



Tail risk management and the skewness premium

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Abstract

In this paper, we analyze how tail risk impacts both asset prices and the optimal asset allocation. For this purpose, we consider an equilibrium model with investors exhibiting an empirically well-justifiable decreasing relative risk aversion (DRRA) and different investment horizons. In contrast to the seminal CAPM, two fund separation does no longer hold, and investors not only regard one risk measure such as the standard deviation but additionally care for the size of tail risk. The shorter the investment period, the more prone they are to negatively skewed returns. In particular, short-term investors not only hold a lower equity ratio than (else equal) long-term investors do, but they also reduce the fraction of assets with negative tail risk. Consistently, the more short-term investors are in a market, the higher the tail risk premium is, i.e., the additional expected return due to skewness beyond a given standard deviation. Consequently, these theoretical findings allow us to draw empirical predictions about (i) the drivers of the skewness premium, (ii) characteristics for markets in which the premium is especially severe, and (iii) the optimal investors' asset allocation.

Keywords Asset allocation · Jump diffusion process · DRRA · Skewness premium · CAPM · Investment horizons

JEL Classification G11 · G12

Introduction

According to the seminal capital asset pricing model (CAPM) by Sharpe (1964), Lintner (1965), and Mossin (1966), optimal investment decisions can be based only on expected returns and standard deviations of the overall portfolio return, but higher moments do not matter at all. To obtain this highly tractable result, the model requires specific assumptions for the return distribution and/or the utility functions of investors. Empirical capital market research [see for example Harvey and Siddique (2000), Ang et al. (2006), Boyer et al. (2010)], however, provides evidence that investors do care about further risk characteristics, which the standard deviation does not fully include. An example of these risk characteristics is tail risk or catastrophe risk. The danger of losing a large part of the portfolio's wealth with

ruinous consequences for the individual with a very small probability is for some investors less attractive than a portfolio with normally distributed returns despite identical means and standard deviations as the other portfolio. Hence, these empirically observed preferences favor an additional risk premium for tail risk or, technically speaking, a skewness premium for (systematic) negatively skewed asset returns.

Since many theoretical applications are based on a CAPM-like notion, skewness premia cannot be analyzed in these approaches as there is no premium for skewness at all due to the consciously imposed set of assumptions. In practice, however, the optimal portfolio choice requires a careful treatment of tail risks and—in case of positive skewness premia—the voluntary selection of skewed returns. Because of a missing established framework for tail risk rather than total market risk, we lack an understanding of the drivers of a potential skewness premium. Moreover, it is doubtful whether each investor should still hold a part of the market portfolio in equilibrium, such as concluded by the CAPM if the extended case that investors not only care for market risk but also tail risk is present.

The aim of this paper is to provide an insightful asset pricing model to explain skewness premia. With the help

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of this model, we want to deal with the questions: (i) what drives a high skewness premium and (ii) which optimal portfolio should heterogeneous investors choose with regard to tail risk.

Our findings are as follows: first, with decreasing relative risk aversion (DRRA) of investors, we can obtain a positive skewness premium that decreases with the investors' investment horizon. Second, allowing for heterogeneous investment horizons, our asset pricing model implies a higher equity fraction of long-term investors than short-term investors. Third, contrary to the conclusion of the CAPM, we explain that the composition of the risky portfolio differs among the heterogeneous investors: Long-term investors invest a higher fraction in assets associated with catastrophe risk than short-term investors do. Fourth, we show that the composition of market participants affects the size of the skewness premium: The skewness premium rises with the proportion of short-term investors in the economy.

The paper is related to the rich literature dealing with the implications of skewed returns in financial applications. Based on Arditti (1967) and Scott and Horvath (1980), who provide evidence under general assumptions that investors have a preference for positive skewness, the role of skewness in the context of asset pricing has been studied in numerous papers. For example, Kraus and Litzenberger (1976) extend the CAPM and stress the relevance of systematic skewness in a three-moment CAPM. Mitton and Vorkink (2007) introduce a one-period model with agents having heterogeneous preferences for skewness and conclude that idiosyncratic skewness is priced. Further, the impact of higher moments on the optimal asset allocation decision of investors is highlighted in several studies. These include among others Liu et al. (2003), Wu (2003), Jondeau and Rockinger (2006), and Harvey et al. (2010).

Our paper contributes to this literature by showing how tail risk affects (i) the portfolio decision of investors who differ in terms of their investment horizon and (ii) the equilibrium prices, which lead to skewness premia depending on investors' investment horizons in a multi-period framework. We deviate from the typical assumption of constant relative risk aversion (CRRA) or a Taylor series expansion to approximate utility functions. Instead, we assume investors with DRRA and thereby capture the often observed behavior of market participants to choose higher equity fractions when their investment periods are longer. In principle, this outcome is in line with the frequently proposed investment rule by practitioners to allocate a percentage of wealth in risky assets equal to 100 minus the investor's age. While this is arguably a somewhat stylized investment advice, it takes up the idea that younger investors with longer investment periods (should) have higher equity fractions.

The remainder of the paper is organized as follows: Section 2 introduces the asset pricing model and specifies the

return generating processes as well as the utility function of investors. The equilibrium characteristics for economies with homogeneous and heterogeneous investment horizons of investors are described in Sect. 3. Section 4 deals with selected parameter modifications as robustness checks. Section 5 concludes.

Model description

Market and time structure

We consider an economy from time t_0 to time T , which consists of a risk-free asset, paying the risk-free rate r_f per annum (p.a.), and two risky stocks. While both stocks have identical expected returns and standard deviations, the returns of Stock 1 are normally distributed, and the returns of Stock 2 exhibit negative skewness. The left-skewed return distribution implies that the probability of large negative returns exceeds the probability of equally large positive returns. Both stocks pay out their respective fundamental values $\tilde{S}_{1,T}$ and $\tilde{S}_{2,T}$ at time T and nothing at other points in time.

Before time T , equilibrium prices of the two stocks are formed under consideration of the known distributions of the two stocks by market participants. At time t_0 , the fundamental values S_{1,t_0} and S_{2,t_0} of Stock 1 and 2, respectively, are normalized to 1, which might differ from the corresponding (trading) prices p_{1,t_0}^{EQ} and p_{2,t_0}^{EQ} in equilibrium. To ensure that the aggregate fundamental value of stocks is equal to 1, the time t_0 supply \bar{X}_k of Stock $k = 1, 2$ is given by

$$\bar{X}_1 = \bar{X}_2 = 0.5. \quad (1)$$

Investors enter the economy at time t_0 , who can either purchase units of the two stocks or invest in the risk-free asset. We assume that the demand for stocks is characterized by perfect competition, which implies that all investors are price takers so that we can use the concept of a representative investor following, e.g., Huang and Litzenberger (1988). This way, we can determine equilibrium prices as if there were only one investor in the economy, endowed with total endowment of (infinitely many) market participants.

To allow for different investment horizons, we consider two types of representative investors: representative long-term Investor L with an investment horizon until time T , and representative short-term Investor S with an investment horizon until time T_S , which is prior to time T . Both investors thereby pursue a buy-and-hold strategy and cannot make any portfolio adjustments throughout their respective investment periods. For tractability, stocks are infinitely divisible and no trading costs exist.

The representative investors at time t_0 are initially endowed with cash, whereby the total monetary



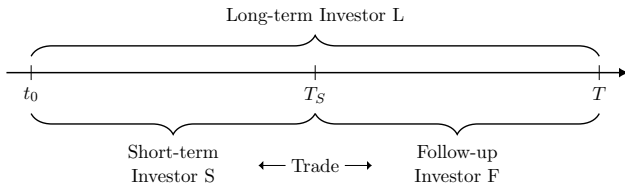


Fig. 1 Investment horizons of the representative investors

endowment \bar{X}_{Cash} equals the aggregated fundamental value of all shares outstanding:

$$\bar{X}_{Cash} = \bar{X}_1 \cdot S_{1,t_0} + \bar{X}_2 \cdot S_{2,t_0} = 1. \tag{2}$$

Against payment of the equilibrium prices p_{1,t_0}^{EQ} and p_{2,t_0}^{EQ} , the investors receive units of the stocks at time t_0 . While Investor L obtains the fundamental values $\tilde{S}_{1,T}$ and $\tilde{S}_{2,T}$ at time T , Investor S leaves the economy at time T_S , offering the stock holdings X_1^S in Stock 1 and X_2^S in Stock 2 without limit.

Analogously to time t_0 , stock supply is hence perfectly inelastic, and we consider representative follow-up Investor F to enter the economy at time T_S . This investor, with residual investment horizon $T - T_S$, is endowed with \bar{X}_{Cash}^F , which equals the sum of fundamental values of shares outstanding at time T_S :

$$\bar{X}_{Cash}^F = X_1^S \cdot S_{1,T_S} + X_2^S \cdot S_{2,T_S}. \tag{3}$$

This treatment assures equal conditions as at time t_0 . Investor F purchases the available units of stocks, paying the time T_S equilibrium prices p_{1,T_S}^{EQ} and p_{2,T_S}^{EQ} to Investor S. Figure 1 summarizes the investment horizons of the investors in the economy.

Before turning to the derivation of the equilibrium prices, we set out the statistical stock properties and specify the utility function of investors in the following.

Statistical stock properties

We consider two well-established stochastic processes for Stock 1 and Stock 2: a pure diffusion model (geometric Brownian motion) and a jump diffusion model (compensated geometric Brownian motion with jumps). The considered return generating models enable us to construct return distributions with equal expected return and standard deviation of returns but which differ in terms of their skewness.

For Stock 1, the well-known stochastic differential equation in continuous time of the standard geometric Brownian motion reads

$$\frac{dS_{1,t}}{S_{1,t}} = \mu \cdot dt + \sigma \cdot dz_t. \tag{4}$$

Equation (4) denotes the instantaneous discrete return of Stock 1 over an infinitesimally small time step dt . μ and σ denote the expected discrete return and standard deviation (p.a.), respectively, and z_t a standard Wiener process. The log return representation follows from Ito's lemma [see, e.g., Hull (2017)] and is given by

$$\ln\left(\frac{\tilde{S}_{1,T}}{S_{1,t_0}}\right) = \left(\mu - \frac{1}{2}\sigma^2\right) \cdot T + \sigma \cdot \tilde{z}_T, \tag{5}$$

where \tilde{z}_T is normally distributed with zero mean and variance T .

Following Merton (1976), we construct the jump diffusion process by adding a Poisson distributed jump component to the stochastic differential equation in Eq. (4). The discrete Poisson probability distribution assigns a probability to the number of events, or jumps, occurring during a fixed period of time and average event rate. For the sake of simplicity, we consider a non-stochastic negative jump of size $\phi < 0$ in the logarithm of the stock price in case a jump occurs, such that the stochastic differential equation of Stock 2, with jump-adjusted drift μ_J and volatility σ_J of the diffusion term, reads

$$\frac{dS_{2,t}}{S_{2,t}} = \mu_J \cdot dt + \sigma_J \cdot dz_t + (e^\phi - 1) \cdot dJ_t. \tag{6}$$

The Poisson process is denoted by J_t and in case a jump occurs, the stock price jumps from $S_{2,t}$ to the lower value $e^\phi \cdot S_{2,t}$. The volatility σ_J denotes the standard deviation (p.a.) of returns conditional on no Poisson event to occur. dJ_t and dz_t are assumed to be independent. To ensure that the expected discrete return $\mathbb{E}\left(\frac{dS_{2,t}}{S_{2,t}}\right)$ equals $\mu \cdot dt$, the jump-adjusted drift μ_J in Eq. (6) needs to be set to

$$\mu_J := \mu - \lambda \cdot (e^\phi - 1),$$

where λ denotes the intensity of the Poisson process, i.e., the instantaneous jump probability is $\lambda \cdot dt$. By subtracting the expected (negative) discrete jump size $(e^\phi - 1)$ from μ , we have $\mu_J > \mu$ and the expected discrete return of the jump diffusion process coincides with the expected discrete return of the geometric Brownian motion in Eq. (4) and equals μ .

Similar to Eq. (5), following the Ito formula for jump diffusion processes given by Cont and Tankov (2003), the log return representation of the process considered for the fundamental value $S_{2,t}$ reads



$$\ln\left(\frac{\tilde{S}_{2,T}}{S_{2,t_0}}\right) = \left(\mu_J - \frac{1}{2}\sigma_J^2\right) \cdot T + \sigma_J \cdot \tilde{z}_T + \phi \cdot \tilde{J}_T, \tag{7}$$

where \tilde{J}_T follows a Poisson distribution with intensity $\lambda \cdot T$ and denotes the number of jump occurrences from time t_0 to time T . The variance of the log return from time t_0 to time T is

$$\text{Var}\left(\ln\left(\frac{\tilde{S}_{2,T}}{S_{2,t_0}}\right)\right) = (\sigma_J^2 + \lambda \cdot \phi^2) \cdot T. \tag{8}$$

As we aim to construct two processes with equal standard deviation, we set the volatility σ_J of the diffusion term such that the total standard deviation of the return of Stock 2, i.e., the square root of Eq. (8) per year, is equal to σ , the annual standard deviation of Stock 1. Solving for the volatility σ_J of Stock 2 conditional on no Poisson event to occur, we get

$$\sigma_J = \sqrt{\sigma^2 - \lambda \cdot \phi^2}, \text{ with } \sigma^2 - \lambda \cdot \phi^2 \geq 0. \tag{9}$$

For given volatility σ , jump intensity λ , and jump size ϕ , Eq. (9) for σ_J ensures equal standard deviations of returns of both Stock 1 and Stock 2. The skewness of the returns generated by the jump diffusion process is

$$\text{Sk}\left(\ln\left(\frac{\tilde{S}_{2,T}}{S_{2,t_0}}\right)\right) = \frac{\lambda}{\sqrt{T}} \cdot \frac{\phi^3}{\sqrt{\sigma_J^2 + \lambda \cdot \phi^2}^3}. \tag{10}$$

Contrary to the normally distributed log returns of the pure diffusion process from Eq. (5), the skewness of the jump diffusion process differs from zero. Due to $\phi < 0$, we always have a negative skewness. Equation (10) shows that the skewness converges to zero with increasing T . Accordingly, the distribution of returns generated by the jump diffusion process converges to the distribution of returns of the pure diffusion process for large T .

Investors

We regard rational investors with decreasing relative risk aversion (DRRA). Contrary to constant relative risk aversion (CRRA), DRRA captures the often observed behavior that investors with a long investment horizon have a higher equity fraction, else equal, than investors with a short horizon. While the availability of closed-form solutions makes CRRA a common choice to model utility in many applications, it can be easily shown that the optimal equity fraction, which results from a static portfolio optimization

problem, is independent of the investment horizon of the investor. DRRA utility, however, implies that the wealthier an investor, the less risk averse the investor is, which results in an increasing equity fraction with the investment horizon [see Thorley (1995)].

Following Merton (1971), we assume one functional form of the hyperbolic absolute risk aversion (HARA) family to model utility of investors. Depending on final wealth W at the end of the investment horizon of arbitrary Investor i , the utility function reads

$$U^i(W) = \frac{(W - \eta^i)^{1-\gamma}}{1-\gamma}, \tag{11}$$

where γ and η^i specify the risk aversion of Investor i with $\gamma > 0 (\neq 1)$. The relative risk aversion (RRA) coefficient for $U^i(W)$ is given by

$$\text{RRA}^i(W) = -\frac{\frac{\partial^2 U^i(W)}{\partial W^2}}{\frac{\partial U^i(W)}{\partial W}} \cdot W = \gamma \cdot \left(\frac{W}{W - \eta^i}\right).$$

It follows that η^i specifies whether Investor i exhibits increasing or decreasing RRA with wealth. While Eq. (11) reduces to CRRA utility for $\eta^i = 0$, utility is characterized by DRRA for $\eta^i > 0$.

Sharpe (2007) refers to η^i as the minimum required level of wealth, which seizes the fact that the utility function in Eq. (11) is only defined for $W > \eta^i$. The condition ensures positive and diminishing marginal utility. He stresses that only values exceeding η^i “should be considered” (p. 23). In a static portfolio optimization problem, the application of the DRRA utility function hence implies an upper bound of possible equity fractions, which assures that $W > \eta^i$. Following Ogaki and Zhang (2001), we refer to η^i as the investor-specific subsistence level of wealth.

We assume investors to have equal γ and set η^i as a fraction of monetary endowment of Investor i according to

$$\eta^i = \eta \cdot \bar{X}_{Cash}^i, \tag{12}$$

with $\bar{X}_{Cash}^i > 0$. Equation (12) assures that investors make identical investment decisions, independent of initial wealth for given subsistence wealth η .

With investors’ utility depending on final wealth at time T , intermediate wealth levels prior to time T are not evaluated by the investor: While the investors require a minimum level of wealth at the end of their investment periods, they are willing to endure substantial portfolio losses in the interim. Specifically, we can assume shorter-orientated investors pursuing a buy-and-hold strategy with capital needs in the nearer future at time T for consumption



purposes or longer-term investors saving for retirement with a specific (minimum) capital requirement at the beginning of their pension period at time T .¹

With utility of investors depending on final wealth, we now turn to the specific expressions for final wealth of the investors in the economy.

Long-term Investor L Final wealth of representative Investor L with investment horizon T reads

$$W_T^L(X_1^L, X_2^L; p_{1,t_0}, p_{2,t_0}) = X_1^L \cdot \tilde{S}_{1,T} + X_2^L \cdot \tilde{S}_{2,T} + X_{Cash}^L \cdot e^{r_f \cdot T}.$$

In this representation, X_1^L and X_2^L denote the number of shares of Stock 1 and Stock 2 purchased at time t_0 and $\tilde{S}_{1,T}$ and $\tilde{S}_{2,T}$ the fundamental stock values at time T , respectively, which follow from Eqs. (4) and (7). The investment X_{Cash}^L at the risk-free rate r_f can be expressed as the difference of initial monetary endowment \bar{X}_{Cash}^L and expenditures for the risky assets with trading prices p_{1,t_0} and p_{2,t_0} . Including this budget constraint, we can write final wealth of Investor L as

$$\begin{aligned} W_T^L(X_1^L, X_2^L; p_{1,t_0}, p_{2,t_0}) &= X_1^L \cdot S_{1,t_0} \cdot e^{(\mu - \frac{1}{2}\sigma^2) \cdot T + \sigma \cdot \tilde{z}_T} \\ &+ X_2^L \cdot S_{2,t_0} \cdot e^{(\mu_j - \frac{1}{2}\sigma_j^2) \cdot T + \sigma_j \cdot \tilde{z}_T + \phi \cdot \tilde{J}_T} \\ &+ \left(\bar{X}_{Cash}^L - X_1^L \cdot p_{1,t_0} - X_2^L \cdot p_{2,t_0} \right) \cdot e^{r_f \cdot T}. \end{aligned} \tag{13}$$

Short-term Investor S Similarly to long-term Investor L, we can write final wealth of representative Investor S with investment horizon $T_S (< T)$ as

$$\begin{aligned} W_{T_S}^S(X_1^S, X_2^S; p_{1,t_0}, p_{2,t_0}) &= X_1^S \cdot \tilde{p}_{1,T_S} + X_2^S \cdot \tilde{p}_{2,T_S} \\ &+ \left(\bar{X}_{Cash}^S - X_1^S \cdot p_{1,t_0} - X_2^S \cdot p_{2,t_0} \right) \cdot e^{r_f \cdot T_S}. \end{aligned} \tag{14}$$

¹ Whether the investor’s utility depends only on wealth at time T or interim shortfalls impact the investor’s utility and/or the investor is not willing to endure substantial losses during the investment period is subject to controversial discussions [see, e.g., Bodie (2015)]. Following the discretionary wealth hypothesis by Wilcox (2003), for example, which builds on the investment criteria by Kelly (1956) and Markowitz (1952), risk aversion of investors is determined by the necessity to avoid “shortfalls, not only at some far-off ending period, but all along the way” (p. 62). Maximizing the expected periodic log return of discretionary wealth, investors do not tolerate a periodic (e.g., monthly or annual) percentage loss exceeding a pre-specified value such that the optimal equity fraction is independent of investors’ investment horizons in a static portfolio optimization problem with repeating periods (Wilcox 2020). For a one-periodic investment horizon, the investment policy by Wilcox is equivalent to the DRRA utility function for the special case of $\gamma = 1$ and $\eta = W_0 \cdot \left(1 - \frac{1}{L}\right)$ in terms of identical asset allocation implications.

While the budget constraint of Investor S reads analogously to the budget constraint of Investor L, Investor S does not receive fundamental values of the stocks at time T_S , but trading prices \tilde{p}_{1,T_S} and \tilde{p}_{2,T_S} , which come from trading with Investor F.

Follow-up Investor F With an investment period from time T_S to time T , representative Investor F receives the fundamental values $\tilde{S}_{1,T}$ and $\tilde{S}_{2,T}$ at time T . The expenditures at time T_S depend on prices \tilde{p}_{1,T_S} and \tilde{p}_{2,T_S} , such that we can write final wealth as

$$\begin{aligned} W_T^F(X_1^F, X_2^F; p_{1,T_S}, p_{2,T_S}) &= X_1^F \cdot \tilde{S}_{1,T} + X_2^F \cdot \tilde{S}_{2,T} \\ &+ \left(\bar{X}_{Cash}^F - X_1^F \cdot \tilde{p}_{1,T_S} - X_2^F \cdot \tilde{p}_{2,T_S} \right) \cdot e^{r_f \cdot (T - T_S)}, \end{aligned} \tag{15}$$

with monetary endowment \bar{X}_{Cash}^F of Investor F given in Eq. (3).

Derivation of the equilibrium

The equilibrium is characterized by all representative investors to make expected utility-maximizing investment decisions and by markets at time t_0 and time T_S for both stocks to clear. Hence, the optimization problem of Investor i at time t_0 for a given price combination p_1 and p_2 of the two stocks at the trading date reads

$$\max_{X_1^i, X_2^i} \mathbb{E}_{t_0} \left[U^i \left(W \left(X_1^i, X_2^i; p_1, p_2 \right) \right) \right]. \tag{16}$$

With DRRA utility from Eq. (11), a closed-form solution for the optimal number of units of stocks bought does not exist so that we solve Eq. (16) by simulation.² In this way, the demand functions $\chi_1^i(p_1, p_2)$ and $\chi_2^i(p_1, p_2)$ of Investor i for Stock 1 and Stock 2 are derived. Else equal, demand for a stock is a decreasing function of the respective price of the stock and increasing in the other stock’s price.

² To generate distributions of final wealth, we simulate 50,000 values for the diffusion term \tilde{z}_T for the two stocks. For numerical stability, we explicitly take into account the number of jumps from zero to \tilde{J}_T of Stock 2, rather than simulating the Poisson-distributed number of jumps \tilde{J}_T for each realization of \tilde{z}_T . The maximum number of jumps considered with respect to time T , \tilde{J}_T , satisfies that the cumulative probability $\sum_{\xi=0}^{\tilde{J}_T} p(\xi; \lambda \cdot T)$ of all jumps included just exceeds 99.9999%, whereby $p(\xi; \lambda \cdot T)$ denotes the probability function of the Poisson distribution with number of jumps ξ and intensity $\lambda \cdot T$:

$$p(\xi; \lambda \cdot T) = \frac{(\lambda \cdot T)^\xi}{\xi!} \cdot e^{-\lambda \cdot T}.$$

We add the residual probability $1 - \sum_{\xi=0}^{\tilde{J}_T} p(\xi; \lambda \cdot T)$ to the respective probability of \tilde{J}_T . The expected utility is then derived as the average of the utilities weighted by the occurrence probability $p(\xi; \lambda \cdot T)$ of the respective jumps ξ .



Table 1 Parameters

Parameter	μ (%)	σ (%)	σ_j (%)	ϕ (%)	λ	r_f (%)	γ	η
Value	10	20	17.61	-30	0.1	2	4	0.8

Table 1 shows the set of parameters utilized in the following. μ denotes the expected return and σ the (total) standard deviation of returns of Stock 1 and Stock 2. σ_j is the standard deviation of returns of Stock 2 conditional on no jump. Jump size ϕ in the logarithm of the price and jump intensity λ characterize the jump component of Stock 2. r_f denotes the risk-free rate (p.a.). The utility function in Eq. (11) is defined by γ and η , whereby the investor's specific subsistence level of wealth η^i depends on initial endowment as given in Eq. (12)

Specifically, since Investor S receives trading prices \tilde{p}_{1,T_S} and \tilde{p}_{2,T_S} at time T_S , which result from trade with follow-up Investor F, a recursive solution of the model is required to determine equilibrium prices p_{1,t_0}^{EQ} and p_{2,t_0}^{EQ} . In line with the notion of perfect competition, it is thereby assumed that Investor S cannot affect prices at time T_S by choosing the number of stocks bought at time t_0 strategically.

At time T_S , Investor F maximizes expected utility depending on final wealth from Eq. (15) by choosing optimal stock holdings for prices p_{1,T_S} and p_{2,T_S} :

$$\max_{X_1^F, X_2^F} \mathbb{E}_{T_S} \left[U^F \left(W_T^F \left(X_1^F, X_2^F; p_{1,T_S}, p_{2,T_S} \right) \right) \right].$$

Thereby, we derive the corresponding demand functions for Stock 1 and Stock 2, $\chi_1^F(p_{1,T_S}, p_{2,T_S})$ and $\chi_2^F(p_{1,T_S}, p_{2,T_S})$. Equilibrium prices p_{1,T_S}^{EQ} and p_{2,T_S}^{EQ} are the price combination such that the demand of Investor F for both stocks at time T_S equals the respective supply of Investor S. The market clearing conditions at time T_S read

$$\chi_1^F(p_{1,T_S}^{EQ}, p_{2,T_S}^{EQ}) - X_1^{S,EQ} \stackrel{!}{=} 0$$

$$\chi_2^F(p_{1,T_S}^{EQ}, p_{2,T_S}^{EQ}) - X_2^{S,EQ} \stackrel{!}{=} 0.$$

In this representation, $\chi_1^F(p_{1,T_S}^{EQ}, p_{2,T_S}^{EQ})$ and $\chi_2^F(p_{1,T_S}^{EQ}, p_{2,T_S}^{EQ})$ are the demand functions evaluated at the equilibrium prices at time T_S and denote the numbers of units of Stock 1 and Stock 2 Investor F purchases in equilibrium. Supply of the two stocks by Investor S in equilibrium is denoted by $X_1^{S,EQ}$ and $X_2^{S,EQ}$ and equal to the priorly chosen units of Investor S:

$$X_1^{S,EQ} := \chi_1^S(p_{1,t_0}^{EQ}, p_{2,t_0}^{EQ})$$

$$X_2^{S,EQ} := \chi_2^S(p_{1,t_0}^{EQ}, p_{2,t_0}^{EQ}),$$

i.e., $\chi_1^S(p_{1,t_0}^{EQ}, p_{2,t_0}^{EQ})$ and $\chi_2^S(p_{1,t_0}^{EQ}, p_{2,t_0}^{EQ})$ are the demand functions of Investor S evaluated at the equilibrium prices at time t_0 .

At time t_0 , Investor S and Investor L maximize expected utility depending on final wealth given in Eqs. (13) and (14), by choosing the optimal stock holdings for price combinations p_{1,t_0} and p_{2,t_0} . The optimization problems of Investor L and Investor S read:

$$\max_{X_1^L, X_2^L} \mathbb{E}_{t_0} \left[U^L \left(W_T^L \left(X_1^L, X_2^L; p_{1,t_0}, p_{2,t_0} \right) \right) \right] \quad (17)$$

$$\max_{X_1^S, X_2^S} \mathbb{E}_{t_0} \left[U^S \left(W_{T_S}^S \left(X_1^S, X_2^S; p_{1,t_0}, p_{2,t_0} \right) \right) \right]. \quad (18)$$

From the utility maximization problems follow the demand functions $\chi_1^L(p_{1,t_0}, p_{2,t_0})$, $\chi_2^L(p_{1,t_0}, p_{2,t_0})$ and $\chi_1^S(p_{1,t_0}, p_{2,t_0})$, $\chi_2^S(p_{1,t_0}, p_{2,t_0})$ for Stock 1 and Stock 2, which depend on prices at time t_0 . For equilibrium prices p_{1,t_0}^{EQ} and p_{2,t_0}^{EQ} , aggregate demand of Investor L and Investor S meets the available quantity from Eq. (1) for both Stock 1 and Stock 2, separately. We can write the market clearing conditions at time t_0 as

$$\chi_1^L(p_{1,t_0}^{EQ}, p_{2,t_0}^{EQ}) + \chi_1^S(p_{1,t_0}^{EQ}, p_{2,t_0}^{EQ}) - \bar{X}_1 \stackrel{!}{=} 0 \quad (19)$$

$$\chi_2^L(p_{1,t_0}^{EQ}, p_{2,t_0}^{EQ}) + \chi_2^S(p_{1,t_0}^{EQ}, p_{2,t_0}^{EQ}) - \bar{X}_2 \stackrel{!}{=} 0. \quad (20)$$

Model results

The results presented in the following are based on the parameter specifications summarized in Table 1. We employ a generally valid set of parameters for the two stocks with an expected discrete return μ of 10% and standard deviation σ of 20%. A jump intensity of $\lambda = 0.1$ implies that, on average, a jump occurs once every ten years with a jump size ϕ of -30%. It follows from Eq. (9) that the volatility σ_j conditional on no jump to occur is equal to 17.61%. The risk-free rate r_f is 2% p.a.

Throughout this paper, we assume that the returns of Stock 1 and Stock 2, conditional on no jump to occur, are governed by the same Wiener process z_t . By construction, we thereby mitigate the possibility of diversification and emphasize the impact of the tail risk of Stock 2 on the optimal portfolio decision of investors.

Regarding the specification of the utility function of investors, we assume that the risk aversion parameter γ is equal to 4 and subsistence wealth η equals 0.8, which implies that the investors require at least 80% of their initial



Required return (p.a.) of Investor L and the skewness premium

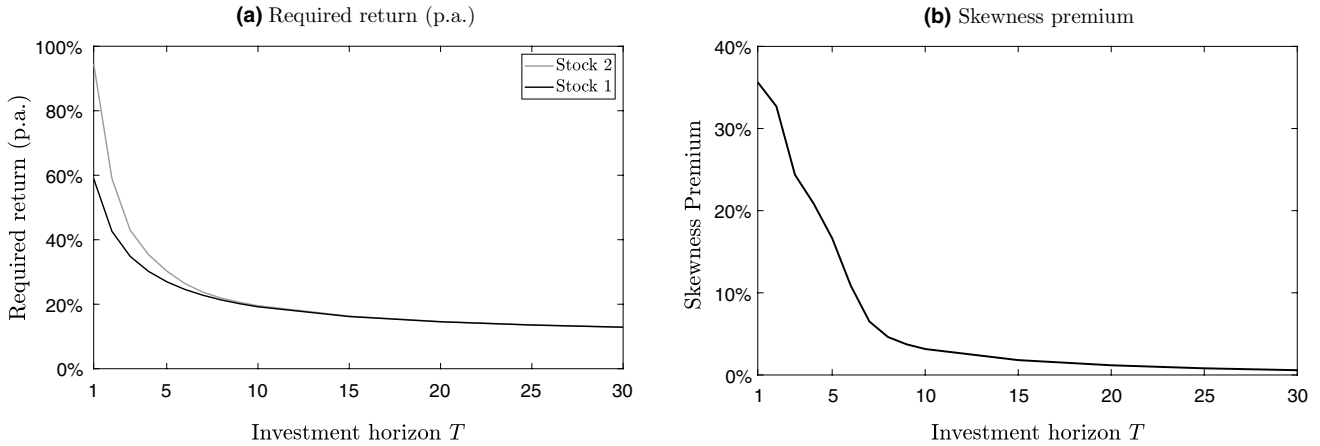


Fig. 2 Fig. 2a shows the required return (p.a.), $y_1^{L,EQ}$ and $y_2^{L,EQ}$, of Investor L for Stock 1 and Stock 2, defined as the annualized logarithmic expected return for each stock in equilibrium according to

Eq. (21). The skewness premium sk^{prem} in Fig. 2b is the logarithmic price difference between Stock 1 and Stock 2 in equilibrium as defined in Eq. (22)

endowment at the end of their investment horizons. Hence, the relative risk aversion is always above 4 for all wealth levels. We provide the results of robustness checks for alternative parameter specifications in Sect. 4.

First, we consider an economy with homogeneous investment horizons of investors represented by long-term Investor L. We derive the equilibrium characteristics for investment horizons T of Investor L ranging from 1 year to 30 years for the parameter specifications described. We then turn to an economy with heterogeneous investment horizons of investors.

Economy with homogeneous investment horizons

With long-term investors only in the economy, it follows from Eq. (2) that representative Investor L is endowed with aggregate monetary endowment \bar{X}_{Cash}^L of

$$\bar{X}_{Cash}^L = 1.$$

The derivation of the equilibrium described in the previous section thereby reduces to the optimization problem of Investor L from Eq. (17) and the market clearing conditions simplify to

$$\chi_1^L(p_{1,t_0}^{EQ}, p_{2,t_0}^{EQ}) - \bar{X}_1 \stackrel{!}{=} 0$$

$$\chi_2^L(p_{1,t_0}^{EQ}, p_{2,t_0}^{EQ}) - \bar{X}_2 \stackrel{!}{=} 0.$$

For the prices in equilibrium, Investor L purchases all available units of stocks at time t_0 according to her demand function, such that

$$\chi_1^L(p_{1,t_0}^{EQ}, p_{2,t_0}^{EQ}) = \chi_2^L(p_{1,t_0}^{EQ}, p_{2,t_0}^{EQ}) = 0.5.$$

In equilibrium, the required return of Investor L for Stock $k = 1, 2$ comes from the logarithmic return of the expected fundamental value $\mathbb{E}_{t_0}(S_{k,T})$ at time T related to the equilibrium trading price p_{k,t_0}^{EQ} at time t_0 . We define $y_k^{L,EQ}$ as the required annual return of Investor L and write the required return for Stock k as

$$T \cdot y_k^{L,EQ} := \ln \left(\frac{\mathbb{E}_{t_0}(S_{k,T})}{p_{k,t_0}^{EQ}} \right). \tag{21}$$

Figure 2a illustrates the required annual returns of Investor L in equilibrium depending on the investment horizon T . $y_1^{L,EQ}$ and $y_2^{L,EQ}$ are monotonously decreasing functions of the investment horizon T . While Investor L requires a return of Stock 2 of 94.6% with an investment horizon of one year, the required return reduces to 12.9% (p.a.) for $T = 30$. The result illustrates the effectiveness of time diversification on the investor’s optimal portfolio allocation: With a longer investment horizon of Investor L, else equal, the investor is willing to accept lower returns (p.a.) for both stocks to purchase the outstanding shares.

Further, the required returns in Fig. 2a indicate that Investor L requires a higher annual compensation for the negatively skewed Stock 2 than for Stock 1, as it holds for all T that

$$y_2^{L,EQ} > y_1^{L,EQ}.$$

The additional annual compensation required for Stock 2 relative to Stock 1, $y_2^{L,EQ} - y_1^{L,EQ}$, thereby decreases with T . Since both stocks have identical means and variances at time T , but Stock 2 suffers from a lower market price in equilibrium due to tail risk, there must be a positive premium for



(negative) skewness. We define the skewness premium as the logarithmic price difference between Stocks 1 and 2 in equilibrium:

$$sk^{prem} := \ln \left(\frac{P_{1,t_0}^{EQ}}{P_{2,t_0}^{EQ}} \right). \quad (22)$$

Figure 2b indicates that the skewness premium is positive for all T . The skewness premium measures the additional logarithmic return of Stock 2 relative to Stock 1 until time T as compensation for a disliked negative jump risk. Thereby, the skewness premium is a decreasing function of T , which declines from 35.6% for $T = 1$ to 0.6% for $T = 30$, and approaches zero for longer investment horizons.

For short investment horizons, Investor L hence requires a relatively high additional compensation for being exposed to the negative jump risk in prices of Stock 2. Even though the probability of one or more jumps to occur is relatively small, already the size $\phi = -30\%$ of a jump leads to substantial losses in wealth from which final wealth hardly recovers over short investment horizons. For longer investment horizons, however, the impact of negative jumps on final wealth diminishes because the return distribution of Stock 2 converges to the return distribution of Stock 1 for long T indicated by the limit of the skewness of Eq. (10). Intuitively speaking, over a shorter investment period, the portfolio return of an investor is hence more strongly affected if a surprising jump in fact occurs.

Accordingly, the impact of catastrophe risk on investors with DRRA utility is less pronounced at longer investment horizons, so that the prices P_{1,t_0}^{EQ} and P_{2,t_0}^{EQ} converge to each other for long T despite the negative jump risk in Stock 2 prices. We summarize the first implication of our model in:

Stylized fact 1 (Skewness premium) The skewness premium is positive and decreases with the investment horizon T of the investors.

Economy with heterogeneous investment horizons

We now consider an economy with long-term investors, who have an investment horizon of 10 years, and short-term investors, who leave the economy after five years. Hence, the investment horizon $T - T_S$ of the follow-up investors also equals five years. With short-term investors being identically endowed as long-term investors, representative Investor L and Investor S hold half of the aggregate monetary endowment \bar{X}_{Cash} each, such that

$$\bar{X}_{Cash}^L = \bar{X}_{Cash}^S = 0.5.$$

In equilibrium, the required return of Investor S for Stock $k = 1, 2$ is the logarithmic return between the expected

equilibrium trading price $\mathbb{E}_{t_0}(P_{k,T_S}^{EQ})$ at time T_S and the equilibrium trading price P_{k,t_0}^{EQ} at time t_0 . With $y_k^{S,EQ}$ defined as the annual return required of Investor S, we can write

$$T_S \cdot y_k^{S,EQ} := \ln \left(\frac{\mathbb{E}_{t_0}(P_{k,T_S}^{EQ})}{P_{k,t_0}^{EQ}} \right). \quad (23)$$

Similarly, the required return of Investor F, conditional on time t_0 expectations, is the logarithmic return of the expected fundamental value $\mathbb{E}_{t_0}(S_{k,T})$ at time T and the expected equilibrium trading price $\mathbb{E}_{t_0}(P_{k,T_S}^{EQ})$ at time T_S :

$$(T - T_S) \cdot y_k^{F,EQ} := \ln \left(\frac{\mathbb{E}_{t_0}(S_{k,T})}{\mathbb{E}_{t_0}(P_{k,T_S}^{EQ})} \right), \quad (24)$$

where $y_k^{F,EQ}$ is the required annual return of Investor F from the perspective of time t_0 .

Since Investor S and Investor F split the investment period of Investor L and trade Stock 1 and Stock 2 at time T_S , we can express the required return of Investor L from Eq. (21) as the sum of the required logarithmic returns of Investor S and Investor F from Eq. (23) and (24):

$$\begin{aligned} \ln \left(\frac{\mathbb{E}_{t_0}(S_{k,T})}{P_{k,t_0}^{EQ}} \right) &= \ln(\mathbb{E}_{t_0}(S_{k,T})) - \ln(P_{k,t_0}^{EQ}) \\ &= \ln(\mathbb{E}_{t_0}(S_{k,T})) - \mathbb{E}_{t_0}(P_{k,T_S}^{EQ}) \\ &\quad + \mathbb{E}_{t_0}(P_{k,T_S}^{EQ}) - \ln(P_{k,t_0}^{EQ}) \\ &= \ln \left(\frac{\mathbb{E}_{t_0}(P_{k,T_S}^{EQ})}{P_{k,t_0}^{EQ}} \right) + \ln \left(\frac{\mathbb{E}_{t_0}(S_{k,T})}{\mathbb{E}_{t_0}(P_{k,T_S}^{EQ})} \right). \end{aligned} \quad (25)$$

This is a result of well-known logarithm computation rules. Substituting the expressions of the required returns in Eq. (25) and dividing by T , we obtain

$$y_k^{L,EQ} = \frac{T_S}{T} \cdot y_k^{S,EQ} + \frac{T - T_S}{T} \cdot y_k^{F,EQ}. \quad (26)$$

The relation states that the required return (p.a.) of Investor L for Stock $k = 1, 2$ is equal to the average of the respective required returns of Investor S and Investor F, weighted by the investor's investment horizon.

In principle, the return required by an investor is affected by two sources: (i) the investment horizon of the investor and (ii) the number of stocks the investor holds in total.

First, from the results of the previous section with homogeneous investment horizons, we know that the required



Table 2 Equilibrium results with heterogeneous investors

Investor	Equity frac- tion (%)	Fraction in Stock 2 (%)	X_1^{EQ}	X_2^{EQ}	sk^{prem}
L	41.8	51.4	0.29	0.33	7.2%
S	25.3	43.0	0.21	0.17	

Table 2 shows the key characteristics of the economy with equal proportion of short-term investors and long-term investors with $T=10$ and $T_S=5$. The equity fraction denotes the risky portfolio value at time t_0 as a fraction of initial wealth. The fraction invested in Stock 2 is the value invested in Stock 2 divided by the risky portfolio value at time t_0 . X_1^{EQ} and X_2^{EQ} denote the equilibrium holdings in Stock 1 and Stock 2. The skewness premium sk^{prem} from Eq. (22) is defined as the logarithmic price difference between Stock 1 and Stock 2 in equilibrium

Table 3 Required returns of the investors

Investor	y_1^{EQ} (%)	y_2^{EQ} (%)
L	20.6	21.3
S	15.7	15.4
F	25.4	27.2

Table 3 shows the annual returns required in the economy with equal proportion of short-term investors and long-term investors with $T=10$ and $T_S=5$. The required returns of representative Investor L, Investor S, and Investor F are given in Eqs. (21), (23), and (24), respectively

returns (p.a.) for both stocks are decreasing functions of the investor’s investment horizon. For Investor L and Investor S to purchase equally many units of both stocks at time t_0 , the returns Investor S requires exceed the required returns of Investor L. The same is true for Investor F, i.e., she requires higher returns than Investor L for identical holdings due to a shorter investment period. Hence, for identical holdings $X_k^S = X_k^L = 0.25$, the right-hand side of Eq. (26) would exceed the left-hand side and is therefore no equilibrium. As a result, Investor L and Investor S must not buy equally many units of stocks in equilibrium.

Second, a reduction (increase) of the units of a stock bought, else equal, reduces (increases) the required return for the stock. Table 2 presents the quantity of stocks Investor S and Investor L purchase in equilibrium, and Table 3 provides the respective annual returns required. In the considered example, Investor S reduces the numbers of stocks purchased to 0.21 units of Stock 1 and 0.17 units of Stock 2, which lowers the required returns on the right-hand side of Eq. (26).

We can interpret $y_k^{F, EQ}$ as the conditional on time t_0 expected discount rate Investor S applies to discount the expected fundamental value $E_{t_0}(S_{k,T})$. Investor S anticipates the annually required returns of Investor F, which equal 25.4% for Stock 1

and 27.2% for Stock 2. For the reduced holdings in equilibrium, Investor S only requires 15.7% and 15.4%, respectively.

Analogously, Investor L increases stock holdings and buys 0.29 units of Stock 1 and 0.33 units of Stock 2 with jump risk in equilibrium, which in turn increases the required returns (p.a.) on the left-hand side of Eq. (26) to 20.6 and 21.3%, respectively. The quantities purchased by Investor S and Investor L in equilibrium ensure that the relation between the required returns in Eq. (26) holds. Table 2 shows that Investor L has 41.8% of wealth invested in the two stocks, while Investor S has an equity fraction of only 25.3%. We summarize the implication in:

Stylized fact 2 (Equity fraction) For investors who differ only in terms of their investment horizon, long-term investors have a higher equity fraction than short-term investors.

Further, we can conclude that the composition of the investors’ portfolios in equilibrium differs: Particularly, Investor L (Investor S) acquires 0.04 units more (less) of Stock 2 with tail risk than of Stock 1 in equilibrium. Accordingly, besides the higher equity fraction of Investor L, the portfolio of Investor L is characterized by a higher fraction invested in Stock 2 of 51.4% compared to 43.0% of Investor S. As a result of the previous section with homogeneous investment horizons, we know that the additional annual compensation required for Stock 2 relative to Stock 1 is a decreasing function of the investor’s investment horizon. Hence, for the skewness premium of 7.2% in equilibrium, Investor L is willing to bear—in relative and absolute terms—more tail risk than Investor S. This is a major difference to the conclusion of the CAPM that all market participants invest a fraction of their wealth in an identical portfolio of risky assets, i.e., the market portfolio (two fund separation theorem). We summarize the implication in:

Stylized fact 3 (Portfolio composition) For investors who differ only in terms of their investment horizon, long-term investors hold a higher portfolio weight in assets with tail risks than short-term investors.

Varying proportions of investors in the economy

For the previous analysis with heterogeneous investment horizons, we assumed an economy with equal proportion of short-term investors and long-term investors, which can be interpreted as Investor L and Investor S with equal monetary endowment. By varying initial monetary endowment \bar{X}_{Cash}^L and \bar{X}_{Cash}^S for given aggregate monetary endowment \bar{X}_{Cash} from Eq. (2), we can modify the composition of investors in the economy. The higher \bar{X}_{Cash}^S at the cost of \bar{X}_{Cash}^L , the higher the proportion of short-term investors relative to long-term ones. Table 4 presents the outcomes of the economies for different proportions of short-term investors.



Table 4 Different composition of investors in the economy

Proportion of Investor S (%)	Investor	Equity fraction (%)	Fraction in Stock 2 (%)	X_1^{EQ}	X_2^{EQ}	sk^{prem} (%)
0	L	39.2	49.2	0.5	0.5	3.2
25	L	40.6	50.2	0.41	0.43	4.9
	S	22.9	41.0	0.09	0.07	
50	L	41.8	51.4	0.29	0.33	7.2
	S	25.3	43.0	0.21	0.17	
75	L	44.9	52.4	0.15	0.19	10.5
	S	28.8	44.8	0.35	0.31	
100	S	29.6	46.9	0.5	0.5	12.6

Table 4 shows the key characteristics of the economy as outlined in Table 2 for proportions of short-term investors in the economy from 0% to 100% by adjusting initial endowment \bar{X}_{Cash}^L and \bar{X}_{Cash}^S of the representative investors for given aggregate monetary endowment \bar{X}_{Cash}

In the absence of short-term investors, Investor L purchases the outstanding 0.5 units of each stock in equilibrium with an equity fraction of 39.2% and a fraction invested in Stock 2 of 49.2%. The skewness premium equals 3.2%.

With 25% of all investors being short-term investors, Investor L is endowed with three times as much cash as Investor S. For equivalent behavior of both investors, Investor S should hold 0.125 units and Investor L 0.375 units of each stock. Investor S and Investor F, however, require higher annual returns for both stocks on average and a higher additional compensation for Stock 2 relative to Stock 1 than Investor L to purchase respective units of each stock. Therefore, the right-hand side of the relation in Eq. (26) would exceed the left-hand side for each stock and respective holdings cannot represent equilibrium.

In equilibrium, to satisfy Eq. (26) for both stocks, Investor S reduces the stock holdings to a volume equal to 0.09 units of Stock 1 and 0.07 units of Stock 2. Correspondingly, Investor L increases the stocks purchased to 0.41 units of Stock 1 and 0.43 units of Stock 2. The equity fraction of Investor L increases from 39.2% with long-term investors only to 40.6% in the considered example. Similarly, the fraction invested in Stock 2 of Investor L rises from 49.2 to 50.2%, and the skewness premium from 3.2 to 4.9%.

With 75% short-term investors, the equity fraction of Investor L increases to 44.9% and the fraction invested in Stock 2 to 52.4%. The skewness premium rises to 10.5%. With short-term investors only, Investor S purchases all outstanding units of stocks in equilibrium and the skewness premium equals 12.6%, which is four times as high as with long-term investors only. We summarize the implication of our model in:

Stylized fact 4 (Market participants) The skewness premium increases with the proportion of short-term investors in the economy. Moreover, long-term investors increase both their equity fraction and their portfolio weight in the asset with tail risk with the proportion of short-term investors in the economy.

Robustness checks

The aim of this section is to analyze the impact of parameter modifications on the key characteristics of the economy with heterogeneous investment horizons. For this purpose, we change one parameter from the previously utilized set of parameters given in Table 1 at a time. The results are presented in Table 5, whereby the first row recapitulates the results from Table 2, which we refer to as the benchmark.

The first three analyses deal with parameter alternations of the stock properties. First, we consider a more negative jump size ϕ of -40% , while keeping the standard deviation of returns σ constant at 20% . From Eq. (10) follows that the resulting skewness of the returns of Stock 2 is more negative than in the benchmark analysis. With identical equilibrium stock holdings as in the benchmark, both Investor L and Investor S bear greater tail risk so that investors require higher compensation. Accordingly, the skewness premium almost doubles to 13.6% .

In the next two analyses, we consider both higher and lower standard deviation σ in the stocks' returns. For fixed jump size $\phi = -30\%$, the standard deviation σ_j conditional on no jump to occur is adjusted according to Eq. (9) for Stock 2. From Eq. (10) follows that a higher (lower) standard deviation σ reduces (increases) the negative skewness of the returns of Stock 2. Hence, a higher standard deviation of 24% decreases the skewness premium to 2.3% . On the contrary, a lower standard deviation of 16% results in a skewness premium of 24.2% , which is more than three times as high as in the benchmark. Therefore, the level of tail risk compared to total risk matters. In case that tail risk relative to total risk is high, the skewness premium rises.

The following two robustness checks deal with the specifications of the utility function of the investors. Specifically, we assume investors who exhibit a higher



Table 5 Robustness checks

	Investor	Equity fraction (%)	Fraction in Sstock 2 (%)	X_1^{EQ}	X_2^{EQ}	sk^{prem} (%)
Benchmark	L	41.8	51.4	0.29	0.33	7.2
	S	25.3	43.0	0.21	0.17	
$\phi = -40\%$	L	41.8	49.2	0.29	0.33	13.6
	S	25.7	42.4	0.21	0.17	
$\sigma = 24\%$	L	33.8	51.6	0.31	0.33	2.3
	S	19.1	45.5	0.19	0.17	
$\sigma = 16\%$	L	49.2	44.9	0.28	0.29	24.2
	S	36.5	42.7	0.22	0.21	
$\gamma = 6$	L	33.9	49.9	0.31	0.33	7.8
	S	18.9	44.8	0.19	0.17	
$\eta = 0.9$	L	34.3	50.7	0.29	0.35	16.0
	S	19.0	37.5	0.21	0.15	

Table 5 shows the equilibrium results as outlined in Table 2 for selected parameter alternations from the set of parameters given in Table 1

RRA, either by increasing the risk aversion parameter γ or the subsistence level of wealth η . First, by increasing γ from 4 to 6, Investor L (Investor S) buys more (less) units of Stock 1, while the holdings in Stock 2 remain unchanged compared to the benchmark. The skewness premium increases by 60 basis points to 7.8%. Accordingly, a higher risk aversion parameter γ results in a greater skewness premium.

Second, with a subsistence level of wealth of $\eta = 0.9$, we find that investors are particularly more averse against skewness risk and require higher additional compensations for Stock 2 relative to Stock 1. Thereby, the effect is more pronounced for short-term investors: For the skewness premium of 16% in equilibrium, Investor L (Investor S) increases (decreases) the number of units bought of Stock 2, while the holdings in Stock 1 remain unchanged compared to the benchmark.

This section illustrates that the derived stylized facts from Sect. 3 do not follow from a specific parameter choice of the DRRA utility function or of the stock properties but hold for a wide range of parameter constellations. We particularly find that:

Robustness check (Skewness premium) The skewness premium increases with the negative skewness in the returns of assets and with the relative risk aversion of the investors specified by γ and η .

Conclusion

In this paper, we introduce a theoretical framework to analyze how tail risk affects both asset prices and the optimal asset allocation of investors. Particularly, we contribute to the understanding of skewness premia with an insightful multi-period

equilibrium model that accounts for heterogeneous investment horizons and investors with DRRA, capturing the empirically observed behavior of investors to allocate more wealth to equities when their investment horizons are longer. The investors' portfolio compositions and endogenous market prices enable a rigorous analysis of investors' tail risk preferences depending on their respective investment horizons.

While only expected returns and standard deviations of the overall portfolio return are relevant for the optimal portfolio allocation according to the CAPM, our empirically justified model explains that investors incorporate assets' distributional properties of returns higher than the second (centralized) moment in their portfolio allocation decision. Notably, since investors require higher compensation to hold the stock with tail risk, there is a positive skewness premium. As a main result, this skewness premium decreases with the length of the investment period of investors as a surprising jump in the stock price more strongly affects final wealth of investors with a shorter investment period. With heterogeneous investment horizons, long-term investors hold both a higher equity fraction and portfolio weight in the asset with negative skewness in equilibrium than short-term investors so that as a major distinction from the seminal CAPM, two fund separation no longer holds.

Our results provide a fruitful foundation by means of testable implications for an empirical analysis of the impact of heterogeneous investment horizons on skewness premia for future research: First, the model's implication that additional premia for assets with tail risk, else equal, exist is not limited to equities but holds for other asset classes with an even stronger negative skewness such as high-yield bonds. Likewise, assets exhibiting positive skewness—such as growth stocks or lottery-like venture capital investments—are supposed to yield lower expected returns, else equal, due to a



preference for positively skewed returns. Second, from an investor's perspective, it may be optimal for longer-orientated investors, who pursue a buy-and-hold strategy, to deliberately select investments with tail risk to benefit from tail risk premia. Third, these tail risk premia are the higher, (i) the more negative the returns' skewness, (ii) the higher the relative risk aversion of market participants, (iii) the more dominated respective markets are by short-term oriented investors, and (iv) the shorter these investment horizons. Consequently, we can expect high premia in stock markets and private equity investments due to a relatively short investment period and/or significant tail risks: The average holding period of US stocks traded at the NYSE has been less than 12 months from 2010 to 2020 [see Chatterjee and Adinarayan (2020)], which indicates that shorter-orientated investors represent a substantial proportion of investors in stock markets. An example for a longer investment horizon is private equity investments with an average holding period of the investments of approximately four years [see, e.g., Jenkinson and Sousa (2015)].

Further, we expect high transaction costs and/or low liquidity of assets to induce investors to hold respective assets for longer periods. With longer-orientated investors selecting higher proportions of those investments, we can conclude from our theoretical model that the ownership structure lowers respective skewness premia. For example, higher entry and exit costs typically characterize mutual funds and real estate investments (or funds). Real estate investments and covered bonds are further examples of (more) illiquid assets, which are hence less attractive to short-term investors.

The stylized facts derived in this paper are intended to raise awareness of the existence of tail risk premia, provide an intuitive understanding of the drivers of these premia, and emphasize the practical relevance for investors to carefully consider tail risks in their investment decisions. Holding the same portfolio of risky assets might not be equally optimal for investors, as being exposed to tail risk can be less or more attractive for investors depending on respective investment horizons.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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