
Original Article

Pure return persistence, Hurst exponents and hedge fund selection – A practical note

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Benjamin R. Auer

is Assistant Professor of Finance at the University of Leipzig and Research Affiliate at CESifo Munich, Germany. He holds a PhD in financial economics and has recently been ranked place 27 among all German-speaking researchers below the age of 40 in the Handelsblatt Research Ranking 2014. His main areas of research are investment management, risk and performance measurement, and applied financial econometrics.

Correspondence: Benjamin R. Auer, University of Leipzig, Grimmaische Straße 12, Leipzig 04109, Germany
E-mail: auer@wifa.uni-leipzig.de

ABSTRACT In this note, we analyse whether measures of pure return persistence can forecast hedge fund performance, that is, whether they have the ability to identify hedge funds with superior performance in the remote future. Using a variety of Hurst exponent estimators (originating from rescaled range, detrended fluctuation, periodogram regression and average wavelet coefficient frameworks) and a rich sample of hedge funds, our study offers two important insights. First, only a small fraction of long-living funds shows significant return persistence whereas the majority is characterised by random behaviour. Second, high Hurst exponents (combined with low negative return ratios) can identify the funds with the highest Sharpe ratios, both in-sample and out-of-sample. Since these results are robust in a variety of settings, they imply that Hurst exponents can act as valuable hedge fund selection tools, not only for individual investors seeking the best funds but also for managers constructing funds of hedge funds.

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INTRODUCTION

For almost two decades, the persistence of hedge fund performance has been subject of a considerable number of academic studies because of its important implications for investors (see Eling, 2009). Investors are naturally interested in selecting the funds with the highest future performance but typically base their decisions on past returns (see Capon *et al.*, 1996). Thus, the identification of the future best managers on the basis of a track record requires a rigorous method to quantify and evaluate persistence in returns (see Géhin, 2005).

Generally, we distinguish between two types of return persistence: relative return persistence (RRP) and pure return persistence (PRP).¹ To analyse RRP, funds are classified as winners or losers by comparing their returns with the median return in a given period. Evidence of return persistence is observed when winners and/or losers remain the same between periods. While the cross-product ratio test (see Agarwal and Naik, 2000), the χ^2 test (see Park and Staum, 1998), the rank information coefficient test (see Herzberg and Mozes, 2003), the Spearman rank correlation test (see Park and Staum, 1998) and the

regression-based method (see Brown *et al.*, 1999) are the most-used two-period approaches, the Kolmogorov–Smirnov test employs a multi-period framework and can be considered the most powerful method of testing RRP (see Agarwal and Naik, 2000; Teo *et al.*, 2003; Eling, 2009). In contrast to RRP methods, which indicate persistence of the outperformance or underperformance relative to other funds, a PRP test allows identification of the funds exhibiting the strongest persistence of their positive returns which is analysed fund by fund.

De Souza and Gokcan (2004), henceforth DSG, propose examining PRP based on the Hurst exponent H because it is a natural indicator of long memory and its fractal estimators are typically superior to more conventional approaches of measuring long memory, such as autocorrelations, variance ratios and spectral decompositions, in terms of sensitivity to highly non-Gaussian series, convergence and non-periodic cycles (see Campbell *et al.*, 1997).² $H < 0.5$ suggests anti-persistence, that is, that positive returns tend to succeed negative returns and vice versa, $H = 0.5$ implies random succession of returns and $H > 0.5$ indicates persistence, that is, that returns tend to be followed by returns of the same sign (see Kantelhardt, 2009). While for high H observations in the remote past are nontrivially correlated with observations in the distant future, no information is given on whether positive or negative returns persist. Thus, DSG suggest a fund selection procedure that combines a high Hurst exponent with a low negative return ratio D because this mix indicates the persistence of positive returns. In an exemplary performance analysis of 314 hedge funds in the period from 1997 to 2002, they show that this procedure can identify funds that significantly outperform the remaining funds out-of-sample and thus can be regarded as a particularly interesting approach for hedge fund selection.³

Despite its high theoretical and empirical appeal, the DSG approach has received little

attention in the literature. This can be attributed to several issues. First, the accuracy of the presented empirical results is limited because Hurst exponent approaches are known to require larger time series to be precise (see Amvella *et al.*, 2010). Second, an extension to a larger number of funds (extracted from one of the standard hedge fund databases) would make the presented evidence more convincing (see Capocci and Hübner, 2004). Finally, we have to consider studies showing that the choice of fractal estimator for the Hurst exponent can crucially influence decision-making (see Taqqu *et al.*, 1995; Clegg, 2006; Fernandez, 2011). Thus, while DSG rely only on the classic rescaled range analysis (RRA) of Hurst (1951) and Mandelbrot (1971), using alternative estimators would give important indications on the robustness of results.

In this article, we address these drawbacks and revisit the DSG approach in a wider setting. On the one hand, we use a richer sample (in the time series as well as in the cross-sectional dimension) of hedge funds obtained from the Center for International Securities and Derivatives Markets (CISDM) database (as used in Auer and Schuhmacher, 2013a, b). On the other hand, we take a detailed look at the sensitivity of results to alternative fractal estimators. In addition to using the standard RRA, we employ three alternative estimators that have received considerable attention in recent years. Specifically, we implement the detrended fluctuation analysis (DFA) of Moreira *et al.* (1994) and Peng *et al.* (1994), the periodogram regression method (PRM) of Geweke and Porter-Hudak (1983) and the average wavelet coefficient method (AWCM) of Simonsen *et al.* (1998). While RRA and DFA have become state-of-the-art techniques not only for measuring long memory in economic time series but also in the related areas of market crash prediction (see Grech and Mazur, 2004), investment strategy development (see Batten *et al.*, 2013) and the evaluation of market efficiency

(see Eom *et al*, 2008), the PRM and the AWCN are less widespread but have proven particularly valuable in the analysis of stock and commodity return dynamics (see Crato and de Lima, 1994; Simonsen, 2003) and order sign forecasting (see Lillo and Farmer, 2004). The primary goal of our study is to evaluate the ability of the Hurst exponent to identify superior hedge funds in-sample and out-of-sample. With respect to the choice of fractal estimator, we are particularly interested in whether it has a crucial influence on the fund selection outcome.

The remainder of the article is organised as follows. The next section provides a brief description of our hedge fund data set. The subsequent section discusses our four fractal estimation procedures for the Hurst exponent. The section after that presents the empirical results and the outcomes of several robustness checks. The final section concludes.

DATA

The hedge fund return data used in this study are obtained from the CISDM hedge fund database that has been subject of many academic studies.⁴ Even though it covers 6854 hedge funds from January 1972 through November 2009, we concentrate on the years 1994–2009 for three reasons.⁵ First, because hedge fund data before 1994 is considered unreliable (see Fung and Hsieh, 2000; Liang, 2000; Li and Kazemi, 2007), the exclusion of this time frame limits the data flaws that plague much of the older hedge fund research (see Capocci and Hübner, 2004; Liang and Park, 2007; Eling, 2009). Second, unlike other studies (see Brown *et al*, 1999; Amenc *et al*, 2003; Baquero *et al*, 2005), our sample contains bullish and bearish markets such that our results are not biased by a concentration on a specific market phase (see Capocci *et al*, 2005; Ding and Shawky, 2007). Third, it is important not to use excessively long time periods in the analysis of hedge fund return persistence because, in this context,

Table 1: Descriptive statistics

	Mean	Standard deviation	Skewness	Kurtosis
Minimum	-1.20	0.16	-11.04	1.77
Maximum	3.88	24.26	5.42	126.36
Mean	0.75	4.25	-0.49	9.26
Standard deviation	0.44	2.96	1.56	10.90

Note: For our sample of 1493 hedge funds, this table shows some descriptive measures (minimum, maximum, mean and standard deviation) characterising the first four moments of the hedge funds' return distributions (mean, standard deviation, skewness and kurtosis). All measures are based on monthly percentage log-returns extracted from the CISDM database.

persistence is associated mostly with the special skill of the fund manager and hedge fund managers usually do not work with the same hedge fund for more than a decade (see Boyson, 2008).

While the time-series dimension of our sample should not be too big because of the aforementioned reasons, we have to consider that our fractal estimators of the Hurst exponent require sufficient sample sizes in order to be precise (see Chamoli *et al*, 2007). Thus, to balance both requirements, only funds with at least 120 monthly return observations are admitted to our sample. This leads to a final cross-sectional sample size of 1493 funds. Furthermore, we have to take into account that Hurst exponent estimates must be obtained based on continuously compounded returns (see Peters, 1992). Thus, we convert the simple returns from the CISDM database to log-returns.

Table 1 presents some descriptive statistics for our sample. It shows the minimum, maximum, mean and standard deviation of the first four moments of the return distributions (mean, standard deviation, skewness and kurtosis). As we can see, the hedge funds in the data set show positive returns of 0.75 per cent on average with an average standard deviation of 4.25 per cent.⁶ On average, returns are skewed to the left and show heavier tails than the normal distribution. However, this is not a concern

for our subsequent analysis because fractal estimators can be applied even to highly non-Gaussian time series (see Mandelbrot and Wallis, 1969; Barunik and Kristoufek, 2010; Kristoufek, 2012).

Even though our sample selection process eliminates periods with unreliable return data, one might argue that the data still suffers from classic survivorship and backfilling biases (see Ackermann *et al.*, 1999; Fung and Hsieh, 2000). These biases influence studies evaluating the performance of the overall hedge fund industry because they lead to an overstatement of the industry's performance. In our study, however, the aim is different. We take the perspective of an investor with the goals to (i) identify the best funds listed in the CISDM database by means of a Hurst exponent approach and to (ii) invest in the resulting selection. This kind of fund-picking is naturally only meaningful when conducted among funds still active at the time the investment decision is made. Furthermore, for this investor, the CISDM database is the only source of return information and he cannot detect whether a fund backfilled data after the incubation period or not. As a consequence, our research question and our results are conditional upon the CISDM data. That is, we answer the question of whether investors can use the Hurst coefficient obtained based on the CISDM data to identify the best hedge funds.

METHODOLOGY

Rescaled range analysis

The RRA of Hurst (1951) and Mandelbrot (1971) is probably the best-known method to estimate the self-similarity parameter or Hurst exponent H . Given a series of continuously compounded returns $\{r_t\}_{t=1}^T$, RRA is basically performed in two steps (see Sánchez Granero *et al.*, 2008; Souza *et al.*, 2008).⁷ We begin the first step by dividing the given time series into d sub-series of length n . Next, for each sub-series $m = 1, \dots, d$, we estimate the

mean μ_m and standard deviation σ_m by means of maximum likelihood and use the resulting values to obtain the rescaled range

$$Q_m = \frac{1}{\sigma_m} \left[\max_{1 \leq i \leq n} \sum_{j=1}^i (r_{j,m} - \mu_m) - \min_{1 \leq i \leq n} \sum_{j=1}^i (r_{j,m} - \mu_m) \right], \quad (1)$$

where the first (second) term in brackets is the maximum (minimum) over i of the partial sums of the first i deviations of $r_{j,m}$ from the sub-series mean μ_m . Finally, we calculate the average $\bar{Q}_n = d^{-1} \sum_{m=1}^d Q_m$ of the rescaled range over all sub-series of length n .

The second step makes use of the fact that the \bar{Q}_n statistic asymptotically follows the relation $\bar{Q}_n \sim cn^H$, where c is a constant and H is the Hurst exponent (see Mandelbrot, 1975). Thus, we can obtain H by running a linear regression over a sample of increasing time horizons, that is, $\log(\bar{Q}_n) = \log(c) + H \log(n) + \epsilon$, where ϵ is the residual of the regression.

Detrended fluctuation analysis

Our second method to estimate H is the DFA proposed by Moreira *et al.* (1994) and Peng *et al.* (1994). Similar to the RRA, the DFA is performed in two steps (see Grau-Carles, 2000; Grech and Mazur, 2004). In the first step, we divide the time series $\{r_t\}_{t=1}^T$ into d sub-series of length n . Next, for each sub-series $m = 1, \dots, d$, we create a cumulative time series $Y_{i,m} = \sum_{j=1}^i (r_{j,m} - \mu_m)$ for $i = 1, \dots, n$, fit a least squares line $Y_{i,m} = a_m + b_m i + \epsilon$ to $\{Y_{1,m}, \dots, Y_{n,m}\}$ ⁸ and calculate the root mean square fluctuation

$$F_m = \sqrt{\frac{1}{n} \sum_{i=1}^n (Y_{i,m} - a_m - b_m i)^2} \quad (2)$$

of the integrated and detrended time series. Finally, we calculate the mean value \bar{F}_n of the root mean square fluctuation for all sub-series of length n . In the second step, a power-law behaviour $\bar{F}_n \sim cn^H$ is expected (see Peng *et al.*, 1994; Taqqu *et al.*, 1995) from which H can be extracted from log-log linear fit similar to RRA.

Periodogram regression method

A semi-parametric procedure to obtain the Hurst exponent has been developed by Geweke and Porter-Hudak (1983). Its basic idea is to estimate the differencing parameter κ of a general fractionally integrated model. Since the spectral density function of such a model is identical to that of a fractional Gaussian noise with Hurst exponent $H = \kappa + 0.5$, the PRM can be used to estimate H . The estimation procedure involves the following calculations (see Weron, 2002; Granger and Hyung, 2004). We begin with calculating the periodogram, which is a sample analogue of the spectral density (see Stoica and Moses, 2005). For a time series $\{r_t\}_{t=1}^T$, it is defined as

$$I_T(\omega_k) = \frac{1}{T} \left| \sum_{t=1}^T r_t e^{-2\pi i(t-1)\omega_k} \right|^2, \quad (3)$$

where $\omega_k = k/T$, $k = 1, \dots, \lfloor T/2 \rfloor$, and $\lfloor x \rfloor$ denotes the largest integer less than or equal to x . The next and final step is to run a linear regression $\log[I_T(\omega_k)] = c + \kappa \log[4\sin^2(\omega_k/2)] + \epsilon_k$ at low Fourier frequencies ω_k , $k = 1, \dots, K \leq \lfloor T/2 \rfloor$. H is then obtained by plugging the estimate of κ into $H = \kappa + 0.5$.

Average wavelet coefficient method

Our last estimation procedure is the AWCN of Simonsen *et al* (1998). This method utilises the wavelet transformation in order to measure the Hurst exponent and is conducted as follows (see Simonsen, 2003; Chamoli *et al*, 2007). First, the given time series is transformed into the wavelet domain. That is, a filter $\psi(\cdot)$ is passed over the series $\{r_t\}_{t=1}^T$. The width of the filter is usually increased by a power two, giving a finite number of filtered signals

$$W_{a,b}(r, \psi) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} r_t \cdot \psi\left(\frac{t-b}{a}\right) dt, \quad (4)$$

where $\psi(\cdot)$ is called the mother wavelet function, a is the scale factor and b is the translation of the origin. The variable $1/a$ gives the frequency scale and b gives the temporal location of an event. Thus, $W_{a,b}(r, \psi)$ can be interpreted as the ‘energy’ of r of scale a at $t = b$ (see Percival and Walden, 2000; Weron, 2006; Serinaldi, 2010). The AWCN consists of finding a representative ‘energy’ or amplitude for a given scale a . This is usually done by taking the arithmetic average of $|W_{a,b}(r, \psi)|$ over all location parameters b corresponding to the same scale a . This yields the spectrum $W_a(r, \psi)$ that depends only on the scale. If r_t is a self-affine process characterised by the exponent H , this spectrum should scale as $W_a(r, \psi) \sim a^{H+0.5}$ (see Simonsen *et al*, 1998). Thus, we can estimate the slope of a double-log regression of $W_a(r, \psi)$ on a and obtain H by subtracting 0.5 from the slope coefficient.

EMPIRICAL ANALYSIS

Basic setting

For the estimation of the Hurst exponent, we use the fractal estimators of the section ‘Methodology’ with the following parameterisations. We set the minimum sub-sample size in RRA and DFA to $n_{\min} = 12$ months. To avoid problems with small n , we apply the Peters (1994) correction to adjust the regression slope in the RRA. In line with the recommendation of Diebold and Rudebusch (1989) and Weron (2002), we choose K in the PRM such that $\lfloor N^{0.2} \rfloor \leq K \leq \lfloor N^{0.5} \rfloor$, that is, $K = \lfloor N^\nu \rfloor$ with $\nu = 0.35$. Finally, we follow the AWCN specification of Weron *et al* (2004).

To select funds based on Hurst exponent estimates, we use the following procedure. In a first step, we estimate the Hurst exponent based on the in-sample return observations $t = 1, \dots, T - T_{\text{oos}} + 1$, where T_{oos} is the number of returns that characterise our out-of-sample period. As some authors regard a number of 24 monthly returns as a minimum

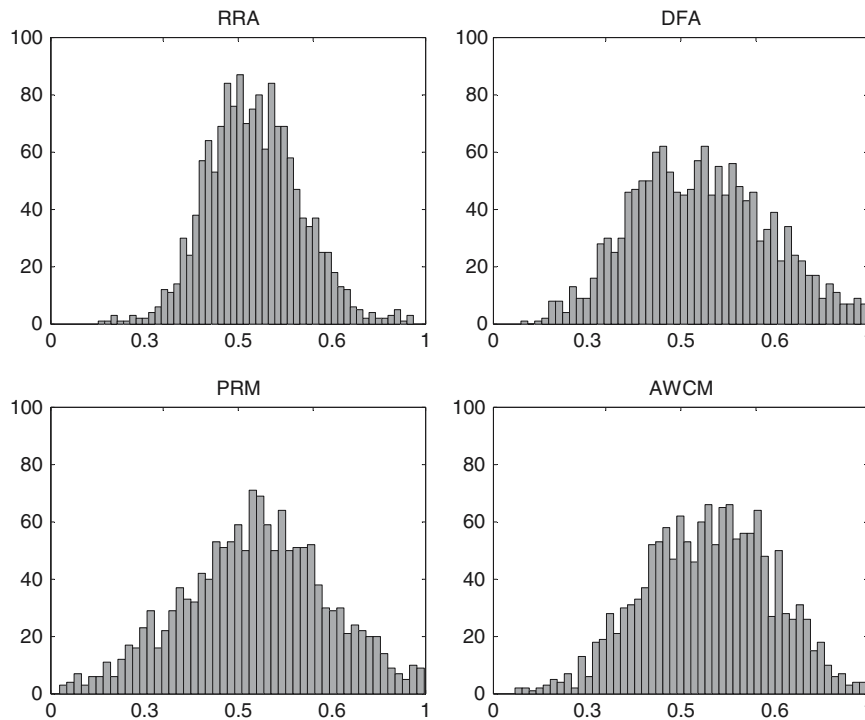


Figure 1: Distributions of hedge fund Hurst exponents. This figure shows the distributions of the Hurst exponents estimated for our hedge fund sample using the RRA, the DFA, the PRM and the AWCM in the specifications outlined in the section ‘Basic setting’.

requirement for evaluating hedge fund performance (see Ackermann *et al*, 1999; Gregoriou, 2002; Capocci and Hübner, 2004; Liang and Park, 2007), we set our out-of-sample phase to this length. In the second step, we calculate the in-sample D ratio, that is, the ratio of negative returns to the total number of returns, and select the funds with the highest H and the lowest D . This is because we are interested in the persistence of positive returns. Finally, we evaluate the average performance of the resulting fund selection by means of its Sharpe ratios in-sample and out-of-sample.⁹

Main results

Figure 1 shows the distributions of the hedge fund Hurst exponents resulting from our four fractal estimators.¹⁰ As the value of H for a random sequence will almost always deviate from 0.5 when the study sample is limited, we need some kind of confidence level that

allows us to interpret this figure, that is, to judge the number of persistent funds. In this respect, we follow the recent empirical study of Hull and McGroarty (2014).¹¹ They consider a Hurst exponent greater (lower) than 0.65 (0.40) to be strong evidence for persistence (anti-persistence). Realisations between 0.40 and 0.65 are interpreted as random behaviour.

With these boundaries in mind, Figure 1 provides the first important result that, even though we analyse a sample of long-lived hedge funds, only a small proportion of them shows very high persistence. The majority of funds is characterised by random return behaviour and thus their good performance may just have been luck (see Fama and French, 2010). Interestingly, this holds irrespective of the choice of fractal estimator.

To obtain a more detailed picture on this issue, Panel A of Table 2 reports the proportions of funds falling into the three groups anti-persistent ($H < 0.4$), random

Table 2: Main results

	RRA	DFA	PRM	AWCM
<i>Panel A: Proportion</i>				
Anti-persistent	0.23	0.32	0.30	0.25
Random	0.59	0.38	0.40	0.41
Persistent	0.18	0.30	0.30	0.34
<i>Panel B: Average Sharpe ratio (in-sample)</i>				
Anti-persistent	0.23	0.23	0.22	0.22
Random	0.33	0.33	0.33	0.30
Persistent	0.58	0.46	0.49	0.50
Persistent (high <i>D</i>)	0.28	0.22	0.24	0.26
Persistent (low <i>D</i>)	0.88	0.69	0.73	0.73
<i>Panel C: Average Sharpe ratio (out-of-sample)</i>				
Anti-persistent	0.08	0.08	0.05	0.09
Random	0.02	0.01	0.06	0.03
Persistent	0.30	0.15	0.12	0.13
Persistent (high <i>D</i>)	0.07	0.04	0.04	-0.01
Persistent (low <i>D</i>)	0.54	0.26	0.20	0.27

Note: For each of our fractal Hurst exponent estimators, Panel A of this table shows the proportions (in decimals) of funds classified as anti-persistent ($H < 0.4$), random ($0.4 \leq H \leq 0.65$) or persistent ($H > 0.65$). Panels B and C report the average in-sample and out-of-sample Sharpe ratios for these groups, respectively. For the group of persistent hedge funds, they also show the average Sharpe ratios for sub-groups built based on high (above the median) and low (below the median) negative return ratios *D*.

($0.4 \leq H \leq 0.65$) and persistent ($H > 0.65$). For all estimators, we find the highest proportion of funds in the random group, that is, 59 per cent for the RRA, 38 per cent for the DFA, 40 per cent for the PRM and 41 per cent for the AWCM. Only 18, 30, 30 and 34 per cent are classified as persistent by RRA, DFA, PRM and AWCM, respectively. Interestingly, the RRA identifies a somewhat lower number of persistent funds than the three alternative approaches.

Panels B and C of Table 2 describe the most important outcome of our study. They show the average in-sample and out-of-sample Sharpe ratios for the funds in the different long memory groups. Furthermore, they subdivide the persistent group into funds with high (above the group median) and low (below the median) *D* ratios.

Looking at the in-sample results first, we find that the persistent funds show the highest Sharpe ratios, followed by the random funds and the anti-persistent funds. In-sample, even

the two latter groups show rather high Sharpe ratios on average. However, this is not surprising because of the overall positive performance of our fund sample (see Table 1). Another straightforward result is that a focus on persistent funds with low *D* ratios yields higher average Sharpe ratios because a low *D* ratio automatically implies a higher Sharpe ratio. Persistent funds with low *D* ratios are characterised by an average Sharpe ratio of 0.88, 0.67, 0.73 and 0.73 for the RRA, DFA, PRM and AWCM, respectively.¹² In contrast, persistent funds with high *D* ratios earn average Sharpe ratios of only 0.28, 0.22, 0.24 and 0.26, respectively.

Turning to the out-of-sample results, we can observe that anti-persistent and random funds do not appear to repeat their good performance because they show low out-of-sample Sharpe ratios that do not crucially differ from 0. A similar argument holds for the persistent funds with high *D* ratios. However, funds with persistent positive returns show not only high average Sharpe ratios in-sample but also out-of-sample. We find average realised Sharpe ratios of 0.54, 0.26, 0.20 and 0.27 for our four approaches, respectively. Even though the Sharpe ratios are somewhat higher in the case of the RRA, all four Hurst exponent approaches appear to have considerable potential for effective hedge fund selection.¹³

Robustness checks

As our main results are subject to issues of arbitrary choice, this section describes a variety of supplementary calculations that verifies the robustness of our results.¹⁴

Influence of outliers

To limit the influence of outliers (either resulting from potentially faulty data or crisis impacts) on our results, we follow Bali *et al* (2011) and perform a 95 per cent winsorization. That is, we set all data below (above) the 5 per cent (95 per cent) percentile to the

Table 3: Robustness checks numbers 1–5

Number	Description	RRA	DFA	PRM	AWCM
1 Winsorization (95%)	Anti-persistent	0.07	0.05	0.08	0.08
	Random	0.02	0.03	0.01	0.03
	Persistent	0.21	0.12	0.09	0.11
	Persistent (high D)	0.07	0.05	0.03	-0.01
	Persistent (low D)	0.34	0.20	0.15	0.22
2 ARMA-GARCH filter (standard specification)	Anti-persistent	0.09	0.06	0.05	0.08
	Random	0.06	0.06	0.09	0.07
	Persistent	0.31	0.26	0.14	0.29
	Persistent (high D)	0.23	0.25	0.24	0.13
	Persistent (low D)	0.39	0.27	0.05	0.45
3 Widening Hurst significance (lower bound = 0.25 and upper bound = 0.85)	Anti-persistent	0.04	0.10	0.04	0.11
	Random	0.07	0.05	0.07	0.06
	Persistent	0.89	0.46	0.27	0.48
	Persistent (high D)	0.39	-0.04	0.02	0.07
	Persistent (low D)	1.40	0.98	0.52	0.88
4 Smaller out-of-sample window (12 months)	Anti-persistent	0.07	0.07	0.07	0.09
	Random	0.15	0.17	0.23	0.17
	Persistent	0.43	0.33	0.25	0.29
	Persistent (high D)	0.29	0.20	0.18	0.20
	Persistent (low D)	0.57	0.47	0.33	0.39
5 Larger out-of-sample window (36 months)	Anti-persistent	0.11	0.10	0.10	0.11
	Random	0.13	0.11	0.11	0.11
	Persistent	0.25	0.23	0.22	0.22
	Persistent (high D)	0.15	0.12	0.12	0.11
	Persistent (low D)	0.35	0.35	0.31	0.34

Note: Similar to Table 2, this table reports the average out-of-sample Sharpe ratios of different hedge fund groups resulting from a variety of modifications in our research design. Numbers 1 and 2 use winsorized data (at the 95 per cent percentile) and ARMA(1,1)-GARCH(1,1) filtered data in the Hurst exponent estimations. Number 3 considers different boundaries that classify a fund as anti-persistent ($H < 0.25$), random ($0.25 \leq H \leq 0.85$) or persistent ($H > 0.85$). Finally, numbers 4 and 5 use smaller ($T_{oos} = 12$) and larger ($T_{oos} = 36$) out-of-sample windows.

5 per cent (95 per cent) percentile. Table 3 reports the resulting average out-of-sample Sharpe ratios for this robustness check. Even though the Sharpe ratios are lower, our main conclusions of the section ‘Main results’ still hold.

Return filters

Several articles have shown that RRA can be sensitive to short memory and heteroskedasticity (see Lo, 1991) and PRM is not always unbiased and not generally consistent in the non-stationary case (see Velasco, 1999; Andrews and Guggenberger, 2003).¹⁵ Thus, recent literature suggests to check the robustness of results by means of ARMA-GARCH filters that eliminate short-range dependencies and heteroskedasticity from the data (see Cajueiro and Tabak, 2004; Batten *et al*, 2013).¹⁶ Fitting a simple ARMA(1,1)-GARCH(1,1) model and obtaining H for the standardised residuals of the fitted model yields the average out-of-sample

Sharpe ratios presented in Table 3.¹⁷ With the exception of the PRM, previous implications on the out-of-sample performance of the Hurst-based selection are confirmed.¹⁸

This filter exercise not only addresses the potential problems with our estimators, it also implicitly rules out a possible explanation for observable return persistence. Henn and Meier (2004) and Kosowski *et al* (2007) argue that funds with smoothed monthly returns (either because of holding illiquid securities or managed returns) might show a high level of performance persistence that does not reflect manager skill. As artificial smoothing of returns becomes visible in the form of short-term autocorrelation (see Gemansky *et al*, 2004) and the filters eliminate this correlation, our identified persistence does not appear to be crucially driven by return smoothing. Furthermore, note that the high out-of-sample performance is a strong

Table 4: Robustness checks numbers 6–9

Number	6	7	8	9
	RRA $n_{\min}=6$	DFA $n_{\min}=6$	PRM $\nu=0.3$	PRM $\nu=0.4$
Anti-persistent	0.10	0.11	0.08	0.06
Random	0.04	0.02	0.05	0.06
Persistent	0.37	0.13	0.11	0.11
Persistent (high D)	0.02	0.01	0.01	0.04
Persistent (low D)	0.72	0.25	0.21	0.18

Note: This table reports the average out-of-sample Sharpe ratios of different hedge fund groups resulting from some final robustness checks. Numbers 6 and 7 use lower minimum sub-sample sizes n_{\min} in RRA and DFA, numbers 8 and 9 use different cut-off rates ν in the PRM.

indication that return persistence actually comes from skill and not from return smoothing.

Hurst significance boundaries

So far, we have interpreted a fund to show significant signs of persistence (anti-persistence) for $H > 0.65$ ($H < 0.40$). As these boundaries may still lead to misleading conclusions (see Weron, 2002), we use more conservative values, that is, a lower bound of 0.25 and upper bound of 0.85. As we can see in Table 3, our conclusions remain unchanged. However, we can now observe even higher out-of-sample Sharpe ratios for the funds with persistent positive returns than in the main analysis. This is quite reasonable because, with higher H , we now concentrate on the most persistent funds.

Size of out-of-sample period

In order to analyse the predictive ability for shorter and longer horizons, we perform two additional robustness checks in which we limit T_{os} to 12 months and extend it to 36 months. Again, Table 3 shows qualitatively similar outcomes. For the RRA, we find a slight tendency of decreasing Sharpe ratio performance (of positively persistent funds) with increasing forecasting horizon.

Basic parameterisations

Finally, Table 4 varies some of the parameter settings in our estimations. First, we use a smaller number of minimum returns $n_{\min} = 6$ to increase the number of regression observations in RRA and DFA. Second, we employ lower and higher cut-off values $\nu = 0.3$ and $\nu = 0.4$ for the PRM. The results are similar either way.

CONCLUSION

Motivated by the need for adequate procedures to identify hedge funds with positively persistent returns (see Géhin, 2005) and the increasing popularity of Hurst exponent approaches to measure long memory (see Kantelhardt, 2009; Batten *et al.*, 2013), we analyse their usefulness in hedge fund selection. We find that, regardless of the estimation procedure, Hurst exponents are highly interesting predictors of future hedge fund performance. Specifically, we show that a combination of a high Hurst exponent and a low proportion of negative returns can identify the funds with the highest out-of-sample Sharpe ratios. Thus, our extended research design provides highly supportive and robust evidence for the suggestion made in earlier studies (see De Souza and Gokcan, 2004) that Hurst exponents might be useful tools for individual investors seeking the ‘best’ hedge fund and for fund managers choosing candidates for a fund of hedge funds.

Future research may extend the results of this article in several important ways. First, there are three main providers of hedge fund data (see Liang and Park, 2007) but we have used only data from one of these providers. Thus, using data from Hedge Fund Research (HFR) and Tremont Advisory Shareholders Services (TASS) might provide further support for the robustness of our results. Second, we have concentrated on return persistence. However, analysing persistence in other performance metrics, that is, Sharpe ratios or fund alphas (see Eling, 2009), may also be fruitful endeavours. Finally, we should

also take a closer look at the negative implications of our results because it is well-known that long memory in returns has severe consequences for financial modelling. Long memory (i) makes optimal consumption/savings decisions extremely sensitive to the investment horizon, (ii) invalidates martingale methods in derivative pricing and (iii) distorts standard asset pricing tests (see Lo, 1991; Beran, 1992). Thus, if persistent hedge funds are part of investment decisions, financial products or asset pricing models, we might have to be careful when applying established standard methods. However, the question of how strong the observed levels of long memory in hedge fund returns would affect these methods still has to be answered.

NOTES

1. We focus on return persistence. Persistence in risk has been studied by Kat and Menexe (2002) and Herzberg and Mozes (2003). For an overview of studies covering persistence in alphas and Sharpe ratios, see Eling (2009).
2. H quantifies long memory because it formally describes the decay in the autocorrelation function $\rho(k) \sim pk^{2H-2}$ as $k \rightarrow \infty$, where $p = H(2H-1)$ (see Beran, 1994; Kristoufek, 2010).
3. For other interesting applications of the Hurst exponent in a hedge fund context, see Amenc *et al* (2003) and Olszewski (2005).
4. For a review of the properties and detailed contents of this database see, for example, Ding and Shawky (2007) and Eling (2009).
5. Unfortunately, we do not have access to an up-to-date version of the database. However, our sample is still richer than the one used by DSG and thus provides important out-of-sample support for their preliminary results.
6. This positive performance does not introduce a sample selection bias because our results in the section 'Empirical analysis' show that a large proportion of these funds are not persistent and do not repeat good past performance out-of-sample.
7. For other forms of conducting RRA, see Mielniczuk and Wojdyło (2007) and Batten *et al* (2013).
8. The order of the polynomial trend is mostly set to the first or the second order because additional variables usually do not add any significant information (see Vandewalle *et al*, 1997).
9. In order to avoid sensitivity to the choice of risk-free rate, we assume it to be zero. However, note that we obtain similar results when we use the 1-month Treasury bill rate (from Ibboston Associates).
10. In the presence of faulty data, our fractal procedures can result in estimates outside the natural boundaries of H , which are given by 0 and 1 (see Abery and Veitch, 1998;

Karagiannis *et al*, 2002; Park *et al*, 2005; Clegg, 2006). As this occurs for a small number of funds, we have excluded them from the ongoing analysis.

11. Their approach of evaluating the significance of H is based on Monte Carlo simulations performed by Qian and Rasheed (2004).
12. Further analysis shows that PRP is not significantly related to the type of fund strategy. This is in contrast to the results of Brown and Goetzmann (2003) and Harri and Brorsen (2004) for RRP.
13. In a stock selection context, non-random patterns in return data do not appear to be good selection criteria (see Freund *et al*, 1997).
14. For full transparency, the source codes of our Hurst exponent approaches, fund selection procedures and robustness checks are available from the authors upon request.
15. In contrast, the DFA is designed to avoid spurious detection of apparent long-range correlation that is an artefact of non-stationarity (see Kantelhardt, 2009).
16. We rule out potential bias in the PRM by conducting the test of Davidson and Sibbertsen (2009).
17. We also used series-specific specifications with the best fit in terms of the criteria suggested by Auer (2014).
18. This is in line with Mishra *et al* (2011) showing that GARCH models cannot fully explain observed dependence in empirical data and, thus, heteroskedasticity does not introduce crucial distortions of long memory results.

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