8179 (1.01)	119.1380 (−1.00)
0.30, 0.30, 0.30	112.6514 (1.22)	112.7600 (1.21)	111.5508 (1.17)	112.7603 (−1.03)	112.5118 (1.01)
0.99, 0.90, 0.70	119.2900 (1.02)	118.7128 (1.09)	120.6695 (1.03)	126.0079 (0.83)	133.9063 (0.77)
0.90, 0.70, 0.50	117.1192 (1.03)	117.6300 (1.11)	117.9400 (−1.09)	118.6603 (0.91)	122.1707 (0.82)
0.70, 0.50, 0.30	111.2807 (1.07)	111.0158 (−1.13)	113.2239 (1.10)	112.2091 (0.95)	115.1014 (0.89)

$$PRE(p) = \frac{MSE\left(\bar{y}_{R(S_tRSS)p}\right)}{MSE\left(\bar{y}_{R(S_tDRSS)p}\right)} \times 100, \quad p = SS, SD, KC, US1, US2.$$

$$PRB(p) = \frac{1}{\bar{Y}}\left[\frac{1}{10000}\sum_{i=1}^{10000}\left(\bar{y}_{R(S_tDRSS)pi} - \bar{Y}\right)\right] \times 100, \quad p = SS, SD, KC, US1, US2.$$

The PRE_s of various stratified double ranked set estimators in comparison with different stratified ranked set estimators are shown in Tables 2, 3, and 4, respectively.

The simulation results showed that with decrease of the correlation coefficients ρ_{yxh}, PRE_s deceases which are expected results. The numerical values given in the first five rows are obtained by assuming equal correlations across the strata, whereas the last three rows

Table 4. The values of *PREs* and (*PRB*) for the simulated data, obtained through bivariate gamma distribution.

L = 3	W_h = (.30, .30, .40)	m_h = (3, 4, 5)	r = 3	n_h = (9, 12, 15)	
ρyxh	*PRE* (*SS*)	*PRE* (*SD*)	*PRE* (*KC*)	*PRE* (*US*1)	*PRE* (*US*2)
0.99, 0.99, 0.99	113.2706 (1.22)	111.2077 (−1.31)	112.0452 (1.29)	111.7003 (1.03)	116.9754 (0.87)
0.90, 0.90, 0.90	111.9140 (1.32)	109.2904 (1.21)	110.9910 (1.30)	108.9109 (1.06)	112.6750 (0.91)
0.70, 0.70, 0.70	110.8904 (1.37)	108.1927 (1.31)	109.6081 (1.29)	106.5339 (−1.07)	111.0386 (0.92)
0.50, 0.50, .50	107.0981 (1.38)	106.0971 (1.33)	107.3802 (−1.30)	105.5009 (1.10)	108.1293 (0.97)
0.30, 0.30, 0.30	105.7746 (1.40)	101.1047 (1.37)	102.9055 (1.30)	102.9007 (1.13)	106.1310 (1.07)
0.99, 0.90, 0.70	106.1008 (1.32)	109.2809 (1.29)	112.9506 (1.10)	111.7900 (0.93)	114.2263 (−0.75)
0.90, 0.70, 0.50	103.9024 (1.33)	107.6302 (1.27)	110.1940 (1.17)	109.1066 (1.01)	110.1037 (0.91)
0.70, 0.50, 0.30	100.2800 (−1.42)	102.9580 (1.31)	104.2203 (1.21)	103.2910 (1.02)	106.1014 (1.00)

Table 5. Summary statistics.

Stratum 1	Stratum 2	Stratum 3
$N_1 = 12$	$N_2 = 30$	$N_3 = 17$
$n_1 = 9$	$n_2 = 9$	$n_3 = 12$
$W_1 = 0.0234$	$W_2 = 0.5085$	$W_3 = 0.02881$
$\bar{X}_1 = 5987.83$	$\bar{X}_2 = 11682.73$	$\bar{X}_3 = 68662.29$
$\bar{Y}_1 = 11788$	$\bar{Y}_2 = 16862.27$	$\bar{Y}_3 = 227371.53$
$\bar{R}_1 = 1.97$	$\bar{R}_2 = 1.44$	$\bar{R}_2 = 3.31$
$S^2_{x_1} = 27842810.5$	$S^2_{x_2} = 760238523$	$S^2_{x_3} = 12187889050$
$S^2_{y_1} = 1538545883$	$S^2_{y_2} = 2049296094$	$S^2_{y_3} = 372428238550$
$S_{y_1x_1} = 62846173.1$	$S_{y_2x_2} = 1190767859$	$S_{y_3x_3} = 27342963562$
$C_{x_1} = 0.8812$	$C_{x_2} = 2.3601$	$C_{x_3} = 1.6079$
$\beta_2(x1) = 14.6079$	$\beta_2(x2) = 10.7527$	$\beta_2(x3) = 8.935$
$\rho_{yx1} = 0.9602$	$\rho_{yx2} = 0.9540$	$\rho_{yx3} = 0.4058$

assume unequal correlations across the strata. Tables 2, 3, and 4 indicate that the proposed estimators have reasonable biases, since the values of *PRB* are all less than 2% in absolute terms. Also, for a given sample size, *PRB* increases with decrease in correlation coefficient. It is very easy to conclude from the results given in Tables 2, 3, and 4 that our proposed estimators performed better than their competitors.

7. Numerical illustration

To observe performances of the estimators, we used a real data set given by Singh (2003). The study variable y is the tobacco production in metric tons and the auxiliary variable x is the area for tobacco in specified countries during 1998. The summary statistics are given in Table 5.

From this population, we took five double-ranked set samples of sizes $m_1 = 3$, $m_2 = 5$, and $m_3 = 4$ from strata 1, 2, and 3, respectively. Further, each double-ranked set sample from each stratum is repeated with number of cycles $r = 3$. Hence sample sizes of stratified double-ranked set samples are equivalent to the stratified ranked set sample of sizes $n_h = m_h r$. The estimated percentage relative efficiencies based upon *MSEs* values of various stratified double-ranked set estimators in comparison with different stratified ranked set estimators are shown in Table 6. This indicates that our proposed ratio-type estimators under S_tDRSS are more efficient than their competitors in S_tRSS.

8. Conclusion

In this study, we proposed different ratio-type estimators in S_tDRSS to estimate the finite population mean following by Mandowara and Mehta (2014). The biases and *MSEs* of these proposed estimators are derived up to the first order of approximation. A simulation

Table 6. Percentage relative efficiencies for the given population.

Samples	*PRE (SS)*	*PRE (SD)*	*PRE (KC)*	*PRE (US1)*	*PRE (US2)*
Sample 1	108.0069	108.0165	108.5050	108.0268	108.9458
Sample 2	102.3903	102.4900	102.9024	102.6900	103.0389
Sample 3	113.0154	113.0437	113.2079	113.5152	113.8109
Sample 4	136.3735	136.3922	136.7719	136.5073	136.97639
Sample 5	106.2402	106.2799	106.5391	106.3101	106.8395

study and a real data set are utilized to observe the performances of estimators. The efficiency conditions for the proposed estimators are obtained both theoretically and numerically. It is shown that all conditions are true for the given data set. On the basis of numerical and simulation studies, our proposed ratio-type estimators under S_tDRSS perform much better as compared to respective competitive estimators in S_tRSS.

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