Research Article

Joint pricing and inventory decisions under resource constraints and price-based substitution

Received (in revised form): 21st August 2014

Man Yu

HKUST Business School, Hong Kong University of Science and Technology, Kowloon, Hong Kong

Man Yu is an assistant professor in the Department of Information Systems, Business Statistics, and Operations Management at the Business School of Hong Kong University of Science and Technology. She received her PhD in business administration from Ross School of Business, University of Michigan. Her research interests include pricing and revenue management with explicit modeling of consumer behavior, interfaces of operations and marketing, and supply chain management.

Correspondence: Man Yu, HKUST Business School, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong
E-mail: manyu@ust.hk

ABSTRACT In this article, we consider a joint pricing-inventory decision problem for horizontally differentiable products that require common resources to produce, procure or hold. The total availabilities of the resources are exogenously fixed. Demands for the products are stochastic, price-based substitutable and lost if unmet. For the case where two products share one resource, we prove the existence of a unique optimal solution to the decision problem. We also characterize the firm's assortment under the optimal solution and identify a distribution-free product index to determine the assortment priority among the products. Furthermore, we present some results regarding the monotonicities of the optimal pricing-inventory policy.
Journal of Revenue and Pricing Management (2015) **14,** 19–27. doi:10.1057/rpm.2014.25; published online 10 October 2014

Keywords: joint pricing and inventory; resource constraint; product substitution

INTRODUCTION

Consider the following situations:

- A brick-and-mortar retail store sells different brands of high-definition televisions (HDTVs). The store has limited storage space on the display shelves and in the back room. Inventory replenishment follows a weekly schedule and no rush order is possible. During a week, customers walk in the store, compare prices and specifications of the brands, and decide whether and which one to buy. If the preferred brand is out of stock, customers will walk away to competing stores. The store maximizes its expected profit by choosing the retail prices and ordering quantities for all the brands, subject to the space-availability constraint.
- A seafood restaurant features a variety of fresh 'catch-of-the-day' seafood delicacies, such as lobsters, jumbo prawns, king crabs and so on. All seafood is locally caught and delivered to the restaurant every morning. Cash payment to the fishermen is due upon delivery.

The restaurant owner decides on the amount to purchase from each kind of seafood, subject to a budget constraint. When making the decision, the owner is unsure about customers' meal choices throughout the day, but (s)he can adjust the entrée prices for today according to the supply (s)he has picked. On the other hand, if the restaurant runs out of the dish that a customer craves the most, the disappointed customer may go to the competing restaurant next door.

- An automobile manufacturing company makes production and pricing plans for all vehicle models on a quarterly basis. Quarterly demand for a car model is uncertain and depends not only on its own price, but also on the prices of all the similar models. The production requires some common resources, such as machine and labor hours, and is constrained by the resource availability.

The examples above illustrate a common problem encountered by many firms in day-to-day operations: a line of horizontally differentiated products need to be jointly priced and procured (or produced) to maximize the firm's total profit. The overall inventory of these products is limited by the availability of some common resources, for example, shelf space, capital and labor hours, which is exogenously determined and difficult to change in short terms. These constraints on resource availabilities can be viewed as limited capacity. Customers carefully compare across all the products before purchase, and consequently, changing one product's price will shift demand toward its substitutes. That is, aggregate demands for the products are price-based substitutable. Owing to rigid replenishment schedules, demands remain uncertain when the pricing-inventory decision is made. Nevertheless, sales will be lost in case of stocking out.

With both resource constraints and demand substitution, products compete with each other for both resources and customers, and are interlinked on both supply and demand sides. Hence, the joint pricing and inventory decisions can be particularly difficult. For example, raising one product's price reduces its own demand, and yet increases the popularity of all its substitutes. To fully take advantage of it, the firm naturally has an incentive to increase the prices and quantities of some of the other products. This, however, is subject to the resource availability and will, in turn, affect the first product's demand and resource allocation. It is unclear as to how these interactions influence the firm's total profit, and whether and when they lead to a unique optimal decision.

In this article, we aim to address this problem. Our work falls within the research area on multi-product pricing. Because of tractability issues, very limited research in this area considers the coordination of multi-product pricing and inventory policies (Elmaghraby and Keskinocak (2003)). Among the existing work, Birge *et al* (1998) examine how to determine the prices or capacity levels for two products. Under different demand models, Aydin and Porteus (2008), Wang and Kapuscinski (2009), Zhu and Thonemann (2009) and Song and Xue (2007) consider joint optimization of pricing and inventory decisions. In particular, Aydin and Porteus (2008) and Wang and Kapuscinski (2009) prove the uniqueness of optimal solution in a lost-sales setting, while Zhu and Thonemann (2009) and Song and Xue (2007) characterize the optimal solution under a backlogging model. In addition to the pricing-inventory models, Krausa and Yano (2003) and Hopp and Xu (2005) address the problem of choosing both price and product variety (assortment). Maddah and Bish (2007) study joint pricing, inventory and assortment decisions. All the aforementioned papers, however, assume unlimited resources, where the interaction among products is only through demand substitution.

Our work is among the few papers that analyze multi-product pricing under resource constraints. When limited resources are shared among products, the pricing problem becomes even more complex, as products are interrelated though both demand substitution and resource competition. In the revenue-management literature, Gallego and van Ryzin (1997) and Cooper (2002) focus on dynamic-pricing strategy and prove asymptotic optimality of several heuristic policies. Maglaras and Meissner (2006) examine a similar dynamic program and investigate ways to reduce the problem dimensionality. None of these papers, however, considers inventory decisions.

Similar to our setting, Tang and Yin (2007) characterize the optimal pricing and inventory decisions for two products with a common resource and deterministic demand. Likewise, Kuyumcu and Popescu (2005) also consider deterministic demand and yet allow arbitrary numbers of products and resources. Our work differs from these two papers by considering stochastic demand. Most closely related to our model, Bertsimas and de Boer (2002), Song and Xue (2008) and Ceryan *et al* (2013) examine joint pricing-inventory decision under both demand uncertainty and resource constraints. Nevertheless, Bertsimas and de Boer (2002) ignore the demand substitution among products, which is considered in our article. Song and Xue (2008) and Ceryan *et al* (2013) assume that all unmet demand are backlogged. With pricing decision, the lost-sales model (considered in our article) is significantly more difficult than the backlogging model, because the profit function is no longer jointly concave in prices. Owing to the technical difficulty, all the existing work on multi-product pricing-inventory control with lost-sales model consider single-period problems.

To summarize, our article, to our knowledge, is the first one to analyze the multi-product joint pricing and inventory problem with resource constraints, product substitution, demand uncertainty and lost sales. The problem is difficult even for a small number of products and resources. We contribute to the literature by solving this problem for a baseline case where two products share one common resource. Specifically, our findings are as follows:

(i) We prove that the uniqueness of the optimal policy, as in Aydin and Porteus (2008) and Wang and Kapuscinski (2009), can be extended to the setting with resource constraint. The proof builds on multi-dimensional strict quasi-concavity of the profit function in a constrained domain.

(ii) Structural results are presented on the firm's optimal assortment. Specifically, the best choice of product(s) to offer is characterized as a function of the resource availability, as well as the relative per-unit resource usage of the two products. Interestingly, we show that the optimal assortment may not always be monotonic in the latter factor. In particular, a decrease in the relative resource usage of one product may enhance the profitability of the other product. We provide an explanation for this counter-intuitive phenomenon. Furthermore, we identify a distribution-free index that the firm can apply to assess the priority of the products. The index is a generalization of the one used by Tang and Yin (2007).

(iii) We prove the monotonicities of the joint optimal policies for some special cases.

In what follows, we define the model formulation in the next section, and present the analysis and results in the subsequent section. The article is then concluded in the final section.

MODEL

The firm's problem

Consider a firm producing and selling two products (indexed by $i = 1, 2$). The two products are horizontally differentiated with price-based substitution. Producing the two products requires a common resource (for example, space, capital, labor hours, machine hours and so on): one unit of product 1 needs A units of the resource, while the per-unit resource usage of product 2 is normalized to 1. Hence, A is also an indicator of the *relative* per-unit resource usage of the two products. The availability of the resource is limited: total amount of resource used in the production cannot exceed y units. The value of y is exogenously given. Customers' demands for the two products, $D_1(p_1, p_2, \varepsilon_1)$ and $D_2(p_1, p_2, \varepsilon_2)$, are uncertain and depend on both prices (details described in the sub-section 'The demand model').

The firm determines prices p_1, p_2 and production quantities q_1, q_2 before the demand uncertainty is resolved, subject to the resource constraint. To preclude the unrealistic case of negative demand, we impose an additional constraint: the firm should set the prices low enough that both products' demands are non-negative almost surely, that is, both $D_1 \geqslant 0$ and $D_2 \geqslant 0$ a.s. (short for almost surely). Similar bounds on prices can be found in, for example, Kuyumcu and Popescu (2005). To understand why this constraint is necessary, consider the case when the firm decides to only offer product 2 (that is, $q_1 = 0$). Without the constraint $D_1 \geqslant 0$ a.s., the firm would have an incentive to boost product 2's demand by setting p_1 infinitely high to take advantage of the demand-substitution effect, such that all units of product 2 can be sold out at an arbitrarily high p_2.

The firm's objective is to maximize its total expected profit. Its decision problem can be formulated as follows:

$$\max_{p_1,p_2,q_1,q_2} \pi(p_1, p_2, q_1, q_2) = p_1\mathrm{E}[\min(q_1, D_1)] + p_2\mathrm{E}[\min(q_2, D_2)] \quad (1)$$

subject to

$$p_1 \geqslant 0, p_2 \geqslant 0, q_1 \geqslant 0, q_2 \geqslant 0,$$
$$Aq_1 + q_2 \leqslant y, D_1 \geqslant 0 \ a.s., D_2 \geqslant 0 \ a.s. \quad (2)$$

Here we consider a static problem without any dynamic stockout-based substitution. Moreover, we assume that, owing to leadtime in production, the firm cannot postpone the inventory decision until after the demand is realized. These two assumptions enable us to focus on the interaction between inventory and pricing decisions with presence of demand uncertainty. Similar settings are considered in several other papers including Zhu and Thonemann (2009) and Aydin and Porteus (2008).

The demand model

The products' demands, D_1, D_2, are functions of prices and some random factors. We assume that demand is linear in prices and subject to additive uncertainty:

$$\begin{aligned} D_1(p_1, p_2, \varepsilon_1) &= z_1(p_1, p_2) + \varepsilon_1 \\ &= a_1 - b_1p_1 + c_1p_2 + \varepsilon_1, \end{aligned} \quad (3)$$

$$\begin{aligned} D_2(p_1, p_2, \varepsilon_2) &= z_2(p_1, p_2) + \varepsilon_2 \\ &= a_2 - b_2p_2 + c_2p_1 + \varepsilon_2, \end{aligned} \quad (4)$$

where $z_1(p_1, p_2)$ and $z_2(p_1, p_2)$ are deterministic components in demand and are linear functions of prices: $a_1, a_2 > 0$ represent base demands, b_1, $b_2 > 0$ denote self-price sensitivities and $c_1, c_2 > 0$ stand for cross-price sensitivities; $\varepsilon_1, \varepsilon_2$ are random factors with mean zero, capturing the combined effect of all the noises not perfectly controllable or observable by the firm. Assume that for $i = 1, 2$, ε_i has a finite support $[-L_i, H_i]$, where $L_i > 0$ and $H_i > 0$.

The linear and additive demand is a simple demand form frequently used in the pricing-inventory literature, for example, Petruzzi and Dada (1999), Tang and Yin (2007), Wang and Kapuscinski (2009) and Ceryan *et al* (2013). Compared to other demand models, such as multiplicative (see, for example, Aydin and Porteus (2008)) or additive-multiplicative (see, for example, Maddah and Bish (2007)), the linear and additive model is widely adopted owing to its desirable analytical tractability. In this article, we show that, even under this simple demand form, the pricing-inventory problem with resource constraint is difficult to solve.

We further assume that the demand functions defined in equations (3) and (4) satisfy the diagonal-dominance condition: $\min(b_1, b_2) > \max(c_1, c_2)$, see Zhu and Thonemann (2009) and Wang and Kapuscinski (2009), and the references therein. The condition implies that the total demand of the two products becomes stochastically smaller when price of either product increases and that one product's demand is more responsive to its own price than to the other product's price, such that if both products' prices increase by the same amount, demand of either product decreases.

By the diagonal-dominance condition, the expected demand $(z_1(p_1, p_2), z_2(p_1, p_2))$ has an inverse:

$$\begin{pmatrix} p_1(z_1, z_2) \\ p_2(z_1, z_2) \end{pmatrix} = \frac{1}{b_1 b_2 - c_1 c_2} \times \left[-\begin{pmatrix} b_2 & c_1 \\ c_2 & b_1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \begin{pmatrix} a_1 b_2 + a_2 c_1 \\ a_2 b_1 + a_1 c_2 \end{pmatrix} \right]. \tag{5}$$

Equations (3)–(5) establish a one-to-one correspondence between (z_1, z_2) and (p_1, p_2). Hence, following the standard approach in literature (see, for example, Gallego and van Ryzin (1997)), we can and shall use the expected demand (z_1, z_2), instead of (p_1, p_2), as the firm's pricing variables. The firm's problem defined in equations (1) and (2) can now be rewritten as

$$\max_{z_1, z_2, q_1, q_2} \pi(z_1, z_2, q_1, q_2) = p_1(z_1, z_2)\mathrm{E}[\min(q_1, z_1 + \varepsilon_1)] + p_2(z_1, z_2)\mathrm{E}[\min(q_2, z_2 + \varepsilon_2)], \tag{6}$$

subject to

$$p_1(z_1, z_2) \geqslant 0, p_2(z_1, z_2) \geqslant 0, q_1 \geqslant 0, q_2 \geqslant 0, \\ A q_1 + q_2 \leqslant y, z_1 \geqslant L_1, z_2 \geqslant L_2. \tag{7}$$

To characterize the optimal solution of the firm's problem, we make several technical assumptions, as listed below.

First, for the extreme case when the resource availability is unlimited, the profit function, denoted by $\pi^U(z_1, z_2) = p_1(z_1, z_2)z_1 + p_2(z_1, z_2)z_2$, has been shown to be bounded, continuous and strictly concave in (z_1, z_2) (Song and Xue (2007)). This further implies that there exists a unique pair (z_1^U, z_2^U) maximizing the profit function in the unlimited-resource case. Denote the corresponding prices by (p_1^U, p_2^U). To avoid any trivial solution, assume that with unlimited resource, it is profitable for the firm to offer both products, that is, $p_1^U > 0$ and $p_2^U > 0$. Furthermore, for consistency, we assume that the condition about almost-surely non-negative demand is satisfied by the optimal solution under unlimited resource, that is, $z_1^U \geqslant L_1$ and $z_2^U \geqslant L_2$.

Second, for the random factors ε_1 and ε_2, assume that they are independent, but not necessarily identically distributed. For $i = 1, 2$, let ε_i's *cdf* be $G_i(\cdot)$ and *pdf* be $g_i(\cdot)$. Throughout this article, we impose the following technical conditions on the distribution function $G_i(\cdot)$:

(1) $G_i(\cdot)$ is twice continuously differentiable on $[-L_i, H_i]$.
(2) $g(\cdot) = G'(\cdot) > 0$ on $(-L_i, H_i)$.
(3) $G_i(x)$ has increasing failure rates (IFR), that is, $g_i(x)/(1-G_i(x))$ is non-decreasing in x, for $x \in [-L_i, H_i]$.

These conditions are standard in literature and are satisfied by many commonly used distributions: for example, uniform, exponential, logistic, normal, extreme-value, power function, Weibull, β, γ, χ and χ^2 (Bergstrom and Bagnoli, 2005). Note that as $G_i(\cdot)$ is twice continuously differentiable, the tail probability function $\overline{G}_i(\cdot) = 1 - G_i(\cdot)$ has an inverse, denoted by $(\overline{G}_i)^{-1}(\cdot)$.

Third, we impose a boundary condition requiring that when both products are offered for free, demand for both products shall always be positive. That is, $z_1(0, 0) > L_1$ and $z_2(0, 0) > L_2$. These two inequalities jointly imply $p_1(L_1, L_2) > 0$ and $p_2(L_1, L_2) > 0$. A similar assumption is employed by Wang and Kapuscinski (2009).

Last, while the two products can differ in their per-unit resource requirement, we limit the extent of such a difference. Specifically, while normalizing product 2's per-unit usage to unity, we impose an upper and an lower bound on product 1's per-unit resource usage A: $\max(c_1, c_2)/b_1 < A < b_2/\max(c_1, c_2)$, that is, the value of A cannot deviate too far from 1 (that is, product 2's per-unit resource usage). This condition is required for technical reasons and is also well in line with the fact that the two products are horizontally differentiated. Essentially, if one product uses a lot more resource (for example, storage space) than the other, for example, a 42-inch HDTV versus a 28-inch HDTV, then the two products are more likely to be vertically differentiated.

CHARACTERIZING THE OPTIMAL SOLUTION

The problem formulated in equations (6) and (7) is a typical multi-product joint pricing and inventory problem with substitutable demand and lost sales. Even without any constraint, the problem is well known for its analytical difficulty, as the profit function is not jointly concave (Aydin and Porteus (2008)). Two recent studies, Aydin and Porteus (2008) and Wang and Kapuscinski (2009), show that the unconstrained problem has a unique optimal solution under either multiplicative or additive demand function. Our model generalizes Wang and Kapuscinski (2009) by incorporating the resource constraint. Next, we prove that the uniqueness of the optimal solution can be extended to the constrained problem. As the profit function is not jointly concave, such a generalization requires a careful examination of the properties of the function in the constrained domain, especially at the boundaries. We also present some structural properties of the optimal solution.

Let $(z_1^\star, z_2^\star, q_1^\star, q_2^\star)$ be the optimal solution to the firm's problem in equations (6) and (7). Let $(p_1^\star, p_2^\star) = (p_1(z_1^\star, z_2^\star),\ p_2(z_1^\star, z_2^\star))$ be the optimal prices. To characterize the optimal solution, we first note a special case. For the unlimited-resource problem, assume that the firm follows the optimal solution (z_1^U, z_2^U) and let $\bar{\gamma}$ be the *maximum* amount of resource needed to satisfy product demand, that is, $\bar{\gamma} = A(z_1^U + H_1) + z_2^U + H_2$. That is, even though resource availability is unlimited, the firm never uses more than $\bar{\gamma}$ units. Hence, for the constrained problem, if the amount of resource γ is higher than $\bar{\gamma}$, the optimal solution for the unconstrained problem remains optimal for the constrained one; otherwise, the resource constraint must be binding. The following lemma is immediate.

Lemma 1: [*Binding Resource Constraint*] *If* $\gamma \geqslant \bar{\gamma} = A(z_1^U + H_1) + z_2^U + H_2$, $q_i^\star = z_i^U + H_i$ *and* $z_i^\star = z_i^U$, *for* $i = 1, 2$; *otherwise (that is, if* $0 < \gamma < \bar{\gamma}$*),* $Aq_1^\star + q_2^\star = \gamma$.

All proofs are available in the supplementary document. For the remainder of the article, we shall focus on the case with limited resource, that is, $0 < \gamma < \bar{\gamma}$. To find the optimal policy, by Lemma 1, we only need to consider those policies satisfying $q_2 = \gamma - Aq_1$. Substitute $q_2 = \gamma - Aq_1$ into the firm's problem and redefine the profit as a function of z_1, z_2, q_1:

$$\max_{z_1, z_2, q_1} \pi(z_1, z_2, q_1) = p_1(z_1, z_2)\mathrm{E}[\min(q_1, z_1 + \varepsilon_1)] + p_2(z_1, z_2)\mathrm{E}[\min(\gamma - Aq_1, z_2 + \varepsilon_2)],$$

subject to

$$z_1 \geqslant L_1, z_2 \geqslant L_2, p_1(z_1, z_2) \geqslant 0,$$
$$p_2(z_1, z_2) \geqslant 0, 0 \leqslant q_1 \leqslant \frac{\gamma}{A}.$$

We first show that the optimal 'safety stock' (that is, inventory in excess of the expected demand) for each product is always within the support of the corresponding random shock. That is, the firm never over- or under-protects itself against demand uncertainty.

Lemma 2: [*Optimal 'Safety Stock'*] *When* $0 < \gamma < \bar{\gamma}$, *the optimal policy* $(z_1^\star, z_2^\star, q_1^\star)$ *satisfies* $-L_1 \leqslant q_1^\star - z_1^\star \leqslant H_1$ *and* $-L_2 \leqslant \gamma - Aq_1^\star - z_2^\star \leqslant H_2$.

Restricting our attention only to the solutions satisfying the conditions in Lemma 2, Theorem 1 below establishes the uniqueness of the optimal policy.

Theorem 1: [*Uniqueness of Solution*]

(i) *for given* $q_1 \in [0, \gamma/A]$, *there exists a unique pair of* (z_1, z_2), *denoted by* $(z_1^\star(q_1), z_2^\star(q_1))$, *which* *maximizes the profit function* $\pi(z_1, z_2, q_1)$ *subject to* $z_1 \geqslant L_1, z_2 \geqslant L_2, p_1(z_1, z_2) \geqslant 0, p_2(z_1, z_2) \geqslant 0$;

(ii) $\pi(z_1^{\star}(q_1), z_2^{\star}(q_1), q_1)$, *is continuously differentiable and strictly quasi-concave in* $q_1 \in [0, \gamma/A]$;

(iii) *there exists a unique solution* $(z_1^{\star}, z_2^{\star}, q_1^{\star})$ *that maximizes the profit function* $\pi(z_1, z_2, q_1)$ *subject to* $z_1 \geqslant L_1, z_2 \geqslant L_2, 0 \leqslant q_1 \leqslant \gamma/A$, $p_1(z_1, z_2) \geqslant 0, p_2(z_1, z_2) \geqslant 0$.

The idea underlying the proof of Theorem 1 is similar to that used in Petruzzi and Dada (1999) for their study of a single-product unconstrained pricing-inventory problem. It was extended to N-product unconstrained problems by Aydin and Porteus (2008) and Wang and Kapuscinski (2009). In its core, the idea is to show that the global maximum is attained at a stationary point and that the Hessian of the profit function is negative-definite in the neighborhood of any stationary point. These facts jointly imply that any stationary point is a strict local maximum and there exists a unique global maximum.

Unfortunately, we cannot directly apply the logic above to the constrained problem. In particular, the global and local maxima may not always be attained at stationary points – they may be at boundaries for some (or all) variables and stationary points for others. In our proof of Theorem 1, we extend the logic and tailor it to the problem by a careful decomposition of the feasible space and a detailed analysis of the higher-order derivatives on all the stationary points and the boundaries.

The significance of the unique solution is twofold. First, it ensures that any gradient-based searching algorithm can find the optimal solution, which is important for practical applications. Second, it forms the basis for further characterization of the optimal policy.

Assume that the firm follows the unique optimal policy shown in Theorem 1. Theorem 2 (depicted in Figure 1) characterizes the firm's product assortment as a function of the products' demand characteristics, per-unit resource requirement and the total resource availability.

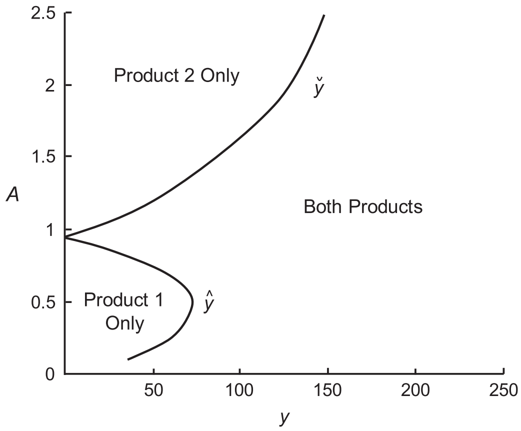

Figure 1: Optimal assortment as a function of resource availability γ and product 1's per-unit resource consumption A: $a_1 = 500$, $a_2 = 400$, $b_1 = 15$, $b_2 = 10$, $c_1 = 1$, $c_2 = 0.5$, $\varepsilon_1, \varepsilon_2 \sim Uniform[-50, 50]$.

Theorem 2: [*Optimal Assortment*]

(i) *Product i's assortment priority is determined by the index* $(a_i - L_i)/(A_i b_i - c_i)$. *Specifically,*

(a) *if* $(a_1 - L_1)/(Ab_1 - c_1) > (a_2 - L_2)/(b_2 - Ac_2)$, *there exists a critical number* $\hat{\gamma} > 0$, *such that the firm should only offer product* 1 *if* $0 < \gamma \leqslant \hat{\gamma}$ *and it should offer both products if* $\gamma > \hat{\gamma}$;

(b) *if* $(a_1 - L_1)/(Ab_1 - c_1) < (a_2 - L_2)/(b_2 - Ac_2)$, *there exists another critical number* $\check{\gamma} > 0$, *such that the firm should only offer product* 2 *if* $0 < \gamma \leqslant \check{\gamma}$ *and it should offer both products if* $\gamma > \check{\gamma}$;

(c) *if* $(a_1 - L_1)/(Ab_1 - c_1) = (a_2 - L_2)/(b_2 - Ac_2)$, *the firm should always offer both products.*

(ii) $\check{\gamma}$ *is non-decreasing in* A, *while* $\hat{\gamma}$ *can be non-monotonic in* A.

As one can expect, when the firm has a lot of resource, it is better off selling both products, whereas with scarce resource, it should only keep one product on the shelf. The intriguing question is which product to offer if there can be only one. Theorem 2(i) provides a distribution-free product index to facilitate the choice. The index combines the effects of demand characteristics and resource requirement. Specifically, a product has a high assortment priority when it has a large worst-case base demand

$(a_i - L_i)$, a low own-price sensitivity (b_i), a high cross-price sensitivity (c_i) and a low resource consumption (A_i). We note that our index is a generalization of the one identified by Tang and Yin (2007). They consider a pricing-inventory decision model with deterministic demand $(\varepsilon_i \equiv 0, i = 1, 2)$ and identical resource requirement by the products $(A_1 = A_2 = 1)$. Hence, their product index, $a_i/(b_i - c_i)$, is a special case of ours.

Theorem 2 also illustrates the effect of per-unit resource consumption on the optimal assortment. One would imagine that, fixing everything else, when a product's per-unit resource usage decreases, it becomes relatively more profitable and thus the firm should be more reluctant to offer the other product. Interestingly, this intuition is not always true. By Theorem 2(ii) and Figure 1, for given resource availability y, a decrease in product 1's per-unit usage A may induce the firm to switch from offering only product 1 to selling both products. To see why, suppose that A is very small and all resource is utilized on product 1. This leads to a huge inventory of product 1 and, as a result, the firm has to significantly lower product 1's price to induce sufficient demand. In such a case, the firm is better off making both products available to segment the market and increase total profit.

Another question of interest is how the firm should adjust production and prices when there is more resource available. When product demand is either cross-price inelastic $(c_i = 0, i = 1, 2)$ or deterministic $(\varepsilon_i \equiv 0, i = 1, 2)$, we prove that the firm should increase the inventory of both products and decrease both prices. Similarly, monotonicity of the optimal policies can be shown for the relative per-unit resource usage A. Theorem 3 follows.

Theorem 3: [*Monotonicities of Optimal Solution*] *When* $c_1 = c_2 = 0$ *or when* $\varepsilon_1 \equiv \varepsilon_2 \equiv 0$,

(i) *Both* $q_1^\star$ *and* $q_2^\star$ *are non-decreasing in* y; $q_1^\star$ *is non-increasing in* A.
(ii) *Both* $z_1^\star$ *and* $z_2^\star$ *are non-decreasing in* y; $z_1^\star$ *is non-increasing in* A.
(iii) *Both* $p_1^\star$ *and* $p_2^\star$ *are non-increasing in* y; $p_1^\star$ *is non-decreasing in* A.

The results in Theorem 3 can be intuitively explained: with a higher resource availability, the firm should fully utilize it by increasing the inventories and lowering the prices; when a product consumes more resource, its price should increase and supply decrease. Although we do not present details here, our numerical study indicates that the intuition appears to also apply to the general case where both demand uncertainty and cross-price elasticities are present.

CONCLUSION

This article formulates and analyzes a joint pricing and inventory problem for two horizontally differentiated products sharing a common resource. Demand is stochastic with additive uncertainty, price-based substitutable and lost if not satisfied immediately.

The joint pricing and inventory problem with additive demand uncertainty and lost sales is known for not having a well-behaved profit function, even in the single-product setting without any resource constraint (Petruzzi and Dada (1999)). We contribute to this literature stream by being the first one to analyze the multi-product pricing-inventory problem with resource constraints, product substitution, additive demand uncertainty and lost sales.

Through the analysis of multi-dimensional strict quasi-concavity of the profit function, we prove that there exists a unique optimal solution to the problem. Building on the solution uniqueness, we characterize the firm's product assortment under the optimal pricing-inventory policy. We discover and explain the non-monotonicity of the assortment in the two products' relative per-unit resource usage. Furthermore, a distribution-free index is proposed to aid the firm in determining the assortment priority of the products. Finally, the monotonicities of the optimal policy in total resource availability, as well as in the products' relative resource usage, are shown for the cases where product demand is either cross-price inelastic or deterministic.

ACKNOWLEDGEMENTS

I am grateful for the support from the Hong Kong Research Grant Council [Grants RGC647312].

REFERENCES

Aydin, G. and Porteus, E.L. (2008) Joint inventory and pricing decisions for an assortment. *Operations Research* 56(5): 1247–1255.

Bergstrom, T. and Bagnoli, M. (2005) Log-concave probability and its applications. *Economic Theory* 26: 445–469.

Bertsimas, D. and de Boer, S. (2002) Joint Network Pricing and Resource Allocation. Massachusetts: Sloan School of Management and Operations Research Center, Massachusetts Institute of Technology. Working Paper.

Birge, J.R., Drogosz, J. and Duenyas, I. (1998) Setting single-period optimal capacity levels and prices for substitutable products. *International Journal of Flexible Manufacturing Systems* 10(4): 407–430.

Ceryan, O., Sahin, O. and Duenyas, I. (2013) Dynamic pricing of substitutable products in the presence of capacity flexibility. *Manufacturing and Service Operations Management* 15(1): 86–101.

Cooper, W.L. (2002) Asymptotic behavior of an allocation policy for revenue management. *Operations Research* 50(4): 720–727.

Elmaghraby, W. and Keskinocak, P. (2003) Dynamic pricing in the presence of inventory considerations: research overview, current practices, and future directions. *Management Science* 49(10): 1287–1309.

Gallego, G. and van Ryzin, G. (1997) A multiproduct dynamic pricing problem and its applications to network yield management. *Operations Research* 45(1): 24–41.

Hopp, W.J. and Xu, X. (2005) Product line selection and pricing with modularity in design. *Manufacturing and Service Operations Management* 7(3): 172–187.

Krausa, U.G. and Yano, C.A. (2003) Product line selection and pricing under a share-of-surplus choice model. *Eoropean Journal of Operational Research* 150(3): 653–671.

Kuyumcu, H.A. and Popescu, I. (2005) Deterministic Price-Inventory Management for Substitutable Products. France: INSEAD. Working Paper 2005/71/DS.

Maddah, B. and Bish, E.K. (2007) Joint pricing, assortment, and inventory decisions for a retailer's product line. *Naval Research Logistics* 54(3): 315–330.

Maglaras, C. and Meissner, J. (2006) Dynamic pricing strategies for multiproduct revenue management problems. *Manufacturing and Service Operations Management* 8(2): 136–148.

Petruzzi, N.C. and Dada, M. (1999) Pricing and the newsvendor problem: A review with extensions. *Operations Research* 47(2): 183–194.

Song, J.-S. and Xue, Z. (2007) Demand Management and Inventory Control for Substitutable Products. Durham, NC: Fuqua School of Business, Duke University. Working Paper.

Song, J.-S. and Xue, Z. (2008) Demand Shaping through Bundling: A Dynamic Multiproduct Inventory-Pricing Model. Durham, NC: Fuqua School of Business, Duke University. Working Paper.

Tang, C. and Yin, R. (2007) Joint ordering and pricing strategies for managing substitutable products. *Production and Operations Management* 16(1): 138–153.

Wang, L. and Kapuscinski, R. (2009) Coordination and Price Flexibility for Multiple Products with Price-Based Substitution. Florida: School of Business, University of Miami. Working Paper.

Zhu, K. and Thonemann, U.W. (2009) Coordination of pricing and inventory control across products. *Naval Research Logistics* 56(2): 175–190.

Supplementary information accompanies this article on the *Journal of Revenue and Pricing Management* website (www.palgrave-journals.com/rpm)