4

Corporate and SME Credit Rating Models

4.1 PD Corporate SME Model Development

This section describes the main activities underlying the developmental steps of a model for the estimation of the PD (see Fig. 4.1). Our focus is mainly on the customer segment of corporate small and medium-sized enterprises (corporate SMEs). We refer the reader to Sect. 6.4 for a description of the main validation tests; these should be performed after the model estimation and before its final functional specification and passage to the production phase.

4.1.1 Step 1: Perimeter of Applicability and Definitions

Whatever the future application of the model to be developed, to establish a firm foundation for the entire process, it is important to pay great attention in the initial phase (Step 1) to the regulatory and operative reference framework, and to the definition of the event to be forecast: the default probability (see Table 4.1).

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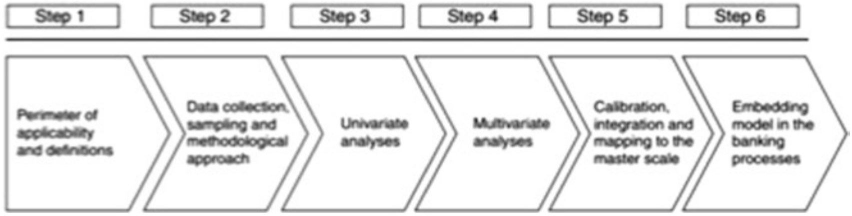

Fig. 4.1 Main steps in developing a rating model

Table 4.1 Main steps in developing a rating model

Step 1: Perimeter of applicability and definitions
Identification of the segment of interest (perimeter of applicability)
Definition of the event to be forecast (the default)
Establishment of the working team
Analysis of the internal and external regulatory framework
Analysis of processes, IT procedures and data to support the credit unit data availability
Analysis of the portfolio
Definition of the modality for dealing with outliers and exceptions
Comparison and discussion with the business and credit experts

The main objective of the model is the estimation of the probability of default within a determined temporal horizon (typically, one year) to classify customers in a portfolio according to their degree of risk.

The central role in the design of a rating model is the definition of default, which allows (future) insolvent customers (defined as the "bads" within the estimation samples) to be distinguished from solvent customers (the "goods"). The definition of default has to be set sufficiently far in advance (far enough from the onset of a problematic situation) to permit the identification of a default before it is too late to take corrective action and, in the meantime, sufficiently close to the moment of default to make an effective distinction between bads and goods.

The default definition used in model development should also be consistent with that used elsewhere in the bank and in line with the default definition required by the regulator. The default definition provided by the New Capital Accord includes bad debts, sub-standard loans, restructured exposures, and past due and overdrawn positions (see Basel Committee 2006).

To develop an effective rating tool, it is essential to establish a heterogeneous working group, characterized by a range of quantitative technical skills (mathematical, statistical and computer science) for:

- descriptive and inferential analysis;
- model design, the architecture of the rating system, the analysis of the origin of existing credit, and monitoring processes;
- the management of databases and implementation of the IT environment for the estimation and validation processes;
- and qualitative skills (economical and juridical) for:
  - the analysis of the enterprises' financial situation and balance sheet data;
  - the assessment of scenario and sector components; and
  - an in-depth knowledge of the bank's internal norms, and national and international rules.

Further requirements are solid experience in the field of the estimation and validation of rating systems, sufficient seniority and knowledge of the main internal processes of a banking group.

The working group should first analyze:

- the internal regulatory framework (of the bank or the banking group) and the external regulatory framework (supervisory regulations, and domestic and international guidelines);
- the credit process underlying the origination of the credit and monitoring of the corporate SME counterparts; and
- the IT procedures that support this process.

The working group should then analyze the corporate SME segment using the most recent data available (for example, up to December 31 of the previous year) with respect to the main classification variables (industry sector, geographic area, company size, and so on) both in terms of position and volumes (that is, credit limit and outstanding debts).

The portfolio analysis represents a central activity within the estimation process: the segment data analyzed in the recent portfolio should be the main reference for the working group in relation to:

- the editing of the data request finalized to the construction of the estimation and model validation samples;
- the definition of existing fields for the indicators; and
- the management of outliers, exceptions and preliminary factor transformations and normalizations, in order to reduce the impact of outliers and to make the multi-factor regression analysis more efficient and factor weights easier to interpret.

4.1.2 Step 2a: Data Collection and Sampling

After analyzing the availability, length of historical series and the quality of the databases underlying the credit processes, the next step is to edit the designated "long list" of potential predictors of default. This list is based on the academic literature, as well as on the input from the experiences of relationship managers and personnel from the credit department of the bank: the so-called "experts" of the working group (see the first activity of Step 2 in Table 4.2). In order to carry out a proper statistic-economic analysis, the indicators included in the initial long list should be grouped into areas and informative categories, obtaining the definition of as many long lists as the number of areas

Table 4.2 Developing a rating model: main activities of Step 2

Step 2: Data collection, sampling and methodological approach
Editing of indicator long list(s)
Comparison with the credit experts and possible enlargement or restriction of the proposed long list(s)
Definition and formulation of the data request
Preliminary explorative data analysis
Data cleaning
Construction of model estimation and validation samples
Validation of representativeness and stability of the identified samples with respect to the recent portfolio
Selection of the methodological approach

of information considered. Typical information areas to be analyzed in the development of an estimation model for probability of default for the corporate SME segment are financial, internal behavioral, external behavioral and qualitative.

The risk indicators belonging to each of the four inquiry areas will be grouped successively into categories for analysis; this is to facilitate the economic interpretation of the subsequent statistical evidence and to verify that, during the reduction that the area's initial long lists will undergo, all the informative categories will be adequately represented.

Table 4.3 presents examples of indicators belonging to the financial area, grouped into information categories.

After finalizing the indicators' long lists and extracting all necessary data, a thorough analysis of the databases must be performed, paying particular attention to:

- the possible presence of duplicated positions for the same analysis key;
- the consistency of elementary variables;
- their economic coherence, both in terms of content and number of expected observations per period (month);
- the variation of indicator values; and
- their stability over time, also with respect to their relative risk by sub-segments of analysis (industry sector, geographic area, company size, and so on).

After carefully carrying out data cleaning, the next step is estimation sample extraction and model validation, ensuring:

- sufficient cardinality and sample depth;
- the correct identification of goods and bads, both in the development and in the model validation samples;
- an adequate proportion of bads and goods, which permits an adequate representation of the event to be forecast within the estimation samples; and
- the stability/representativeness of the samples with respect to the reference portfolio.

Table 4.3 Financial indicators grouped by categories: an illustrative example

Category	Indicator
Size	Capital employed; Cash. Equity; Fixed assets; Inventory; Net margin; Net sales. Operating cash flow; Profit or loss; Provision funds; Total assets; Turnover. Value added
Profitability	(Gross margin)/(Capital employed) (Net margin)/Equity (Net margin)/(Total assets) (Operating cash flow)/Sales (Profit after interest expenses)/(Capital employed) (Profit before interest expenses)/Sales (Profit or loss)/(Total assets)
Debt service capacity	(Commercial debt)/Turnover (Financial debt)/(Gross margin) (Financial debt)/Turnover (Fiscal debt)/Turnover (Gross margin)/(Current liabilities) (Interest expenses)/(Total debts) (Long-term debt)/Turnover (Net margin)/(Interest expenses) (Net margin)/(Long-term debt) (Operating cash flow)/(Total debts) (Profit after tax)/(Financial debt) [(Short + Long term debt)−Cash]/Equity (Total debt)/Turnover
Liquidity	Accounts receivable Cash/Turnover Cash/Equity Cash/(Total current liabilities) Cash/(Total debt) (Current liabilities)/Sales (Debt to suppliers)/(Raw materials) Inventory/Turnover Revaluation/Sales (Total credits)/Turnover (Total credits)/(Capital employed) (Total credits)/(Total assets) (Total credits)/(Total current liabilities) (Total credits)/(Total debt) (Total current assets)/(Total current liabilities) Working capital (Working capital)/(Net sales) (Working capital)/Turnover

(*continued*)

Table 4.3 (continued)

Category	Indicator
Gearing	(Book equity)/(Total assets)
	(Capital employed)/(Fixed assets)
	(Current liabilities)/(Total assets)
	Equity/(Long-term debt)
	Equity/(Total assets)
	Equity/(Fixed assets)
	[Equity−(Issued shares)]/(Total assets)
	(Issued shares)/(Total assets)
	(Issued shares)/(Total liabilities)
	(Long term debt)/(Fixed assets)
	(Short + Long-term bank debt)/(Book equity)
	(Total debt)/Equity
	(Total debt)/(Total assets)
Activity	(Direct cost)/(Total assets)
	(Direct cost)/Turnover (Labor cost)/Sales
	(Operating cash flow)/(Interest expenses)
	(Provision reserves)/Turnover
	(Raw materials)/(Commercial debt)
	Sales/(Fixed assets)
	Sales/(Total assets)
Stability	Change in capital employed
	Change in current assets
	Change in fixed assets Change in cash
	Change in [(Financial debt)/(Gross margin)]
	Change in long-term debt
	Change in [(Net margin)/(Interest expenses)]
	Change in [(Operating cash flow)/(Sales)]
	Change in return on investment (ROI)
	Change in [Sales/(Fixed assets)] Change in turnover
	Change in total assets

Generally, for the construction of the estimation samples of a rating model, all the positions that went into default in the observation horizon (bad customers) and a sub-set of the positions that never went into default in the observation horizon (good customers) are adopted. In certain cases, the samples could be balanced – that is, the same number of bads and goods.

One possible sampling methodology is the random extraction of positions, without repetition, stratified with respect to the representative

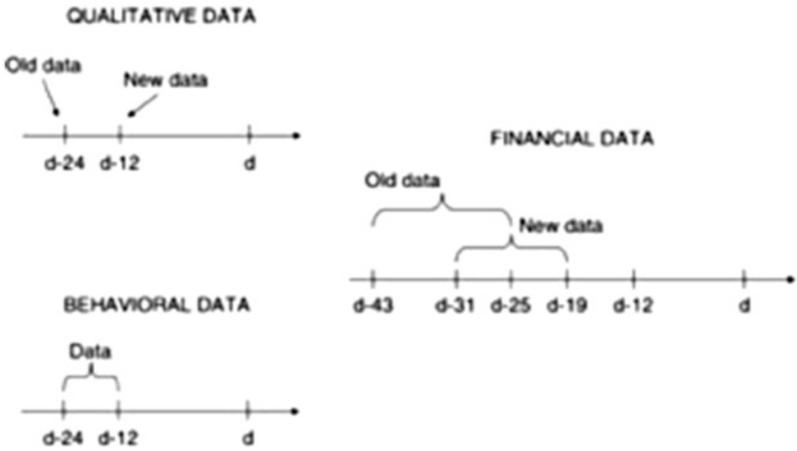

Fig. 4.2 Information-gathering rules: an illustrative example

variables and to the year of default, with constant sampling probability (simple sampling) within layers. Of the extracted samples, one must verify carefully the completeness of information and the existing fields (ranges) observed in the recent portfolio. The possible infeasibility of one of the above conditions requires the re-extraction of the sample.

The linking of information (financial, behavioral and qualitative) to the sample positions must be performed in a manner coherent with the effective availability of the information (updating time, source, and so on). This allows for the construction of the indicators defined in the long lists to be carried out early enough to respect the time of default, both for the single bad position and for the corresponding (twin) good positions in the sample.

A possible information-linking rule is depicted in Fig. 4.2.

If “d” denotes the instant (month) of entrance into default of a generic bad position, the period of data observation of the bad position and of the corresponding good one varies between:

- “d-12” and “d-24” for the information of a qualitative nature – to evaluate the possible variation of this kind of information across the interval of 12 months;

- "d-12" and "d-24" for the behavioral information – to build relevant derived indicators such as quarterly, semi-annual and annual averages/variations;
- "d-19" and "d-43" for the financial variables – to simulate the effective availability of at least two balance sheets in the production phase.

Once a preliminary sample analysis has been performed (quality, numeracy and observation depth), it is possible to design the model structure and define the best methodological approach to be followed during the model development.

4.1.3 Step 2b: Model Structure

The most widespread rating model structure is modular, with the number of modules equal to the number of information areas that feed the model – in this case, four: one financial module, two behavioral modules and a qualitative module. Each module, according to the chosen methodology, produces as output a score that expresses, in numerical terms, the credit merit of the counterpart, depending on the type of information computed: the accounting data (financial module); the borrower behavior with the bank (internal behavioral module), or with the banking system (external behavioral module); and the qualitative judgment expressed by the relationship manager (qualitative module).

Depending on the practical availability of data (financial, behavioral and qualitative), it is possible to develop models on a statistical basis (in the presence of sufficient robust data) or an expert basis (judgmental).

As shown in Fig. 4.3, the score produced by a module developed on a statistical base is transformed, successively, into a default probability that is expressed on a scale from 0 (minimal risk) to 1 (maximum risk) to the likelihood that, during a period of 12 months, the borrower will become insolvent, according to the default definition adopted. The (modular) PDs obtained separately are then integrated, according to an algebraic formula, in a unique default probability, associated successively with a rating class of the bank's master scale.

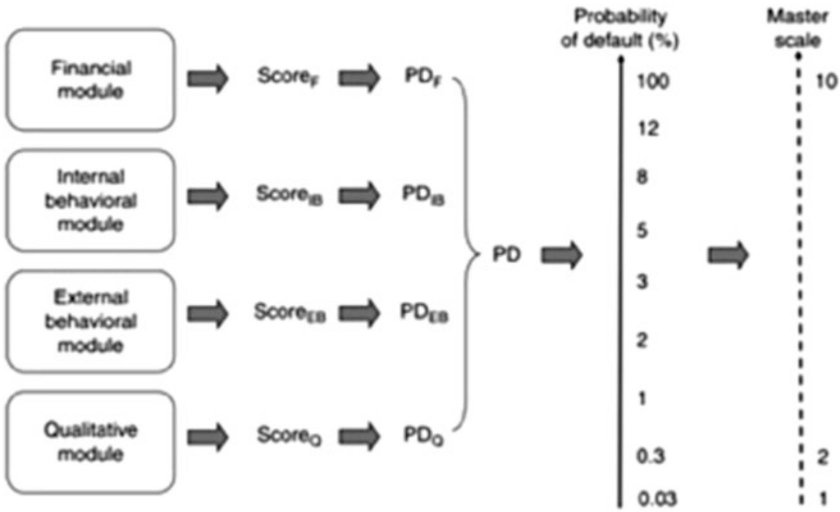

Fig. 4.3 Main steps in the development of statistical models

The score produced by the modules developed on a judgmental basis (inside the upper dotted line in Fig. 4.4) is generally not transformed into a default probability but, rather, is used to correct – upward (upgrading) or downward (downgrading) – the rating class assigned by the statistical component of the model (inside the lower dotted line shown in Fig. 4.4).

Finally, in the presence of modules and components developed only on an expert basis, the judgmental score can be employed to correct (upward or downward) the rating class corresponding to the default probability assigned (ex ante) to the portfolio segment, following the analysis of its current and historical default rates in the medium to longer term (see Fig. 4.5).

4.1.4 Step 2c: Methodological Approach

As far the methodological approach is concerned, for the segments characterized by databases that are sufficiently broad and stable and that have an adequate number of defaults (called a “high default portfolio”), it is

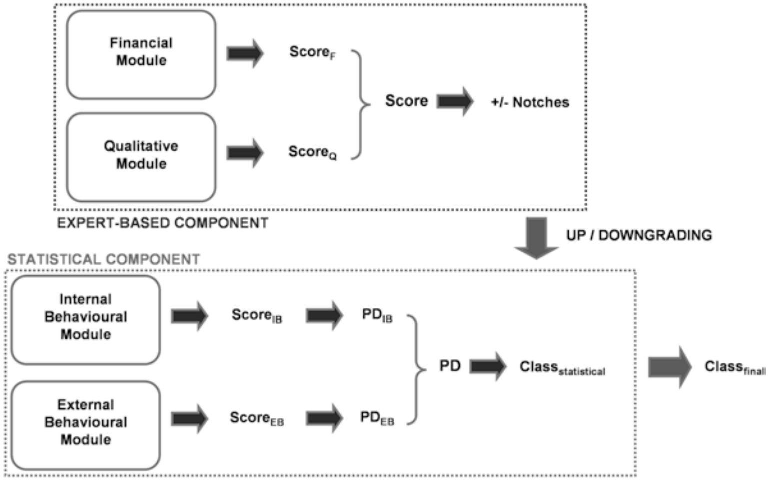

Fig. 4.4 Main steps in the development of statistical/expert-based models

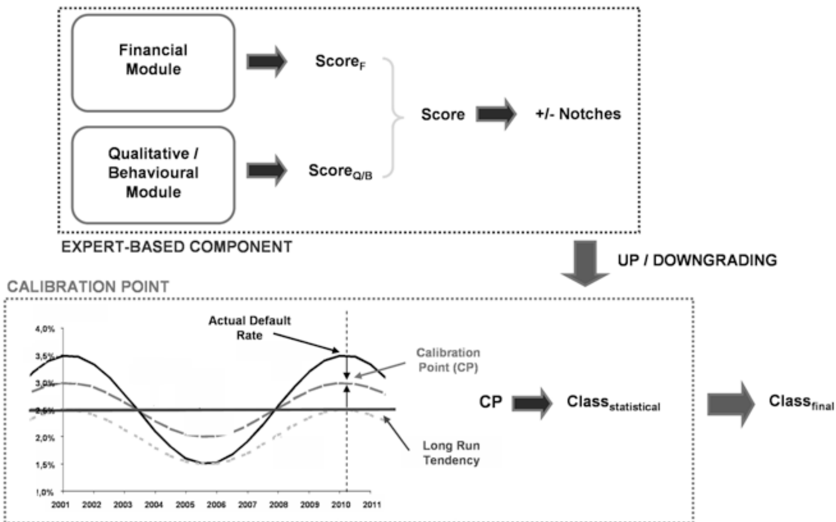

Fig. 4.5 Main steps in the development of purely expert-based models

possible to adopt a statistical approach for the assessment of qualitative information in cases supported by judgmental techniques .

The most frequently adopted statistical technique for the corporate SME segment is logistic regression: alternative techniques are discriminant analysis; probit models; and the more recent inductive models of a heuristic nature, such as genetic algorithms and neural networks.

For insights regarding the listed approaches, see Resti and Sironi (2007). Next, we describe the development of a default probability estimation model based on the logit method.

4.1.5 Statistical Methodology

In the literature, it is recognized that logistic regression is one of the best methodologies for the estimation of a function capable of linking the probability of the possession of a dichotomous attribute (in this case, bad = 1; good = 0) to a set of explicative variables (financial, behavioral or qualitative).

The logistic regression represents a specific case of regression analysis: the dependent variable, Y, is dichotomous, its distribution is binomial and the estimation of Y, varying from 0 to 1, assumes the meaning of a probability: $P\{Y = 1 | x\} = \pi(x)$ that is:

$$Y = \begin{cases} 1, & \text{with probability } \pi(x) \\ 0, & \text{with probability } 1 - \pi(x) \end{cases}$$

The logistic regression function has the form:

$$\text{logit}(\pi(x)) = \beta_i + \sum_{i=1}^{n} \beta_i \cdot x_i = x \cdot \beta$$

where logit ($\pi(\boldsymbol{x})$) denotes the natural logarithm of the ratio of the probability of "success" (that is, the probability that the analyzed position defaults in the 12 months successive to the evaluation) and the probability of "no success" (solvent) given the vector **x** of *n* predictive variables (for example, the vector **x** could contain behavioral variables of the customer):

$$\operatorname{logit}\left(\pi\left(x\right)\right)=\ln\left[\frac{\pi\left(x\right)}{1-\pi\left(x\right)}\right]$$

As $\pi(\boldsymbol{x})$ denotes the probability that Y is 1, conditional to the explicative variables **x**, the probability of Y can be expressed as a logistic function:

$$\pi\left(x\right)=\frac{\mathrm{e}^{x}\cdot\beta}{1+\mathrm{e}^{x\cdot\beta}}$$

The choice of the logit to describe the function that links the probability of Y to the combination of predictive variables is determined by the observation that the probability gets gradually close to the limits "0" and "1", describing an "S" shape (called a "sigmoid").

While it is not a unique function that permits the modeling of the probability of a phenomenon, the logit is privileged with respect to the others as it represents a transformation of the ratio of two complementary probabilities (a quantity known as "odd"); that is, the *ratio* of the number of successes over each failure of the examined phenomenon.

4.1.5.1 Expert-based Methodology

The modules developed according to an expert approach are generally inspired by a multi-attribute value theory such as the Analytical Hierarchical Process™ (AHP) proposed by Saaty at the end of the 1970s. The AHP method allows the modeling of a decision problem by means of a hierarchy of levels (see Fig. 4.6) and by the conversion of qualitative and quantitative information in a uniform manner by means of the concept of relative importance in a finite set of alternatives.

The choice of a hierarchical approach for the definition of the expert-based components is often preferred to alternative techniques; this is for reasons of conceptual and implementable simplicity, methodological

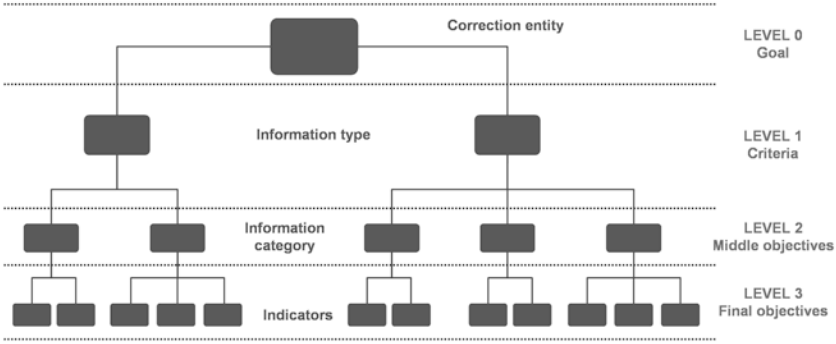

Fig. 4.6 Schematic view of the proposed hierarchy

transparency and the possibility of performing fine-tuning on all the parts of the structure, also in an independent manner.

Following a top-down approach, the main objective of the analysis – that is, the determination of the quantity of the improvement/worsening of the counterparty risk estimated by the statistical component of the model – is decomposed according to a hierarchy of sub-objectives at lower levels of the hierarchy specifically for the segment to which the borrower belongs.

Such decomposition allows us to design a sort of "conceptual map" of the expert-based component and, at the same time, to formalize the basic hierarchical structure.

Following this method, it is possible to define the mathematical formalization of one or more (expert-based) modules of a rating model in parallel with the definition of the conceptual map(s), with these main objectives:

- to establish the criteria to be used for dealing with differing information, according to its type (continuous or categorical) to ensure the correct transformation of indicators into model variables;
- to assure the uniqueness of the variables' value range;
- to define the criteria for dealing with missing values;
- to identify the model variables to which to assign a weight;
- to establish the criteria for the computation of weights to manage possible diversity in the "discriminant capability" of some risk indicators.

At the highest level of the hierarchy, the total risk function is computed – the score (integrated if it results from more than one module) which determines the size of the correction of the statistical rating class – whose value depends on the nodes at the lower hierarchy level.

The hierarchy proposed consists of four levels.

- "Level 0" (or the "starting level") contains the main objective (or "goal") of the evaluation: the risk expert-based score to be assigned to the examined positions.
- "Level 1", containing the evaluation criteria (financial and/or qualitative) that specify the content and meaning of the goal: the Level 1 criteria are divided into more specific objectives.
- The objectives of "Level 2" (the categories of information to be analyzed which, in case of a qualitative module, can be: demand/offer in the reference market; competitive position of the company; proprietary structure/account quality; and so on) that are themselves subdivided in Level 3.
- The single terminal objectives of "Level 3" of the hierarchy, originated from single module variables.

A value is assigned to each modality of the variables that feed the expert-based component – continuous for continuous variables and discrete for categorical variables in the interval – for example, from 0 (maximum risk) to 10 (minimum risk).

To each objective of the structure, a "local weight" is assigned ranging from 0 to 1, which determines the relative importance with reference to the objective of the higher level.

The importance of each terminal objective in relation to the goal is determined by the "hierarchy composition rule":

- the local weights assigned to the different terminal objectives are multiplied by the value of the corresponding variables;
- the values so computed are summed up to obtain the values of the objectives of the higher level; and moving from the bottom to the top, the weighted sums of the variables, first, and then the categories/types of information lead to the determination of the score (integrated,

where more than one module is present) of the expert-based model component.

4.1.6 Step 3: Univariate Analysis

The aim of the univariate analysis is to investigate the link between the single variable (financial, behavioral, qualitative) and the default, and the consequent reduction of the factors' long lists to medium lists that are logically and methodologically sound, removing factors that do not perform well or that show a high percentage of missing values (see Table 4.4).

The univariate analysis follows the preliminary explorative sample analysis (data quality and representativeness) and after the rebuilding of the factor algebra (by association with all the sample observations the indicators defined in the long lists).

The aims of the univariate analysis – performed separately for each informative category of the single areas of enquiry – are:

Table 4.4 Developing a rating model: main activities of Step 3

Step 3: Univariate analyses
Univariate statistical analysis (for continuous variables) and analysis of the distribution (for categorical variables) of the single indicators of the long lists
Analysis of the economic meaning of indicators and analysis of their relation to the default
Definition of the modality to deal with missing values
Management of missing data, outliers and exceptions
Exclusion of the variables characterized by a rate of missing data higher than a predetermined threshold (vertical missing analysis)
Exclusion of observations characterized by missing information greater than a predetermined threshold (horizontal missing analysis)
Analysis of the discriminant power of the stand-alone indicators
Transformation and normalization of indicators at univariate level
Definition of the medium lists of indicators made for a single inquiry area by the transformed variables, which result, at the end of the transformation, in being more predictive than the others
Verification, on the validation sample, of the stability of the chosen transformations and of the predictivity of the medium lists' variables
Comparison with the credit experts and possible enlargement/reduction of the individuated medium lists

- to analyze the distribution (in classes or quantiles according to the type) of all the variables in their fields of existence;
- to verify the economic soundness of the factors; and their proper relationship with the default.

As an example, in Figs. 4.7, 4.8 and 4.9 three variables are characterized by identical distributions for a range of values (shaded bars), but by three different relations with the risk (default rate of the population in the eight ranges, shown by the curve on the graph). Figure 4.7 shows a trend growing with the risk, Fig. 4.8 shows a decreasing trend and Fig. 4.9 illustrates uncertainty.

In the first two cases, if the trend with respect to the risk is confirmed by the economic interpretation of the indicators under consideration, the two variables will be included in the factors' medium list(s) to be analyzed, at multivariate level, in Step 4.

The variable represented in Fig. 4.9 will be excluded from the successive analysis process because of its undetermined relation with respect to the event to be forecast – the default.

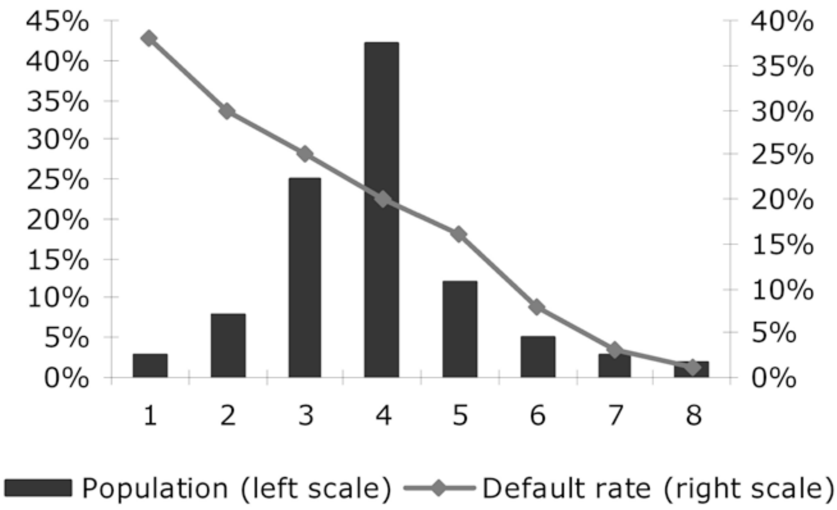

Fig. 4.7 Example of a variable growing monotonically with the risk

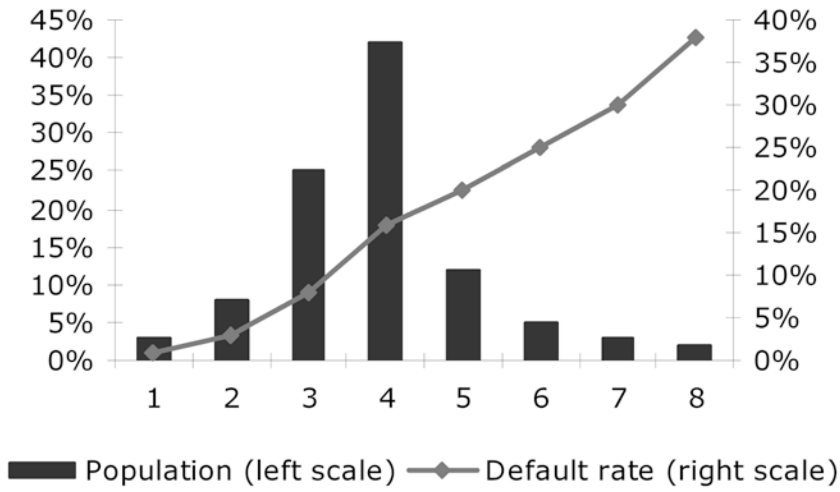

Fig. 4.8 Example of a variable decreasing monotonically with the risk

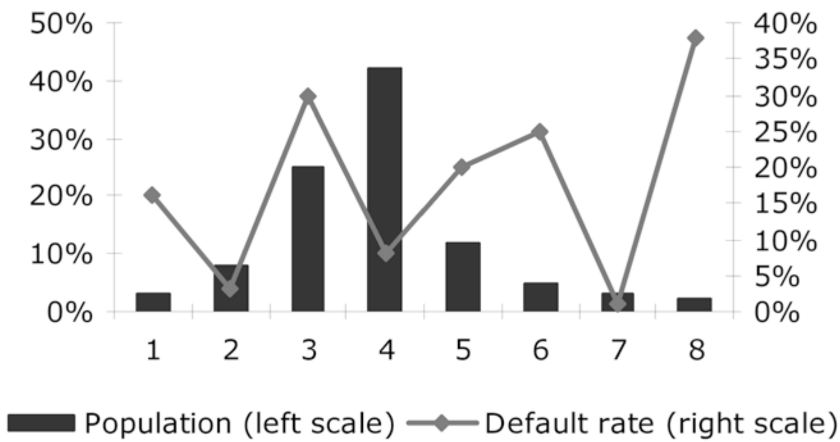

Fig. 4.9 Example of an uncertain relation with the risk

It is necessary to work out the analysis of distribution and its relation with the default, both before and after the preprocessing of data. This is intended to eliminate problems such as missing data, outliers and exceptions (for example, "0/0", "missing/0" and so on).

There are a number of ways to manage missing data: elimination of the indicators not available for a significant percentage of observations (vertical missing data), substitution of the missing data with predefined values, or the elimination of observations for which a significant number of indicators from the long lists are not available (horizontal missing data).

A common approach to the management of outliers is to define their data variability in order to assess their economic and statistical feasibility ranges and the consequent substitution of values outside the range of pre-fixed thresholds. Definition of these feasibility ranges requires special attention; if the ranges are too narrow, this could lead to models the fit of which is biased by an arbitrary variance reduction of the input data.

As with the missing data and the outliers, the exceptions also require specific treatment.

In the construction of variables derived across time horizons of three, six, twelve months and so on – as minimum, maximum, correlation, coefficient of variation and so forth – it is necessary to define the minimum thresholds for the presence of information; below such thresholds, the value obtained for the indicator should be considered to be missing.

Generally, for indicators built on a number of n months, it is it may be necessary to have at least $n + 1$ information if n is odd, or n if n is even.

There are two other important activities related to univariate analysis: the management of the "U-shaped" factors; and their transformation, inside the feasibility interval, to emphasize their relation with the default.

The first of these two analyses, performed separately on each factor of the long lists, is devoted to identifying the possible "U" relation – which must also be confirmed by the economic analysis – between the range of values assumed by the indicator and the default rate (see Fig. 4.10, upper chart).

The analysis is carried out by dividing the interval of assumed values into quantiles, from which the default rate is computed.

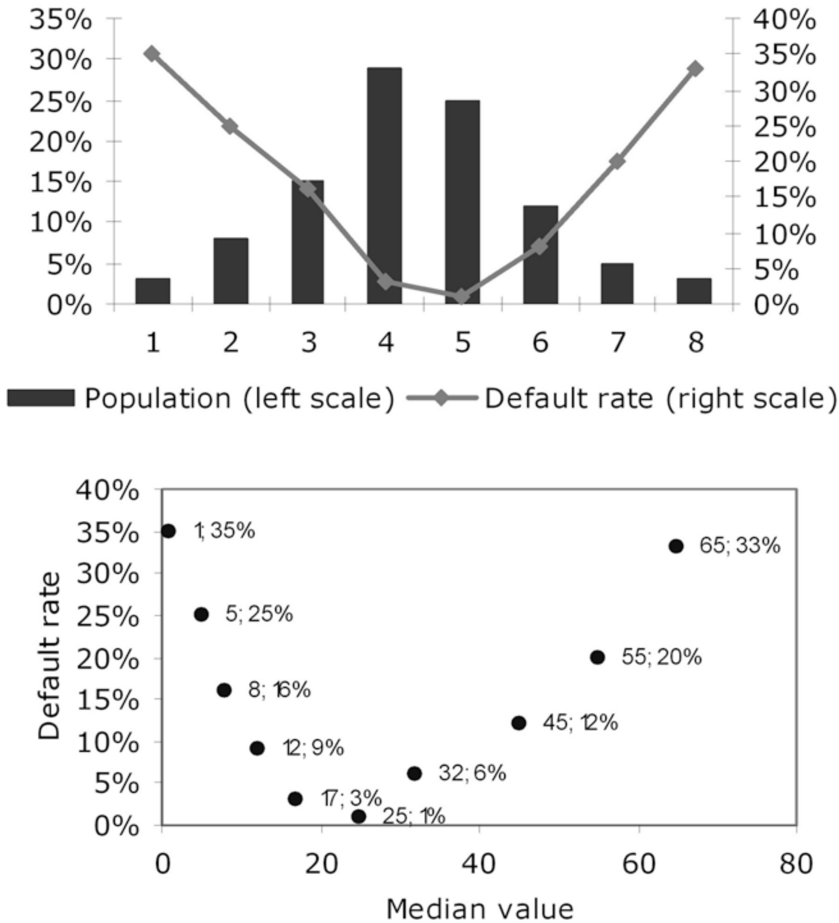

Fig. 4.10 Example of a "U-shaped" factor

The median value of each quantile and the corresponding default rate are identified, respectively, on the *x* and *y* axes of the Cartesian plane, allowing the graphical representation of the relation of each indicator with the default (see Fig. 4.10, lower chart).

In the event of a "U-shaped" pattern, once the point (x_0; y_0) of the derivative sign change has been set – that is, the minimum of the function, ideally a parabola with the two branches going upward – it is possible to identify the best preliminary transformation that ensures a cross near the point (x_0; y_0) and, simultaneously, to minimize the deviation between the interpolating curve and the observed values.

At the end of such transformation, the most significant factors of the long lists will show a monotonous trend (increasing or decreasing, according to their economic meaning) with respect to the default. They may also be subjected to a final phase of (deterministic) transformation and normalization to reduce the impact of outliers, and to make the multifactor regression analysis more efficient and the factor weights easier to interpret.

As an example, for continuous variables, one can identify, for each indicator, the value interval [x_l; x_u], where a significant portion of observations falls (equal, e.g., to 75–80 %) and, at the same time, the monotonic relation with the default event appears with specific evidence.

Then, the upper and lower bounds are denoted, respectively, as *xu* and *xl* – and it is possible, by means of a deterministic transformation (e.g. logit) to enhance the discriminatory capability of the single factor in the interval [*xl*; *xu*] and flatten it outside the interval, where the relation with the default is less important. Following this transformation, the analysis of the ordering capability of individual indicators at univariate level is carried out using a discriminatory power test on both the developing sample and the validation sample.

By setting the minimum level of acceptability for the discriminatory power tests required for the variables belonging to the same types of information (financial, behavioral or qualitative) and by assessing the coherence of the indicators' behavior (values and relation to the default) with respect to their economic significance, it becomes possible to select from the corresponding long list the three sub-sets of factors (financial, behavioral and qualitative) that are:

- most predictive of the default event;
- intuitive from the economic point of view; and
- capable of ensuring coverage of the main risk categories, which the panel of experts considers to be the determinants in the evaluation of creditworthiness.

Such sub-sets of indicators are usually referred to as the "medium" list. It is very important to eliminate factors with low predictive power before initiating the multifactor analyses: including a factor with no ability to differentiate between bad and good clients creates unwanted noise and increases the risk of over-fitting the model to the sample data.

4.1.7 Step 4: Multivariate Analysis

The aim of the multivariate analysis is to determine the optimal variable selection and weight of each indicator (see the main activities in Table 4.5). First, a further reduction of indicators is carried out, to eliminate from the medium lists those that are highly correlated with other, more predictive indicators.

In this phase of the analysis, the indicators are compared at multivariate level inside the informative categories to which they belong, applying techniques such as cluster analysis and logistic regression inside the identified clusters.

In this way, the single short lists of indicators can be defined, one for each information category analyzed (see Table 4.6).

Table 4.5 Developing a rating model: main activities of Step 4

Step 4: Multivariate analyses
Correlation analysis separated by information category and area
Cluster analysis by information category and area
Identification of the short lists, containing the most predictive and least correlated variables of each information category
Comparison with the credit experts and verification of the coverage of the main risk drivers
Integration of variables' category according to the selected techniques: purely statistic (e.g. logit analysis), statistical-judgmental or purely judgmental
Definition of one or more alternative modules for each information area
Assessment, on the validation sample, of the statistical robustness and discriminatory power of the identified modules
Comparison with the credit experts for the selection of the best module for each information area that satisfies the criteria of coverage of relevant risk variables and statistical robustness

Table 4.6 From the long list to the final model indicators

	Single modules						Integrated model
Information area	Long list	Medium list	Short list		Input to the regression analysis	Final list	Model indicators
Financial	Financial long list (unique to all the information categories of the financial area	Financial medium list (unique to all the information categories of the financial area, obtained after the univariate analyses)	Short list of "size" Short list of "profitability" Short list of "debt service capacity" Short list of "stability"	One of each information category of the financial area, obtained after the multivariate analyses performed on each information category	Unique list of financial indicators, obtained after the multivariate analyses performed on the merging of the short lists of the area	Variables selected after the final regression analysis performed on the financial area	
Internal behavioral	Internal behavioral long list (unique to all the information categories of the internal behavioral area)	Internal behavioral medium list (unique to all the information categories of the internal behavioral area, obtained after the univariate analyses)		One of each information category of the internal behavioral area, obtained after the multivariate analyses performed on each information category	Unique list of internal behavioral indicators, obtained after the multivariate analyses performed on the merging of the short lists of the area	Variables selected after the final regression analysis performed on the internal behavioral area	Set of all the variables feeding the developed modules

(*continued*)

Table 4.6 (continued)

	Single modules					Integrated model
Information area	Long list	Medium list	Short list	Input to the regression analysis	Final list	Model indicators
External behavioral	External behavioral long list (unique to all the information categories of the internal behavioral area)	External behavioral medium list (unique to all the information categories of the internal behavioral area, obtained after the univariate analyses)	One of each information category of the external behavioral area, obtained after the multivariate analyses performed on each information category	Unique list of external behavioral indicators, obtained after the multivariate analyses performed on the merging of the short lists of the area	Variables selected after the final regression analysis performed on the external behavioral area	
Qualitative	Qualitative long list (unique to all the information categories of the internal behavioral area)	Qualitative medium list (unique to all the information categories of the internal behavioral area, obtained after the univariate analyses)	One of each information category of the qualitative area, obtained after the multivariate analyses performed on each information category	Unique list of qualitative indicators, obtained after the multivariate analyses performed on the merging of the short lists of the area	Variables selected after the final regression analysis performed on the qualitative area	

Successively, the short lists of the same enquiry area are merged, obtaining, in this case, four lists of variables to be tested jointly through the logistic regression analysis performed by:

- applying the step-by-step selection technique – without setting the maximum number of predictors;
- according to the cluster analysis identified in the hierarchical manner – where each class (cluster) of variables belongs to a larger cluster, which is again contained in a larger one and so on until the cluster that contains the whole set of analyzed factors is reached; and
- relying on identification through logical-economic considerations, starting with the short list, the sub-set of "best" variables – in relation to their economic interpretation, capability of covering the main risk categories, forecasting power and in relation to the correlation matrix – to be provided as input to the regression analysis for the enquiry area.

The final list of factors of each module is chosen from among the optimal candidates and constructed using both statistical and experience-based criteria. The factor weights of the single module and significance level of each factor are then calculated through a statistical regression (typically, a logistic regression). In general, for each area of analysis, there are several modules that are near optimal and present only minor differences in terms of performances: to select a final model, it is necessary to consult the bank experts, to make sure that all the above-mentioned criteria have been satisfied.

Four illustrative modules are presented in Tables 4.7, 4.8, 4.9, 4.10: (financial, external behavioral, internal behavioral, and qualitative); these could potentially be employed in the evaluation of the creditworthiness of corporate SME counterparties. (Table 4.11)

The coefficients of the first three modules, estimated by means of logistic regression, are expressed as percentages.

Indeed, setting the existing monotonic relation between the logistic function:

Table 4.7 Financial module: an illustrative example

Code	Description	Weight (%)
D1	Gross margin/Interest expenses	9.6
D2	Interest expenses/Turnover	23.8
G1	(Equity–Book equity–Intangible assets)/(Total assets–Intangible assets)	9.2
G2	(Long-term debt + Total current liabilities)/Total assets	14.6
L1	Cash/Total assets	6.2
L2	(Total current assets–Inventory)/(Total current liabilities–Advanced payments by clients)	10.2
P1	Gross margin/Total assets	13.8
ST1	Turnover {t}/Turnover {t–1}–1	12.6

Table 4.8 External behavioral module: an illustrative example

Code	Description	Weight (%)
EB1	Six months' average of the ratio: Withdrawn facilities outstanding toward the banking system (evaluating bank excluded)/Withdrawn facilities limit toward the banking system (evaluating bank excluded)/	83.5
EB2	Three months' average of: Unauthorized drawn toward the banking system (evaluating bank excluded)	16.5

Table 4.9 Internal behavioral module: an illustrative example

Code	Description	Weight (%)
IB1	Six months' average of the ratio: Average balance/Withdrawn facilities limit	41.5
IB2	Three months' average of the ratio: Withdrawn facilities outstanding/Withdrawn facilities limit	58.5

Table 4.10 Qualitative module: an illustrative example

Code	Description	Weight (%)
Q1	For how many years has the company been a customer of the bank?	5.56
Q2	What percentage of assets/investments is not linked strategically to the company's business?	5.56
Q3	Has the company's top management developed a business plan?	5.56
Q4	If a business plan has been developed, has the proposed strategy been implemented?	5.56
Q5	Has the company been involved in any extraordinary operations (mergers, acquisitions, divisions and so on) with negative effects?	5.56
Q6	Overall, how have you evaluated the management with reference to the level of knowledge, experience, skills and competences?	5.56
Q7	Is the future of the company dependent on a few key managers?	5.56
Q8	Is there an investor (or a group of investors) holding a share of the company's stock sufficient to influence the company's strategies?	5.56
Q9	What is the evaluation of the market in which the company operates?	5.56
Q10	What is the expected production trend for the current year?	5.56
Q11	What is the quality of the company's market references?	5.56
Q12	Does the company's official financial forecast appear realistic?	5.56
Q13	What is the quality of the official financial information that the company communicates to the market?	5.56
Q14	What is the company's geographical business concentration?	5.56
Q15	To what extent is the company's business diversified?	5.56
Q16	What is the level of liquidity of the company's inventories?	5.56
Q17	What is the quality of the company's customers?	5.56
Q18	Has the company required deferred payments to the bank (interests, capital)	5.56

Table 4.11 Developing a rating model: main activities of Step 5

Step 5: Calibration, integration and mapping to the master scale
Estimate of the average default probability (calibration point) against which to calibrate the output of every module
Integrate the different modules
Comparison with the credit experts' opinion for the verification of the correct weight of each information area (module) inside the integrated model
Definition of the master scale
Mapping of the calibrated default probability into the master scale
Identification of the events that determine the assignment of positions to the administrated rating classes, independently of the model risk forecast
Complete validation of the selected model
Possible tuning of the model following the outcomes of the validation activity
Documentation of the model estimation process to ensure the complete replicability of obtained results

$$\pi(x) = \frac{e^{x \cdot \beta}}{1 + e^{x \cdot \beta}}$$

and the exponential function argument:

$$x \cdot \beta = \beta_0 + \sum_{i=1}^{n} \beta_i \cdot x_i$$

it is possible to compute the weights p_1 , p_2 , … , p_n of the n variables of each module as:

$$p_i = \frac{\beta_i}{\sum_{i=1}^{n} \beta_1}$$

with

$$\sum_{i=1}^{n} p_{1=1}$$

and

$$0\,''\; p_i\,''\; 1 \;\; for \;\; any$$
$$i = 1,\ldots,n$$

and postpone, to the following phase of calibration, the transformation of the risk score into a default probability.

Put differently, the weights assigned to the variables (questions) of the qualitative module have been assigned in a directly judgmental way, as an alternative to the proposed multi-attribute value theory method.

4.1.8 Step 5: Calibration, Integration and Mapping to the Master Scale

The output of the logistic regressions assumes values in the interval [0; 1] and could be interpreted as a default probability. Yet, the regression output is correctly "calibrated" when bank's risk manager estimates the average probability on the perimeter under consideration close to the one-year forecast default rate (the so-called "calibration point") and not by the average frequency of the default of the sample.

The calibration process, which allows the transformation of the logistic regression output in a default probability to 12 months, can be represented in the steps shown in Table 4.11:

- estimation of the calibration point (*CP*), which represents the level of average PD considered coherent with the portfolio under examination;
- computation of the default rate of the sample used for the calibration DR^{sample};
- sub-division of the sample in *n* quantiles, ordered with respect to the regression output (the score);
- computation of the median score associate with each quantile ($i = 1, \ldots, n$);
- computation of the default rate relative to each quantile, $DR_i (i = 1, \ldots, n)$;

- re-apportionment of the default rate of each quantile with respect to the *CP*, by applying Bayes theorem:

$$DR_i^{\text{calibrated}} = \frac{DR_i \cdot \dfrac{AP}{DR^{\text{sample}}}}{DR_i \cdot \dfrac{AP}{DR^{\text{sample}}} + (1-DR) \cdot \dfrac{(1-AP)}{1-DR^{\text{sample}}}}$$

where $\text{DR}^{\text{calibrated}}$ denotes the re-apportioned default rate of the *i* quantile, constrained to the interval [0; 1]; and

- the estimation of the (*a*) and (*b*) parameters which specify the exponential curve equation that relates to the score and the (re-apportioned) default rate observed in the quantiles:

$$\ln\left(DR_i^{\text{calibrated}}\right) = \alpha \cdot s_i + b$$

so obtaining the punctual (granular) values of default probability for each sample position contained in the interval [0; 1], and such that the average PD estimated on the whole sample will be equal to the calibration point.

The re-calibrated (and standardized) output of every module can eventually be integrated using both statistical methodologies (if a sufficiently large sample is available on which all the model indicators are computed; see Table 4.6), and internal bank experience alone. Table 4.12 presents

Table 4.12 Module integration weights

Type of customer	Financial PD (%)	Internal behavioral PD (%)	External behavioral PD (%)	Qualitative PD (%)
New (without internal behavioral information)	38.00	–	57.00	5.00
Old (with internal behavioral information)	33.25	28.50	33.25	5.00

examples of integration weights for the default probabilities estimated (and calibrated) separately for every module.

It is a reasonable suggestion initially to assign a limited weight to the qualitative module (in this case, 5 %) and to increase it progressively after comparing the judgment assigned by the relationship managers (by means of a questionnaire) with the quantitative model components (financial, external and internal behavior) and testing their correctness.

The integrated default probability is then associated with a rating class; that is, to one (and only one) of the ordered and disjoint sets that determines the partition of the possible values that the probability can assume.

The table on the left-hand side of Fig. 4.11, representing the so-called "master scale" of a generic rating system, illustrates the method for associating a default probability with a corresponding rating class.

For the definition of the master scale, the numerosity and amplitude of the rating classes should be set so that the scale:

- divides the portfolio customers into a sufficient number of risk classes;
- avoids excessive concentrations (both in terms of the number of positions and outstanding debts) in single rating classes; and
- allows a direct comparison with the final assessment (rating class) expressed, with the same counterparties, and the main external agencies and banking groups adopting a comparable master scale both in terms of average PDs and default definition.

Risk	Rating class	Medium PD	Minimum PD	Maximum PD
Low risk	1	**0,01%**	0,00%	0,02%
	2	**0,04%**	0,02%	0,07%
	3	**0,13%**	0,07%	0,22%
	4	**0,39%**	0,22%	0,52%
Medium risk	5	**0,70%**	0,52%	0,90%
	6	**1,17%**	0,90%	2,02%
	7	**3,51%**	2,02%	6,08%
High risk	8	**10,55%**	6,08%	18,29%
	9	**31,73%**	18,29%	48,78%
	10	**75,00%**	48,78%	100,00%

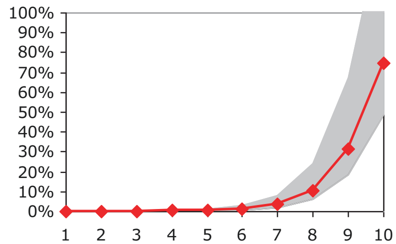

Fig. 4.11 An illustrative master scale

Risk	Rating class	Medium PD	Population distribution
Low risk	1	**0,01%**	3%
	2	**0,04%**	8%
	3	**0,13%**	15%
	4	**0,39%**	25%
Medium risk	5	**0,70%**	20%
	6	**1,17%**	14%
	7	**3,51%**	6%
High risk	8	**10,55%**	5%
	9	**31,73%**	3%
	10	**75,00%**	1%

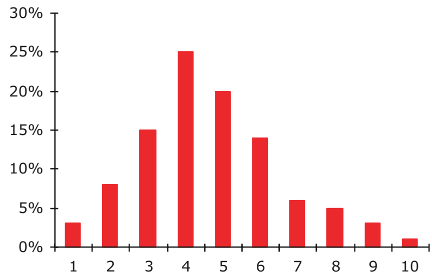

Fig. 4.12 Rating class distribution

Figure 4.12 shows, for the purposes of illustration, a possible portfolio distribution analyzed by rating class.

The risk judgment expressed by the integrated model can be corrected (in general, worsening the outcome) in the presence of events/behavior that represent eminent risk to the counterparty or its risk group. Corrections following policy rules or discriminatory events, even if they do not modify the default probability estimated by the algorithm, increase the attention level of the counterparty during the origination phase. This may lead the counterparty to assign its credit evaluation to higher power delegation, and, in the monitoring phase, the counterparty may move to a dedicated management unit. Before releasing the model into production, it is necessary to submit it to a thorough validation, correcting/integrating it and documenting the whole estimation process to ensure that the nature of the results is replicable.

4.1.9 Step 6: Embedding the Model in the Banking Processes

The model release happens, generally, by means of a preliminary prototype development, which allows us to test the calibration impact on bank credits and commercial policies (see Table 4.13).

As stated in Table 4.13, among the main uses of a rating model within the banking processes are:

Table 4.13 Developing a rating model: main activities of Step 6

Step 6: embedding model in the banking process
Estimated model prototype development
Definition of the risk parameter weights to identify delegation powers
Embedding of risk parameters inside the credit management process
Embedding of risk parameters inside pricing policies
Optimization of the risk/return profile of the bank's capital requirement computation

- the definition of delegation powers in relation to the expected loss associated with the single risk position;
- the definition of the pricing for the required facility;
- the cost of risk computation; and
- the optimization of the risk/return profile of the bank.

Some of these will be detailed in later chapters of this book.

4.2 PD Corporate SME Sub-segment Models

In relation to the practical availability of data (financial, behavioral and qualitative), it is possible to estimate the different modules of a PD model either on a statistical basis (in the presence of sufficiently robust data) or on an expert basis. Also, in the presence of company samples that fall into the good/bad type, representative of the bank's portfolio and statistically robust, expert evaluation always plays a part, both in the selection of final financial and behavioral modules, and in the development of the qualitative module (Tables 4.14, 4.15, 4.16).

Table 4.14 Start-up model: an illustrative financial module

Category	Code	Weight (%)	Indicator
Gearing	G1	30	Equity/Initial investment
Profitability	P1	20	Initial investment/EBITDA steady
Debt service capacity	D1	30	(Financial debts – Subordinate debts to partners)/(Book equity + Subordinate debts to partners)
	D2	20	(Financial debts + Interests outflow)/EBITDA steady

Table 4.15 Consortia model: an illustrative financial module

Category	Code	Weight (%)	Indicator
Size	SZ1	5	Net sales
Debt service capacity	D1	5	(Financial debts – Subordinate debts to partners with residual life of less than five years)/(Equity + Subordinate debts to partners with residual life of less than five years)
	D2	15	(Net margin + Tangible depreciations and amortizations)/Interest expenses
	D3	15	Interest expenses/Net sales
Liquidity	L1	5	Cash/Total assets
	L2	10	(Total current assets – Inventories)/(Total current liabilities – Advanced payments by clients)
Gearing	G1	10	(Equity – Intangible fixed assets)/(Total assets – Intangible fixed assets)
	G2	15	(Equity – Issued shares)/Total Assets
Stability	ST1	10	Net sales {*t*}/Net sales
	ST2	10	{t – 1} – 1 Capital employed {t}/Capital employed {t – 1} – 1

Table 4.16 Financial company model: an illustrative financial module

Category	Code	Weight (%)	Indicator
Profitability	P1	8	(Extraordinary profit or loss + Revaluations)/Total assets
	P2	8	(Profit or loss)/Equity
Debt service capacity	D1	15	Financial liabilities/Equity
Gearing	G1	24	Equity/Total assets
	G2	15	(Equity – Intangible fixed assets)/Financial liabilities
Activity	A1	15	Credit risk provision funds/(Extraordinary profit or loss + Revaluations)
	A2	15	Operating costs/Operating incomes

In the absence of robust databases, the expert-based component simply assumes a more relevant role in the framework of the definition of the whole structure of the model.

In particular, models composed from expert-based modules refer to customer sub-segments characterized by portfolios that are:

- rarefied in terms of counterparts (for example, insurance companies); or

- constituted by a reduced number of defaults (non-profit organizations); or
- lacking a historical database of clearly codified balance sheets (non-profit organizations) or sufficiently reliable.

The release of models with expert-based modules also aims to make known the rating discipline in terms of number of positions/default rates for portfolios/sub-segments that are less relevant than others.

This contributes to the settling down of a data collection process on a systematic base on these bank portfolios.

As soon as a reliable database is available for these modules, it will be possible to start the "objectivization" phase of weights and variables following statistical techniques.

4.2.1 Statistical Expert-based Models

Possible models constituted both by statistical components and by expert-based modules are devoted to the evaluation of corporate SME counterparties belonging, for example, to the following segments: farmers, start-ups, consortia and financial companies.

In the case of farmers, the expert-based component could be represented by the qualitative module; in the remaining three models (devoted to start-ups, consortia and financial companies), one could assume that the expert-based score would be the result of the weighted average of the scores produced by the financial and qualitative modules.

The following two sub-sections present a brief description of the process of derivation of the financial and qualitative expert-based modules, as illustrated earlier in the chapter.

As explained in Figure 4.3, such modules/components will be allowed to modify, in a limited manner (in terms of notches), the behavioral (or behavioral and financial) evaluation expressed by the model's statistical component.

4.2.1.1 Qualitative Modules

In the definition of the qualitative modules of the models devoted to the evaluation of farmers, start-ups, consortia and financial companies, all the variables suggested by the expert are generally inserted into the final components, with a weight variable from 0 to 1 in relation to its recognized importance to the insolvency forecast capability.

The weights indicated by the experts are differentiated according to their "vintage", assuming that, for "new" customers, no answer could be found for certain questions (variables): in a first approximation, the relative weights could simply be redistributed proportionally over the remaining questions.

The score assigned to each indicator included in the interval [0; 1] must be obtained according to the examined variable type:

- for indicators similar to continuous variables, a score can be assigned by means of linear regression, analogous to what was undertaken for the variables of a financial nature; or
- for indicators of a categorical type, the expert team must identify the possible outcomes and set the relative risk score.

Tables 4.17, 4.18, 4.19 and 4.20 describe the structure of four possible quantitative modules for the evaluation of, respectively, farmers, start-ups, consortia and financial corporate SMEs.

4.2.1.2 Integration of the Statistical and Expert-based Components

As mentioned earlier in the chapter, the rating class of a counterparty in the sub-segments of farmers, start-ups, consortia and financial companies, estimated by means of the statistical component of the corresponding rating model, can be corrected upward or downward, according to the score level assigned to the same counter party from the expert-based component.

As every variable of the expert-based component has a value between 0 and 1, as well as other possible intermediate expert-based scores, according to the hierarchical structure, the final score will also be included in the interval [0; 1].

Having sub-divided the score variation range into seven risk sub-intervals, the magnitude of correction upward or downward of the

Table 4.17 Farmers model: an illustrative qualitative module

Category	Variable	Weight new customer (%)	Weight old customer (%)
Competitive position/ business image	Company life-cycle and growth perspectives	9	8
	Existence of trade agreements for purchasing raw materials (seeds, fertilizers and so on)	13	11
	Existence of trade agreements for sale of final products	13	11
	Product quality	13	11
	Does the company benefit from government contributions?	4	4
	Is the company subject to government obligations which limit production capabilities?	4	4
	Does the company respond positively to requirements to benefit from interbanking insurance funds?	10	8
Business characteristics/ credit portfolio	Geographical concentration of sales	9	8
	Is there any procedure to manage and monitor the credit risk of trade activities?	4	4
Management/ sponsor characteristics/ business plan/ property	For how many years has the entrepreneur operated in the sector?	9	8
	Entrepreneur's reputation	4	4
	Ethical behavior of the entrepreneur	4	4
	Entrepreneur's attitude to safety and environmental issues	4	4
Relation with the bank	Bank manager's opinion of the fiduciary relationship with the customer (for old customers only)	–	11

rating class, estimated statistically, could be defined, agreeing with the expert team, as shown in Table 4.21, or be further differentiated in relation to the rating class estimated by means of the model's statistical component.

Table 4.18 Start-up model: an illustrative qualitative module

Category	Variable	Weight (%)
Sector characteristics	Existence of entry barriers	5
	Growth perspectives of the sector	5
	Risk level of the sector	8
	Niche differentiation	5
	Costs leadership	5
	Level of competition	3
Management/Sponsor characteristics/Business plan/Property	Capital and economic strength of the entrepreneur (of the partners)	5
	Enterpreneur's (partners') reputation	3
	For how many years has the entrepreneur (partners) operated in the sector?	5
	Ethical behavior of the entrepreneur (of the partners)	3
	Enterpreneur's (partners') attitude to safety and environmental issues	3
	Management capability to produce a business plan	8
	Completeness and level of detail of the business plan	8
	Business plan's objective reachability	5
	Stress analysis	5
Business characteristics/ Credit portfolio	Percentage of medium/long-term loans for which the interest rate risk is hedged	5
	Existence of trade agreements which stabilize the costs	5
	Existence of trade agreements which stabilize the sales	5
	Has enterprise already obtained the concessions to make the investments?	5
	Is there any procedure to manage and monitor the credit risk of trade activities?	4

Following such a correction, it is possible to associate the counterparties belonging to particular corporate SME sub-segments, such as farmers, start- ups, consortia, financial companies, with a final rating class and a default probability to be employed for both regulatory and management purposes (delegation powers, remuneration and pricing).

Table 4.19 Consortium model: an illustrative qualitative module

Category	Variable	Weight new customer (%)	Weight old customer (%)
Business characteristics/ credit portfolio	Level of standardization of products/services offered	13	10
	Production differentiation level and geographical sales concentration	18	16
	Production growth forecasts with respect to the previous year	7	6
	Is there any procedure to manage and monitor the credit risk of trade activities?	7	6
Management/ Sponsor characteristics/ Business plan/ Property	For how many years has the consortium operated in the sector?	13	10
	Consortium's reputation	7	6
	Ethical behavior of the consortium	7	6
	Capital and economic strength of the consortium	7	6
	Consortium's attitude to safety and environmental issues	7	6
	Management's capability to produce a business plan	7	6
	Business plan's objective reachability	7	6
Relation with the bank	Bank manager's opinion of the fiduciary relationship (for old consortia only)	–	16

4.2.2 Pure Expert-based Models

Pure expert-based models are, for example, those that can be developed for the corporate SME counterparties in the sub-segments of insurance companies, holding companies and non-profit organizations.

As illustrated in Fig. 4.5, the model structure is still modular: the financial module and the qualitative/behavioral module compute, separately, two scores that express in numerical terms the creditworthiness of the counterparty.

Table 4.20 Financial company model: an illustrative qualitative module

Category	Variable	Weight new customer (%)	Weight old customer (%)
Relation with the bank	Bank manager's opinion on the fiduciary relationship (for old customers only)	–	12
Management/Sponsor characteristics/Business plan/Property	For how many years has the management operated in the sector?	8	8
	Management's reputation	5	4
	Ethical behavior of the management	5	4
	Operational risk management	5	4
	Existence of internal control bodies/procedures	5	4
	Management's capability to produce a business plan	5	4
	Business plan's objective reachability	9	8
	Level of completeness/ reliability of official financial information (balances, quarterly/ semi-annual reports, financial plans)	8	8
Business characteristics/ Credit portfolio	Geographical differentiation level of the credit portfolio	5	4
	Sector differentiation level of the credit portfolio	8	8
Competitive position/ Business image	Company's competitive position in the domestic market	13	12
	Company's market share	9	8
	Differentiation and diffusion level of distribution channels	5	4
	Diversification level of offered products/services	5	4
Risk management	Effectiveness of risk management strategies	5	4

Table 4.21 Expert-based correction entity

Score	Up/downgrading
0	+3
[1;2]	+2
[3;4]	+1
5	0
[6;7]	−1
[8;9]	−2
10	−3

The scores generated by the two modules are combined, adopting a weighted average, in a final score variable between 0 (maximum risk) and 10 (minimum risk), expressing the size of upward correction (upgrading) or downward correction (downgrading) to be applied to the rating corresponding to the average risk of the segment under examination, possibly corrected in a through-the-cycle perspective (the calibration point).

For the correction, one can refer to a structure similar to that proposed in Table 4.21.

4.2.2.1 Financial Modules

Tables 4.22, 4.23 and 4.24 summarize the structure of three possible financial modules for the evaluation, respectively, of insurance companies, holding companies and non-profit organizations.

4.2.2.2 Qualitative/Behavioral Modules

Tables 4.25, 4.26 and 4.27 describe the structures of three possible qualitative/behavioral models for the evaluation of insurance companies, holding companies and non-profit organizations, respectively.

4.2.2.3 Integration of Pure Expert-based Modules

As anticipated at the beginning of this section, the scores generated separately by the financial and qualitative/behavioral modules are integrated according to a weighted average (convex combination) in a final score variable, which is also in the interval [0; 10].

Table 4.22 Insurance companies model: an illustrative financial module

Category	Code	Weight (%)	Indicator
Size	SZ1	30	Operative result
	SZ2	20	Ln (Total assets)
Profitability	P1	10	(Profit or loss)/Equity
	P2	10	Loss ratio + (Administrative costs/Profit before taxes)
	P3	10	Profit before taxes/Net premium
Gearing	G1	20	Net technical reserves/Equity

Table 4.23 Holding companies model: an illustrative financial module

Category	Code	Weight (%)	Indicator
Profitability	P1	17	Dividends and income from investments/ Fixed assets in investments
Debt service	D1	17	Cash/Equity
Capacity	D2	17	(Financial income + Revaluations)/(Interest expenses + Depreciation)
Gearing	G1	24	(Financial liabilities – Cash)/Investment value
Activity	A1	8	Depreciation/Income from investment
	A2	17	Depreciation/Fixed assets in investments

Table 4.24 Organizations model: an illustrative financial module

Category	Code	Weight (%)	Indicator
Profitability	P1	18	Loss/Equity
Debt service	D1	10	Interest expenses/Turnover
capacity	D2	18	(Net financial debts - Sub. debt to affiliates)/ (Equity + Sub. debt to affiliates)
Liquidity	L1	18	Liquidity/Financial debts
Gearing	G1	18	(Fixed assets market value + Liquidity)/ Financial debts
	G2	18	Financial debts/Total assets

Table 4.28 proposes possible integration weights for the two modules, differentiated for types of counterpart (insurance companies, holding companies and non-profit organizations).

The integrated score, when divided, for example, into the seven classes presented in Table 4.20, can be used to establish whether the risk of the single counterparty is greater or smaller than the average of a sub-segment, and to assign to these a specific default probability.

Table 4.25 Insurance companies model: an illustrative qualitative/behavioral module

		Weight new customer (%)				Weight old customer (%)			
		Listed company		Non-listed company		Listed company		Non-listed company	
Category	Indicator	With parent company	Without parent company	With parent company	Without parent company	With parent company	Without parent company	With parent company	Without parent company
Relation with the bank	Bank manager's opinion on the fiduciary relationship with the customer)					8	10	9	11
	Bank (banking group) percentage of overall activity (bank limit/banking system limit	6	7	7	8	6	8	6	7
	Undrawn credit amount (Limit-Outstanding limit)/Limit versus Banking system	9	11	10	14	8	10	9	11
Competitive position/ Business image	Company market share (damage+ life)	9	10	10	14	7	10	9	11
	Diversification level of products/ services	3	4	3	4	3	3	3	4
	Differentiation and diffusion level of distribution channels	3	4	3	4	3	3	3	4
Business characteristics/ Credit portfolio	Prudential parameters of investment policy	6	7	7	8	6	7	6	7

(*continued*)

Table 4.25 (continued)

		Weight new customer (%)				Weight old customer (%)			
		Listed company		Non-listed company		Listed company		Non-listed company	
Category	Indicator	With parent company	Without parent company	With parent company	Without parent company	With parent company	Without parent company	With parent company	Without parent company
Management/ sponsor characteristics/ business plan/ Property	Management capability to produce a business plan	3	4	3	4	3	3	3	4
	Business plan objective reachability	3	4	3	4	3	3	3	4
	Level of completeness/ reliability of the official financial information (Balances, quarterly/ semi-annual reports, financial plans)	3	4	3	4	3	3	3	4
	For how many years has the management operated in the sector?	6	7	7	8	6	7	6	7
	Management reputation	3	4	3	4	3	3	3	4
	Ethical behavior of the management	3	4	3	4	3	3	3	4
	Operational risk management	3	4	3	4	3	3	3	4
	Existence of internal control bodies/procedures	6	7	7	8	6	7	6	7
Risk management	Effectiveness of risk management strategies	7	7	8	8	6	8	7	7

Market stock trend	Stock growth rate in the previous year	3	4		3	3		
	Stock growth rate in the previous year with respect to the main competitors	3	4			3	3	
	Analysts’ assessment in recent equity researches	3	4			3	3	
		9		10		7		9
Relationship with the parent company	Strategic importance of the parent company	9		10		7		9
	Capital and economic strength of the parent company	9		10		7		9

Table 4.26 Holding companies model: an illustrative qualitative/behavioral module

Category	Indicator	Weight new customer (%)	Weight old customer (%)
Business characteristics/ Credit portfolio	Geographical diversification level of the investment portfolio	7	7
	Sector diversification level of the investment portfolio	7	7
	Liquidity of the investment portfolio	11	10
	Volatility of the subsidiaries' economic results	7	7
	Percentage of holding investments in the overall portfolio	11	10
Management/ sponsor characteristics/ business plan/ Property	Management's capability to produce business plan	7	7
	Business plan's objective reachability	7	8
	Level of completeness/reliability of the official financial information (balances, quarterly/semi-annual reports, financial plans)	4	3
	For how many years has the management operated in the sector?	7	8
	Management reputation	4	3
	Ethical behavior of the management	4	3
	Operations risk management	4	3
	Existence of internal control bodies/procedures	4	3
Risk management	Effectiveness of risk management strategies	4	3

Table 4.27 Example of default data

Year	Number of companies at start of year	Defaults per year	Cumulative defaults
1	100	1	1
2	99	2	3
3	97	3	6
4	94	4	10
5	90	5	15

Table 4.28 Mapping of suggested master scale to S&P grades

Suggested master scale grade	S&P equivalent grade	S&P grade used
1	AAA	AAA
2	AA+	AA+
3	AA	AA
4	AA−	AA−
5	A+	A+
6	A	A
7	A−	A−
8	BBB+	BBB+
9	BBB	BBB
10	BBB−	BBB−
11	BB+	BB+
12	BB+/BB	BB+
13	BB	BB
14	BB/BB−	BB
15	BB−	BB−
16	BB−/B+	BB−
17	B+	B+
18	B+/B	B+
19	B	B
20	B/B−	B
21	B−	B−
22	CCC	CCC

4.3 Term Structure of Probability of Default

The effects of grade migration over a period of time create a term structure of PDs. For example, an AAA-rated borrower cannot improve in rating over time and so, on average, is likely to deteriorate. However, a CCC-credit rated borrower, if it survives, can only improve.

4.3.1 Observed Term Structures

Figure 4.13 shows the term structure observed for Standard & Poor's (S&P) rated companies. It can be seen from this figure that higher-quality credits tend to deteriorate over time and lower-quality credits improve.

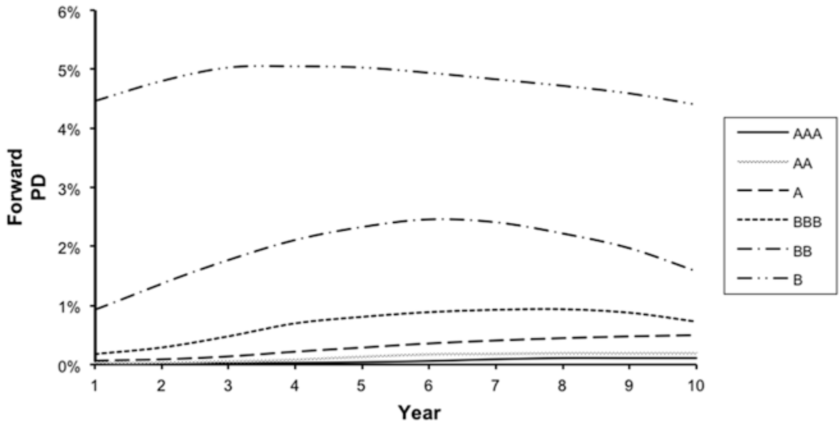

Fig. 4.13 Observed term structure of S&P rated companies (based on one-year forward PD) (Source: Internal Rating Model Development Handbook – Capitalia Banking Group)

4.3.2 Marginal, Forward, and Cumulative Probability of Default

The PDs for each year shown in Fig. 4.10 are *forward* PDs; they are the PDs that would be expected that year expressed as a percentage of companies that have survived. The number of companies that survive can be determined from the *cumulative* default rate. To illustrate these concepts, consider the simple example in Table 4.27.

Consider three different questions. What is the probability that:

1. a company will default over a four-year period?
2. a company in year four will default over the next year?
3. a company will default in the fourth year of a facility?

The answers require different combinations of the numbers presented in Table 4.27:

1. Of 100 companies, 10 default in the first four years: 10 %.
2. The *Cumulative Default Rate* in year four is 10 %.

3. Of the 94 companies that survived until year four, 4 will default in year four: 4.2 % is the *Forward Default Rate* in year four.
4. Of the 100 companies, 4 that have been granted loans default in the fourth year of their life: 4.0 % is the *Marginal Default Rate* in year four.

The pricing model requires both the cumulative PD and forward PD for the discounted cash flow calculation. The cumulative PD is required to determine the probability of which revenues and costs are incurred in any given year (that is, to account for survivorship) and the forward PD is required to calculate expected loss and regulatory capital.

4.3.3 Mapping PD Ratings to Observed Term Structures

Once the marginal PDs have been calculated (Fig. 4.14), it is then possible to calculate the forward PDs using the following equation:

$$PD_{\text{forward,year }n} = \frac{PD_{\text{marginal,year }n}}{1 - \sum_{\text{year}=0}^{n} PD_{\text{marginal}}}$$

As not all grades of the suggested 22-point grade system master scale can be mapped directly onto the S&P grade system (as some of them are intermediate grades), the simplified mapping shown in Table 4.28 can be used to determine the forward PDs. The result based on the suggested 22-point rating system master scale is shown in Table 4.29.

4.4 Transition Matrix State – Dependent

In the previous sections, an analysis was used that was indifferent to the phases of the economic cycle. This section approaches the production of European transition matrices based on the different phases of the cycle

$$
\begin{array}{ccccc}
\begin{array}{c}\text{Marginal}\\ \text{PD}\\ \text{Year n+1}\end{array} & & \begin{array}{c}\text{1 Year}\\ \text{Migration}\\ \text{Matrix}\end{array} & & \begin{array}{c}\text{Marginal}\\ \text{PD}\\ \text{Year n}\end{array} \\
\begin{bmatrix} AAA \\ AA+ \\ AA \\ AA- \\ A+ \\ A \\ A- \\ BBB+ \\ BBB \\ BBB- \\ BB+ \\ BB \\ BB- \\ B+ \\ B- \\ CCC \end{bmatrix}
& = &
\begin{bmatrix} AAA \ldots CCC & & & & & \\ AAA & & & & & \\ AA+ & & & & & \\ AA & & & & & \\ AA- & & & & & \\ A+ & & & & & \\ A & & & & & \\ A- & & & & & \\ BBB+ & \cdots & \cdots & \cdots & \cdots & \cdots \\ BBB & & & & & \\ BBB- & & & & & \\ BB+ & & & & & \\ BB & & & & & \\ BB- & & & & & \\ B+ & & & & & \\ B- & & & & & \\ CCC & & & & & \end{bmatrix}
& \times &
\begin{bmatrix} AAA \\ AA+ \\ AA \\ AA- \\ A+ \\ A \\ A- \\ BBB+ \\ BBB \\ BBB- \\ BB+ \\ BB \\ BB- \\ B+ \\ B- \\ CCC \end{bmatrix}
\end{array}
$$

Fig. 4.14 Calculating marginal PD from the migration matrix

itself. The type of transition matrix states of the economy dependent on each business segment are summarized in Table 4.30. The average downgrading and upgrading probability states of the economy dependent on all of the business segments are shown in Table 4.31.

Downgrading probabilities are, on average, increasing from recovery to hard landing.

Upgrading probabilities decrease from recovery (higher probabilities) to hard landing.

Tables 4.32, 4.33, 4.34 and 4.35 show state-dependent transition matrices for large corporates, corporates, SME corporates and SME retail.

4.5 Validation of Internal Credit Rating Models

A credit rating system undergoes a "validation process". This consists of a formal set of activities, instruments and procedures aimed at ensuring that the design of a model is conceptually sound; that its implementa-

Table 4.29 Forward PD for suggested master scale with 22-point ratings (illustrative, (%))

	Year									
	1	2	3	4	5	6	7	8	9	10
1	0.010	0.014	0.018	0.022	0.028	0.035	0.042	0.051	0.061	0.072
2	0.020	0.023	0.028	0.033	0.040	0.049	0.059	0.070	0.082	0.096
3	0.030	0.034	0.039	0.047	0.057	0.068	0.082	0.096	0.111	0.127
4	0.040	0.045	0.054	0.065	0.080	0.096	0.114	0.133	0.152	0.171
5	0.050	0.060	0.075	0.094	0.115	0.139	0.163	0.187	0.210	0.232
6	0.070	0.087	0.111	0.140	0.171	0.203	0.234	0.264	0.291	0.315
7	0.090	0.125	0.167	0.212	0.258	0.301	0.340	0.374	0.404	0.429
8	0.130	0.195	0.264	0.330	0.391	0.445	0.491	0.528	0.558	0.580
9	0.220	0.330	0.430	0.519	0.594	0.655	0.702	0.737	0.760	0.774
10	0.390	0.542	0.678	0.792	0.880	0.943	0.985	1.009	1.018	1.015
11	0.670	0.904	1.086	1.216	1.302	1.352	1.371	1.368	1.349	1.317
12	0.670	0.904	1.086	1.216	1.302	1.352	1.371	1.368	1.349	1.317
13	1.170	1.480	1.690	1.816	1.875	1.884	1.856	1.804	1.737	1.660
14	1.170	1.480	1.690	1.816	1.875	1.884	1.856	1.804	1.737	1.660
15	2.030	2.419	2.619	2.685	2.661	2.579	2.462	2.328	2.186	2.045
16	2.030	2.419	2.619	2.685	2.661	2.579	2.462	2.328	2.186	2.045
17	3.510	3.869	3.941	3.840	3.643	3.399	3.139	2.882	2.638	2.413
18	3.510	3.869	3.941	3.840	3.643	3.399	3.139	2.882	2.638	2.413
19	6.080	6.114	5.797	5.322	4.798	4.285	3.811	3.388	3.018	2.697
20	6.080	6.114	5.797	5.322	4.798	4.285	3.811	3.388	3.018	2.697
21	10.540	9.404	8.134	6.924	5.860	4.964	4.226	3.625	3.136	2.737
22	18.270	13.862	10.447	7.934	6.132	4.843	3.914	3.231	2.718	2.324

Source: Internal Rating Model Development Handbook – Capitalia Banking Group

Table 4.30 List of transition matrix states of the economy dependent on each business segment

	Recovery	Overheat	Hard landing	Soft landing
Large corporate	√	√	√	√
Corporate	√	√	√	√
SME corporate	√	√	√	√
SME retail	√	√	√	√

Table 4.31 Transition probabilities in terms of stability, downgrading and upgrading (%)

	Recovery	Overheat	Hard landing	Soft landing
Stability	77.17	75.13	73.80	75.97
Downgrading	13.46	18.59	19.12	15.97
Upgrading	14.55	13.69	12.53	13.79

tion is accurate and consistent with the theory; and to assess the accuracy of the estimates of all material risk components and the regular operation, predictive power and overall performance of the internal rating system.

A model validation process will be triggered whenever a new model is developed, or when any significant changes are made to one that has been previously approved. Models are also subject to periodic reviews, which aim to reassess the adequacy of their performance over time (e.g. the verification of the validity of their assumptions under different market conditions; investigation of mismatches between realized and model-predicted values; and comparisons with competitors' best practice).

Hence, model validation must be seen as an ongoing process: at least once a year, banks have to verify the reliability of the results generated by the rating system on an ongoing, iterative basis and also its continued consistency with regulatory requirements, operational needs and changes in the reference market.[2]

The rating system validation process is complementary to the developmental process (see Fig. 4.15).

The initial validation, before a model's implementation, aims to consolidate all new models; the ongoing validation ensures the reliability and robustness of the regulatory parameters over time.

Table 4.32 Large corporate transition matrices

	AAA	AA+	AA	AA–	A+	A	A–	BBB+	BBB	BBB–	BB+	BB	BB–	B+	B	B–	CCC	Default
AAA	83.9	3.3	6.2	1.7	2.5	1.7	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AA+	2.0	92.1	3.3	2.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AA	0.0	1.1	82.2	11.2	4.5	0.0	0.7	0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AA–	0.2	0.2	2.4	80.5	12.7	3.0	0.8	0.0	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
A+	0.1	0.1	0.0	2.4	87.6	7.4	1.3	1.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
A	0.0	0.0	0.1	0.3	3.2	82.7	9.8	2.9	0.9	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
A–	0.0	0.2	0.0	0.0	1.3	8.8	78.9	7.1	2.3	0.9	0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
BBB+	0.0	0.0	0.1	0.0	0.0	1.1	7.7	79.4	8.1	2.6	0.3	0.1	0.1	0.3	0.0	0.0	0.0	0.0
BBB	0.0	0.0	0.0	0.1	0.1	0.4	1.6	6.2	80.8	7.5	1.6	0.4	0.5	0.6	0.1	0.1	0.0	0.0
BBB–	0.0	0.0	0.0	0.0	0.6	0.0	0.8	2.4	9.5	75.7	5.0	3.1	1.6	0.1	0.7	0.4	0.1	0.0
BB+	0.0	0.0	0.0	0.0	0.2	0.0	0.4	0.7	2.0	9.0	73.5	6.8	4.5	1.6	0.9	0.2	0.0	0.4
BB	0.0	0.0	0.0	0.0	0.2	0.4	0.2	0.9	0.9	2.9	12.0	70.2	5.6	1.1	2.9	1.3	1.3	0.0
BB–	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.0	0.4	0.3	1.8	8.8	73.9	5.1	6.1	1.6	0.4	1.3
B+	0.1	0.0	0.0	0.0	0.1	0.4	0.1	0.1	0.6	0.1	0.1	3.0	8.8	72.9	5.1	4.7	1.6	2.2
B	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.4	1.3	2.9	9.4	67.7	9.6	5.4	3.1
B–	0.0	0.0	0.2	0.0	0.0	0.0	0.0	0.2	0.2	0.2	0.0	0.6	0.6	5.8	7.3	65.2	9.6	10.0
CCC	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.6	0.0	0.6	0.6	1.5	6.8	68.5	21.4
Large corporate – overheat (%)																		
AAA	86.6	3.9	8.7	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AA+	2.6	77.6	2.0	17.1	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AA	1.8	4.3	79.1	11.0	1.2	2.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AA–	0.0	0.0	1.2	86.4	6.5	4.3	1.0	0.5	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.0	0.0
A+	0.0	0.0	0.4	3.2	81.1	9.8	3.6	1.6	0.0	0.2	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.0
A	0.4	0.0	0.1	0.4	3.2	81.8	7.1	4.5	1.0	0.1	0.3	0.0	0.7	0.0	0.3	0.0	0.0	0.0
A–	0.0	0.0	0.0	0.2	1.3	5.6	75.5	9.7	4.5	1.6	0.5	0.3	0.3	0.2	0.0	0.0	0.0	0.3
BBB+	0.0	0.0	0.0	0.0	0.2	0.3	4.4	75.7	9.6	4.7	2.3	0.6	0.0	0.9	0.2	0.6	0.3	0.2

(*continued*)

Table 4.32 (continued)

	AAA	AA+	AA	AA–	A+	A	A–	BBB+	BBB	BBB–	BB+	BB	BB–	B+	B	B–	CCC	Default
BBB	0.1	0.3	0.0	0.0	0.3	0.3	1.1	5.6	76.9	9.7	1.4	1.3	0.6	0.4	0.5	0.0	1.0	0.5
BBB–	0.2	0.0	0.0	0.0	0.0	0.5	0.2	0.8	8.3	74.2	7.5	2.4	2.4	1.0	0.5	0.3	0.8	0.8
BB+	0.2	0.0	0.0	0.0	0.2	0.7	0.0	0.5	2.3	9.7	68.1	7.6	4.2	2.3	0.2	1.6	0.7	1.6
BB	0.0	0.0	0.3	0.3	0.0	0.3	0.0	0.0	0.0	2.2	9.4	70.2	8.3	4.1	2.8	0.6	0.8	0.8
BB–	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.6	0.2	0.8	2.4	8.5	73.2	5.4	4.6	1.4	1.3	1.4
B+	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.3	0.0	1.0	2.5	6.3	72.1	8.7	4.5	2.5	2.2
B	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.5	0.0	0.5	0.2	0.3	1.9	9.1	68.4	8.8	5.3	5.0
B–	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.4	0.0	0.4	0.2	1.5	2.7	11.0	64.2	13.5	5.8
CCC	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.4	0.2	0.4	0.4	1.2	2.8	8.7	69.8	15.8
Large corporate – hard landing (%)																		
AAA	93.2	1.0	5.2	0.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AA+	2.0	77.6	6.8	13.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AA	1.5	3.0	79.0	11.4	2.6	1.8	0.4	0.0	0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AA–	0.2	1.3	1.6	83.1	7.8	2.9	1.8	0.4	0.0	0.2	0.2	0.0	0.4	0.0	0.0	0.0	0.0	0.0
A+	0.0	0.2	0.5	4.0	81.9	9.7	3.4	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
A	0.2	0.0	0.3	2.8	4.2	76.8	10.9	2.2	1.2	0.3	0.3	0.2	0.0	0.0	0.2	0.0	0.1	0.2
A–	0.1	0.3	0.3	0.6	0.8	5.5	74.8	9.8	4.4	1.4	0.3	0.3	0.6	0.4	0.3	0.1	0.1	0.0
BBB+	0.0	0.3	0.1	0.0	0.0	1.6	4.3	76.9	10.6	3.7	0.7	0.9	0.3	0.1	0.0	0.0	0.1	0.4
BBB	0.1	0.1	0.0	0.4	0.3	0.5	2.4	5.4	75.4	10.6	2.3	1.0	0.4	0.1	0.5	0.4	0.0	0.0
BBB–	0.0	0.0	0.0	0.2	0.0	0.5	1.2	2.6	9.6	69.9	5.2	3.4	3.3	1.9	0.3	0.0	0.9	1.0
BB+	0.0	0.0	0.0	0.0	0.0	0.2	1.9	0.8	1.9	8.8	71.0	3.5	5.6	1.9	2.1	1.2	0.0	1.2
BB	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.7	2.7	7.5	67.9	9.0	2.3	5.4	1.8	0.9	1.6
BB–	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	1.0	2.8	3.9	70.2	3.2	8.2	4.2	1.4	5.0
B+	0.0	0.0	0.1	0.0	0.1	0.2	0.0	0.1	0.0	0.7	0.7	1.5	4.0	69.4	5.8	7.9	3.4	6.1
B	0.0	0.0	0.1	0.1	0.0	0.0	0.1	0.4	0.3	0.0	0.5	0.5	1.7	4.3	67.9	6.8	7.0	10.3
B–	0.0	0.0	0.2	0.0	0.2	0.0	0.2	0.0	0.0	0.2	0.0	0.4	0.5	2.1	2.3	62.3	12.8	18.9
CCC	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.0	2.5	67.7	29.3

Large corporate – soft landing (%)

AAA	91.7	3.6	2.3	1.0	1.0	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AA+	2.4	83.2	5.6	2.8	3.6	0.8	0.4	0.8	0.0	0.0	0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AA	0.5	4.6	81.7	4.9	4.1	2.0	1.7	0.0	0.0	0.0	0.0	0.0	0.2	0.2	0.0	0.0	0.0	0.0
AA–	0.1	0.7	3.6	81.5	6.1	5.5	1.2	0.4	0.1	0.4	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0
A+	0.1	0.1	0.7	5.2	81.4	6.1	2.6	0.7	0.7	0.3	0.8	0.8	0.2	0.3	0.0	0.0	0.0	0.0
A	0.0	0.1	0.3	0.5	4.5	84.5	5.3	2.2	1.5	0.5	0.1	0.1	0.2	0.1	0.1	0.0	0.1	0.0
A–	0.0	0.0	0.1	0.0	1.5	7.9	78.5	5.9	2.5	2.4	0.4	0.3	0.1	0.2	0.1	0.2	0.0	0.0
BBB+	0.1	0.0	0.0	0.1	0.3	3.3	6.8	78.7	5.8	2.6	1.2	0.0	0.4	0.5	0.1	0.0	0.0	0.0
BBB	0.0	0.2	0.0	0.1	0.0	1.2	3.1	6.6	80.5	4.9	1.7	0.2	0.5	0.5	0.4	0.2	0.0	0.0
BBB–	0.0	0.0	0.0	0.0	0.0	0.7	0.5	3.7	9.2	75.1	6.7	1.9	0.9	0.5	0.2	0.3	0.0	0.3
BB+	0.0	0.0	0.0	0.1	0.1	0.0	0.7	0.9	5.0	9.2	70.1	6.1	3.9	1.0	1.6	1.0	0.1	0.2
BB	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.7	0.8	2.8	9.1	67.9	9.6	2.9	3.3	1.4	0.6	0.6
BB–	0.0	0.1	0.0	0.0	0.1	0.1	0.2	0.5	0.2	0.6	3.0	4.7	68.5	10.5	5.9	3.0	0.8	1.8
B+	0.0	0.0	0.0	0.1	0.1	0.0	0.2	0.0	0.2	0.0	0.5	2.5	7.1	67.9	12.3	4.2	1.9	3.1
B	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.1	0.0	0.1	0.3	0.8	2.4	7.9	65.8	10.7	6.5	5.3
B–	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.4	1.0	4.5	7.6	66.7	11.4	8.0
CCC	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.2	0.2	0.0	0.7	1.1	2.3	4.5	74.8	15.9

Table 4.33 Corporate transition matrices

	AA	AA–	A+	A	A–	BBB+	BBB	BBB–	BB+	BB	BB–	B+	B	B–	CCC	Default
Corporate – recovery (%)																
AA	54.9	35.8	8.0	0.0	1.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AA–	18.1	26.8	39.0	7.5	3.2	0.0	5.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
A+	0.0	11.3	35.8	31.0	6.6	15.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
A	2.2	6.5	14.4	21.9	31.1	16.4	4.8	0.0	2.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
A–	0.0	0.0	9.6	22.8	23.6	25.8	13.4	2.7	2.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
BBB+	0.0	0.0	0.0	3.6	21.5	28.2	33.3	11.8	1.2	0.0	0.2	0.1	0.0	0.0	0.0	0.0
BBB	0.0	0.6	0.0	1.1	3.4	16.4	28.1	30.9	12.0	6.5	0.8	0.2	0.0	0.0	0.0	0.0
BBB–	0.0	0.0	0.0	0.0	3.0	4.9	22.9	31.4	18.7	13.2	5.2	0.2	0.5	0.1	0.0	0.0
BB+	0.0	0.0	0.1	0.0	0.3	1.6	2.6	18.1	40.6	23.1	8.7	3.3	0.3	0.0	0.0	1.3
BB	0.0	0.0	0.0	0.1	0.4	0.8	2.5	7.3	34.3	37.6	12.9	2.6	1.3	0.3	0.1	0.0
BB–	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.8	5.6	44.2	33.9	7.5	4.7	0.2	0.0	2.0
B+	0.0	0.0	0.0	0.0	0.0	0.0	1.4	0.0	1.2	13.4	35.1	35.5	5.8	3.1	0.1	4.2
B	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.2	8.2	16.4	31.7	28.6	5.7	1.4	4.8
B–	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.7	4.9	24.7	34.4	13.0	2.9	15.5
CCC	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.4	9.8	23.0	30.3	15.4	16.1
Corporate – overheat (%)																
AA	54.2	36.3	2.2	7.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AA–	11.5	37.0	25.7	13.8	4.9	7.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
A+	5.2	11.1	24.0	29.8	13.7	16.0	0.0	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
A	2.7	8.0	13.2	19.9	20.9	23.2	4.6	1.1	6.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
A–	0.0	0.0	8.2	12.5	19.3	29.9	22.1	3.9	2.6	0.5	0.9	0.0	0.0	0.0	0.0	0.0
BBB+	0.0	0.0	2.1	0.9	10.7	23.7	34.8	18.9	8.6	0.0	0.0	0.3	0.0	0.0	0.0	0.0
BBB	0.0	0.0	0.0	0.6	2.1	12.4	22.4	33.7	8.9	18.9	0.9	0.1	0.0	0.0	0.0	0.0
BBB–	0.0	0.0	0.0	0.5	0.6	1.6	19.5	29.8	27.4	10.1	7.8	1.1	0.3	0.1	0.0	1.2
BB+	0.0	0.0	0.1	0.0	0.0	1.0	2.9	18.4	35.3	24.3	7.6	4.4	0.1	0.1	0.3	5.5
BB	0.0	0.0	0.0	0.1	0.0	0.0	0.0	5.4	26.1	36.7	18.6	9.3	1.2	0.1	0.0	2.5

BB–	0.0	0.0	0.0	0.0	0.0	0.2	0.4	2.2	7.2	42.4	33.4	8.0	3.5	0.2	0.1	2.2
B+	0.0	0.0	0.0	0.0	0.0	0.0	0.6	0.0	9.7	11.2	25.4	35.6	10.2	3.0	0.2	4.2
B	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.5	1.4	2.1	11.8	33.6	31.7	5.7	1.5	8.6
B–	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.5	10.8	11.3	51.1	12.5	3.9	8.8
CCC	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.7	15.0	32.8	28.8	11.8	8.9
Corporate – hard landing (%)																
AA	52.7	36.7	4.6	5.3	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AA–	14.1	33.4	29.0	8.7	8.6	6.2	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
A+	7.4	15.3	26.8	32.6	14.4	3.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
A	4.0	34.6	11.2	12.1	20.5	7.3	3.6	1.6	4.9	0.1	0.0	0.0	0.0	0.0	0.0	0.0
A–	5.5	0.0	5.2	12.1	19.0	30.0	21.3	3.3	1.5	0.4	1.5	0.0	0.0	0.0	0.0	0.0
BBB+	0.0	0.0	0.0	4.6	11.0	25.3	40.3	15.7	2.8	0.0	0.4	0.0	0.0	0.0	0.0	0.0
BBB	0.0	1.6	0.0	1.2	4.0	11.0	20.3	33.8	13.5	14.1	0.5	0.0	0.0	0.0	0.0	0.0
BBB–	0.0	0.0	0.0	0.5	3.8	4.7	20.9	26.0	17.5	13.2	9.7	2.0	0.2	0.0	0.0	1.4
BB+	0.0	0.0	0.0	0.0	1.4	2.0	2.6	18.9	41.5	12.5	11.4	4.0	0.8	0.1	0.0	4.7
BB	0.0	0.0	0.0	0.0	0.0	0.2	1.9	6.8	21.2	36.1	20.8	5.2	2.5	0.4	0.0	4.9
BB–	0.0	0.0	0.0	0.0	0.0	0.0	0.4	3.3	10.2	23.6	38.8	5.7	7.6	0.7	0.2	9.4
B+	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8.1	7.9	18.3	38.7	7.6	5.9	0.3	13.3
B	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.2	3.9	11.7	17.6	35.0	4.9	2.2	19.5
B–	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.6	5.6	12.7	15.2	17.2	5.3	40.4
CCC	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	14.4	0.0	19.3	27.1	39.2
Corporate – soft landing (%)																
AA	61.6	17.7	8.3	6.3	3.4	0.6	1.5	0.4	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AA–	26.8	26.7	18.5	13.5	4.6	4.9	3.8	0.2	0.6	0.2	0.0	0.0	0.0	0.0	0.0	0.1
A+	10.9	19.0	25.8	19.7	10.8	7.5	4.7	0.5	0.7	0.2	0.1	0.0	0.0	0.0	0.0	0.0
A	6.0	10.3	19.7	21.8	16.6	12.1	7.4	4.3	1.8	0.1	0.0	0.0	0.0	0.0	0.0	0.0
A–	2.0	4.4	10.6	18.9	21.7	19.7	13.4	6.2	2.1	0.4	0.3	0.0	0.0	0.0	0.0	0.2
BBB+	1.0	1.6	4.2	9.8	17.4	25.9	21.8	11.2	4.7	1.4	0.5	0.2	0.0	0.0	0.0	0.3
BBB	0.4	0.5	1.2	3.6	6.9	18.1	29.4	21.5	13.5	3.3	1.0	0.2	0.0	0.0	0.0	0.4

(*continued*)

Table 4.33 (continued)

	AA	AA–	A+	A	A–	BBB+	BBB	BBB–	BB+	BB	BB–	B+	B	B–	CCC	Default
BBB–	0.0	0.2	0.4	0.7	1.9	7.4	21.9	30.7	24.7	7.8	2.9	0.6	0.2	0.1	0.0	0.5
BB+	0.0	0.0	0.1	0.2	0.5	2.0	6.8	18.8	39.3	21.0	7.7	2.0	0.6	0.1	0.1	0.9
BB	0.0	0.0	0.0	0.0	0.2	0.6	2.2	6.7	24.6	34.6	21.2	6.4	1.5	0.3	0.0	1.6
BB–	0.0	0.0	0.0	0.0	0.1	0.2	0.6	2.0	10.3	26.0	34.8	17.3	5.0	0.5	0.1	3.2
B+	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.9	4.5	11.2	28.0	32.5	14.0	2.7	0.1	5.7
B	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	2.5	5.5	14.5	28.8	30.1	6.9	1.8	8.9
B–	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.4	2.2	3.3	7.9	19.7	36.6	13.5	3.4	12.6
CCC	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.3	5.6	16.9	31.8	18.2	15.3	10.9

Table 4.34 SME corporate transition matrices

	BBB	BBB−	BB+	BB	BB−	B+	B	B−	CCC	Default
SME corporate – recovery (%)										
BBB	33.9	53.4	9.9	2.5	0.0	0.0	0.0	0.3	0.0	0.0
BBB−	14.0	55.3	19.2	8.1	3.1	0.1	0.3	0.0	0.0	0.0
BB+	0.3	14.4	50.4	22.9	6.8	3.6	0.4	0.0	0.0	1.2
BB	0.1	2.1	31.7	47.9	13.3	2.1	2.0	0.3	0.4	0.0
BB−	0.0	0.1	2.6	38.2	43.2	8.2	4.7	0.4	0.3	2.0
B+	0.0	0.0	0.4	7.4	32.2	46.2	6.4	3.4	0.7	3.2
B	0.0	0.0	1.1	4.7	12.0	35.4	31.4	8.9	2.2	4.3
B−	0.0	0.0	0.0	1.9	2.3	21.3	33.5	17.6	8.8	14.7
CCC	0.0	0.0	0.0	0.0	1.2	7.6	12.1	31.8	14.5	32.8
SME corporate – overheat (%)										
BBB	27.1	58.3	7.3	7.3	0.0	0.0	0.0	0.0	0.0	0.0
BBB−	11.4	50.2	26.9	5.9	4.4	0.5	0.2	0.0	0.0	0.4
BB+	0.3	14.7	44.0	24.2	5.9	4.8	0.1	0.2	0.4	5.2
BB	0.0	1.5	23.3	45.1	18.6	7.2	1.8	0.1	0.2	2.1
BB−	0.0	0.4	3.4	37.0	43.1	8.8	3.6	0.4	1.0	2.3
B+	0.0	0.0	3.1	6.3	23.9	47.5	11.4	3.4	1.1	3.3
B	0.0	0.0	0.5	1.2	8.5	36.9	34.3	8.7	2.3	7.6
B−	0.0	0.0	0.0	0.6	4.9	9.5	48.5	16.6	11.9	8.1
CCC	0.0	0.0	0.0	0.0	0.7	13.1	19.4	34.1	12.4	20.4
SME corporate – hard landing (%)										
BBB	24.4	58.4	11.1	5.4	0.0	0.0	0.0	0.7	0.0	0.0
BBB−	13.9	49.8	19.5	8.8	6.3	1.0	0.2	0.0	0.0	0.6
BB+	0.3	15.3	52.6	12.7	9.1	4.4	0.9	0.2	0.0	4.5
BB	0.1	1.9	19.2	45.1	21.1	4.1	3.7	0.4	0.3	4.2
BB−	0.0	0.5	4.7	20.2	48.8	6.1	7.6	1.4	1.2	9.5
B+	0.0	0.0	2.5	4.3	16.6	50.0	8.3	6.5	1.7	10.1
B	0.0	0.0	1.9	2.2	8.6	19.8	38.7	7.7	3.4	17.6
B−	0.0	0.0	0.0	1.4	2.4	10.1	13.7	21.7	15.2	35.5
CCC	0.0	0.0	0.0	0.0	0.0	8.2	0.0	14.9	18.6	58.3
SME corporate – soft landing (%)										
BBB	41.5	43.5	13.0	1.5	0.0	0.0	0.0	0.5	0.0	0.0
BBB−	13.3	54.0	25.5	4.8	1.7	0.3	0.1	0.0	0.1	0.2
BB+	0.7	15.7	51.3	21.9	6.3	2.3	0.7	0.2	0.1	0.9
BB	0.1	1.9	22.8	44.0	21.9	5.1	2.3	0.3	0.2	1.4
BB−	0.0	0.3	4.7	22.3	44.1	18.7	5.0	0.9	0.7	3.3
B+	0.0	0.0	1.4	6.3	25.9	42.7	15.5	3.1	0.8	4.4
B	0.1	0.0	0.9	3.1	10.5	31.7	32.6	10.5	2.8	7.9
B−	0.0	0.3	0.3	1.3	3.7	17.1	35.9	18.5	10.7	12.0
CCC	0.0	0.0	0.0	0.0	1.4	15.2	19.3	22.1	16.6	25.5
BB	0.0	1.5	23.3	45.1	18.6	7.2	1.8	0.1	0.2	2.1
BB−	0.0	0.4	3.4	37.0	43.1	8.8	3.6	0.4	1.0	2.3
B+	0.0	0.0	3.1	6.3	23.9	47.5	11.4	3.4	1.1	3.3

(*continued*)

Table 4.34 (continued)

	BBB	BBB–	BB+	BB	BB–	B+	B	B–	CCC	Default
B	0.0	0.0	0.5	1.2	8.5	36.9	34.3	8.7	2.3	7.6
B–	0.0	0.0	0.0	0.6	4.9	9.5	48.5	16.6	11.9	8.1
CCC	0.0	0.0	0.0	0.0	0.7	13.1	19.4	34.1	12.4	20.4
SME corporate – hard landing (%)										
BBB	24.4	58.4	11.1	5.4	0.0	0.0	0.0	0.7	0.0	0.0
BBB–	13.9	49.8	19.5	8.8	6.3	1.0	0.2	0.0	0.0	0.6
BB+	0.3	15.3	52.6	12.7	9.1	4.4	0.9	0.2	0.0	4.5
BB	0.1	1.9	19.2	45.1	21.1	4.1	3.7	0.4	0.3	4.2
BB–	0.0	0.5	4.7	20.2	48.8	6.1	7.6	1.4	1.2	9.5
B+	0.0	0.0	2.5	4.3	16.6	50.0	8.3	6.5	1.7	10.1
B	0.0	0.0	1.9	2.2	8.6	19.8	38.7	7.7	3.4	17.6
B–	0.0	0.0	0.0	1.4	2.4	10.1	13.7	21.7	15.2	35.5
CCC	0.0	0.0	0.0	0.0	0.0	8.2	0.0	14.9	18.6	58.3
SME corporate – soft landing (%)										
BBB	41.5	43.5	13.0	1.5	0.0	0.0	0.0	0.5	0.0	0.0
BBB–	13.3	54.0	25.5	4.8	1.7	0.3	0.1	0.0	0.1	0.2
BB+	0.7	15.7	51.3	21.9	6.3	2.3	0.7	0.2	0.1	0.9
BB	0.1	1.9	22.8	44.0	21.9	5.1	2.3	0.3	0.2	1.4
BB–	0.0	0.3	4.7	22.3	44.1	18.7	5.0	0.9	0.7	3.3
B+	0.0	0.0	1.4	6.3	25.9	42.7	15.5	3.1	0.8	4.4
B	0.1	0.0	0.9	3.1	10.5	31.7	32.6	10.5	2.8	7.9
B–	0.0	0.3	0.3	1.3	3.7	17.1	35.9	18.5	10.7	12.0
CCC	0.0	0.0	0.0	0.0	1.4	15.2	19.3	22.1	16.6	25.5

It is possible to select the three most relevant areas for analysis:

- validation of the rating model;
- validation of the rating process; and
- validation of the dedicated IT system.

This chapter selects and describes the main set of analyses and statistical tests to be performed in order to assess, the appropriate aspects of a rating model for each relevant risk component (PD, LGD and EAD):

- the model design;
- the estimation of the risk parameters; and
- the model's performance beyond the evaluation of the impact of company processes and the evaluation of the judgmental revisions of in relation to the performance of the statistical components of the rating models.

Table 4.35 SME retail transition matrices

	BBB	BBB–	BB+	BB	BB–	B+	B	B–	CCC	Default
SME retail recovery (%)										
BBB	33.9	53.4	9.9	2.5	0.0	0.0	0.0	0.3	0.0	0.0
BBB–	14.0	55.3	19.2	8.1	3.1	0.1	0.3	0.0	0.0	0.0
BB+	0.3	14.4	50.4	22.9	6.8	3.6	0.4	0.0	0.0	1.3
BB	0.1	2.1	31.7	47.9	13.3	2.1	2.0	0.3	0.4	0.0
BB–	0.0	0.1	2.6	38.2	43.2	8.2	4.7	0.4	0.3	2.2
B+	0.0	0.0	0.4	7.4	32.1	46.1	6.4	3.4	0.7	3.6
B	0.0	0.0	1.1	4.6	12.0	35.3	31.3	8.8	2.1	4.8
B–	0.0	0.0	0.0	1.9	2.2	20.9	32.9	17.3	8.7	16.1
CCC	0.0	0.0	0.0	0.0	1.1	7.3	11.6	30.4	13.8	35.7
SME retail – overheat (%)										
BBB	27.1	58.3	7.3	7.3	0.0	0.0	0.0	0.0	0.0	0.0
BBB–	11.4	50.2	26.9	5.9	4.4	0.5	0.2	0.0	0.0	0.5
BB+	0.3	14.7	43.8	24.1	5.9	4.8	0.1	0.2	0.4	5.7
BB	0.0	1.5	23.2	45.0	18.6	7.2	1.8	0.1	0.2	2.3
BB–	0.0	0.4	3.4	37.0	43.0	8.8	3.6	0.4	1.0	2.5
B+	0.0	0.0	3.1	6.3	23.8	47.3	11.4	3.4	1.1	3.6
B	0.0	0.0	0.5	1.2	8.4	36.6	34.0	8.7	2.3	8.3
B–	0.0	0.0	0.0	0.6	4.8	9.4	48.0	16.4	11.7	9.0
CCC	0.0	0.0	0.0	0.0	0.6	12.7	18.9	33.1	12.1	22.5
SME retail – hard landing (%)										
BBB	24.4	58.4	11.1	5.4	0.0	0.0	0.0	0.7	0.0	0.0
BBB–	13.9	49.8	19.5	8.8	6.3	1.0	0.2	0.0	0.0	0.6
BB+	0.3	15.3	52.3	12.6	9.0	4.4	0.9	0.2	0.0	5.0
BB	0.1	1.9	19.1	44.9	21.0	4.0	3.7	0.4	0.3	4.6
BB–	0.0	0.5	4.6	20.0	48.3	6.1	7.5	1.3	1.2	10.4
B+	0.0	0.0	2.5	4.3	16.5	49.5	8.2	6.5	1.7	11.0
B	0.0	0.0	1.8	2.2	8.5	19.4	38.0	7.5	3.4	19.2
B–	0.0	0.0	0.0	1.3	2.3	9.7	13.2	20.8	14.6	38.0
CCC	0.0	0.0	0.0	0.0	0.0	7.6	0.0	13.8	17.2	61.4
SME retail – soft landing										
BBB	41.5	43.5	13.0	1.5	0.0	0.0	0.0	0.5	0.0	0.0
BBB–	13.3	54.0	25.5	4.8	1.7	0.3	0.1	0.0	0.1	0.2
BB+	0.7	15.7	51.2	21.9	6.3	2.3	0.7	0.2	0.1	1.0
BB	0.1	1.9	22.7	44.0	21.9	5.1	2.3	0.3	0.2	1.6
BB–	0.0	0.3	4.7	22.2	44.0	18.6	5.0	0.9	0.7	3.6
B+	0.0	0.0	1.4	6.2	25.8	42.5	15.4	3.0	0.8	4.9
B	0.1	0.0	0.9	3.1	10.4	31.4	32.3	10.4	2.8	8.7
B–	0.0	0.3	0.3	1.3	3.6	16.9	35.4	18.2	10.6	13.2
CCC	0.0	0.0	0.0	0.0	1.3	14.7	18.7	21.3	16.0	28.0

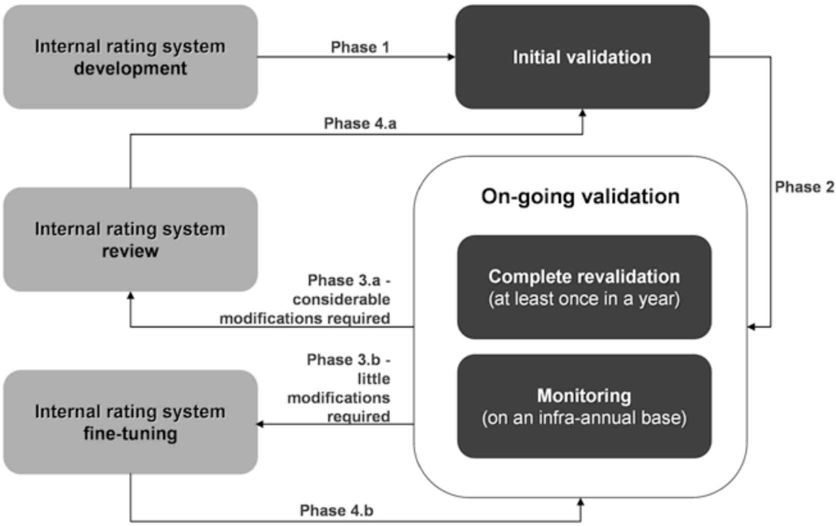

Fig. 4.15 Rating system life-cycle

4.6 Validation of the PD Model

As we can infer from Fig. 4.16 and Fig. 4.17, the validation of a PD model requires the use of both qualitative and quantitative analyses.

The main relevant areas of a PD qualitative validation are:

- the model's design (model type, model architecture, default definition);
- the rating process (attribution of the rating, IT requirements of the rating system); and
- the use test (relevance of the rating information across the credit/reporting processes).

Conversely, a quantitative validation analysis focuses on:

- the model's discriminatory power; that is, the ability of the rating model to discriminate ex ante between defaulting and non-defaulting borrowers (rank ordering and separation tests);
- the stability of the model and representativeness of the development samples over time; and

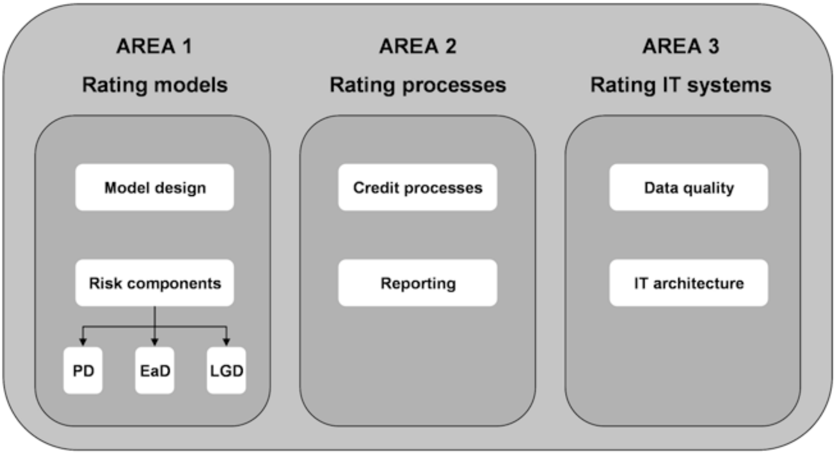

Fig. 4.16 Rating system validation: areas of analysis

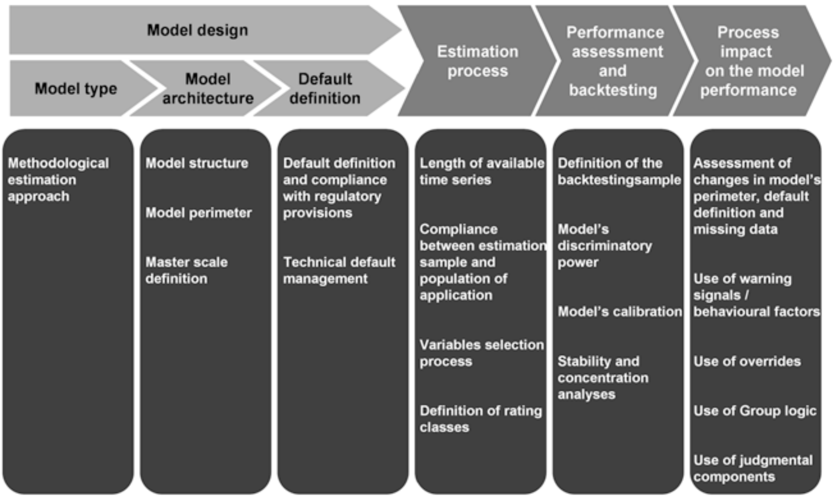

Fig. 4.17 PD model validation: areas of assessment

- the model's adequacy in associating a PD with each rating grade, which gives a quantitative assessment of the likelihood that graded obligors will default (concentration and calibration tests).

The following sections summarize the main analysis to be performed in the PD validation.

4.6.1 PD Model Design Validation

Model design validation is essentially about investigating the methodological approach selected to assess the credit risk profile of obligors assigned to the portfolio under consideration, the rationales supporting the choice, underlying architectural features and the definition of default addressed in the model.

Table 4.18 presents a possible checklist of analyses related to the area of model design validation, grouped by the three dimensions listed in Fig. 4.17: model type, model architecture and default definition.

4.6.2 PD Estimation Process Validation

Table 4.36 illustrates a list of analyses that should be executed during the estimation process validation.

For the dynamic properties of a rating system, refer to: Bangia et al. (2002), Lando and Skodeberg (2002), Bardos (2003) and Basel Committee on Banking Supervision (2005b). For the purposes of estimating risk parameters, banks may elect not to classify so-called "technical defaults" as defaulted – that is, positions that do not reflect a state of financial difficulty on the part of the obligor, such as to generate losses – so long as this is consistent with reference to the various risk parameters (see Bank of Italy 2006) (Table 4.37).

Table 4.36 Model design validation analyses: PD parameter

Dimension	Topic	Main analyses
Model type	Methodological estimation approach	Description of the selected methodological approach
		Assessment of reasons behind the choice: strengths versus weaknesses
		Validation of the comparative analysis carried out during the development stage, to consider possible alternative approaches
		Check the model's adequacy in respect of the portfolio of application
Model architecture	Model structure	Assessment of model's structure: modular versus integrated
		Assessment of number of models: single model versus multiple sub-models
		Rating philosophy point-in-time (PIT) versus through-the-cycle (TTC)
		Definition of relevant input data sources
		Assessment of the adequacy of the model's input variables to predict the borrower's default risk, irrespective of the specific nature of any underlying transaction
		Assessment of the model's functional requisites: updating frequency, validity of rating for operating purposes, computational rules, minimum information requirements, customer/exposure unique rating value
		Assessment of model's main assumptions
	Model perimeter	Definition of relevant variables for model's perimeter scoping
		Definition of relevant segmentation variables for definition of sub-models
		Definitions of exclusions
		Development samples' definition and reconciliation
		Definition and IT acquisition of the model's population (last available date)
		Definition and IT acquisition of a backtesting sample
		Compliance check: IT segmentation rules versus model's development perimeter versus commercial segmentation versus regulatory exposure classes
		Assessment of new clients and management of start-up enterprises
		Assessment of management of group connections

(*continued*)

Table 4.36 (continued)

Dimension	Topic	Main analyses
	Master scale definition	Definition of adopted master scale
		Assessment of the presence of at least 7 grades for non-defaulted obligors and 1 for defaulted obligors
		Analysis of distribution of rates obligors among various rating classes, in terms of both position and exposure
		Assessment of the absence of excessive concentrations within a single rating grade Assessment of empirical evidence supporting high concentration within a single rating grade
Default definition	Default definition and compliance with regulatory provisions	Regulatory compliance of adopted default definition
		Default assessment for temporal generations
		Duplication check
		Transition status assessment
		Analysis of positions management (exceptions/discriminating events)
		Default contagion (intergroup versus intercompany)
	Technical default management	Assessment of technical default definition
		Validation of selected identification criteria
		Assessment of technical default exclusion from the development sample

Table 4.37 Estimation process validation analyses: PD parameter

Topic	Main analyses
Length of available time series	Verify that PD estimates are not based solely on judgmental considerations, but rely consistently on the long-run default experience and on empirical evidence Verify that PD estimates are based on updated, relevant and representative data of the portfolio under analysis Verify the compliance of the development sample's observation period with regulatory provisions
Compliance between estimation sample and population of application	Assess the presence of a fair number of exposures in the development sample Assess the representativeness over time of the development samples with respect to the bank's most recent portfolio of application (distribution of portfolio and sample by segmentation variables: macro-geographical area, macro-industrial sector, turnover, and so on)
Variables selection process	Definition of explanatory variables' long list(s) Analysis of the economical relevance of long lists' variables with respect to the event of default (coherence of information value's sign) Description of variables' selection process and criteria (univariate versus multivariate analysis, cluster and correlation analyses, regression analysis and so on) Missing values, outliers and exceptions management Assessment of the degree of correlation among selected explanatory variables Assessment of model's output replicability PIT versus TTC adjustment
Definition of rating classes	Definition of internal rating master scale Assignment of obligors to internal rating grades (calibration) Distributive analysis

4.7 PD Performance Assessment and Backtesting

The performance assessment and backtesting consists in analyses such as those listed in Table 4.38.

4.7.1 Process Impact on the PD Model's Performance

Finally, regarding the process impact on the performance of the statistical model, Table 4.39 offers a possible analysis checklist. The quantitative

Table 4.38 Performance assessment and backtesting: PD parameter

Topic	Main analyses
Definition of the backtesting sample	Definition of a backtesting sample univariate analysis on model's short list(s) Assessment of short lists' variables distribution Analysis of default distribution along the sample Comparison with model's portfolio
Model's discriminatory power	Descriptive statistics (in bonis versus defaults average PD/score and variance) Graphical assessment of cumulative accuracy profile (CAP) and receiver operating characteristic (ROC) curves Calculation of accuracy ratio (AR) and area under the ROC curve (AUROC) at univariate, multivariate and sub-segment levels Calculation of corrected Gini coefficient (denoted as Gini[a] in the following) Calculation of contingency tables: false alarm rate (FAR), hit rate (HR) and misclassification rate (MR) Calculation of Kolmogorov–Smirnov distance (KS) Calculation of Pietra index
Calculation of conditional information entropy ratio (CIER)	Calculation of information value Calculation of mean difference Calculation of divergence statistic Calculation of Brier score Calculation of other discriminatory power indicators Comparison with model's performance at development stage
Model calibration	Descriptive statistics (in bonis versus default distributions) Graphical assessment of realized default rates compliance with estimated PD confidence interval for each rating grade Graphical assessment of cumulative default curve Chi-square test (Hosmer–Lemeshow, HSLS) Binomial test (with and without asset correlation) Traffic light test Calculation of other calibration measures Comparison with model's performances at development stage
Stability and concentration analyses	Analysis of obligors' distribution by rating grades Portfolio's composition by stratification variables Calculation of the population stability index (PSI) at univariate, multivariate and sub-segment levels Herfindahl–Hirschman index test Transition matrices assessment: persistence rate (PR), migration rate within 1 notch (M1C), migration rate within 2 notches (M2C), rating reversal analysis (RR) Calculation of other stability/concentration measures Comparison with model's performances at development stage

[a]See Brier (1950)

valuation anlysis of PD estimation models are finalized to evaluate, on a ongoing basis:

- the ability of a model to discriminate the in bonis positions from the future defaults (ordering and separation tests);
- its adequacy in representing the correct risk profile of the reference portfolio (calibration); and
- the model's stability and the development samples' representativeness with respect to the current portfolio.

Table 4.39 Process impact on the model's performance: PD parameter

Topic	Main analyses
Assessment of changes in model's perimeter, default definition and missing data	Assessment of changes in model's perimeter during the implementation stage, with respect to the development stage Alignment of default definition adopted during model's implementation with that used for development purposes Assessment of potential impact of missing data on model's performance
Use of warning signals/ behavioral factors	Assessment of the presence of internal processes that may have a direct influence on the rating score Impact on model's performance of irregular positions (so-called "administrative positions")
Use of overrides	Assessment of changes in overrides policy from model's development to implementation phase Allowed overrides typologies Frequency and size of overrides Information gain through overrides Impact of overrides' powers on model's performance
Use of group logic	Use of group mapping for rating purposes Assessment of changes in group logic from model's development to implementation phase Group logic and overrides relationship Frequency and size of changes on rating because of group logic Impact of group logic on model's performance
Use of judgmental components	Use of judgmental components for rating purposes Assessment of changes in judgmental components from model's development to implementation phase Judgmental components and overrides relationship Impact of judgmental components on model's performance

Next, we offer a brief description of the most common default probability validation tests on portfolio segments characterized by an enough number of defaults.

4.7.2 PD Discriminatory Power Tests

The accuracy ratio (AR) or Gini coefficient is the most common rank ordering power test: it measures the model's ability to order a sample/population according to its level of risk.

The indicator assumes values between 0 and 1: the higher the AR, the greater the model's discriminant power. A model that does not discriminate at all has a null AR, while the perfectly discriminating model is characterized by an AR (in absolute value) equal to 1. The Lorenz curve or cumulative accuracy profile (CAP) is the graphical analysis tool with which to evaluate the efficacy of a model's ordering power.

The x-axis in Figure 4.21 shows the counterparts subject to evaluation rates from more to less risky according to the model's score; the y-axis identifies the cumulative percentage of the insolvencies.

From this, we can obtain the CAP curve corresponding to the analyzed model; this is compared graphically with the curve of the perfect model and of the random model. The curve of the perfect model is obtained by assuming a model capable of assigning the worst possible scores to future insolvents; the random model – represented by the diagonal – corresponds to a model with no discriminant ability that uniformly distributes both in bonis and defaulted customers.

A "real" model falls unavoidably between the two curves: the better its discriminant ability, the closer its CAP curve will be to that of the perfect model.

The receiver operating curve (ROC) is a graphical representation of the "false alarm rate" (FAR) and "hit rate" (HR); this is obtained by letting the separation of solvent and future insolvent customers' cut-off "C" vary from 0 to 1. The false alarm rate identifies the frequency of effectively solvent subjects that have been incorrectly classified as in default; the hit rate identifies the percentage of correct classification of future insolvents (see Fig. 4.18).

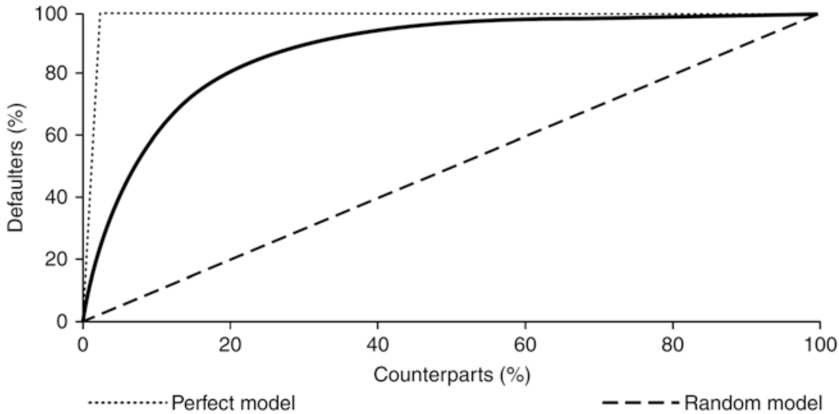

Fig. 4.18 Cumulative accuracy profile: an illustrative example

The information contained in the ROC can be synthesized in the measure denoted as the area under the receiver operating curve (AUROC). The AUROC assumes a value of 0.5, corresponding to a random model with no discriminatory capabilities, and 1 in the event of a perfect model: the higher the value, the better the model.

The AUROC and the AR parameters are linked by the relation: AR = 2 AUROC - 1

The corrected Gini coefficient (Gini*) is defined as: Gini* = AR · (1 – DR) where DR represents the sample default rate.

In Table 4.40, the contingency tables synthesize, within the four possible quadrants illustrated, the information relative to the:

- percentage of counterparties correctly foreseen in bonis by the model (Specificity);
- percentage of bad counterparties incorrectly foreseen in bonis (Type I error);
- percentage of good counterparties incorrectly foreseen in default (Type II error or FAR); and
- percentage of bad counterparties correctly classified (Sensitivity or HR).

Table 4.40 Contingency table: an illustrative example

Forecast status (%)			
Actual status	Good	Bad	
Good	80	20	Type II error (%): 20
Bad	30	70	Type I error (%): 30

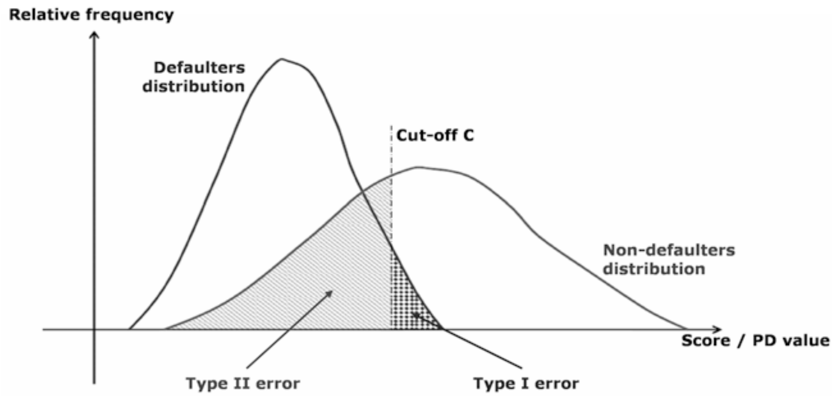

Fig. 4.19 Score distribution of good and bad positions of the sample

As shown in Fig. 4.20, the number of errors of the first and second type depend strongly on the cut-off value (C), settled as a separator of future default (counterparties characterized by a score value equal or less than C) from the futures in bonis (score value greater than the cut-off value).

In general, an error of the first type generates a loss corresponding to the capital and the interest lost due to the insolvency of a counterparty having been incorrectly classified as "healthy" and, hence, approved.

An error of the second type, conversely, produces a more limited loss (at least, in the corporate segment), originating from lost earnings in terms of fees and interest margin due to the incorrect classification of the healthy customer as a future insolvent. Once the cut-off has been defined, the following indicators are determined:

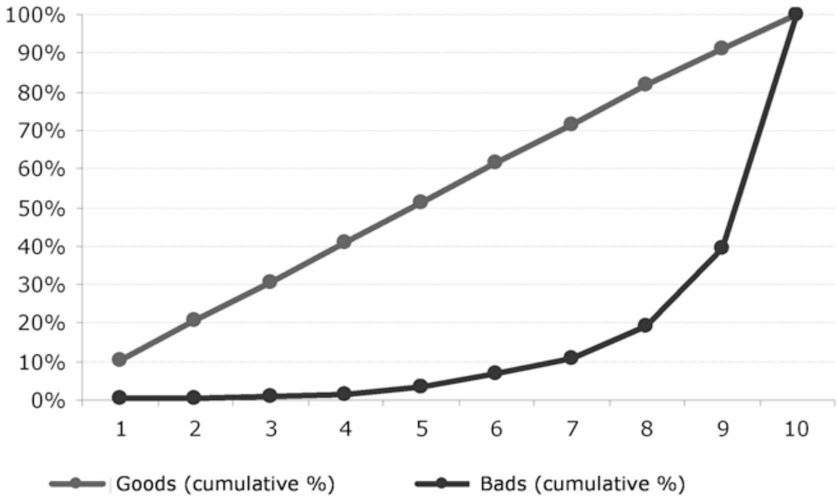

Fig. 4.20 The cumulative distribution of bads and goods per score decile: an illustrative example

- the misclassification rate (MR) – the percentage of counterparties wrongly classified (good as future default; bad as future solvent) over the whole sample positions set; and
- the hit rate (HR) – the percentage of correct classifications of bads over the total of the defaulted positions.

Table 4.41 shows the two rates of correct (HR) and incorrect (MR) classification, coherent with the illustrative contingency table proposed in Table 4.40.

The Kolmogorov–Smirnov distance (KS) evaluates the degree of separation between the solvent and defaulted positions, measuring the maximum vertical distance (in absolute values) between the empirical cumulative distributions of goods and bads. The variation in its values is the [0; 1] interval: the greater the index, the better the model's separation ability.

On the basis of the KS computation, Figure 4.20 illustrates the cumulative distribution of goods and bads in the same sample; Fig. 4.21 compares the trends of the KS test on two different samples: development and validation.

Table 4.41 Hit rate and misclassification rate: an illustrative example

Test	Value (%)
Hit rate	70
Misclassification	25

For further insights into discriminant power tests, see Brier (1950), Bamber (1975), Lee (1999), Engelmann et al. (2003), Sobehart and Keenan (2004) and Basel Committee (2005b).

4.7.3 PD Calibration Tests

The aim of calibration analysis is to evaluate the accuracy of the estimated (and calibrated) PDs with respect to the default rates effectively observed per rating class. Such analysis has particular importance: a rating system that underestimates the probability of insolvency of one or more credit portfolio segments requires careful monitoring (and, in some cases, a deep revision), because the estimation of capital requirements could be not aligned with the risks effectively assumed by the bank. (Fig. 4.23)

Before beginning the calibration test, a series of descriptive analyses (both graphical and tabular) must be conducted to represent and compare by quantiles and rating classes:

- the distributions, joint and separate, of the bads and goods of the estimation and validation samples; and
- the trend and the level of the observed default rate, with respect to the PD forecast by the model.

Tables 4.42 and 4.43, and Figs. 4.21, 4.23 and 4.24 give some examples.

Generally, three types of tests are used to check the adequacy of the model to represent the correct risk profile of the reference portfolio, :

- binomial (with and without asset correlation);
- Hosmer–Lemeshow χ^2 (chi-square); and
- the traffic lights approach.

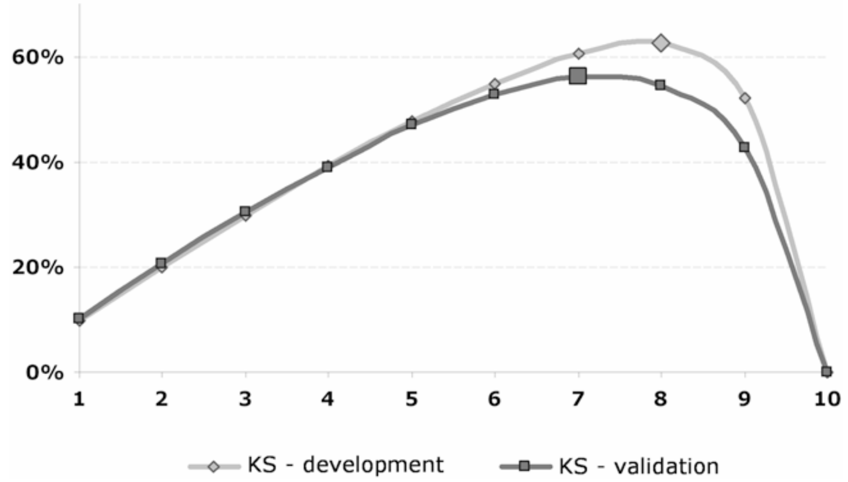

Fig. 4.21 The Kolmogorov–Smirnov statistic per score decile: an illustrative example

Table 4.42 The Kolmogorov–Smirnov statistic per score decile: an illustrative example

Decile development sample (%)				Validation sample (%)		
Percentage of bad		Default rate		Percentage of bad		Default rate
1	0.2	4.0	0.06	0.4	7.4	0.20
2	0.4		0.11	0.7		0.33
3	0.5		0.15	1.0		0.47
4	0.9		0.29	2.1		1.00
5	2.0		0.61	3.1		1.47
6	3.1	96.0	0.97	5.0	92.6	2.33
7	4.7		1.44	7.7		3.60
8	8.4		2.58	12.7		5.93
9	18.9		5.82	24.3		11.33
10	61.0		18.81	42.9		20.00
Total	100.0	100.0	3.08	100.0	100.0	4.67

The binomial test is based on a comparison, for every rating class, of the default rate observed values with the estimated PD. It is a "conservative", unidirectional test applied to single classes and – in its original formulation – based on the default independence within the risk classes.

Table 4.43 An illustrative example of risk and distribution per rating class: validation sample

Rating class	Total	Good	Bad	Default rate (%)	PD (%)
1	4819	4816	3	0.06	0.03
2	11,245	11,210	35	0.31	0.12
3	19,277	19,170	107	0.56	0.45
4	28,916	28,612	304	1.05	1.24
5	40,161	39,400	761	1.89	2.01
6	53,012	50,800	2212	4.17	3.87
7	24,096	22,000	2096	8.70	7.49
8	11,245	9500	1745	15.52	15.08
9	4819	3620	1199	24.89	23.22
10	2410	1540	870	36.09	40.17
Total	200,000	190,668	9332	4.67	

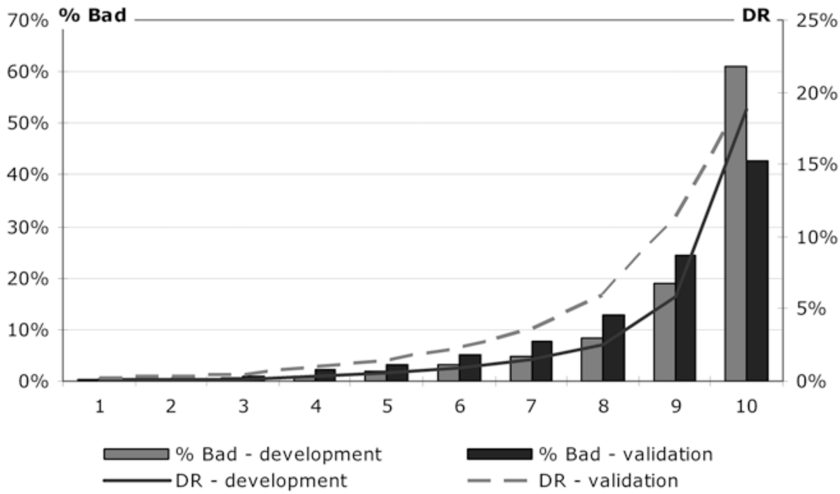

Fig. 4.22 An illustrative example of the percentage distribution of bad and default rates per score decile: development versus validation sample

For a given level of confidence, the null hypothesis (H_0) underlying the test is: "the PD estimated for single rating class is correct"; and the alternative hypothesis (H_1) is: "the PD is underestimated". As outlined in Basel Committee on Banking Supervision (2005b), the default independence hypothesis is not adequately confirmed by the empirical evidence.

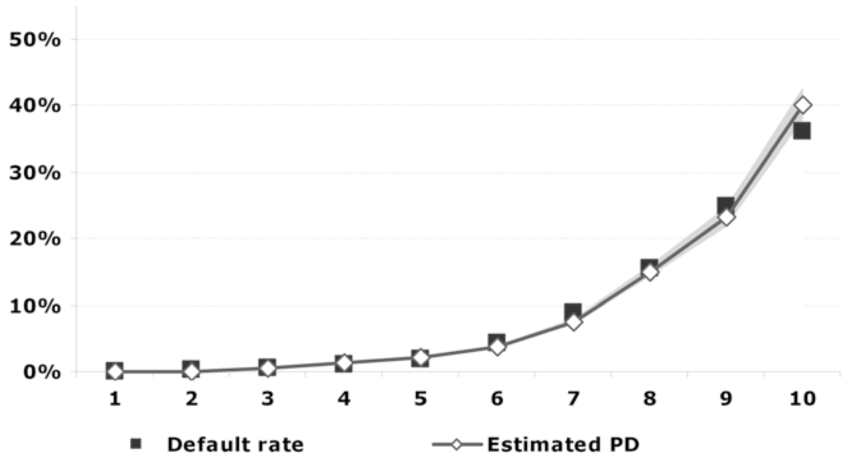

Fig. 4.23 An illustrative example of a comparison between default rate and PD per rating class

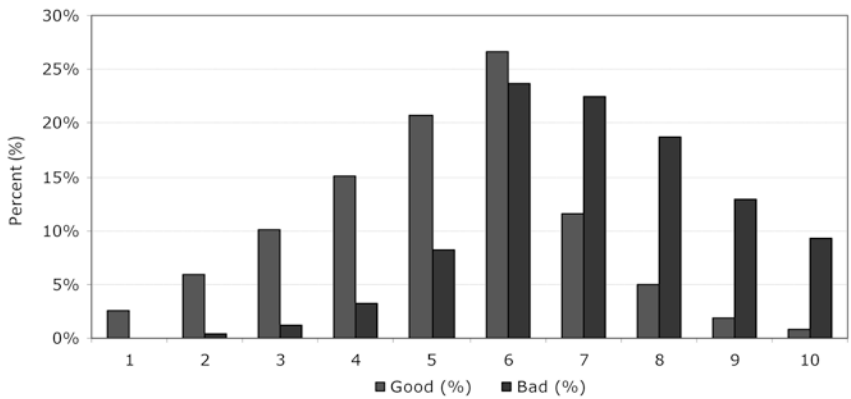

Fig. 4.24 An illustrative example of the percentage distribution of bads and goods per rating class: validation sample binomial test usually includes in its workings the regular asset correlation with respect to different levels of confidence

For this reason, the Hosmer–Lemeshow χ^2 (chi-square) test consists of overriding one of the binomial test limits: the verification of the model's capacity at a single class level separated from the synthetic indication of the whole model calibration. The Hosmer–Lemeshow test applied to the whole portfolio presumes a default independence within and among the rating classes.

Setting a determined level of confidence, the test verifies the alignment between the estimated PDs and the number of observed defaults in the classes: a null hypothesis rejection can imply, therefore, both an underestimation, and an overestimation of the effective number of defaults. Finally, the traffic lights approach – applied to single rating classes – is a parametric test of a conservative type. Setting a determined level of confidence, it is possible to identify two thresholds – lower (PD^{inf}) and upper (PD^{sup}) for each rating class (i = 1, ... , 10).

If the default rate observed in the class i (DR_i) is lower than PD^{inf}, the test outcome is "green for go" (overestimation of the effective insolvency rate); if it is "red for stop" (underestimation) a re-calibration action is needed; otherwise the outcome is "yellow" (coherent estimation).

For further insights on calibration tests, see Blochwitz et al. (2003), Tasche et al. (2003) and Basel Committee on Banking Supervision (2005b).

4.7.3.1 PD Stability Tests

Stability analysis checks the alignment over time between the distributions of the development and validation samples, in order to identify possible differences that could originate future possible model instabilities.

Internal stability is evaluated by means of (i) the computation of the population stability index, and (ii) the transition matrix analysis.

The population stability index (PSI), is a synthetic indicator used to measure the representativity of the estimation sample with respect to the current portfolio, and for the stability of a single indicator or of the entire model, respectively, for bands of assumed values or for rating classes.

Once the variable subject to examination (e.g. the rating class), its possible modality (the 10 classes effectively evaluated) and the percentage distribution of the variable (with respect to the rating classes) of the estimation and validation samples have been identified, it is possible to define the PSI as follows:

$$PSI = \sum_{i=1}^{k} \left(P_i - C_i\right) \cdot \log\left(\frac{P_i}{C_i}\right)$$

where k is the number of modalities subject to analysis (in this example, the 10 evaluated classes), $P_i(i = 1, \ldots, k)$ denotes the percentage of the validation sample assigned to the class i, while $C_i(i = 1, \ldots, k)$, the percentage of the estimation sample.

The indicator defined in this way assumes a value of between zero and $+\infty$: the small values of PSI are expressions of a good level of stability/representativeness of the sample used for the model estimation; high values are a symptom of instability.

Transition matrices allow us to examine the evolution of the portfolio over time, highlighting possible variations in the positions of the different rating classes, both upgrading and downgrading.

The population stability degree is evaluated through the calculation of the permanence rate in the same class (persistence rate, or PR), the migration rates within one or two classes (migration rates M1C or M2C) with respect to the rating assigned initially and at the rating reversal analysis.

Table 4.44 shows figures and percentages of the class changes of opposite signs, inferred by the observation of the rating assigned across a consecutive three-year horizon, confirming the stability over time of the PD model adopted for illustrative purposes.

Table 4.44 An illustrative example of rating reversal analysis over three consecutive years

Type of rating reversal	Number	Percentages	
Reverse	1,491	12.4	
downgrade – upgrade	774	6.5	12.4
upgrade – downgrade	717	6.0	
Stable	10,509	87.6	
upgrade – stable	1,516	12.6	
stable – upgrade	937	7.8	24.3
upgrade – upgrade	463	3.9	
stable – stable	3,499	29.2	29.2
downgrade – stable	1,332	11.1	
stable – downgrade	1,559	13.0	34.1
downgrade – downgrade	1,203	10.0	
Total	12,000	100.0	