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ORIGINAL ARTICLE

Wealth Inequality and the Financial Accumulation Process

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Abstract

Previous theoretical and computational analyses demonstrate that uncorrelated variations in individual asset returns promote extreme inequality in financial wealth. This paper describes a standard individual-based computational model of this financial accumulation process and then extends it in order to expose other key influences on wealth inequality. We find large effects of individual behavior, cultural practices, tax policy, and technological change. Specifically, we present simulation experiments with heterogeneous saving rates, a stylized marriage institution, a wealth tax structured to mirror contemporary policy proposals, and variations in wage growth. These experiments demonstrate that modest concessions to realism have large effects on long-run wealth inequality in models of the financial accumulation process.

Keywords Financial accumulation process · Wealth inequality · Saving · Wealth tax · Growth

JEL Classifications D31 · E17 · E21

Introduction

Economists have long recognized that the distribution of wealth is remarkably unequal (Pareto 1897; Lorenz 1905). In the USA, the Gini coefficient of inequality currently exceeds 80%, and the top 5% of the wealth distribution hold more than half of total wealth (Cagetti and De Nardi 2008; Díaz-Giménez et al. 2011). Although high wealth inequality at the country level is common, there is substantial variability across countries. The smooth kernel histogram in Fig. 1 summarizes the distribution of 148 country Gini coefficients, thereby providing a nonparametric window on the variation across countries. Such divergent wealth inequality experiences have intrigued researchers and policy makers, who wish to understand or influence the level of wealth inequality.

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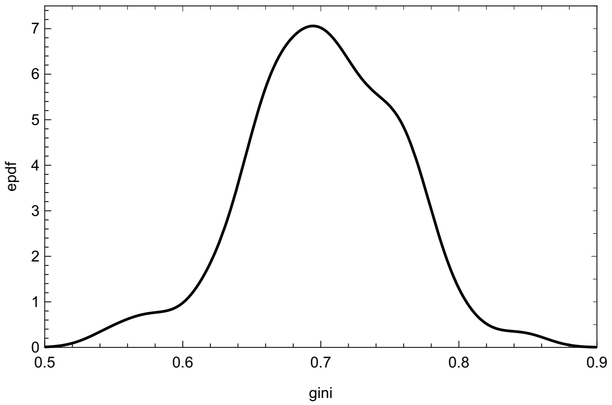

Fig. 1 Wealth inequality across countries. An unweighted kernel density plot of wealth Gini coefficients for 148 countries exposes substantial cross-country diversity in wealth inequality. *Data Source*: Davies et al. (2009)

Mainstream economists have produced a large theoretical literature on wealth accumulation. This is typically rooted in refinements of the 1960s life-cycle model of consumption and saving (Ando and Modigliani 1963). From a life-cycle perspective, wealth accumulation is a by-product of intertemporal consumption smoothing: saving during peak-earning years buffers consumption in low-income years. Economists quickly recognized that the resulting models struggle to explain observed wealth inequality: they typically predict levels of wealth inequality that are too low (Atkinson 1971; Oulton 1976; Huggett 1996). The gap between these predictions and the empirical evidence is the *wealth concentration puzzle*, and its challenge to the life-cycle model persists (Cagetti and De Nardi 2008; Hubmer et al. 2016). Consumption smoothing does not appear to offer an adequate explanation of observed levels of wealth inequality.

An alternative approach to understanding wealth inequality traces to the 1950s. Models of the *financial accumulation process* focus on the evolution of wealth that is implied by the compounding of random investment returns (Sargan 1957; Wold and Whittle 1957). In these models, high wealth is more likely to produce high investment income that when saved further increases wealth. (The term *investment income* denotes the returns generated by wealth.) This financial accumulation process underpins many individual-based computational models of the distribution of wealth (Yunker 1999; Levy and Levy 2003; Isaac 2008; Biondi and Righi 2019). These models discard life-cycle saving behavior in favor of simpler rule-based saving behaviors. Saving rules more easily accommodate some evidence on wealth accumulation, such as wealth increases by high-income households long after retirement (Land and Russell 1996; Smith et al. 2009).

In stark contrast to life-cycle approaches, the simplest models of the financial accumulation process tend to predict levels of wealth inequality that *exceed* observed values. However, computational research has demonstrated that taxing and redistributing investment income can substantially dampen the wealth inequality generated by the financial accumulation process (Yunker 1999; Isaac 2008; Biondi and Righi 2019). Our paper adds to the computational research on the financial accumulation process. It extends a simple individual-based computational model in order to incorporate additional behavioral, cultural, institutional, and technological influences on long-run wealth inequality. Our simulation experiments demonstrate the potential importance of each of these influences.

The paper is organized as follows. The "Computational Model" section presents our computational model of the financial accumulation process. It then introduces a baseline parameterization of the model that reproduces the traditional results, described above. This baseline model provides the point of comparison for the computational experiments of the "Simulation Experiments" section. This paper describes four experiments, which demonstrate the sensitivity of the baseline results to individual behavior, cultural institutions, fiscal policy, and technological change. Specifically, we extend the baseline model to consider the consequences for wealth inequality of heterogeneous saving behavior, alternative marriage practices, wealth taxes, and the growth of labor income. We relate these extensions to the existing literature and demonstrate that each can strongly affect wealth inequality. The final section summarizes the experimental results and draws conclusions.

Computational Model

This section provides a brief overview of the computational model. A preliminary subsection reviews the algebraic approach to the financial accumulation process. The next subsection provides a computational description of the individual wealth holder and a summary of the simulation schedule. A final subsection proposes a baseline parameterization that hews closely to the algebraic approach and can reproduce results that are familiar from the literature on the financial accumulation process. The subsequent section experiments with alternative parameterizations in order to elucidate some key mechanisms underpinning the emergence and persistence of wealth inequality.

Algebra for a Financial Accumulation Process

Before approaching the computational model, consider an algebraic characterization of a basic financial accumulation process. This will provide context for the simulation model and for the computational experiments of the present paper. For the simplest alebra, let wealth accumulation equal the interest income on financial assets (Yunker 1999; Levy and Levy 2003; Isaac 2008; Biondi and Righi 2019). Let $k_{i,t}$ be the wealth of individual i at time t, and let $R_{i,t}$ be the individual's one-period gross return on this wealth. Given an initial wealth $k_{i,0}$, the wealth of individual i after T periods is $k_{i,T} = k_{i,0} \prod_{t=0}^{T-1} R_{i,t}$.

Standard models of the financial accumulation process do not assume that wealth inequality simply reflects heterogeneous investment acumen. Instead, the returns on financial assets are subject to idiosyncratic random shocks. Gross returns

$(R_{i,0}, \ldots, R_{i,T-1})$ are independently drawn from a common distribution of returns with mean $\bar{R} > 0$. This implies that wealth is expected to grow exponentially. For example, at time 0 the expected wealth in period T is $\mathcal{E}_0 k_{i,T} = k_{i,0}\bar{R}^T$.[1] When net returns ($r_{i,t} = R_{i,t} - 1$) are expected to be positive, wealth is expected to grow without bound. This process underpins a simple mathematical intuition for the emergence of wealth inequality.

Consider an economy that comprises a fixed number of wealth owners, who for now may be thought of as dynastic households. At any point in time, the distribution of wealth in the economy reflects the historical experiences of the individual households. In particular, it reflects their differential luck, embodied in their idiosyncratic net returns received over time. These vagaries of fortune underpin the emergent wealth inequality.

In the long run, a simple financial accumulation process with many agents tends to produce a lognormal distribution of agent wealths. A well-known heuristic argument provides good intuition for this outcome (Kalecki 1945; Steindl 1965; Sutton 1997). Suppose gross returns are normally distributed and are small enough that the log of the gross return is a good approximation of the net return ($\ln[R_{i,t}] \approx r_{i,t}$). Logarithmically transform the earlier expression for period T wealth to produce $\ln[k_{i,T}] \approx \ln[k_{i,0}] + \sum_{t=0}^{T-1} r_{i,t-1}$, and then recall that a sum of independently distributed normal variables is itself normally distributed.

Even from an initially egalitarian distribution, this financial accumulation process leads to high wealth inequality. For example, consider a completely egalitarian economy, with a corresponding Gini coefficient of wealth inequality that is 0. As shown in the next section, substantial inequality emerges within a few generations, and eventually this Gini coefficient will approach the maximum possible value of 1.[2] In this sense, one may say that even when comparable individuals continually face comparable opportunities, chance differences in individual luck produce high wealth inequality.

Overview of the Computational Model

The algebraic model is simple and powerful, but it is prohibitively difficult to extend it to encompass many economically relevant considerations. This section therefore advances to a computational model.[3] As a model of the financial accumulation process, it must be true that individuals save out of investment income, which is subject to idiosyncratic returns. Saving remains the only source of wealth accumulation. However, agents in a computational model are easily endowed with a rich set of

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[1] In this literature, it is common to impose a zero-wealth boundary, which effectively truncates R at 0. Whether zero wealth is then likely to be an absorbing boundary for wealth accumulation depends on the model parameterization. For example, it is in Yunker (1999) but it is not in Biondi and Righi (2019).

[2] Biondi and Righi (2019) draw on (Theorem 2 Fernholz and Fernholz 2014) to provide a heuristic proof that this convergence is almost certain, and they provide simulation evidence that the speed of convergence depends on the standard deviation of the individual returns.

[3] Python code for this model is available upon request.

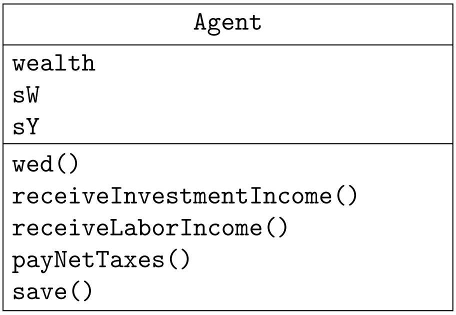

Fig. 2 Features of an individual agent

features. These permit an exploration of the influence of many behavioral, cultural, institutional, and technological features. As shown in the next section, these can have large effects on the evolution of wealth inequality.

Figure 2 summarizes the features of the `Agent` in our model in a simplified classifier diagram (Object Management Group 2017, p.101). Features may be divided into attributes and operations, listed in separate compartments. Conventionally, this diagram additionally signifies an operation by appending parentheses to its name. The diagram displays only those features that are crucial to understanding the individual in this model.

The `wealth` attribute holds the financial wealth of an `Agent`. (Discussions of the computational model will denote individual wealth by `wealth` instead of k.) Individual wealth earns an idiosyncratic gross real return, as in the algebraic model. In the computational model, an individual may also earn labor income (see the "Labor Income and Wealth Inequal" section for details). The computational model also allows for taxes and public assistance (see the "Wealth Taxes and Wealth Inequality" section for details). As in the algebraic model, individual wealth naturally changes due to saving out of investment income, but the computational model additionally allows saving out of other income. The saving rates out of after-tax investment income (s_W) and other income (s_Y) may differ.

Operations are actions performed by or on the individual. In the simulation literature on the financial accumulation process, key operations are `receiveInvestmentIncome()` and `save()`. Although these operations are often left implicit, they incorporate functionality that must be present. As described above, the investment income received by an individual depends on personal wealth, and changes in the individual's wealth depend on personal saving. Additionally, in the present model, an `Agent` may `receiveLaborIncome()`, `payNetTaxes()`, or `wed()`. The next section will provide details of these extensions to the baseline model.

Models of the financial accumulation process focus not on the wealth of any single individual but rather on the distribution of wealth across individuals. Outcomes in the economy determine the individual incomes, but in line with the simulation literature on the financial accumulation process, the model remains agnostic about

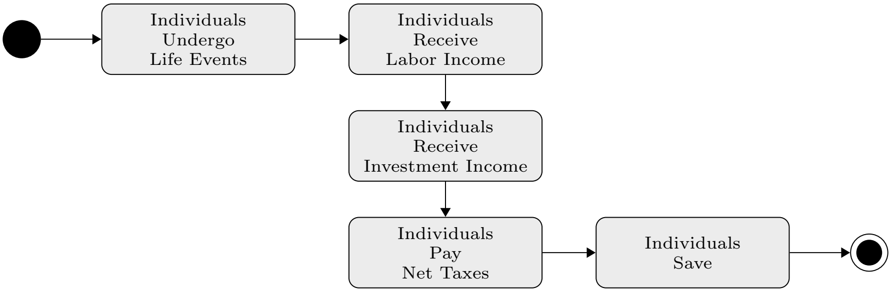

Fig. 3 Simulation schedule

the interactions between individuals.[4] Uncorrelated stochastic returns to wealth underpin the emergence of wealth inequality. The computational model produces an evolving wealth distribution by simulating the changes in wealth experienced by many individual agents over many periods.

Figure 3 presents the simulation schedule for the computational model, inclusive of the extensions of this paper. At the beginning of a year, agents undergo any life events, such as death or marriage. Life events may redistribute income (see the "Marriage Practices and Wealth Inequality" section for details). Agents then earn investment income (as described above) and any labor income (as described in the "Labor Income and Wealth Inequal" section). Agents may pay net taxes on their earnings (as described in the "Wealth Taxes and Wealth Inequality" section). Finally and crucially, each agent saves a fraction of net after tax income. During a simulation, wealths update iteratively based on this saving, using a time scale of one year per iteration.

Baseline Parameterization

This subsection presents a baseline parameterization of the computational model. In order to facilitate easy comparison with the existing literature on the financial accumulation process, the baseline suppresses all model features except investment income and saving. (The subsequent section explores additional model features via key parameter changes.) The resulting computational model simply implements the algebraic model above, for a fixed number of agents.[5] This resembles existing

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[4] This paper does not engage the debate over the boundaries between agent-based models, individual-based models, and econophysics models. Readers should adopt their preferred classification.

[5] Empirically, households appear to experience persistent differences in asset returns (Cao and Luo 2017). Levy and Levy (2003) suggest that individual investment talent may influence investment income, and they therefore extend their baseline model of the financial accumulation process so that $R_{i,t}$ has an agent-specific mean. In this context, however, they end up arguing that chance (i.e., the stochastic component of $R_{i,t}$) must be more important than skill in producing observed wealth inequality. Our baseline implementation of the financial accumulation process therefore focuses on the role of chance. (However, see the "Labor Income and Wealth Inequal" section.)

Table 1 Baseline parameterization

Parameter	Baseline value
Number of agents	5000
Number of iterations	5000
Initial wealth	10
Net-return distribution	$N[0.05, 0.05]$
Mean saving rate from asset income	1.0
Mean saving rate from other income	0.0
Std. saving rate from asset income	0.0
Std. saving rate from other income	0.0
Rate of random marriage (%)	0
Wealth-tax rate (%)	0
Wealth-tax exempted (%)	100
Initial annual wage	0.0
Annual growth rate of labor income	0.0

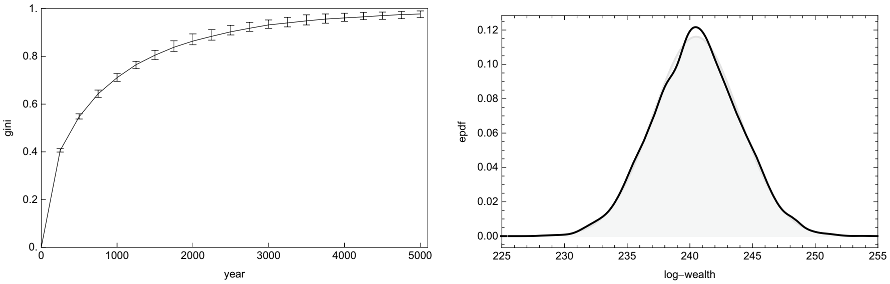

Fig. 4 Evolution of Gini with final log-wealth distribution. Baseline parameterization (see Table 1). First subfigure: median Gini (with 90% bounds) over time; 100 replicates. Second subfigure: kernel density plot for typical final log-wealths, overlaid on a fitted normal distribution

computational models of the financial accumulation process. To drive home this point, Table 1 presents a baseline parameterization developed to align with a recent paper on the financial accumulation process (Biondi and Righi 2019). As shown in the first panel of Fig. 4, under this restricted parameterization, our model yields a close replication of their results.

The first panel of Fig. 4 illustrates the evolution of wealth inequality with a time-series plot of the Gini coefficient. Using results from the baseline parameterization given in Table 1, this chart reports median Gini coefficients along with 90% bounds for 100 replicates. (These results are robust to the changes in the number of agents, the initial level of wealth, the rate of saving from investment income, and the number of replicates.) This Gini coefficient initially rises rapidly, and eventually it closely approaches complete inequality. This reflects the convergence of the wealth distribution to a lognormal distribution, as expected from the algebraic discussion of the "Algebra for a Financial Accumulation Process" section. The second panel of

Fig. 4 confirms this by displaying a smooth kernel histogram for end-of-simulation log-wealths; the shaded region illustrates the fitted normal distribution for the same data. Even without any statistical tests, it is clear that the distribution is approximately lognormal. Correspondingly, a Jarque-Bera test on the log-transformed data fails to reject normality even at the .01% level.

Despite the completely egalitarian initial distribution of wealth, extreme wealth inequality quickly emerges. This is solely due to random variations in the returns to wealth, which are uncorrelated across time and across individuals. Emergent inequality is a key reason that the financial accumulation process attracted the attention of social scientists. Individuals who behave identically and face identical opportunities nevertheless experience dramatically different wealth outcomes. For economists, this calls into question models that rely on the representative-agent assumption, even for researchers who are amenable to assumptions of implausible behavioral uniformity.

Simulation Experiments

The baseline parameterization of the previous section incorporates the two key assumptions of any model of the financial accumulation process: idiosyncratic returns to financial wealth, and positive saving rates from investment income. If rates of return were common across the population, the financial accumulation process would not promote inequality. If savings from investment income were nil, the financial accumulation process would disappear. Under the baseline parameterization, the simulation model illustrates the classic result that a basic financial accumulation process produces extreme wealth inequality.

The baseline parameterization produces results with two clear problems. First, the level of predicted inequality is too high. Second, the level of predicted inequality is not variable. As shown in Fig. 1, country-level inequality outcomes exhibit substantial variability, which is not implied by the baseline model of the financial accumulation process. This section explores the sensitivity of the baseline results to modest increases in the realism of the model. From the diverse array of possible experiments, this section chooses four that have particular economic interest. We will consider one experiment each on the role of idiosyncratic behavior, cultural norms, policy institutions, and technological change. The results demonstrate that wealth inequality is extremely sensitive to the inclusion of any of these factors.

Saving Rates and Wealth Inequality

The baseline simulations demonstrate that extreme wealth inequality emerges in a population of identical individuals due to the financial accumulation process. Naturally, behavioral differences across individuals can also influence the distribution of wealth. The first simulation experiment demonstrates this by introducing idiosyncratic saving rates. This aligns with the turn towards idiosyncratic preferences in the life-cycle literature, which has constituted one effort to coerce life-cycle models to

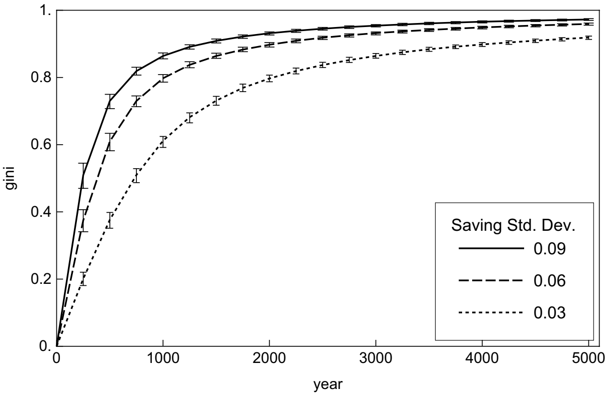

Fig. 5 Cross-sectional saving variability and inequality outcomes. Idiosyncratic saving rates produce high wealth inequality even when asset-return shocks are always shared. The plots display median Gini coefficients (with 90% bounds) over time, for 100 replicates

generate plausible levels of wealth inequality (Hendricks 2007). Intuitively, a household that saves more of its interest income ought on average to experience a more rapid growth of household wealth.

In order to sharpen the results, consider the following modification of the baseline parameterization: introduce idiosyncratic saving rates, but let asset returns be shared across individuals. (Assuming shared asset returns affects the results only modestly but sharpens the intuition for the emergent inequality.) Additionally, in order to facilitate comparisons across scenarios, fix the mean saving rate out of investment income at 0.5 across scenarios. Figure 5 illustrates the outcomes of three scenarios, each with a different dispersion of individual saving rates.[6]

In order to facilitate comparison with the baseline results, this experiment once again begins with a completely egalitarian distribution of wealth. All three scenarios again imply the rapid emergence of wealth inequality, as illustrated in Fig. 5. Since this experiment suppressed idiosyncratic investment returns, the inequality is due entirely to idiosyncratic saving rates. As Fig. 5 illustrates, even a small dispersion of individual saving rates contributes to rapidly rising inequality. This experiment demonstrates that the work on the determinants of wealth inequality must attend to the role of behavior variation as well as to the role of luck.[7]

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[6] For this experiment, each agent's saving rate is time invariant. Individual saving rates are distributed uniformly in an interval. The three intervals producing the illustrated results, as implied by the standard deviations reported in the figure legend, are roughly [0.45, 0.55], [0.40, 0.60], and [0.35, 0.65]. In terms of the algebra of the "Algebra for a Financial Accumulation Process" section, period T wealth becomes $k_{i,T} = k_{i,0} \prod_{t=0}^{T-1} (1 + s_{W,i} r_t)$, where $s_{W,i}$ is the saving rate out of investment income for individual i and r_t is the *shared* net return on investments in period t.

[7] Not evident when comparing Fig. 5 with Fig. 4 is the difference in wealth mobility across time. When inequality is induced by the baseline financial accumulation process, there is always some individual mobility in the economy's wealth distribution. In contrast, relative wealth rankings induced by idiosyncratic saving rates persist. As an aside, note that in contrast to Benhabib et al. (2019), these idiosyncratic saving rates are *not* responses to changes in wealth.

Marriage Practices and Wealth Inequality

Under the baseline parameterization, wealth owners perdure through the entire simulation. This is a typical assumption in both theoretical and computational models of the financial accumulation process (Levy and Levy 2003; Biondi and Righi 2019). Recall from Table 1 that the baseline parameterization specifies 5000 iterations per simulation, which proved more than enough to reveal long-run inequality trends. Given the baseline distribution of investment returns, the time scale of these simulations must be roughly one year per iteration.

In this context, the baseline parameterization may be classified with dynastic models of wealth inequality. Dynastic models of wealth accumulation may be roughly linked to models of wealth accumulation in the presence of assortative marriage, since these models allow persistence of familial wealth (Blinder 1973; Isaac 2014). In this sense, the baseline parameterization of the financial accumulation process represents outcomes under an assortative-marriage cultural practice. In a simple and stylized fashion, the next experiment explores the consequences of a *random-marriage* cultural practice—that is, of marriages that are uncorrelated with wealth.

This experiment is motivated by a computational model of inheritance with dispersive bequests, which argues that marriage practices are an important determinant of long-run wealth inequality outcomes (Isaac 2008). The current experiment demonstrates that this claim can be supported by our simple model of the financial accumulation process, with only a small modification of the baseline parameterization. Of course marriage and bequest institutions are purely implicit in the present model, but this allows a particularly simple stylized implementation: each period, each household has a one 1% chance of averaging its wealth with another randomly chosen household. This requires only minimal changes to the baseline model. Algorithmically, the computational implementation proceeds as follows.

- Randomly select 1% of households.
- Randomly pair up the selected households.
- For each pair, set the wealth of each household to the pair's mean wealth.

Although this is a radically implicit characterization of a marriage institution that allows marriage across class boundaries, it suffices for the current experiment. Figure 6 illustrates the consequences for wealth inequality—both its evolution, and its long-run tendency.[8] The core intuition is simple: when marriages are independent of wealth, large fortunes eventually disperse. A random-marriage cultural practice counteracts the wealth-concentration produced by dynasties and causes wealth to disperse. This experiment uses a 1% chance of dispersal each year, so fortunes have a mean duration of a century. One might therefore expect the effects on wealth inequality to be modest, but in fact they are substantial. These results lend support to the view that marriage practices are an important influence on long-run wealth

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[8] Accordingly, estate taxes with a mean household lifespan of 100 years can have a similar dampening effect on inequality. (Results not shown.)

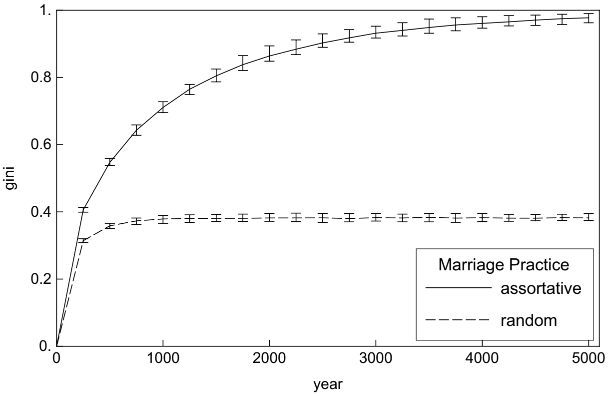

Fig. 6 Assortative versus random marriage. The *assortative* marriage practice retains the baseline parameterization. The *random* marriage practice implements a 1% chance of random household-wealth recombination, which strongly dampens long-run wealth inequality. The plots display median Gini coefficients (with 90% bounds) for 100 replicates

inequality. More specifically, this experiment supports the claim of Isaac (2008) that random marriage strongly limits long-run wealth inequality.

Wealth Taxes and Wealth Inequality

Recall that under the baseline parameterization, saving and the return to wealth are the primary drivers of wealth inequality. The previous experiment demonstrates that marriage practices also affect wealth inequality. A rich literature on the financial accumulation process suggests that tax institutions can also influence wealth inequality (Yunker 1999; Isaac 2008; Biondi and Righi 2019). The next experiment contributes to that literature by exploring the distributional effects of a small wealth tax on the wealthiest 1% of households. Intuitively, wealth taxes should provide some counterforce to wealth concentration, but the size of this effect is difficult to anticipate.[9]

In countries that have tried them, wealth taxes often have proved unpopular and difficult to enforce. Nevertheless, interest in wealth taxes has been growing, even in countries that previously tried and abandoned them.[10] In the USA, a wealth tax has attracted contemporary policy interest: multiple presidential candidates proposed a federal wealth tax in 2020, and two of them subsequently introduced the

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[9] Discussion of these effects by economists frequently have turned to the language of ethics. For example, Mill (1861) urged that it was "fair and reasonable that the general policy of the State should favour the diffusion rather than the concentration of wealth." This section eschews the ethical, pragmatic, and legal questions in favor of a descriptive approach.

[10] Germany eliminated its wealth tax amid arguments that it would have to be confiscatory in order to raise substantial revenues. However, recent policy discussion has shifted towards restoring the wealth tax.

Table 2 Wealth-tax experiment

Parameter	Value(s)
Wealth-tax rate (%)	2
Wealth-tax exempted (%)	99
Mean saving rate from other income	(0.0, 0.5, 1.0)

The wealthiest 1% pay a 2% tax on wealth above the exemption; the bottom 99% are exempt. In this experiment, the other income is the national dividend

Ultra-Millionaire Tax Act in 2021.[11] The following wealth-tax experiment responds to the current policy discussions by choosing a parameterization of the tax that is linked to these discussions. Specifically, it explores whether a modest wealth tax of 2%, structured to exempt all but the top 1% of wealth holders, has any appreciable effect on wealth inequality.

In macroeconomic models that include tax collection, the handling of tax revenues varies widely. The simplest treatment is for the revenues to simply vanish. This may represent a maximally inefficient government, for which the activity of tax collection consumes in resources the entirety of revenues. Next simplest is to redistribute the tax revenues equally to all agents, representing a maximally efficient redistributional program. In between lie leaky-bucket approaches (Okun 1975).

The present experiment adopts the second approach, but the core results are robust across the alternatives. In order to link this to contemporary policy discussions, call it the national-dividend model of redistribution.[12] This experiment introduces two new model parameters: the percentage of the population that is exempt from the wealth tax, and the rate at which wealth not exempted is taxed. Algorithmically, the introduction of taxation and redistribution proceeds as follows.

- Determine the threshold for wealth taxation by exempting 99% of agents.
- Tax wealth beyond the threshold at a rate of 2%.
- Redistribute the tax revenues equally among all agents.

Table 2 specifies the parameter values for the wealth-tax experiment, listing only the deviations from the baseline parameterization. In addition to adding the wealth tax, this experiment considers variations in the proportion of assistance income that is saved. As documented in Table 2, the experiment specifies three different values

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[11] Not only are these proposals controversial, but their constitutionality remains debatable: as a direct tax, a wealth tax may be subject to an apportionment rule (Johnsen and Dellinger 2018).

[12] This accords with the citizenship model of Isaac (2008) and public-service redistribution model of Biondi and Righi (2019). In terms of the algebra in the "Algebra for a Financial Accumulation Process" section and footnote 6, at time t an individual i not subject to the wealth tax now saves $s_W r_{i,t-1} k_{i,t-1} + s_Y a_t$ where a_t is the period t national dividend per capita. Contrast with the insurance model of Isaac (2008) or the welfare model of Biondi and Righi (2019), wherein redistribution targets the poorest agents. Since (as shown below) even a national dividend proves very effective at reducing inequality, this paper does not additionally report results for means-tested redistribution.

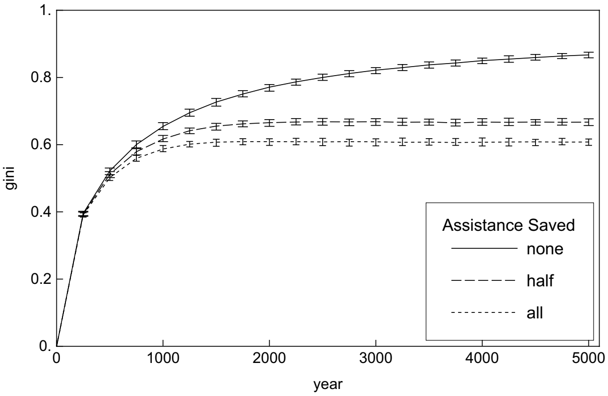

Fig. 7 Wealth inequality with a wealth tax. A wealth tax of 2% applied to the wealthiest 1% of agents dampens wealth inequality, but saving out of assistance influences the size of the result. The plots display median Gini coefficients (with 90% bounds) for 100 replicates

for this parameter. This proves to be an important influence on the effectiveness of public assistance in altering the wealth distribution.

Figure 7 illustrates the core results of this policy experiment. Comparison to the baseline results of Fig. 4 reveals strong effects on the emergence and persistence of wealth inequality. This traces to two sources. First, the wealthiest individuals encounter a reduced after-tax rate of return on wealth. Second, the lognormal distribution has a heavy tail, potentially subjecting a substantial portion of total wealth to the tax. As a result of the wealth tax, the financial accumulation process is mitigated, particularly in the long run.

However, the extent of mitigation depends on the saving behavior of the recipients of public assistance. (This experiment varies only the saving out of assistance, not the saving out of investment income.) If at least half of public assistance is saved, long-run wealth inequality is very strongly mitigated. This is true even though most of agents are exempt from the wealth tax and the tax rate is modest. This experiment suggests that recent wealth-tax proposals may prove effective in promoting their professed goal of reducing wealth inequality, especially if the resulting revenues are efficiently redistributed as public assistance.[13]

Labor Income and Wealth Inequality

The final experiment explores the influence of labor income on wealth inequality. Labor income is ignored in many models of the financial accumulation process. An exception is the model of Biondi and Righi (2019), who find that labor income does not influence long-run wealth inequality. This subsection presents an experiment

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[13] Reducing the efficiency of redistribution has the same effect on individual wealth accumulation as reducing the saving rate from the national dividend.

Table 3 Labor-income parameters

Parameter	Value(s)
Initial annual wage	1.0
Saving rate from other income	25%
Annual growth of labor income	(4.9, 5.0, 5.1%)

Given the initial annual wage, labor income grows at the specified rates. Agents save a quarter of other income, which in this experiment is labor income.

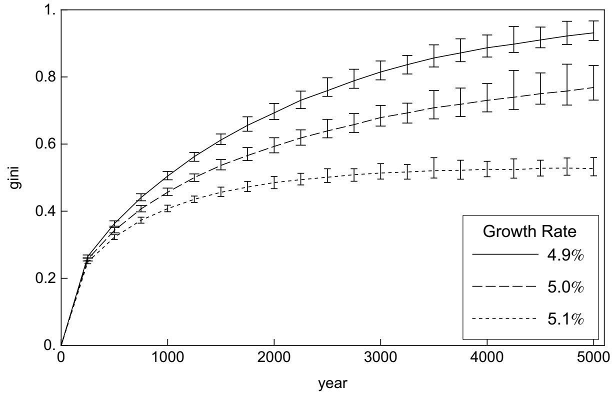

Fig. 8 Labor income growth can constrain wealth inequality. The plots display median Gini coefficients (with 90% bounds) for 100 replicates, given three different annual growth rates of labor earnings and mean asset returns of 5%. Labor earnings effectively limit long-run inequality when wage growth exceeds average investment returns

that overturns this finding. The experiment demonstrates that their result depends on an implicit assumption that, relative to investment income, labor income eventually becomes negligible. After removing that assumption, we find that labor income can strongly influence the distribution of wealth.

As indicated in Table 3, three model parameters support this experiment. The initial level of labor income must be positive, but otherwise it has negligible effect on simulation outcomes. The rate of saving out of labor income similarly must be nonzero: if there is no saving out of labor income, then labor income cannot affect the accumulation of wealth. As long as saving out of labor income is positive, the long-run distribution of wealth is not very sensitive to the particular saving rate. (The speed of convergence is affected, however.) In the reported experiment, individuals save a quarter of their labor income.

Finally, the rate of growth of wage income is the focus of this experiment. This parameter proves crucial to simulation outcomes. To demonstrate the sensitivity of long-run wealth inequality to this parameter, the reported experiment considers

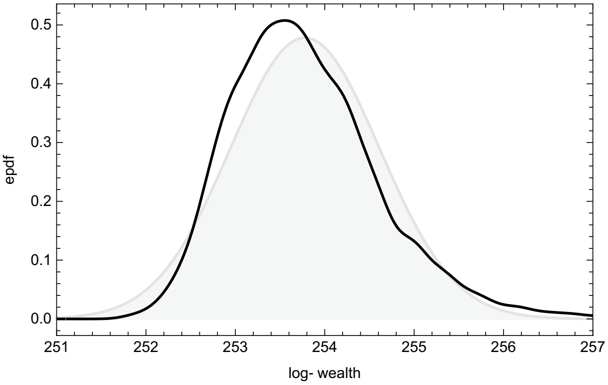

Fig. 9 Wealth distribution with earnings growth. When the labor-earnings growth rate (0.051) slightly exceeds the mean asset return (0.050), the observed wealth distribution diverges from the lognormal. The line plot is a smooth histogram for typical end-of-simulation log-wealths; the shaded area represents a fitted normal distribution

three very similar growth rates of labor income.[14] These are chosen to be very close to the mean return on financial assets, which remains at its baseline value of 5%. Table 3 lists the values considered, and Fig. 8 illustrates the corresponding results.

When the growth rate of labor income keeps up with the asset-return driven growth rate of wealth, wealth inequality increases much more slowly than in the baseline. When it falls even slightly short, the dynamics of wealth inequality resemble those under the baseline. However, if the growth rate of labor income is even slightly greater than the mean return on wealth, the dynamics of wealth inequality change substantially, and long-run inequality is strongly moderated.

The results of this experiment comport nicely with claims in the growth literature. The extensive monograph of Piketty (2014) argues that wealth inequality must increase when real asset returns exceed the economy's real growth rate. The current experiment demonstrates that such effects emerge even in an extremely simple model of the financial accumulation process.

This result may be usefully compared to another result in the literature on the financial accumulation process. Levy and Levy (2003) add an unexplained exogenous rising lower bound on wealth to their model of the financial accumulation process. They show that this addition affects the distribution of individual wealths, which no longer trends towards a lognormal distribution. Instead, the distribution of individual wealths eventually approximates a Pareto distribution. In light of their result, one may anticipate that the addition of ongoing income growth will similarly push the wealth distribution away from the lognormal, particularly when wage growth exceeds the mean return on investments.

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[14] For the reported experiment, the labor-income growth rates are deterministic. Allowing transient shocks produces similar results.

Figure 9 confirms this conjecture for a wage growth rate (0.051) that is just above the mean asset return (0.050). The chart is a smooth histogram for a typical end-of-simulation log-wealth distribution. When compared to the baseline results in Fig. 4, this figure demonstrates that the labor-income growth contributes a definite skew to the wealth distribution. The shaded area displays the PDF of a normal distribution, fitted to the observed data. The best fit normal distribution displays substantial deviation from the observed distribution. Correspondingly, a Jarque-Bera test rejects normality even at the 10% level.

Conclusion

Modern economies exhibit substantial wealth inequality; most countries have wealth Gini coefficients between 0.6 and 0.8. The gap between this empirical evidence and the much lower predictions of life-cycle savings models is the *wealth concentration puzzle*. In contrast, models of the financial accumulation process predict very high long-run wealth inequality, even higher than the observed levels. This paper augments a baseline computational model of the financial accumulation process in order to explore some key influences on long-run wealth inequality.

The financial accumulation process is the stochastic, multiplicative process governing financial assets. We develop a baseline parameterization of our model that reproduces classic results for the financial accumulation process. Under the baseline parameterization, the long-run distribution of wealth reflects saving out of the idiosyncratic investment returns received by households. Over time, ex ante identical households experience very different levels of wealth accumulation. As explicated in the "Algebra for a Financial Accumulation Process" section, this necessarily engenders the highly unequal distribution of wealth associated with the financial accumulation process.

However, as shown in Fig. 1, country-level inequality outcomes exhibit substantial variability. This is not implied by the baseline model of the financial accumulation process. The computational experiments in this paper demonstrate that small, plausible extensions of the baseline model strongly affect the distribution of wealth. Specifically, this paper considers idiosyncratic saving behavior, cultural practices in the form of stylized marriage institutions, public policy in the form of a modest wealth tax, and technological change as proxied by wage growth. We find that these modest concessions to realism have strong implications for the level of wealth inequality produced by the financial accumulation process.

By means of a sequence of computational experiments, this paper demonstrates that the baseline results are fragile. The first experiment suppresses idiosyncratic returns, which on its own would suppress the emergence of wealth inequality. However, this experiment extends the baseline model by allowing heterogeneous saving rates. This produces a comparable, highly unequal, distribution of wealth. Empirically, saving rates do differ across agents, but asset returns are also partially idiosyncratic. The results of this simulation experiment suggest that future applied work may profit from attempts to sort out the relative contribution of each.

In the baseline simulation model, the wealth ownership perdures perpetually. As explicated in the "Marriage Practices and Wealth Inequality" section, we may interpret this as a stylized dynastic model of wealth accumulation, which links the baseline model to models of wealth accumulation in the presence of assortative marriage (Blinder 1973). Our second experiment extends the baseline model so as to implement a stylized representation of random marriage. The results support Isaac (2008), who argues that dispersive bequests combine with random marriage practices to disrupt the wealth accumulation process. Marriage and bequest practices in our model are intentionally implicit, but this experiment nevertheless provides a qualitative confirmation of previous results that marriage practices influence long-run wealth inequality.

A third experiment considers the inequality consequences of introducing a modest wealth tax, which is redistributed. It is unsurprising that redistributionist policy can affect inequality. However, this experiment demonstrates a surprising sensitivity to very modest policy interventions. This experiment structures the wealth tax to resemble contemporary policy proposals, and it finds large effects even when the tax is apparently modest in scope and size. A substantial reduction in long-run wealth inequality results from a 2% wealth tax, even when it falls on wealth only on wealth above a very high threshold. Such a tax effectively reduces the after-tax return on wealth above the threshold, which increases the likelihood that less fortunate agents can eventually catch up. When the resulting revenues are redistributed, the extent of private saving out of public assistance payments also matters for long-run wealth inequality.

A final experiment contests a recent, puzzling claim that, labor income has little effect on the distribution of wealth (Biondi and Righi 2019). This experiment demonstrates that wage growth has the potential to powerfully mitigate wealth inequality. We show how to align this experiment with a well-known argument of Piketty (2014) that wealth inequality increases when asset returns exceed the growth rate of labor income. This experiment supports that claim in what may be the simplest possible setting: simply adding labor income growth to the baseline model of the financial accumulation process produces the Piketty result. This experiment also finds that this is a knife-edge proposition. As a result, small changes in the growth rate of labor income can produce large long-run effects on wealth inequality.

The inequality implications of the financial accumulation process depend heavily on behavioral, cultural, institutional, and technological substrata. By varying each of these separately, this paper finds that saving behavior, marriage practices, taxation institutions, and labor-income growth each substantially influence long-run inequality. Recent evidence suggests that marriage is becoming increasingly assortative by wealth and that wage income is falling as a share of GDP (Greenwood et al. 2014; Piketty 2014). If these trends continue, our model suggests that wealth inequality will continue to rise. However, it also suggests that a modest wealth tax can strongly constrain long-run inequality.

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