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Dufour efect on unsteady MHD OPEN fow past a vertical plate embedded in porous medium with ramped temperature

Subhrajit Sarma* **& NazibuddinAhmed**

The present investigation aims to fnd an exact solution to the problem of a free convective, viscous, radiating, chemically reacting, optically thick, non-gray, and incompressible MHD fow past an exponentially accelerated semi-infnite vertical plate in presence of a transverse magnetic feld. The medium of fow is porous. Arbitrary ramped temperature and difusion thermo efects are also considered. Rosseland approximation method is used to describe the fux that appears in the energy equation. The efects of diferent parameters on fow and transport characteristics are discussed with the help of suitable graphs. It is noticed that velocity feld and concentration feld decreases but temperature feld increases with an upsurge in Schmidt number. Also, Nusselt number and skin friction rise with increasing chemical reaction parameter but lowers with increasing radiation parameter. Faster consumption of chemical substances decelerates both concentration and velocity but accelerates temperature of the fuid. An interesting outcome outcome of our investigation is that both Dufour efect and arbitrary ramped temperature diminishes fuid velocity.

List of symbols

- $\frac{a}{B}$ Surface acceleration parameter

→ Magnetic flux density
-
- B_0 Strength of the applied magnetic field $\left(\frac{\text{Weber}}{\text{m}^2}\right)$
- C Molar species concentration $\left(\frac{\text{mol}}{\text{m}^3}\right)$ \setminus
- C_p Specific heat at constant pressure $\left(\frac{J}{\text{kgK}}\right)$
- C_s Concentration susceptibility
- C_{∞} Concentration far away from the plate $\left(\frac{\text{mol}}{\text{m}^3}\right)$
- C_w Concentration at the plate $\left(\frac{\text{mol}}{\text{m}^3}\right)$
- D_M Mass diffusivity $\left(\frac{m^2}{s}\right)$
-
- $\frac{Du}{g}$ Dufour number
 $\frac{v}{g}$ Gravitation acceleration vector
- *g* Gravitational acceleration $\left(\frac{m}{s^2}\right)$
- Gr Thermal Grashof number
- Gm Solutal Grashof number
 K_T Thermal diffusion ratio
- K_T Thermal diffusion ratio
 $K*$ Porosity parameter
-
- \overrightarrow{f} Current density vector $m²$ \setminus
- \overline{K} Chemical reaction rate $\left(\frac{\text{mol}}{\text{m}^2\text{s}}\right)$ \setminus
- K Chemical reaction parameter
 M Magnetic parameter
- M Magnetic parameter
N Radiation parameter
- Radiation parameter
- *p* Pressure $\left(\frac{N}{m^2}\right)$

Department of Mathematics, Gauhati University, Guwahati, Assam 781014, India. [⊠]email: sarmasj021@gmail.com

-
- Pr Prandtl number
 $\frac{\overrightarrow{q}}{\overrightarrow{q_r}}$ Fluid velocity vector

Radiation heat flux vector
-
- q_r Radiation heat flux $\left(\frac{W}{m^2}\right)$
- Sc Schmidt number
- t' ₀ Time (s)
- t_0 Critical time for rampedness (s)
 t_1 Non-dimensional critical time
- t_1 Non- dimensional critical time for rampedness T Fluid temperature (K)
- T_w Fluid temperature (K)
 T_w Temperature at the pla
- T_w Temperature at the plate (K)
 T_{∞} Undisturbed temperature (K
- Undisturbed temperature (K)
- u' X-component of fluid velocity $\left(\frac{m}{s}\right)$
- U_0 Plate velocity $\left(\frac{m}{s}\right)$

Greek symbols

- μ Coefficient of viscosity $\left(\frac{kg}{ms}\right)$
- *σ* Electrical conductivity $\left(\frac{S}{m}\right)$
- σ∗ Stefan-Boltzmann constant $\left(\frac{W}{m^2K^4}\right)$
- ρ Fluid density $\left(\frac{\text{kg}}{\text{m}^3}\right)$
- ρ_{∞} Fluid density far away from the plate $\left(\frac{\text{kg}}{\text{m}^3}\right)$
- κ Thermal conductivity $\left(\frac{W}{mK}\right)$
- κ∗ Mean absorption constant $\left(\frac{1}{m}\right)$
- β Volumetric coefficient of thermal expansion $(\frac{1}{\beta})$
- $\overline{\beta}$ Volumetric coefficient of solutal expansion $\left(\frac{P}{K_{\text{mol}}}\right)$
- *ν* Kinematic viscosity $\left(\frac{m^2}{s}\right)$

Subscripts

- w Refers to physical quantity at the plate
- ∞ Refers to physical quantity far away from the plate

The branch of physics that deals with the interaction of the magnetic field with electrically conducting fluid are termed Magnetohydrodynamics (MHD). Saltwater, liquid metals, plasmas, electrolytes are some common examples of such fluids. Noted Swiss scientist Hannes Alfven¹ initiated the field of MHD for which he received the Noble prize in physics in the year 1970. But, due to substantial contributions from other authors like Cowling², Shercliff³, Ferraro and Plumpton^{[4](#page-29-3)}, Roberts⁵, Crammer and Pai⁶, MHD is at present form. There are several applications of MHD in modern technologies. Geophysical and astrophysical applications of MHD are nicely elaborated by Dormy and Nunez⁷. Dynamo, motor, fusion reactors, dispersion of metals, metallurgy, etc. are some engineering applications of MHD. Aeronautical applications of MHD were studied exclusively by Li et al. 8 . Farrokhi et al.⁹ studied biomedical applications of MHD. Rana et al.¹⁰ investigated how microbes swim in blood flow of nano- bioconvective Williamson fluid.

Change in fuid temperature and species concentration generates density variation in the fuid mixture. Tis variation develops buoyancy forces that act on the fluid. The flow produced due to the buoyancy force is termed free convection or natural convection. Manh et al.¹¹, Das and Ahmed^{[12](#page-29-11)}, Kafoussias¹³, Kumar and Singh^{[14](#page-30-1)}, etc. studied the efect of free convection on various MHD problems.

The porous medium contains holes or voids that are filled with solid particles which let the fluid pass through it. The mechanism of porous flow finds its applications in inkjet printing, nuclear waste disposal, electro-chem-istry, combustion technology, etc. Dwivedi et al.^{[15](#page-30-2)} studied MHD flow through the vertical channel in a porous medium while Raju et al[.16](#page-30-3) observed the MHD fow through horizontal channel taking viscous dissipation and Joule heating into account. Free convection in the porous media was investigated by Helmy¹⁷, Raju and Varma^{[18](#page-30-5)}, Pattnaik and Biswal¹⁹, Sinha et al.²⁰, Basha and Nagarathna^{[21](#page-30-8)}.

Radiation is a form of heat transfer by electromagnetic waves. Many environmental and industrial procedures encounters with radiative convective fows. Flows of this kind take crucial role in space technology and high temperature activities. This influence many authors to perform model research on free convection with thermal radiation in several hydrodynamic and magnetohydrodynamic problems under various physical and geometrical conditions. Mbeldogu et al.^{[22](#page-30-9)}, Makinde²³, Samad and Rahman^{[24](#page-30-11)}, Orhan and Ahmet^{[25](#page-30-12)}, Prasad et al.^{[26](#page-30-13)}, Ahmed and Dutta²⁷, Takhar et al.²⁸, Seth et al.²⁹, Balla and Naikoti³⁰, Siviah et al.^{[31](#page-30-18)} are some worth mentioning researchers in this area.

The effect of chemical reaction carries a great practical significance in heat and mass transfer problems. So, many researchers studied applications of chemical reaction in different MHD flow problems. Apelblat^{[32](#page-30-19)} investigated chemical reaction effect in a mass transfer problem with axial diffusion. Mahapatra et al.³³ examined the efects of chemical reaction in a free convective fow in a porous media surrounded by a vertical surface. Andersson et al.³⁴ and Takhar et al.^{[35](#page-30-22)} considered the diffusion of a chemically reactive species from a stretching sheet while Ganesan and Rani³⁶ studied the diffusion of chemically reactive species through a vertical cylinder. Muthucumaraswamy and Ganesan³⁷, Kandasamy et al.^{[38](#page-30-25)}, Raptis and Perdikis³⁹, etc. investigated the effects of

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chemical reaction in various MHD problems. Arifuzzaman et al.⁴⁰ studied chemically reactive and naturally convective high speed MHD flow through an oscillating vertical porous plate.

If two non-reacting and chemically diferent fuids are allowed to difuse into each other at the same temperature, the system produces a heat fux. Efect of fux due to composition gradient is defned as Dufour efect or difusion thermo efect. Renowned Swiss scientist L. Dufour discovered this efect in 1873. Tis efect is nicely elaborated by Eckert and Drake^{[41](#page-30-28)}. Swetha et al.⁴² analyzed Dufour and radiation effects on a free convective flow in a porous medium. Reddy et al.⁴³ studied both Soret and Dufour effects of an MHD flow past a moving vertical plate immersed in a porous medium taking Hall current and rotating system into account. Oyekunle and Agunbiade^{[44](#page-30-31)} explored the consequences of the Dufour and Soret effect of MHD flow on an inclined magnetic feld. Kumaresan et al.[45](#page-30-32) analytically investigated the Dufour efect on unsteady free convective fow past an accelerated vertical plate. Vijaya Kumar et al.[46](#page-30-33) studied Dufour and radiation efects on a free convective MHD flow past an infinite vertical plate in presence of chemical reaction. Shateyi et al.^{[47](#page-30-34)} studied the effects of Soret, Dufour, Hall current and radiation of a mixed convective flow in a porous medium. Postelnicu⁴⁸ examined the consequences of both Soret and Dufour efects on a vertical surface embedded in a porous medium.

The present investigation aims to analyze the role of the diffusion thermo effect in a free convective, radiative, and chemically reacting fuid in a porous medium with arbitrary ramped temperature. Reviewing the existing literature, we found that no work has been done taking Dufour efect and ramped temperature with arbitrary characteristic time simultaneously in a flow past an exponentially started vertical plate. The governing equations are frst converted to non-dimensional partial diferential equations using some dimensionless quantities. A closed-form of the Laplace transform technique is adopted to solve the equations. Efects of diferent fow parameters like Prandtl number, Schmidt number, magnetic parameter, thermal Grashof number, solutal Grashof number, Dufour number, chemical reaction parameter, radiation parameter, porosity parameter, etc. on temperature feld, concentration feld, velocity feld, Nusselt number, Sherwood number, and skin friction are discussed graphically. The obtained results are also verified with previously published work. It is hoped that the present paper will be useful in designing cooling systems, fow meters, MHD generators, etc. In the feld of life science, this investigation can be helpful in magnetic drug treatment, devices for cell separation, magnetic endoscopy etc. This paper will also help scientists and researchers in the field of heat and mass transfer.

Mathematical model of the problem

Equations that govern the convective fow of an electrically conducting, incompressible, viscous, chemically reactive, and radiating fuid in a porous medium in presence of a magnetic feld having constant mass difusivity and thermal difusivity taking the difusion- thermo efect into account are.

Continuity equation:

$$
\vec{\nabla} \cdot \vec{q} = 0 \tag{1}
$$

Magnetic feld continuity equation:

$$
\vec{\nabla} \cdot \vec{B} = 0 \tag{2}
$$

Ohm's Law:

$$
\overrightarrow{f} = \sigma \left(\overrightarrow{E} + \overrightarrow{q} \times \overrightarrow{B} \right)
$$
 (3)

Momentum equation:

$$
\rho \left[\frac{\partial \overrightarrow{q}}{\partial t'} + \left(\overrightarrow{q} \cdot \overrightarrow{\nabla} \right) \overrightarrow{q} \right] = -\overrightarrow{\nabla} p + \overrightarrow{f} \times \overrightarrow{B} + \rho \overrightarrow{g} + \mu \nabla^2 \overrightarrow{q} - \frac{\mu \overrightarrow{q}}{K*}
$$
(4)

Energy equation:

$$
\rho C_p \left[\frac{\partial T}{\partial t'} + \left(\vec{q} \cdot \vec{\nabla} \right) T \right] = \kappa \nabla^2 T - \vec{\nabla} \cdot \vec{q}_r + \frac{\rho D_M K_T}{C_S} \nabla^2 C \tag{5}
$$

Species continuity equation:

$$
\frac{\partial C}{\partial t'} + \left(\vec{q} \cdot \vec{\nabla}\right) C = D_M \nabla^2 C + \overline{K}(C_{\infty} - C) \tag{6}
$$

Equation of state as per Boussinesq approximation:

$$
\rho_{\infty} = \rho \left[1 + \beta (T - T_{\infty}) + \overline{\beta} (C - C_{\infty}) \right]
$$
\n(7)

The radiation heat flux as per Rosseland approximation is given by

$$
\overrightarrow{q_r} = -\frac{4\sigma^*}{3\kappa^*} \overrightarrow{\nabla} T^4
$$

Now,

$$
T^{4} = (T - T_{\infty} + T_{\infty})^{4} = 4TT_{\infty}^{3} - 3T_{\infty}^{4}, \text{as } |T - T_{\infty}| \ll 1
$$

Figure 1. Flow configuration.

So,

$$
\overrightarrow{\nabla} \cdot \overrightarrow{q_r} = -\frac{16\sigma^* T_{\infty}^3}{3\kappa^*} \nabla^2 T
$$

Therefore, Energy Eq. (5) (5) (5) reduces to

$$
\rho C_p \left[\frac{\partial T}{\partial t'} + \left(\vec{q} \cdot \vec{\nabla} \right) T \right] = \kappa \nabla^2 T + \frac{16\sigma^* T_{\infty}^3}{3\kappa^*} \nabla^2 T + \frac{\rho D_M K_T}{C_S} \nabla^2 C \tag{8}
$$

We now consider a transient MHD free convection flow of a viscous incompressible electrically conducting fuid through a porous medium past a semi-infnite vertical plate in presence of a uniform magnetic feld applied normal to the plate, directed into the fuid region. Initially, the plate and the surrounding fuid were at rest with uniform temperature T_{∞} and concentration C_{∞} at all points in the fluid. At time $t' > 0$, the plate is exponentially accelerated with velocity $U_0e^{a't'}$. The plate temperature is instantaneously elevated to $T_{\infty} + (T_w - T_{\infty})\frac{t'}{t_0}$, for $0 < t' \le t_0$, and thereafter T_w when $t' > t_0$. The concentration is raised to C_w and maintained thereafter.

To idealize the mathematical model, we enforce the following constraints-

- I. Except the variation in density in the buoyancy force term, all the fluid properties are constant.
II. Energy dissipation occurring from friction and Joule heating is negligible.
- II. Energy dissipation occurring from friction and Joule heating is negligible. III. Compared to applied magnetic field, induced magnetic field is negligible.
- Compared to applied magnetic field, induced magnetic field is negligible.
- IV. Flow is one- dimensional which is parallel to the plate.
- V. The plate is electrically insulating.
- VI. Polarization voltage is negligible because no external electric feld is applied.

We now consider a tri- rectangular Cartesian co-ordinate system (x', y', z', t') with X-axis vertically upwards along the plate, Y-axis normal to the plate directed into the fuid region, and Z-axis along the width of the plate as displayed in Fig. [1.](#page-3-0) Let $\vec{q} = (u', 0, 0)$ be the fluid velocity and $B = (0, B_0, 0)$ be the magnetic induction vector at the point (x', y', z', t') in the fluid.

Equation [\(1\)](#page-2-1) yields,

$$
\frac{\partial u'}{\partial x'} = 0
$$

i.e., $u' = u'(y', t')$ (9)

Equation [\(2\)](#page-2-2) is trivially satisfied by $\vec{B} = (0, B_0, 0)$. Equation [\(4\)](#page-2-3) reduces to

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$$
\rho \left[\frac{\partial u'}{\partial t'} \hat{i} + 0 \right] = -\hat{i} \frac{\partial p}{\partial x'} - \hat{j} \frac{\partial p}{\partial y'} - \rho g \hat{i} - \sigma B_0^2 u' \hat{i} + \mu \frac{\partial^2 u'}{\partial y'^2} \hat{i} - \frac{\mu u'}{K \ast} \hat{i}
$$
\n(10)

Equation [\(10\)](#page-4-0) gives

$$
\rho \frac{\partial u'}{\partial t'} = -\frac{\partial p}{\partial x'} - \rho g - \sigma B_0^2 u' + \mu \frac{\partial^2 u'}{\partial y'^2} - \frac{\mu u'}{K*}
$$
(11)

And

$$
0 = -\frac{\partial p}{\partial y'}\tag{12}
$$

Equation ([12\)](#page-4-1) shows that pressure near the plate and pressure far away from the plate are the same along the normal to the plate.

For fluid region far away from the plate, Eq. (11) (11) takes the form

$$
0 = -\frac{\partial p}{\partial x'} - \rho_{\infty} g \tag{13}
$$

Eliminating $\frac{\partial p}{\partial x'}$ from Eqs. [\(11](#page-4-2)) and [\(13\)](#page-4-3), we get,

$$
\rho \frac{\partial u'}{\partial t'} = (\rho_{\infty} - \rho)g - \sigma B_0^2 u' + \mu \frac{\partial^2 u'}{\partial y'^2} - \frac{\mu u'}{K*}
$$
(14)

Now, Eq. ([7\)](#page-2-4) gives,

$$
\rho_{\infty} - \rho = \rho \left[\beta (T - T_{\infty}) + \overline{\beta} (C - C_{\infty}) \right]
$$
\n(15)

Putting value of Eq. [\(15\)](#page-4-4) in Eq. [\(14](#page-4-5)),

$$
\rho \frac{\partial u'}{\partial t'} = \rho \left[\beta (T - T_{\infty}) + \overline{\beta} (C - C_{\infty}) \right] g - \sigma B_0^2 u' + \mu \frac{\partial^2 u'}{\partial y'^2} - \frac{\mu u'}{K*}
$$

$$
i.e., \frac{\partial u'}{\partial t'} = g \beta (T - T_{\infty}) + g \overline{\beta} (C - C_{\infty}) - \frac{\sigma B_0^2 u'}{\rho} + \nu \frac{\partial^2 u'}{\partial y'^2} - \nu \frac{u'}{K*}
$$
(16)

Equation [\(8\)](#page-3-1) yields,

$$
\rho C_p \frac{\partial T}{\partial t'} = \kappa \frac{\partial^2 T}{\partial y'^2} + \frac{16\sigma^* T_{\infty}^3}{3\kappa^*} \frac{\partial^2 T}{\partial y'^2} + \frac{\rho D_M K_T}{C_S} \frac{\partial^2 C}{\partial y'^2}
$$
(17)

Equation [\(6\)](#page-2-5) becomes,

$$
\frac{\partial C}{\partial t'} = D_M \frac{\partial^2 C}{\partial y'^2} + \overline{K}(C_{\infty} - C)
$$
\n(18)

The relevant initial and boundary conditions are:

$$
u' = 0, T = T_{\infty}, C = C_{\infty} : \forall y' \ge 0; t' \le 0
$$

\n
$$
u' = U_0 e^{a't'}, C = C_W : y' = 0, t' > 0
$$

\n
$$
T = T_{\infty} + (T_w - T_{\infty}) \frac{t'}{t_0} : \overline{y} = 0; 0 < t' \le t_0
$$

\n
$$
T = T_w : y' = 0; t' > t_0
$$

\n
$$
u' \to 0, T \to T_{\infty}, C \to C_{\infty} : y' \to \infty; t' > 0
$$
\n(19)

For the sake of normalization of the mathematical model of the problem, we introduce the following nondimensional quantities-

$$
Du = \frac{D_M K_T (C_w - C_{\infty})}{C_S C_P (T_w - T_{\infty}) \nu}, N = \frac{\kappa \kappa^*}{4\sigma^* T_{\infty}^3}, u = \frac{u'}{U_0}, y = \frac{U_0}{\nu}, y = \frac{U_0^2}{\nu}, t = \frac{U_0^2}{\nu}, G = \frac{\nu g \beta (T_w - T_{\infty})}{U_0^3}, a = a' \frac{\nu}{U_0^2},
$$

\n
$$
Gm = \frac{\nu g \overline{\beta} (C_w - C_{\infty})}{U_0^3}, \theta = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \phi = \frac{C - C_{\infty}}{C_w - C_{\infty}}, \text{Pr} = \frac{\mu C_p}{\kappa}, M = \frac{\sigma B_0^2 \nu}{\rho U_0^2}, Sc = \frac{\nu}{D_M}, \Lambda = 1 + \frac{4}{3N},
$$

$$
K = \frac{\nu \overline{K}}{U_0^2}, t_1 = \frac{U_0^2}{\nu} t_0, M_1 = M + \frac{1}{K*}
$$

The non-dimensional governing equations are

$$
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi - M_1 u \tag{20}
$$

$$
\frac{\partial \theta}{\partial t} = \frac{\Lambda}{\text{Pr}} \frac{\partial^2 \theta}{\partial y^2} + D u \frac{\partial^2 \phi}{\partial y^2}
$$
(21)

$$
\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - K\phi \tag{22}
$$

Subject to the initial and boundary conditions

$$
u = 0, \theta = 0, \phi = 0 : \forall y \ge 0; t \le 0
$$

\n
$$
u = e^{at}, \phi = 1 : y = 0, t > 0
$$

\n
$$
\theta = \frac{t}{t_1} : y = 0; 0 < t \le t_1
$$

\n
$$
\theta = 1 : y = 0; t > t_1
$$

\n
$$
u \to 0, \theta \to 0, \phi \to 0 : y \to \infty; t > 0
$$
\n(23)

Method of Solution

On taking Laplace transform of the Eqs. ([22](#page-5-0)), ([21](#page-5-1)), and [\(20](#page-5-2)) respectively, we get the following equations:

$$
s\overline{\phi} = \frac{1}{Sc} \frac{d^2 \overline{\phi}}{dy^2} - K\overline{\phi}
$$
 (24)

$$
s\overline{\theta} = \frac{\Lambda}{\text{Pr}} \frac{d^2 \overline{\theta}}{dy^2} + Du \frac{d^2 \overline{\phi}}{dy^2}
$$
 (25)

$$
s\overline{u} = \frac{d^2\overline{u}}{dy^2} + Gr\overline{\theta} + Gm\overline{\phi} - M_1\overline{u}
$$
 (26)

Subject to the initial and boundary conditions:

$$
y = 0: \overline{\theta} = \frac{2}{s^2 t_1} (1 - e^{-st_1}), \overline{\phi} = \frac{1}{s}, \overline{u} = \frac{1}{s - a}
$$

\n
$$
y \to \infty : \overline{\theta} \to 0, \overline{\phi} \to 0, \overline{u} \to 0
$$
\n(27)

Solving equations from Eqs. [\(24\)](#page-5-3) to [\(26\)](#page-5-4) subject to the conditions (Eq. [27](#page-2-1)) and taking inverse Laplace transform of the solutions, the expression for temperature field θ , concentration field ϕ , and velocity field u are as follows:

$$
\phi = \psi_1 \tag{28}
$$

$$
\theta = \begin{cases} \theta_{1,1} + \theta_{1,2} - \theta_{1,3} : \Delta Sc \neq Pr \\ \theta_{2,1} + \theta_{2,2} - \theta_{2,3} : \Delta Sc = Pr \end{cases}
$$
 (29)

$$
u = \begin{cases} u_{1,1} + u_{1,2} + u_{1,3} + u_{1,4} + u_{1,5} : \Pr \neq \Lambda, \text{Sc} \neq 1, \Pr \neq \Lambda \text{Sc} \\ u_{2,1} + u_{2,2} + u_{2,3} + u_{2,4} + u_{2,5} : \Pr = \Lambda, \text{Sc} \neq 1 \\ u_{3,1} + u_{3,2} + u_{3,3} + u_{3,4} + u_{3,5} : \Pr \neq \Lambda, \text{Sc} = 1 \\ u_{4,1} + u_{4,2} + u_{4,3} + u_{4,4} + u_{4,5} : \Pr = \Lambda, \text{Sc} = 1 \\ u_{5,1} + u_{5,2} + u_{5,3} + u_{5,4} + u_{5,5} : \Pr \neq \Lambda, \text{Sc} \neq 1, \Pr = \Lambda \text{Sc} \end{cases}
$$
(30)

where

$$
\psi_1 = \psi\big(Sc, K, y, t\big), a_1 = \frac{\text{Pr}}{\Lambda}, a_2 = \frac{\Lambda ScK}{\Lambda Sc - \text{Pr}}, a_3 = \frac{Du\,\text{Pr}\,Sc}{\Lambda Sc - \text{Pr}}, \theta_{1,1} = \frac{1}{t_1}\Delta\lambda_1, \lambda_1 = \lambda(a_1, y, t),
$$

$$
\theta_{1,2} = a_3(A_1E_1 + A_2E_3), E_1 = erf(\frac{y\sqrt{a_1}}{2\sqrt{t}}), E_2 = erf(\frac{y\sqrt{a_1 - a_2}}{2\sqrt{t}}), E_3 = e^{-a_2t}E_{3,}A_1 = \frac{K}{a_2},
$$

\n
$$
A_2 = \frac{a_2 - K}{a_2}, \theta_{1,3} = a_3(A_1\psi_1 + A_2\psi_2), \psi_2 = \Psi(Sc, K, -a_2, y, t), \theta_{2,1} = \theta_{1,1}, a_4 = \frac{D u Pr}{\Lambda K},
$$

\n
$$
\theta_{2,2} = a_4(K\lambda_4 + P_1), P_1 = P(a_1, y, t), \theta_{2,3} = a_4(K\psi_1 + l_1), l_1 = l(Sc, K, y, t),
$$

\n
$$
u_{1,1} = u_{1,1,1} + u_{1,1,2} + u_{1,1,3} + u_{1,1,4} + u_{1,1,5}, u_{1,1,1} = h_2, h_2 = e^{iH}h_1, h_1 = h(M_1 + a, y, t), a_5 = \frac{M_1}{a_1 - 1},
$$

\n
$$
a_6 = \frac{Ksc - M_1}{Sc - 1}, a_7 = \frac{Gr}{t_1(a_1 - 1)}, u_{1,1,2} = a_7(A_3\Delta h_5 + A_4\Delta h_5 + A_5\Delta r_1), A_3 = \frac{1}{a_2}, A_4 = -A_3, A_5 = \frac{1}{a_5}.
$$

\n
$$
h_3 = h(M_1, y, t), h_5 = e^{-a_5t}h_4, h_4 = h(M_1 - a_5, y, t), n = r(M_1, y, t), a_8 = \frac{Gra_3}{a_1 - 1},
$$

\n
$$
u_{1,1,3} = a_8(A_6b_7 + A_7b_5 + A_8b_3), A_6 = \frac{a_2 - K}{a_2(a_5 - a_1)}, A_7 = \frac{a_3 - K}{a_5(a_2 - a_3)}, A_8 = \frac{K}{a_2, h_7} = e^{-a_7t}h_6,
$$

\n
$$
h
$$

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$$
u_{5,1,3} = a_{18}(A_{16}h_5 + A_{17}h_3), a_{18} = \frac{GrDuSc}{(a_1 - 1)K}, A_{16} = \frac{a_5 - K}{a_5}, A_{17} = \frac{K}{a_5}, u_{5,1,4} = a_{18}(A_{18}h_3 + A_{19}h_9),
$$

$$
A_{18} = \frac{K}{a_6}, A_{19} = \frac{a_6 - K}{a_6}, u_{5,1,5} = u_{1,1,5}, u_{5,2} = u_{1,2}, u_{5,3} = -a_{18}(A_{16}\psi_4 + A_{17}\psi_1), \psi_4 = \Psi\big(Sc, K, -a_5, y, t\big),
$$

$$
u_{5,4} = -a_{18}(A_{18}\psi_1 + A_{19}\psi_3), u_{5,5} = u_{1,5}
$$

Nusselt number

The heat flux q^* at the plate $y' = 0$ is obtained by Fourier's law of conduction is given by

$$
q^* = -\kappa_0^* \frac{\partial T}{\partial y'}\bigg]_{y'=0} \tag{31}
$$

where $\kappa_0^* = \kappa + \frac{16\sigma^* T_{\infty}^3}{3\kappa^*}$ is the modified thermal conductivity.
Equation [\(31\)](#page-5-5) yields

$$
Nu = -\frac{\partial \theta}{\partial y}\bigg]_{y=0} \tag{32}
$$

where $Nu = \frac{q^*v}{\kappa_0^* U_0(T_w - T_\infty)} = \frac{3Nq^*v}{\kappa (4+3N)(T_w - T_\infty)U_0}$ is called the Nusselt number which is concerned with the rate of heat transfer at the plate.

Equation [\(32\)](#page-7-0) gives,

$$
Nu = -\begin{cases} Nu_{1,1} + Nu_{1,2} - Nu_{1,3} : \Lambda Sc \neq Pr \\ Nu_{2,1} + Nu_{2,2} - Nu_{2,3} : \Lambda Sc = Pr \end{cases}
$$
(33)

where

$$
Nu_{1,1} = \frac{1}{t_1} \Delta v_1, v_1 = v(a_1, t), Nu_{1,2} = a_3(A_1\alpha_1 + A_2\alpha_3), \alpha_1 = \alpha \left(\frac{\sqrt{a_1}}{2\sqrt{t}}\right), \alpha_2 = \alpha \left(\frac{\sqrt{a_1 - a_2}}{2\sqrt{t}}\right),
$$

$$
\alpha_3 = e^{-a_2t}\alpha_2, Nu_{1,3} = a_3(A_1\Omega_1 + A_2Z_1), \Omega_1 = \Omega(\mathcal{S}\mathcal{C}, K, t), Z_1 = Z(\mathcal{S}\mathcal{C}, K, -a_2, t), Nu_{2,1} = Nu_{1,1},
$$

$$
Nu_{2,2} = a_4(Kv_1 + I_1), I_1 = I(a_1, t), Nu_{2,3} = a_4(K\Omega_1 + T_1), T_1 = T(Sc, K, t)
$$

Sherwood number

The mass flux M_w at the plate $y' = 0$ is specified by Fick's law of diffusion is given by

$$
M_w = -D_M \frac{\partial C}{\partial y'}\bigg]_{y'=0} \tag{34}
$$

Equation [\(34\)](#page-7-1) gives

$$
Sh = -\frac{\partial \phi}{\partial y}\bigg]_{y=0} \tag{35}
$$

In Eq. [\(35](#page-7-2)), $Sh = \frac{M_w v}{D_M U_0 (C_w - C_\infty)}$ is called the Sherwood number which is associated with the rate of mass transfer at the plate.

Equation (35) yields

$$
Sh = -\Omega_1 \tag{36}
$$

Skin friction

The viscous drag at the plate $y' = 0$ is determined by Newton's law of viscosity is given by

$$
\overline{\tau} = -\mu \frac{\partial u}{\partial y'}\bigg]_{y'=0} \tag{37}
$$

Equation [\(37\)](#page-7-3) gives

$$
\tau = -\frac{\partial u}{\partial y}\bigg|_{y=0} \tag{38}
$$

In Eq. [\(38](#page-7-4)), $\tau = \frac{\bar{\tau}v}{\mu U_0^2}$ is called the skin friction or coefficient of friction which is associated with the rate of momentum transfer at the plate.

Equation [\(38\)](#page-7-4) yields,

$$
\tau = -\begin{cases} \tau_{1,1} + \tau_{1,2} + \tau_{1,3} + \tau_{1,4} + \tau_{1,5} : \Pr \neq \Lambda, Sc \neq 1, \Pr \neq \LambdaSc \\ \tau_{2,1} + \tau_{2,2} + \tau_{2,3} + \tau_{2,4} + \tau_{2,5} : \Pr = \Lambda, Sc \neq 1 \\ \tau_{3,1} + \tau_{3,2} + \tau_{3,3} + \tau_{3,4} + \tau_{3,5} : \Pr \neq \Lambda, Sc = 1 \\ \tau_{4,1} + \tau_{4,2} + \tau_{4,3} + \tau_{4,4} + \tau_{4,5} : \Pr = \Lambda, Sc = 1 \\ \tau_{5,1} + \tau_{5,2} + \tau_{5,3} + \tau_{5,4} + \tau_{5,5} : \Pr \neq \Lambda, Sc \neq 1, \Pr = \Lambda Sc \end{cases} \tag{39}
$$

where

$$
\tau_{1,1} = \tau_{1,1,1} + \tau_{1,1,2} + \tau_{1,1,3} + \tau_{1,1,4}, \tau_{1,1,1} = N_2, N_2 = e^{at} N_1, N_1 = N(M_1 + a, t),
$$

$$
\tau_{1,1,2} = a_7(A_3 \Delta N_5 + A_4 \Delta N_3 + A_5 \Delta O_1), N_3 = N(M_1, t), N_4 = N(M_1 - a_5, t), N_5 = e^{-a_5 t} N_4,
$$

$$
O_1 = O(M_1, t), \tau_{1,1,3} = a_8(A_6N_7 + A_7N_5 + A_8N_3), N_7 = e^{-a_2t}N_6, N_6 = N(M_1 - a_2, t),
$$

 $\tau_{1,1,4} = a_9(A_9N_7 + A_{10}N_9 + A_{11}N_3),$ $N_9 = e^{-a_6t}N_8$, $N_8 = N(M_1 - a_6, t)$, $\tau_{1,1,5} = a_{10}(A_{12}N_9 + A_{13}N_3)$,

$$
\tau_{1,2} = -a_7(A_3 \Delta \alpha_5 + A_4 \Delta \alpha_1 + A_5 \Delta \nu_1), \alpha_5 = e^{-a_5 t} \alpha_4, \alpha_4 = \alpha \left(\frac{\sqrt{a_1 - a_5}}{2\sqrt{t}} \right),
$$

$$
\tau_{1,3} = -a_8(A_6 \alpha_3 + A_7 \alpha_5 + A_8 \alpha_1), \tau_{1,4} = -a_9(A_9 Z_1 + A_{10} Z_2 + A_{11} \Omega_1), Z_2 = Z(Sc, K, -a_6, t),
$$

$$
\tau_{1,5}=-a_{10}(A_{12}Z_2+A_3\Omega_1),\tau_{2,1}=\tau_{2,1,1}+\tau_{2,1,2}+\tau_{2,1,3}+\tau_{2,1,4}+\tau_{2,1,5},\tau_{2,1,1}=\tau_{1,1,1},\tau_{2,1,2}=a_{11}\Delta O_1,
$$

 $\tau_{2,1,3} = a_{12}(A_{14}N_7 + A_{15}N_3), \tau_{2,1,4} = \tau_{1,1,4}, \tau_{2,1,5} = \tau_{1,1,5}, \tau_{2,2} = -a_{11}\Delta v_2, v_2 = v(1,t),$

$$
\tau_{2,3} = -a_{12}(A_{14}\alpha_8 + A_{15}\alpha_6), \alpha_6 = \alpha\left(\frac{1}{2\sqrt{t}}\right), \alpha_8 = e^{-a_2t}\alpha_7, \alpha_7 = \alpha\left(\frac{\sqrt{1-a_2}}{2\sqrt{t}}\right), \tau_{2,4} = \tau_{1,4}, \tau_{2,5} = \tau_{1,5}, \tau_{1,6} = \tau_{1,6}, \tau_{1,7} = \tau_{1,8}, \tau_{1,8} = \tau_{1,9}, \tau_{1,10} = \tau_{1,11} = \tau_{1,12} = \tau_{1,13} = \tau_{1,14} = \tau_{1,15} = \tau_{1,16} = \tau_{1,17} = \tau_{1,18} = \tau_{1,19} = \tau_{1,10} = \tau_{1,10} = \tau_{1,10} = \tau_{1,11} = \tau_{1,10} = \tau_{1,11} = \tau_{1,10} = \tau_{1,11} = \tau_{1,10} = \tau_{1,11} = \tau_{1,11} = \tau_{1,10} = \tau_{1,11} =
$$

 $\tau_{3,1} = \tau_{3,1,1} + \tau_{3,1,2} + \tau_{3,1,3} + \tau_{3,1,4} + \tau_{3,1,5}, \tau_{3,1,1} = \tau_{1,1,1}, \tau_{3,1,2} = \tau_{1,1,2}, \tau_{3,1,3} = \tau_{1,1,3},$

$$
\tau_{3,1,4}=a_{13}(A_{14}N_7+A_{15}N_3),\tau_{3,1,5}=a_{14}N_3,\tau_{3,2}=\tau_{1,2},\tau_{3,3}=\tau_{1,3},\tau_{3,4}=-a_{13}(A_{14}N_{11}+A_{15}N_{12}),
$$

 $\tau_{3,5} = -a_{13}\Omega_1, N_{10} = N(K - a_2, t), N_{11} = e^{-a_2t}N_{10}, N_{12} = N(K, t),$ $\tau_{4,1} = \tau_{4,1,1} + \tau_{4,1,2} + \tau_{4,1,3} + \tau_{4,1,4} + \tau_{4,1,5},$ $\tau_{4,1,1} = \tau_{1,1,1}$, $\tau_{4,1,2} = \tau_{2,1,2}$, $\tau_{4,1,3} = a_{15}(KN_3 + Y_1)$, $Y_1 = Y(M_1, t)$,

$$
\tau_{4,1,4}=a_{16}(KN_3+Y_1),\tau_{4,1,5}=a_{17}N_3,\tau_{4,2}=\tau_{2,2},\tau_{4,3}=-a_{15}\alpha_6,\alpha_6=\alpha\left(\frac{1}{2\sqrt{t}}\right),\tau_{4,4}=-a_{16}(KN_{12}+Y_2),
$$

$$
Y_2 = Y(K, t), \tau_{4,5} = -a_{17}N_{12}, \tau_{5,1} = \tau_{5,1,1} + \tau_{5,1,2} + \tau_{5,1,3} + \tau_{5,1,4} + \tau_{5,1,5}, \tau_{5,1,1} = \tau_{1,1,1}, \tau_{5,1,2} = \tau_{1,1,2},
$$

$$
\tau_{5,1,3} = a_{18}(A_{16}N_5 + A_{17}N_3), \tau_{5,1,4} = a_{18}(A_{18}N_3 + A_{19}N_9), \tau_{5,1,5} = \tau_{1,1,5}, \tau_{5,2} = \tau_{1,2},
$$

$$
\tau_{5,3}=-a_{18}(A_{16}Z_3+A_{17}\Omega_1),Z_3=Z(Sc,K,-a_5,t),\tau_{5,4}=-a_{18}(A_{18}\Omega_1+A_{19}Z_2),\tau_{5,5}=\tau_{1,5}
$$

Result and discussion

The effects of various flow parameters associated with the flow and transport properties are examined by assign-ing some specific values. The results are demonstrated from Figs. [2](#page-9-0), [3,](#page-9-1) [4,](#page-10-0) [5](#page-10-1), [6](#page-10-2), [7,](#page-11-0) [8](#page-11-1), [9](#page-12-0), [10,](#page-12-1) [11](#page-13-0), [12](#page-13-1), [13,](#page-14-0) [14](#page-14-1), [15,](#page-15-0) [16,](#page-15-1) [17](#page-16-0), [18,](#page-16-1) [19](#page-17-0), [20](#page-17-1), [21](#page-18-0), [22,](#page-18-1) [23,](#page-19-0) [24](#page-19-1), [25](#page-20-0), [26](#page-20-1), [27,](#page-21-0) [28,](#page-21-1) [29](#page-22-0), [30](#page-22-1), [31](#page-23-0), [32,](#page-23-1) [33,](#page-24-0) [34](#page-24-1) and [35.](#page-25-0)

Figures [2](#page-9-0), [3](#page-9-1) and [4](#page-10-0) display the variation of concentration feld versus normal co-ordinate y. Figure [2](#page-9-0) admits that the concentration feld keeps on increasing with time. Figure [3](#page-9-1) reveals that there is a comprehensive fall in the concentration feld for increasing chemical reaction parameter. A faster chemical reaction consumes chemical substances present in the fluid rapidly and as a result concentration of the fluid declines. The behaviour of concentration profles for various fuids such as hydrogen (Sc=0.22), helium (Sc=0.30), water vapour (Sc=0.60) and ammonia ($Sc = 0.78$) are demonstrated in Fig. [4](#page-10-0). It suggests that a higher Schmidt number lowers the concentration field. Thus higher mass diffusivity hikes the concentration field.

Figures [5](#page-10-1), [6,](#page-10-2) [7](#page-11-0), [8,](#page-11-1) [9](#page-12-0) and [10](#page-12-1) illustrate the variation of temperature feld versus normal co-ordinate y. Figure [5](#page-10-1) suggests that the temperature feld escalates with time. Figure [6](#page-10-2) shows that the temperature feld upsurges with

Figure 2. versus *y* for different *t* and $Sc = 0.22, K = 1$.

Figure 3. ϕ versus γ for different *K* and $t = 1$, *Sc* = 0.22.

increment in chemical reaction parameter. Increasing chemical reaction parameter upsurges collision between fuid molecules and as a result temperature of fuid hikes. Figure [7](#page-11-0) displays that increasing the Dufour number hikes temperature feld. An increment in the Dufour number indicates a comprehensive rise in concentration gradient over temperature gradient. Hence, increasing concentration gradient upsurges the temperature feld. Figure [8](#page-11-1) suggests that the temperature feld elevates with an uplif in Schmidt number. Tus, the temperature feld decreases with increasing mass diffusivity. The temperature field decelerates with increasing radiation parameter as noticed in Fig. [9](#page-12-0). It is in agreement with the fact that radiation tends to decline temperature. The nature of temperature profiles for various fluids such as oxygen (Pr=0.60), air (Pr=0.71), ammonia (Pr=1.38) etc. are demonstrated in Fig. [10](#page-12-1). It shows that the temperature feld falls with ascending values of the Prandtl number. This informs that the temperature field accelerates with higher thermal diffusivity.

Figures [11,](#page-13-0) [12](#page-13-1), [13,](#page-14-0) [14](#page-14-1), [15,](#page-15-0) [16](#page-15-1), [17,](#page-16-0) [18](#page-16-1), [19](#page-17-0) and [20](#page-17-1) depict the variation of velocity feld versus normal co-ordinate y. Figure [11](#page-13-0) reveals that as time progresses, the velocity feld increases. Figure [12](#page-13-1) admits that the velocity feld declines considerably as the Dufour number rises. Consequently, a large concentration gradient relative to the temperature gradient results in a dip in the velocity feld. Figure [13](#page-14-0) shows that velocity reduces with increasing chemical reaction parameter. Tis is because increasing chemical reaction parameter accelerates the process of collision between fuid molecules and as a result, kinetic energy is lost. Velocity falls with increasing magnetic parameter as noticed in Fig. [14](#page-14-1). Application of transverse magnetic feld produces a resistive force known as

Figure 4. ϕ versus y for different *Sc* and $t=1$, $K=0.22$.

Figure 5. θ versus *y* for different *t* and *Sc* = 0.22, *K* = 0.5, *N* = 5, *Pr* = 0.71, *Du* = 1, *t*₁ = 0.5.

Figure 6. θ versus *y* for different *K* and *t*=0.8, *Sc*=0.22, *N*=4, *Pr*=0.71, *Du*=1, *t*₁=0.5.

Figure 7. θ versus *y* for different *Du* and $t = 1$, *Sc* = 0.22, *K* = 0.5, *N* = 5, *Pr* = 0.71, t_1 = 0.5.

Figure 8. θ versus *y* for different *Sc* and *t* = 1, *K* = 0.5, *N* = 2, *Pr* = 0.71, *Du* = 1, *t*₁ = 0.5.

Lorentz force, which slows down fuid velocity. Figure [15](#page-15-0) exhibits that increasing Schmidt number decrease velocity feld. Tus, high mass difusivity escalates fuid velocity. Velocity feld upsurges in a thin layer adjacent to the plate and its nature take reverse turn outside the layer as thermal Grashof number upsurges as demonstrated in Fig. [16.](#page-15-1) So, thermal buoyancy force hikes velocity in a small layer surrounding the plate but lowers velocity outside the layer. Velocity rises with increment in solutal Grashof number as noticed in Fig. [17](#page-16-0). Tus, solutal buoyancy force upsurges velocity. Hence higher mass difusivity raises velocity feld but increasing thermal difusivity reduces velocity. Increasing porosity parameter means the fuid gets more free space to fow. As a result fuid velocity hikes. Tis phenomenon is refected in Fig. [18.](#page-16-1) Increasing radiation parameter accelerates fluid velocity as observed in Fig. [19.](#page-17-0) The reason behind it is that when the radiation increases, chemical bonding between the fuid molecules becomes weak so that velocity hikes. Figure [12](#page-13-1) shows that ascending values of Prandtl number uplif velocity. Tus, higher thermal difusivity diminishes velocity.

Figures [21](#page-18-0) and [22](#page-18-1) demonstrate the variation of Sherwood number versus time *t*. Sherwood number increases with increment in chemical reaction parameter as noticed in Fig. [11.](#page-13-0) From Fig. [22,](#page-18-1) it is observed that increasing Schmidt number upsurges Sherwood number. This result establishes the fact that higher mass diffusivity accelerates the process of mass transfer from the plate to the fuid.

Figures [23](#page-19-0), [24,](#page-19-1) [25](#page-20-0), [26](#page-20-1) and [27](#page-21-0) exhibit the variation of Nusselt number versus time t. Nusselt number increases for a small time but decreases thereafer for increasing radiation parameter as noticed in Fig. [24.](#page-19-1) Tus, radiation increases the rate of heat transfer from the plate to the fluid for a small time and decreases afterward. Figure [26](#page-20-1) shows that the Nusselt number hikes for a small time but declines thereafer with ascending values of the Prandtl

Figure 9. θ versus *v* for different *N* and $t = 1$, $Sc = 0.22$, $K = 0.5$, $Pr = 0.71$, $Du = 1$, $t_1 = 0.5$.

Figure 10. θ versus *y* for different *Pr* and *t*=1, *Sc*=0.22, *K*=0.5, *N*=3, *Du*=1, *t*₁=0.5.

number. So, higher thermal difusivity lessens the rate of heat transfer for a small time but increases as time progresses. From Fig. [25](#page-20-0) and Fig. [27,](#page-21-0) it is observed that the Nusselt number declines for a small time but upsurges thereafer with increment in Dufour number and Schmidt number respectively. Figure [26](#page-20-1) shows that higher chemical reaction parameter hikes Nusselt number. Increasing chemical reaction parameter suggests a hike in heat generation. So, the process of heat transfer is accelerated.

Variations of skin friction versus time t are demonstrated in Figs. [28](#page-21-1), [29,](#page-22-0) [30,](#page-22-1) [31](#page-23-0), [32](#page-23-1), [33,](#page-24-0) [34](#page-24-1) and [35](#page-25-0). Figure [28](#page-21-1) admits that there is a comprehensive rise in skin friction as Dufour number hikes. Tus, the concentration gradient generates more frictional resistance compared to the temperature gradient. Skin friction uplifs with increment in thermal Grashof number as noticed in Fig. [29.](#page-22-0) Tus, thermal buoyancy force hikes frictional resistivity at the plate. Skin friction hikes with an upsurge in both chemical reaction parameter and porosity parameter as shown in Fig. [30](#page-22-1) and Fig. [31](#page-23-0) respectively.Fig. [32](#page-23-1) reveals that increasing magnetic parameter raises skin friction. Hence Lorentz force accelerates frictional resistivity of the plate. Higher Schmidt number hikes skin friction as displayed in Fig. [33.](#page-24-0) Terefore, increasing mass difusivity lowers the frictional resistance of the plate.Fig. [34](#page-24-1) and Fig. [35](#page-25-0) give us an idea that enhancement in radiation parameter and Prandtl number lowers skin friction.

Figure [36](#page-25-1) and Fig. [37](#page-26-0) reveal that ascending critical time for rampedness lowers both temperature and velocity of the fuid respectively. Tus, arbitrary ramped temperature has inverse efect on both temperature and velocity fields. It is observed from Fig. [38](#page-26-1) that increasing critical time for rampedness hikes Nusselt number. This means

Figure 11. *u* versus *y* for diferent *t* and *Sc*=0.22, *K*=2, *N*=5, *Pr*=0.71, *Du*=0.5, *M*=0.5, *K**=1, *Gr*=1, *Gm* = 10, $a=1$, t_1 = 0.5.

Figure 12. *u* versus *y* for different *Du* and $t = 1$, $Sc = 0.22$, $K = 2$, $N = 5$, $Pr = 0.71$, $M = 0.5$, $K^* = 1$, $Gr = 1$, $Gm = 5$, $a=1, t_1=0.5.$

that arbitrary ramped temperature efect has a tendency to accelerate the rate of heat transfer from the plate to the fluid. Figure [39](#page-27-0) shows that increasing critical time for rampedness declines skin friction. Thus arbitrary ramped temperature weakens the rate of momentum transfer from the plate to the fuid.

Numerical values of Nusselt number *Nu* against diferent time *t*, Dufour number *Du* and radiation parameter are analyzed in Table[1](#page-27-1). It is observed that for a small time, the Nusselt number decreases with increment in Dufour number but its behavior reverses as time progresses. An opposite behavior is noticed for increasing radiation parameter. Tis asserts that a high concentration gradient decelerates but radiation accelerates the process of heat transfer from the plate to the fuid. Tis is in complete agreement with our results from Fig. [23](#page-19-0) and Fig. [25](#page-20-0).Numerical values of skin friction τ against diferent time *t*, chemical reaction parameter *K*, radiation parameter *N*, Dufour number *Du*, thermal Grashof number *Gr* and solutal Grashof number *Gm* are demonstrated in Tabl[e2.](#page-28-0) It is noticed that ascending values of time, chemical reaction parameter, Dufour number, and thermal Grashof number hike skin friction whereas ascending values of radiation parameter and solutal Grashof number declines the value of skin friction. This is in accordance with our result from Fig. [30](#page-22-1) and Fig. [28](#page-21-1), Fig. [29](#page-22-0) and Fig. [34](#page-24-1) respectively.

Figure 13. *u* versus *y* for different *K* and $t=1$, $Sc=0.22$, $N=5$, $Pr=0.71$, $Du=0.5$, $M=0.5$, $K^*=1$, $Gr=1$, *Gm* = 10, $a=1$, $t_1=0.5$.

Figure 14. *u* versus *y* for diferent *M* and *t*=1, *Sc*=0.22, *K*=3, *N*=5, *Pr*=0.71, *Du*=0.5, *K**=2, *Gr*=1, *Gm*=20, $a=1, t_1=0.5.$

Comparison of result

To check the validity of our result, we have compared one of our results with Seth et al[.49](#page-30-36) who considered the unsteady free convective MHD flow of a chemically reactive, radiative flow past a moving vertical plate immersed in a porous medium. In absence of Dufour and chemical reaction efects and for vanishing Schmidt number (i.e., $Du=0$, $K=0$ and $Sc=0$), expression of temperature field of the present problem is

 $\theta = \theta_{1,1}$

Figure [40](#page-28-1) and Fig. [41](#page-29-12) display the temperature field versus normal co- ordinate y for different t_1 obtained by Seth et al[.49](#page-30-36) and present authors respectively. Both fgures uniquely expresses the fact that temperature feld declines for ascending values of critical time of rampedness. Hence, an excellent agreement of results between present authors and Seth et al.⁴⁹ is observed.

Table [3](#page-29-13) display the variation of Sherwood number for diferent *K*, *Sc* and *t* obtained by Asogwa et al[.50](#page-30-37), Seth et al.^{[51](#page-31-0)}, Kataria and Patel⁵² and present authors respectively. This table indicates that current study is in line with the results obtained by these authors.

Figure 15. *u* versus *y* for different *Sc* and $t = 1$, $K = 3$, $N = 5$, $Pr = 0.71$, $Du = 0.5$, $M = 0.5$, $K^* = 1$, $Gr = 1$, $Gm = 10$, $a=1, t_1=0.5.$

Figure 16. *u* versus *y* for different *Gr* and $t = 1$, $Sc = 0.22$, $K = 2$, $N = 5$, $Pr = 0.71$, $Du = 0.5$, $M = 0.5$, $K^* = 1$, *Gm* = 10, $a=1$, t_1 = 0.5.

Conclusion

The prime purpose of the present work was to study exclusively the effects of radiation, chemical reaction and Difusion thermo efect of an unsteady MHD fow past a moving vertical plate embedded in a porous medium with ramped temperature. The behavioral study of flow and transport characteristics under the action of different parameters was carried out with aid of graphs. The prominent outcomes of the present work are as follows:

- i. Velocity feld, concentration feld, and temperature feld accelerate with time.
- ii. Fluid gets thinner rapidly as chemical reaction parameter and Schmidt number hikes.
- iii. Radiation and Lorentz force resists fuid velocity.
- iv. Higher mass difusivity results in a fall in Nusselt Number, Sherwood number, and skin friction.
- v. Radiation slow down rate of momentum transfer.

Figure 17. *u* versus *y* for diferent *Gm* and *t*=1, *Sc*=0.22, *K*=2, *N*=5, *Pr*=0.71, *Du*=0.5, *M*=0.5, *K**=1, *Gr*=1, $a=1, t_1=0.5.$

Figure 18. *u* versus *y* for diferent *K** and *t*=1, *Sc*=0.22, *K*=3, *N*=5, *Pr*=0.71, *Du*=0.5, *M*=0.5, *Gr*=1, $Gm = 20$, $a=1$, $t_1 = 0.5$.

The solution of the present work also validates with the previous result obtained by Seth et al.⁴⁷, Asogwa et al.⁴⁸, Seth et al.^{[49](#page-30-36)} and Kataria and Patel^{[50](#page-30-37)} in particular case.

The governing equations of the present problem are solved using Laplace transform technique. The problem is idealized by imposing some realistic constraints (e.g., viscous dissipation, Joule heating, efect of suction, induced magnetic field are neglected for mathematical simplicity). The same problem may be re- investigated by removing or reducing number of constraints. In this context, some numerical and computational techniques like Runge- Kutta method, shooting method, Crank- Nicolson method etc. may be suggested.

Figure 19. *u* versus *y* for diferent *N* and *t*=1, *Sc*=0.22, *K*=3, *Pr*=0.71, *Du*=0.5, *M*=0.5, K*=1, *Gr*=1, *Gm*=5, $a=1, t_1=0.5.$

Figure 20. *u* versus *y* for diferent *Pr* and *t*=1, *Sc*=0.22, *K*=3, *N*=5, *Du*=0.5, *M*=0.5, K*=1, *Gr*=1, *Gm*=5, $a=1, t_1=0.5.$

Figure 21. *Sh* versus *t* for diferent *K* and *Sc*=0.22.

Figure 22. *Sh* versus *t* for diferent *Sc* and *K*=1.

Figure 23. *Nu* versus *t* for different *N* and *Sc* = 0.22, *Pr* = 0.71, *Du* = 0.5, *K* = 0.5, t_1 = 0.5.

Figure 24. *Nu* versus *t* for different *Pr* and *Sc* = 0.22, *N* = 3, *Du* = 0.5, *K* = 0.5, t_1 = 0.5.

Figure 25. *Nu* versus *t* for different *Du* and *Sc* = 0.22, *N* = 5, *Pr* = 0.71, *K* = 0.5, t_1 = 0.5.

Figure 26. *Nu* versus *t* for different *K* and *Sc* = 0.22, *N* = 5, *Pr* = 0.71, *Du* = 0.5, t_1 = 0.5.

Figure 27. *Nu* versus *t* for different *Sc* and $N = 5$, $Pr = 0.71$, $Du = 0.5$, $K = 0.5$, $t_1 = 0.5$.

Figure 28. τ versus *t* for diferent *Du* and *Sc*=0.22, *K*=1, *N*=5, *Pr*=0.71, *M*=0.5, *K**=5, *Gr*=1, *Gm*=5, *a*=1, $t_1=0.5$.

Figure 29. τ versus *t* for diferent *Gr* and *Sc*=0.22, *K*=1, *N*=5, *Pr*=0.71, *Du*=0.5, *M*=0.5, *K**=5, *Gm*=5, *a*=1, $t_1=0.5$.

Figure 30. τ versus *t* for diferent *K* and *Sc*=0.22, *N*=5, *Pr*=0.71, *Du*=1, *M*=0.5, *K**=3, *Gr*=1, *Gm*=5, *a*=1, $t_1 = 0.5$.

Figure 31. τ versus *t* for diferent *K** and *Sc*=0.22, *K*=1, *N*=3, *Pr*=0.71, *Du*=0.5, *M*=0.5, *Gr*=1, *Gm*=10, $a=1, t_1=0.5.$

Figure 32. τ versus *t* for diferent *M* and *Sc*=0.22, *K*=2, *N*=2, *Pr*=0.71, *Du*=0.5, *K**=1, *Gr*=1, *Gm*=1, *a*=1, $t_1=0.5$.

Figure 33. τ versus *t* for diferent *Sc* and *K*=2, *N*=3, *Pr*=0.71, *M*=0.5, *Du*=0.5, *K**=3, *Gr*=1, *Gm*=5, *a*=1, $t_1=0.5$.

Figure 34. τ versus *t* for diferent *N* and *Sc*=0.22, *K*=1, *Pr*=0.71, *M*=0.5, *Du*=0.5, *K**=5, *Gr*=5, *Gm*=10, $a=1, t_1=0.5.$

Figure 35. τ versus *t* for diferent *Pr* and *Sc*=0.22, *K*=3, *N*=5, *M*=0.5, *Du*=0.1, *K**=5, *Gr*=5, *Gm*=5, *a*=1, $t_1=0.5$.

Figure 36. θ versus γ for different t_1 and $t = 1.5$, $Sc = 0.22$, $K = 0.5$, $N = 3$, $Pr = 0.71$, $Du = 1$.

Figure 37. *u* versus *y* for different t_1 and $t = 1$, $Sc = 0.22$, $K = 3$, $N = 5$, $Pr = 0.71$, $Du = 0.5$, $M = 0.5$, $K^* = 1$, $Gr = 1$, *Gm*=5, *a*=1.

Figure 38. *Nu* versus *t* for different t_1 and $N = 5$, $Sc = 0.22$, $K = 0.5$, $Pr = 0.71$, $Du = 0.5$.

Figure 39. τ versus *t* for diferent t1 and *Sc*=0.22, *K*=2, *N*=3, *Pr*=0.71, *Du*=0.5, *M*=0.5, *K**=3, *Gr*=1, *Gm*=5, $a=1$.

Table 1. Computational values of Nusselt number for various *t, Du* and *N* when *Pr*=0.71, *Sc*=0.22, K=0.5, $t_1=0.5$.

t	K	\boldsymbol{N}	Du	Gr	Gm	τ
$\mathbf{1}$	1	5	0.5	1	1	9.1499
1.5						13.2425
$\overline{2}$						20.4193
1	$\overline{2}$	5	0.5	1	1	9.9629
	3					11.2420
	5					16.8475
1	1	\overline{c}	0.5	1	1	9.7355
		5				9.1499
		7				8.9585
1	1	5	1		1	10.1642
			$\overline{2}$	1		14.2214
			3			18.2786
1	1	5	0.5	1	1	9.1499
				3		21.4736
				5		33.7974
1	$\mathbf{1}$	5	0.5	$\mathbf{1}$	$\mathbf{1}$	9.1499
					3	7.9392
					5	6.7286

Table 2. Computational values of skin friction for various *t, K, N, Du, Gr* and *Gm* when *Pr*=0.71, *Sc*=0.22, $a=1, M=0.5, K^*=0.5, t_1=0.5.$

Figure 40. Scanned graph of temperature field versus y for different t_1 when $t = 1.2$, $N = 2$, $Pr = 0.71$ drawn by Seth et al.⁴⁹.

Figure 41. temperature field versus y for different t_1 when $t = 1.2$, $N = 2$, $Pr = 0.71$, $Sc = 0$, $Du = 0, K = 0$.

Table 3. Comparison of computational values of Sherwood number for various *K, Sc* and *t* obtained by Asogwa et.al⁵⁰, Seth et. Al⁵¹, Kataria and Patel⁵² and present authors.

Data availability

All data generated or analysed during this study are included in this published article and its supplementary information fles.

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Author contributions

S.S. has conducted the theoretical and graphical investigation of the problem and prepared the manuscript. Prof. N.A. supervised the whole investigation and reviewed the manuscript.

Competing interests

The authors declare no competing interests.

Additional information

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Correspondence and requests for materials should be addressed to S.S.

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