Original Article

The application of fuzzy sets theory in eco-city classification

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ABSTRACT Fuzzy sets concept can solve the problem of imprecise data or varied scales of measurement in the attributes of the city theme. This article highlights this advantage of fuzzy sets theory by discussing how it can be appropriately applied to a research problem related to city characterization. Using eco-city as a hypothetical case, this article demonstrates a fuzzy sets application and its three basic fuzzy sets operations, namely complementation or negation, union and intersection in eco-city classification. Arbitrarily selected eco-city attributes include green coverage, recycling ratio and the rate of vehicle increase. This article finally suggests important research areas, possible policy recommendations and noticeable difficulties in spatial and regional study that can utilize the proposed fuzzy sets approach.

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INTRODUCTION

Branding of cities involves complex spatial elements, structuring of spaces and relationships among stakeholders (Hankinson, 2010; Pike, 2011). When branding requires subjective interpretations of cities by people, it inevitably treats spaces as complex entities that carry meanings (Brown, 2006; Giovanardi, 2012). This necessitates the identification of the attributes of a city, and thus, the thematization and classification of different types of cities (Richards and Wilson, 2006; Firat and Ulusoy, 2009). These are all geographical problems that include an imprecision of human cognition and evaluation (Leung, 1982), and the decision-making based on multiple fuzzy criteria (Chen and Tan, 1994; Hong and Choi, 2000).

In the resolution of geographical problems, fuzzy sets application is useful because the real world being studied often contains imprecision. Fuzzy sets theory and the frameworks constructed are proven to be effective in handling problems that are difficult, if not impossible, to be analyzed and solved by conventional methods (Leung, 1988). In the context of spatial analysis and regional planning in geography, researchers deal with spatial problems that are always relevant to some fundamental concepts of spatial analysis, such as distance, direction and connection. How these concepts become languages and how these languages are used to represent or interpret the complex real

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world situations containing an imprecision of human cognition and valuation inside. This has led to the importance of studying the approximate characterization of spatial concepts through fuzzy sets theory (Leung, 1982).

Key geographical problems that have encountered this difficulty include, but not limited to, the analysis of environmental regionalization of spatial units (Leung, 1984), natural resource or environmental assessment (Cornelissena *et al*, 2001; Braimoh *et al*, 2004; Xu and Liu, 2009), and identification of geographical boundaries (Leung, 1987; Guesgen, 2005). The variables characterizing spaces or geographical units might not include precise numerical data. Imprecise information often appears in the form of fuzzy measurements (for example, 'approximately 80 per cent') and linguistic variations (for example, 'high', 'quite long', 'very noisy' or 'overcrowded'). These variations provide ambiguous meanings of the spatial characteristics for researchers. The use of these linguistic hedges, such as the application of the term 'very', has revealed complexity and differences in connotation to interpreters and the resultant different magnitudes of effect on the membership functions (Leung, 1981). However, comparing with some conventional statistical techniques and methods which require an exact database for analysis, the fuzzy sets theory can offer a more flexible, natural and informative approach for studying regionalization and other geographical problems (Leung, 1983, 1984, 1987; Kilr and Folger, 1988).

The usefulness of fuzzy sets concept also extends to city thematization and classification. Cities, by their nature as a set of geographical entities, may be classified according to different attributes such as their physical boundaries, population sizes and other attributes. Cities can also be themed based on their projected image and directions of development. In determining the position of a city or cities under a particular theme (for example, eco-, creative or smart city), fuzzy sets concept can solve the problem of imprecise data or varied scales of measurement in attributes. However, there is a dearth of knowledge and understanding of applying fuzzy sets in city characterization especially in the regional context. Using eco-city and a hypothetical case as a focus, this article aims at: (i) discussing how cities can be classified according to their eco-city attributes through fuzzy sets application, (ii) illustrating three basic fuzzy sets operations: complementation (or negation), union and intersection in eco-city classification and (iii) proposing important research areas and possible policy recommendations in regional development that can utilize fuzzy sets theory.

APPLICATIONS OF FUZZY SETS IN CITY CLASSIFICATION

The concept of fuzzy sets

Fuzzy sets theory originated in early 1960s (Zimmermann, 2001). Fuzzy sets play an important role in decision-making by providing precise representation under imprecise information. This approach offers an alternative and distinctive form of uncertainty representation out of the traditional probability theory (Gaines *et al*, 1984).

Fundamentally, a fuzzy set contains the membership function as a component. This membership function allows a variety of degrees of membership for the elements in a given fuzzy set (Zimmermann, 2001; Deschrijver and Kerre, 2003). Thus, a fuzzy set can be defined as 'any set that does not require a strictly crisp membership, but instead allows its members to have various degrees of membership ranging from 0 to 1' (Vert, Iyengar, & Phoha, 2011, p. 79). For instance, the traditional description of the feeling of 'hot' are represented by a binary indication of 'hot' or 'not hot', which is characterized by an exact degree of cutting point and a crisp function of 'hot' as shown in Figure 1 (upper diagram). As an alternative fuzzy approach, the measurement allows a flexible degree of membership function (Figure 1, lower part).

There are more possibilities of operating fuzzy sets in different contexts through an extension of different relationships between fuzzy sets concepts (see a review from Deschrijver and Kerre, 2003). Three main operations of fuzzy sets theory were discussed by Vert *et al* (2011) and depicted in

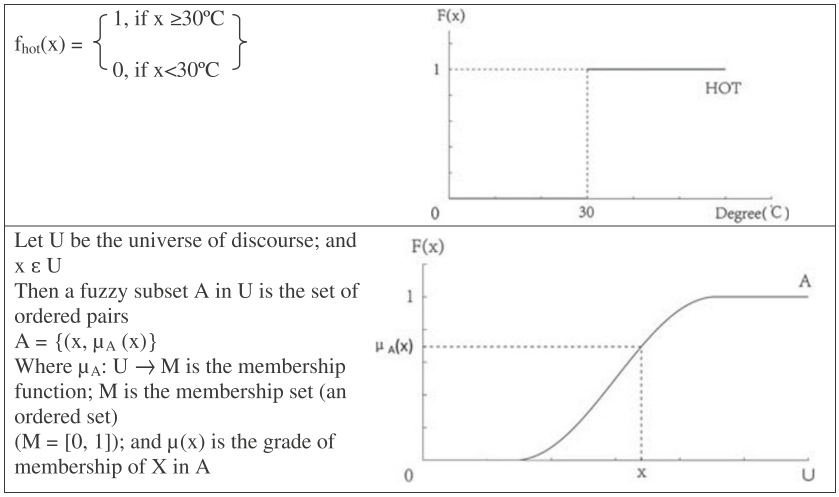

Figure 1: A crisp and a fuzzy subset A representing a function of 'hot'.

Figure 2. These operations are basic to be applied in a simpler circumstance. First, the complementation or negation operation of a fuzzy subset is a set of 'NOT' algebra, which represents mutually exclusive membership functions as illustrated in Definition 1 of Figure 2. In the grade of membership of X in the fuzzy subset A, for example, the 'Not A' function equals to 1 minus the degree of 'μ_A'. Second, the union operation represents an 'OR' algebra of two fuzzy subsets (A or B in the figure) with membership functions μ_A and μ_B (Definition 2). The represented function can fall within either subset A or B. Last, the intersection operation is an 'AND' algebra of the subsets and membership functions μ_A and μ_B (Definition 3). The represented function must fulfill both subsets A and B for this category of operation.

Fuzzy sets application in product and brand categorization

In the study of product marketing, fuzzy set-based measures or analytical tools have been used in the areas of product benefit segmentation and market structuring in early years (for example, Leung, 1980; Wedel and Steenkamp, 1991) or more recently (for example, Orriols-Puig *et al*, 2013; Marchi *et al*, 2014; Wu and Nguyen, 2014). Studies on leisure and tourism also applied fuzzy sets theoretical basis (for example, Moharrer *et al*, 2010; Chang and Chang, 2015; Chu and Guo, 2015) though many knowledge gaps in this area are still present. The development of models for tourism destination sustainability indicators (for example, Stojanović, 2011), for example, appears more relevant to looking at cities as characterized places, but related studies are in a shortage. The use of fuzzy sets approach in brand categorization was apparent (Abimbola and Kocak, 2007; Ahmad and Richard, 2014), but its application is still missing in the study of place branding.

Benefits sought by consumers or visitors, classifications of product or place brands, and the consumer segments themselves are believed to be non-clear-cut concepts. Market areas are believed to be separated into a certain degree only between organizations or firms (Leung, 1980). In the context of product categorization, the fuzzy sets theory is considered applicable because products can have different degrees of membership on specific attributes, instead of a crisp set of features for categorization (Viswanathan and Childers, 1999). A product can possess a partial membership in a category. Take an example of products made in different countries (that is, country-or-origin or country brand effect on a product) as an illustration. For instance, one of the attributes that characterizes a particular commodity is regarded to be 'having high-product quality', while a

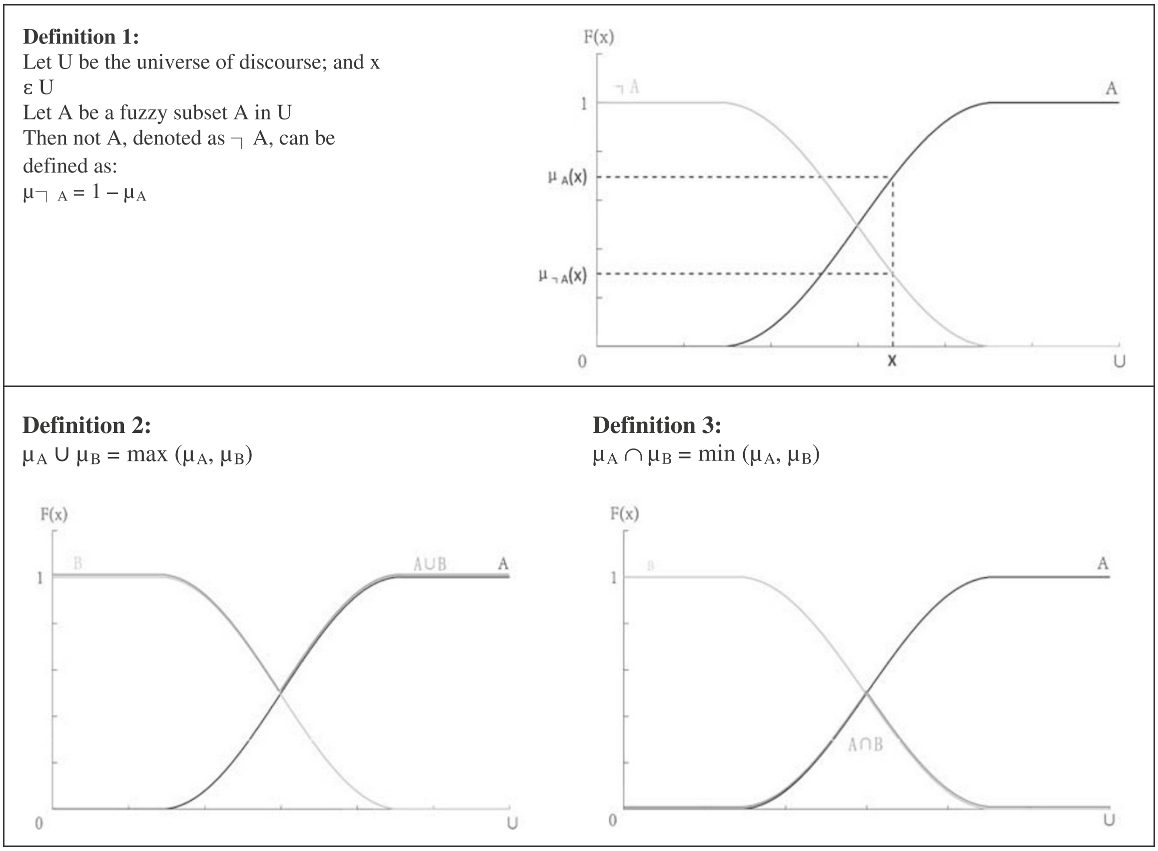

Figure 2: Three basic operations of a fuzzy subset. (Figures in colour are available from the author upon request.)

presumed or assigned level of quality, for example, ⩾3 years of use without repair, is regarded as 'high-product quality'. When a certain brand of product from a country named 'C' has found to offer customers 4 years of use without repair, this brand 'C' is defined as 'high-product quality', which is then classified into a promising brand. This brand 'C' has a full membership (*m*) in the promising brand membership set (that is, denoted as $m = 1$). Another brand of product from a country named 'B' has only offered customers 2 years of use before the first-time repair, which is considered not belong to promising brand category and therefore has a 0 membership (that is, denoted as $m = 0$). The circumstance is depicted in Figure 3a. If product-to-category relationships along attributes are crisp sets such that all attributes take on a value of 0 or 1 only, any brand of product would be assigned a full membership or $m = 1$ when a brand exceeds the assigned value (year of use without repair) in

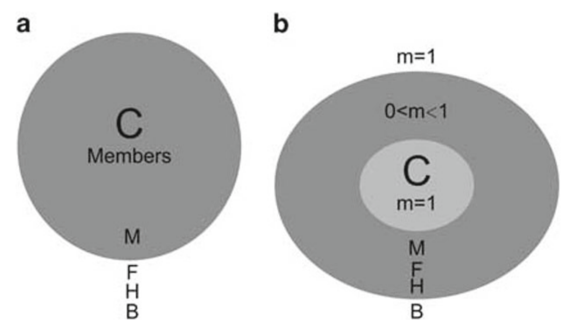

Figure 3: (**a**) A crisp set of category. (**b**) A fuzzy set of category.
Adapted from Viswanathan and Childers (1999, p. 77).

an attribute, no matter if there is any difference between attributes under each brand. In this situation, a crisp set of category is based on precise information.

However, as articulated by Ahmad and Richard (2014), a commodity often possesses both positive and negative attributes for consumption decision.

Under a fuzzy set logic for categorization, a product from a certain country can have an in-between membership or various degrees of membership for a particular attribute, for example, year of use without repair. The difference in degrees of membership of attributes will result in different product categorization, for example, the category of promising brand which can be defined by an intersection of attribute memberships. Taking an earlier example of a situation of a promising brand characterized by an attribute of 'long' period of usage, there can be a fuzzy subset that defines a membership function of 'long' usage, instead of either a full or zero membership. A brand of product named '*M*' may have a certain degree of membership (for examples, 0.8 membership or $m = 0.8$) in this 'long' usage fuzzy subset. Similarly, different brands of various countries can have different degrees of membership in the category of promising brands along a particular attribute, such as year of use without repair, country of origin or price (Figure 5b). With the fuzzy set-based approach, the difference in membership in a specific product category (for example, promising brands) can be explained by membership functions along any attributes.

The angle of consumer product marketing regards the nature of cities as much more complex, carrying multi-layered amalgam and packages of service experiences for consumption or usage (Govers and Go, 2009; Hankinson, 2010). As a result, it is necessary to classify or thematize these 'city products' before establishing specific marketing or brand building strategies.

Cities as products

In 2009, the percentage of urban world population has reached 50.1 per cent, while this indicator of urbanization will continue to grow to 56.6 and 68.7 per cent in 2025 and 2050, respectively (UNDESA, 2010). Urbanization is an inevitable global trend that brings a number of fundamental changes in spatial, demographic, economic, environmental, social, cultural and institutional dimensions to cities (United Nations – HABITAT, 2008). Cities, instead of taking a passive role, have proactively attempted to maximize the benefits of urbanization brought about because cities are important leaders for their respective countries to become wealthy and prosperous (United Nations – HABITAT, 2008). Cities, on one hand, provide economies of scale and high proximity of clustering activities to enhance productivity. Different types of migrants and visitors utilize resources and services offered by cities. On the other hand, cities have increasingly competed for various types of capitals and attention in a globalized world. In attempt to attract tourists, investors and professionals from other regions, cities have begun to study how to promote, market and even brand their competitive advantages, and make these advantages sustainable in the long run (Boisen, 2007; Hankinson, 2010; Sepe, 2010). From a marketing perspective, scholars agreed that cities can be viewed and branded as similar as consumer products and services (Kotler and Gertner, 2002; Olins, 2002), or can have characteristics that large corporations or corporate images often possess (Dinnie, 2004; Hankinson, 2010; Kavaratzis, 2010).

When considering the appropriate ways to market cities, the fuzzy sets theory can shed some lights on city product development and market segmentation. As aforementioned, the applications of fuzzy sets are viable in solving geographical regionalization and product categorization problems. Having a similar approach, cities being viewed as clusters of characteristics or attributes may be classified by some sorts of criteria before thematizing a city, developing its brand and marketing it to the world.

Cities themed and classified as eco-cities

It is proposed by scholars that cities can become more competitive by becoming or branding 'green' (for example, Benson *et al*, 1998; Lerner and Poole, 1999; Konijnendijk, 2008; Braiterman, 2011). These 'green' concepts may be associated with a number of issues such as greenspace planning and provision, strategies and actions of tree planting, energy saving and sustainability of cities (Konijnendijk, 2008; Cilliers, 2011).

There were some growing concepts such as 'green city', 'green infrastructures' and 'eco-city', following a growing need for an improved urban environments and quality of life in cities (Newman, 2010). One of the vigorous discussions was an 'eco-city' development. Originated in 1980s, the concept of 'eco-city' was a proposal for building cities based on ecological principles (Register, 2002) that view cities as living creations. Such development pattern should support and maintain a healthy relationship with nature (Roseland, 2001; Wong and Yuen, 2011).

The concept of 'eco-city' represents a more holistic view of depicting a city as an ecosystem that contains multiple and interrelated components (Roseland, 2001; Register, 2002; Wong and Yuen, 2011), which demonstrate the multiple functions of various urban environmental metabolism (Roberts *et al*, 2009). These components and the resultant discussion about the eco-city concept surround a number of areas from micro-scale aspects such as tree planting and waste management, to meso-scale areas like greenspace and urban planning, and then to macro-ones such as climate change, energy use and sustainable development (Low *et al*, 2005; Birch and Wachter, 2008; Roberts *et al*, 2009). This article undertakes to use the concept of 'eco-city' in the classification of cities, that is, a city may be classified as eco-city when it satisfies some criteria or possesses some prescribed attributes.

In practice, scholars and some municipal governments had identified a number of eco-city components which adopted an ecological approach and combined management strategies for sustainable urban development. Some of these components are summarized as follows:

1. Adaptation and mitigation of climate change (Head and Lam, 2011)
2. Conservation of urban forest and greenspaces (Lipkis, 1992; Newman and Kenworthy, 1999; Kahn, 2006; Newman, 2010)
3. Development of urban agriculture (Roseland, 2001; Head and Lam, 2011)
4. Energy usage and innovations (Crenshaw, 1992; Walter, 1992; Newman and Kenworthy, 1999; Devuyst, 2001)
5. Construction of liveable housing (Roseland, 2001; Lehmann, 2011)
6. Resolution of pollution problems (Newman and Kenworthy, 1999)
7. Protection of sanitation and public health (Newman and Kenworthy, 1999; van Dijk, 2011)
8. Social justice (Roseland, 2001)
9. Sustainable urban governance (Devuyst, 2001)
10. Transportation and communication networks (Siembab and Walter, 1992; Snyder, 1992; Woodhill, 1992; Newman and Kenworthy, 1999; Devuyst, 2001; Head and Lam, 2011)
11. Waste reduction and management (Clements, 1992; Westley, 1992; Newman and Kenworthy, 1999; Devuyst, 2001; Roberts *et al*, 2009; Head and Lam, 2011)
12. Water resources management (Roley, 1992; Newman and Kenworthy, 1999; Head and Lam, 2011)

FUZZY SETS APPLICATION IN ECO-CITY CLASSIFICATION

Different ecological performance levels among cities in different regions of the world can be measured and evaluated based on a collection of indicators and their statistics (Head and Lam, 2011). It is in principle possible to classify each city into different classes of eco-city. When applying a fuzzy sets approach in eco-city identification, cities should not be classified in a clear-cut manner or by a crisp set of category based on precise information of attributes. In other words, for instance, a crisp set of classification defines a city as an 'eco-city' if it has over 60 per cent of land covered by greenspace or vegetation, whereas another city is not an 'eco-city' if its greenspace coverage is lower than 30 per cent. Such prescribed eco-city 'standards' can be problematic because cities carry diversified backgrounds, resources and characteristics, which impose difficulties in setting a universal and precise standard for city classification worldwide. Such subjectivity problem also applies to many city ranking exercises (Kavaratzis and Ashworth, 2008; Wong, 2012).

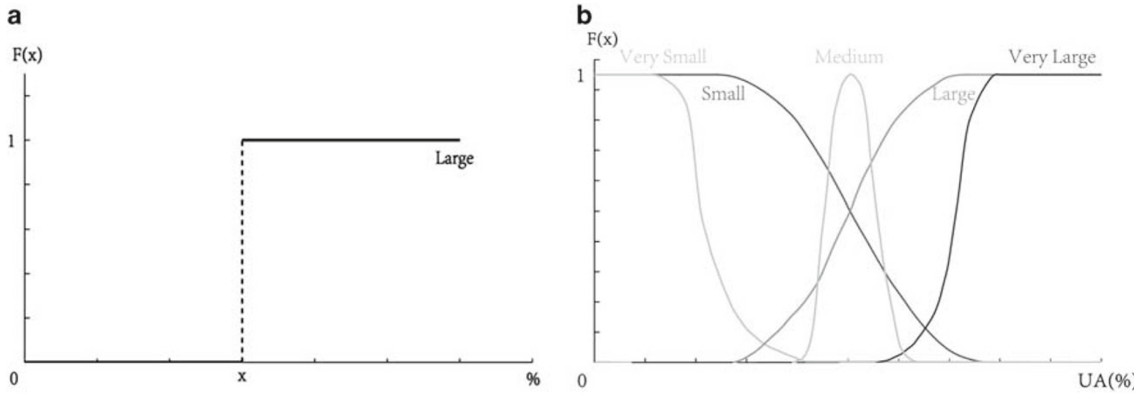

Figure 4: (**a**) A crisp subset of 'large' green coverage. (**b**) Fuzzy subsets of 'very large', 'large', 'medium', 'small' and 'very small' green coverage. (Figures in colour are available from the author upon request.)

Instead, linguistic terms are used to identify the city ecological status, thus there can be classes of eco-city like 'high-level eco-city' or 'low-level eco-city'. This implies impreciseness in classification and decision-making that justifies the application of fuzzy sets. To simplify the illustrations of how fuzzy sets are applied in the classification of eco-cities, this article assumes that an eco-city is consisted of three key attributes: (i) conservation of urban forest and greenspaces, (ii) waste reduction and management and (iii) transportation and communication networks.

For each key attribute, a measurable indicator is assigned to assess the performance of a city, which in turn can be regarded as the class of eco-city. The three indicators are: (i) the percentage of greenspace coverage in a city area, denoted as 'green coverage' (for conservation of urban forest and greenspaces) (ii) recycling ratio of the total solid wastes collected in a city, denoted as 'recycling ratio' (for waste reduction and management) and (iii) rate of increase in the number of private vehicles registered per period in a city, denoted as 'vehicle increase' (for transportation and communication networks). The 'rate of increase in the number of private vehicles registered per period in a city' is an indicator that reflects the demand of transportation facilities and infrastructure on the whole. The growth of private transport may partially indicate the decline in the use of public transport, which is a negative condition to the sustainable city development (Kenworthy, 2006). The selection of these three indicators from the list of components is arbitrary because the main objective of this article is to propose the idea of fuzzy sets application to city classification. Nevertheless, it is believed that conservation of urban greenspaces, waste reduction and transportation are important foci addressing various aspects across eco-city discussions (for example, Newman and Kenworthy, 1999; Devuyst, 2001; Kahn, 2006; Head and Lam, 2011; Newman, 2010). The author believes the three indicators as measurable and comparable parameters for demonstrating the fuzzy sets application.

Let U_A, U_B, U_C the universe of discourse of each attribute set (green coverage, recycling ratio, vehicle increase). Then, there can be a number of subsets using linguistic descriptions (for example, 'very large', 'high' and 'medium') in each of the universe of discourse. In this article, five subsets are assigned for each attribute.

Unlike a clear-cut crisp set approach as shown in Figures 4a, 5a and 6a, fuzzy sets theory allows the use of linguistic measurements to define the three attributes. First, 'green coverage' can have five fuzzy subsets to indicate namely 'very large', 'large', 'medium', 'small' and 'very small' coverage (Figure 4b). Second, 'recycling ratio' can have another five fuzzy subsets to indicate 'very large', 'large', 'medium', 'small' and 'very small' recycling ratios (Figure 5b). Finally, five fuzzy subsets for depicting the level of 'vehicle increase' can range from 'very high' to 'very low' through 'high',

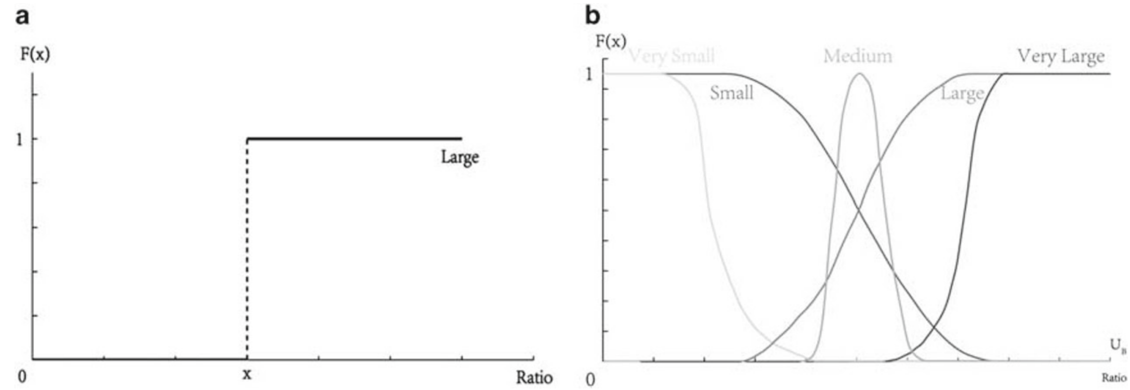

Figure 5: (**a**) A crisp subset of 'large' recycling ratio. (**b**) Fuzzy subsets of 'very large', 'large', 'medium', 'small' and 'very small' recycling ratios. (Figures in colour are available from the author upon request.)

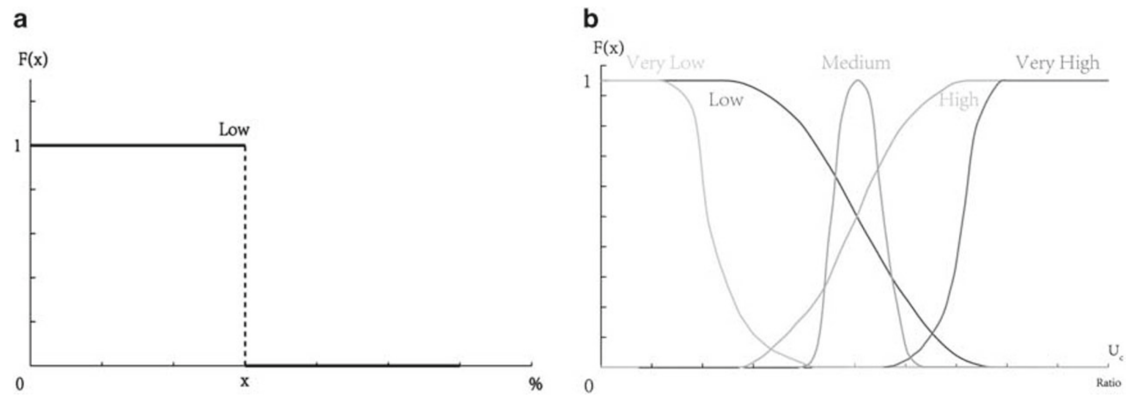

Figure 6: (**a**) A crisp subset of 'low' rate of vehicle increase. (**b**) Fuzzy subsets of 'very low', 'low', 'medium', 'high' and 'very high' rates of vehicle increase. (Figures in colour are available from the author upon request.)

'medium' and 'low' in between. These are the rates of increase in number of vehicle (Figure 6b). When a researcher had conducted a measurement or collected data of an attribute (for example, 60 per cent of green coverage of a city is recorded), this measurement or data can then be assigned in the corresponding fuzzy set to determine whether this city has a 'very large', 'large', 'medium' or other levels of green coverage.

Cities are then classified into different classes of eco-city by assigning different levels of performance in the attributes. In this article, a classification scheme is proposed in Table 1 based on the three hypothetical attributes selected earlier. In practice, the cut-off points of the classes of eco-city (from 'very high' to 'very low') and the range of each level of the three attributes (between 'very large/high' and 'very small/low') are to be determined by policy makers. A city, for example, with a 'very large' green coverage, a 'very large' recycling ratio and a 'very low' rate of vehicle increase will be classified as a 'very high' level eco-city. In principle, varied levels of the attributes may combine to represent a certain class of eco-city if the combined contribution of the attributes is pre-defined. However, there is a lack of

Table 1: Classification of eco-cities according to levels of attributes

Class of eco-city	*Green coverage*	*Recycling ratio*	*Vehicle increase*
Very high	Very large	Very large	Very low
High	Large	Large	Low
Medium	Medium	Medium	Medium
Low	Small	Small	High
Very low	Very small	Very small	Very high

recognition of which level should each attribute have so as to attain a higher class of eco-city status. The following section will demonstrate how to apply the three major fuzzy sets operations in an eco-city classification.

OPERATIONS OF FUZZY SETS THEORY TO ECO-CITY CLASSIFICATION

Complementation or negation operation

When a specific attribute (for example, green coverage of 35 per cent) of a particular city is reported, it is possible to study to what extent (degree of membership) this data (35 per cent) represents a 'large' green coverage, and also in opposite of a 'not large' green coverage. In the latter condition, instead of taking the data to a fuzzy subset of 'large' green coverage, it is necessary to use a complementation or negation operation under fuzzy sets theory.

By referring to the Definition 1 in Figure 2, a complementation or negation operation of fuzzy sets is defined as $\mu_{\neg A} = 1 - \mu_A$. In the eco-city example of a fuzzy subset of 'large' green coverage (μ_{large}), a 'not large' green coverage subset can be generated (Figure 7), that is, $\mu_{\text{not large}}$ or $\mu_{\neg\,\text{large}} = 1 - \mu_{\text{large}}$. In this case, a reported eco-city attribute of a particular city (in this example, a green coverage of 35 per cent) can have two related definitions from the normal and complement fuzzy subsets, respectively. This approach similarly applies to the other subsets of all the attributes. For a city with a measurement of green coverage of 35 per cent, in a fuzzy subset of 'large' green coverage, let it has an assumed grade of membership of 0.4. Then in a fuzzy subset of 'not large' green coverage, this city will have a grade of membership of 0.6 as shown in equation (1).

$$\begin{aligned} \mu\rceil_{\text{large}}(x) &= 1 - \mu_{\text{large}}(x) \\ &= 1 - \operatorname*{Sup}_{x} \mu_{\text{large}}(35\,\%) \\ &= 1 - 0.4 \\ &= 0.6 \end{aligned} \tag{1}$$

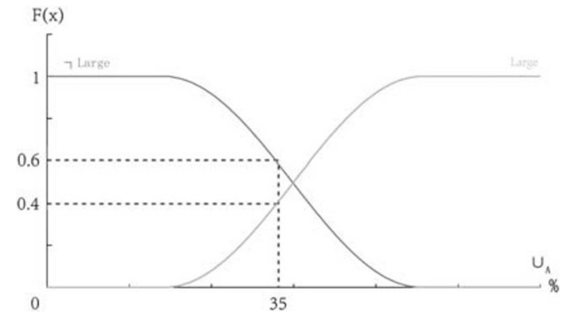

Figure 7: A complementation or negation operation of a 'large' green coverage subset. (Figures in colour are available from the author upon request.)

(The grade of membership of level of 'not large' green coverage is 0.6)

Union operation

A union operation of fuzzy sets can be applied in principle though its applicability is less relevant under a multi-criteria condition of eco-city classification. Referring to the Definition 2 in Figure 2, a union operation of two fuzzy sets is defined by $\mu_A \cup \mu_B = \max(\mu_A, \mu_B)$. Given that two fuzzy subsets define the 'large' (μ_{large}) and 'medium' (μ_{medium}) green coverage, respectively (Figure 4b), the loosened and thus union operation is defined by: $\mu_{\text{large}} \cup \mu_{\text{medium}} = \max(\mu_{\text{large}}, \mu_{\text{medium}})$. For instance, let a city has a measurement of green coverage of 60 per cent, in a fuzzy subset of 'large' green coverage, it assumes to carry a grade of membership of 0.5, whereas in a fuzzy subset of 'medium' green coverage, it has a grade of membership of 0.8 as illustrated in equation (2) (Figure 8).

$$\begin{aligned} &\text{Given, } \mu_{\text{large}\,\cup\,\text{medium}}(x) \\ &= \max\left[\mu_{\text{large}}(x), \mu_{\text{medium}}(x)\right] \\ &= \mu_{\text{large}}(x) \vee \mu_{\text{medium}}(x), \ \forall\, x \varepsilon U \\ &= \operatorname*{Sup}_{x}\left(\mu_{\text{large}}(60\,\%) \vee \mu_{\text{medium}}(60\,\%)\right) \\ &= 0.5 \vee 0.8 \end{aligned} \tag{2}$$

The level of eco-city is 0.8.

The conclusion in equation (2) is based on an assumption that an eco-city is determined by one

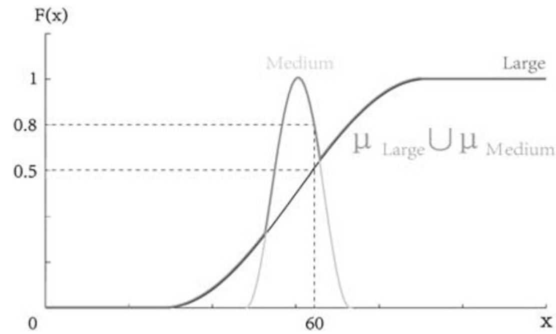

Figure 8: A union operation of subsets of 'large' and 'medium' green coverage. (Figures in colour are available from the author upon request.)

key factor, green coverage of a city. It is understood that this is an unrealistic situation, especially in forms of applicability and relevance of multi-criteria eco-city classification. However, the above hypothetical example is used to show an union operation of fuzzy sets.

In an application of a union operation of fuzzy sets, it is necessary to define the membership functions of each class of eco-city in Table 1. Denoting the three attributes of green coverage, recycling ratio and vehicle increase as GC, RR and VI, respectively. Consequently, the three attributes should have their own degree of membership in each level. For example, μ_{large} (GC) represents a 'large' green coverage; μ_{medium} (RR) represents a 'medium' level of recycling ratio; and μ_{low} (VI) represents a 'low' rate of vehicle increase. A combination of these degrees of membership will constitute a grade of membership of eco-city class correspondingly.

Following this logic, for instance, a 'high' eco-city class has a membership function of having either a medium or large level of green coverage, a medium or high level of recycling ratio, *or* a low or medium level of vehicle increase. The resultant membership function of a 'high' eco-city class should be:

$$\mu_{\text{high}}(\text{GC}, \text{RR}, \text{VI}) = min[\max\left(\mu_{\text{large}}(\text{GC}),\ \mu_{\text{medium}}(\text{GC})\right), \max\left(\mu_{\text{large}}(\text{RR}), \mu_{\text{medium}}(\text{RR}\right), \min\left(\mu_{\text{low}}(\text{VI}),\ \mu_{\text{medium}}(\text{VI})\right].$$

Intersection operation

The last fuzzy sets operation is an intersection of attribute functions. In Table 1, a city with a 'large' green coverage, a 'large' recycling ratio, *and* a 'low' rate of vehicle increase will be classified as a 'high'-level eco-city. This can demonstrate an intersection operation of fuzzy sets approach.

By the Definition 3 in Figure 2, an intersection operation of two fuzzy sets is defined as $\mu_A \cap \mu_B = \min\ (\mu_A, \mu_B)$. Similarly, an intersection operation of three fuzzy sets can be defined as $\mu_A \cap \mu_B \cap \mu_C = \min\ (\mu_A, \mu_B, \mu_C)$. Again, use the earlier same denotations to distinguish the three attributes (that is, GC, RR and VI). Given that the above-mentioned three fuzzy sets represent 'large' green coverage ($\mu_{\text{large GC}}$), 'large' recycling ration ($\mu_{\text{large RR}}$), *and* 'low' rate of vehicle increase ($\mu_{\text{low VI}}$), respectively (Figures 4b, 5b and 6b), the intersection operation is defined by: $\mu_{\text{large GC}} \cap \mu_{\text{large RR}} \cap \mu_{\text{low VI}} = \max\ (\mu_{\text{large GC}}, \mu_{\text{large RR}}, \mu_{\text{low VI}})$.

For instance, let a city has measurements of green coverage of 60 per cent, recycling ratio of 0.3 and rate of vehicle increase of 1 per cent per period. In a fuzzy subsets of the three attributes, a 'large' green coverage indicates that this city has a grade of membership of 0.5. A 'large' recycling ratio offers this city a grade of membership of 0.3, and a 'low' rate of vehicle increase represents a grade of membership of 0.8 for the city. Again, the grades of memberships are assumed value only. As a result, the membership function of a 'high' eco-city class is illustrated in equation (3) (Figure 9).

$$\begin{aligned}
\mu_{\text{high}}&(\text{GC, RR, VI}) \\
&= min\left[\mu_{\text{largeGC}}(\text{x}_1),\ \mu_{\text{largeRR}}(\text{x}_2),\ \mu_{\text{lowVI}}(\text{x}_3)\right] \\
&= \mu_{\text{largeGC}}(\text{x}_1) \wedge \mu_{\text{largeRR}}(\text{x}_2) \wedge \mu_{\text{lowVI}}(\text{x}_3), \forall\ \text{x}\varepsilon\text{U} \\
&= \operatorname*{Sup}_{\text{x}}\left(\mu_{\text{largeGC}}(60\,\%),\ \mu_{\text{largeRR}}(0.3),\ \mu_{\text{lowVI}}(1\,\%)\right) \\
&= 0.5 \wedge 0.3 \wedge 0.8 \\
&\quad \text{(The level of eco-city is 0.3)} \qquad (3)
\end{aligned}$$

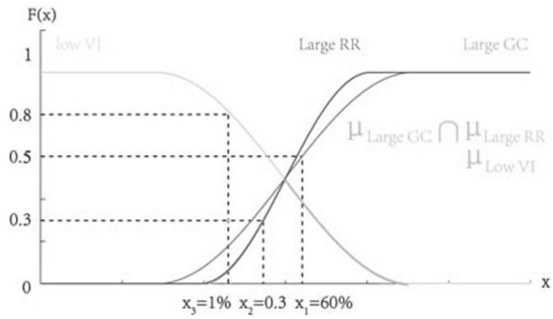

Figure 9: An intersection operation of subsets of 'large' green coverage, 'large' recycling ratio and 'low' vehicle increase. (Figures in colour are available from the author upon request.)

CONCLUSION AND IMPLICATIONS

This article discusses how fuzzy sets can be appropriately applied to identify and justify a research problem related to eco-city classification through three basic operations: complementation or negation (in a fuzzy subset of one attribute), union and intersection (in two or three fuzzy subsets of one or more attributes), under hypothetical conditions.

The fuzzy sets application provides some theoretical and practical contributions to topical areas such as city ranking, theme identification, urban destination product development and market segmentation, and city branding. First, city ranking is pervasive nowadays where these exercises usually include subjectively determined multi-disciplinary and multi-dimensional indicators (Kavaratzis and Ashworth, 2008; Ooi and Stöber, 2010). The fuzzy sets theory and its operations provide an appropriate theoretical framework for analyses of human spatial behavior in an environment which is full of uncertainty, imprecision and complexity (Leung, 1980; Kilr and Folger, 1988). These obstacles are especially difficult in the context of city classification and in turn making comparison across competitive places and regions. Instead of asserting an exact rank of each city based on a number of measurable but non-compatible indicators, the fuzzy sets approach is useful to describe those imprecise attributes or levels of these attributes of cities. An example of such opportunity is to utilize the intersection operation of fuzzy sets, which had already demonstrated by the case example of this article.

Table 2: Proposed classification of eco-cities according to intersected grades of membership

Class of eco-city	*Proposed intersected grades of membership*
Very high	0.9–1.0
High	0.7–0.8
Medium	0.5–0.6
Low	0.3–0.4
Very low	0.1–0.2
Unclassified	0.0

Second, these fuzzy sets operations can identity how a city is distinctive and to what level it has achieved in a specific theme (in this case, eco-city). It is possible to further extend the intersection operation of fuzzy sets by classifying cities into various levels of eco-cities according to each of their intersected grades of membership. On the basis of the classification matrix in Table 1, for example, a city has 0.1–0.2 intersected grade of membership that will be classified as 'very low' class of eco-city, whereas other cities with intersected grades of membership ranged from 0.3 to 1 can be classified as 'low' to 'very high' eco-city class accordingly (Table 2). This is a proposed way of classification for eco-city, whereas the numbers are only one set of suggested ranges or cut-off points. Similar city categories (for example, cultural city, creative city, smart city, sustainable city and so on) are also applicable when the specific attributes and classes of these city themes can be established. It is recommended that the classification scheme and the intersected grades of membership are established through the consultation of a relevant panel of expert and, where possible, the involvement of stakeholders and the public.

Urban destination product development involves the determination of physical products, people, package and programming (Morrison, 2013). In this regard, fuzzy sets are useful as the concept applied to product categorization and benefit study rigourously (for example, Wedel and Steenkamp, 1991; Viswanathan and Childers, 1999; Orriols-Puig *et al*, 2013). One possible extension in research is the exploration of market segments through the fuzzy measurement of destination product categories, and the study of destination image perception by tourists.

These connects to the field of place branding. City branding requires an thorough understanding of the spatial identity perceived by local citizens and non-local city consumers (Govers and Go, 2009; Hankinson, 2010). Spatial identity is a fuzzy concept (Kalandides, 2011), and so is brand measurement (Abimbola and Kocak, 2007; Ahmad and Richard, 2014). All the above opportunities of research potential have shown some applicability of fuzzy sets concept in the social science and tourism disciplines, especially in the fields that involves imprecise or subjective measurement of attributes.

In reality, however, the attributes and conditions that constitute to a well-recognized eco-city identity can be much more complex. The operation of the three fuzzy sets scenarios (negation, union and intersection) in this article is only a conceptualized and hypothetical illustration. The decision of different classes of an eco-city is often made based on the intersection or sometimes negation of a number of attributes, but the applicability of an union operation is in principle possible though less relevant. The relevance and function of each operation in a city should be determined by the actual circumstance and the decision by the authority. In addition to the problem of imprecise information and measurements that fuzzy sets theory had already provided an effective improvement, there are at least two big hurdles that must be crossed over. The foremost difficulty is the selection of attributes of measurement. Regional development requires consensus in selecting appropriate indicators of sustainability which should be widely agreed upon the group of cities concerned. This missing consensus is not about the applicability and effectiveness of fuzzy sets approach, but rather the arenas of individual contexts, such as how to define an eco-city, a smart city and so on. Furthermore, it is very complex to determine an agreed grade of membership to each type of measurement attributes across stakeholders. Different assumptions of assigning a grade of membership to a specific attribute can generate very different membership functions of classes, and thus in turn affect the resultant classes of cities.

The case examples elaborated in this article are hypothetical, which incur a limitation but a potential for empirical study to apply real world data from (eco-)cities to test the fuzzy sets operations in theory. The grades of membership in the examples were also assigned by the author, though served an illustrative purpose, also caused a shortcoming to the conceptual applicability.

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