Journal of Russian Laser Research, Volume 25, Number 6, 2004

MEASUREMENT OF THE CHARACTERISTICS OF OPTICAL SIGNALS VARYING IN TIME BASED ON REGISTRATION OF THE DOUBLED SPECTRUM WITH GEOMETRICAL DISPLACEMENT IN SPECTRAL PLANE

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Abstract

A solution to the phase problem in optics is considered within the context of the analysis of time-dependent signals. The analysis concerns, in particular, determination of the amplitude and phase structure of signals and processes of ultrashort duration. The operation of the scheme is based on the registration of two spectra of the investigated radiation separated spatially. The first spectrum corresponds to the signal directly, while the other one is formed by summation of two spectra shifted geometrically with respect to each other by a distance of the order of a spectral device resolution. The description of the summarized spectrum contains the frequency derivative. The information obtained allows one to determine the amplitude and phase structure of the signal.

Keywords: structure of optical signals, spectral analysis.

1. Introduction

Nowadays optical investigations (in particular, the spectral ones) based on ultrashort radiation pulses, especially on signals of ultrashort duration $10^{-10} - 10^{-14}$ s, are of great interest (see, e.g., [1–4]). These studies are basically done in such fundamental fields as matter state analysis and kinetics of atoms and molecules in physical, chemical, and biological processes, analysis of nonstationary processes of interaction of radiation with matter, formation and study of wave packets, very-high-speed optoelectronic systems, and a number of others. In solving physical problems, one needs, along with analysis of the structure and spectrum of a signal changing in time, to determine the amplitude and phase structures of the influence of the substance or object studied on the probing optical radiation.

Among measurement methods one should first of all distinguish direct methods of signal registration with the use of high-speed image tubes [5–11]. The intensity distribution of a signal is measured. The time resolution is limited by the dynamical resolution of electron optics and by the maximum speed of electronic scanning and is about one picosecond. Among indirect measurement tools, correlation methods provide the highest time resolution [5, 6, 8]. The structure of the correlation function is directly obtained by these methods. Moreover, the amplitude structure of a signal can be obtained as well. In this case some additional information about the signal is required to interpret the results unambiguously. Methods based on the Kerr effect, the Pockels effect, or the saturating absorption [5] give similar results.

Translated from a manuscript submitted July 13, 2004.

To obtain information on the amplitude and phase structures of signals, holographic methods are used, which allow one to record data on separate spectral components. Methods based on the nonstationary reference wave [12–15] were developed further in spectral holography [16–19]. We would also like to note a physically attractive approach based on hole burning in the absorption spectra of spectral-selective media [20–22]. Media with a narrow line and homogeneous line broadening and a wide absorption band caused by inhomogeneous broadening are used for the registration. However, holographic methods as well as registration in selective media require a single pulse as a reference signal. The duration of the pulse should be less than the typical scale of time variations in the analyzed signal, because for registration of the total spectrum one needs an additional spectrum of the same width or wider. The analysis of such an additional pulse naturally reduces to a similar problem of its amplitude-phase structure.

The spectral methods [23–25] give partial information about a signal.

Thus, all the methods mentioned above have some limitations or present some difficulties in their realization. Either only a part of the information is registered (the duration, the correlation function, the spectrum), or a specially formed reference wave or specific low-temperature recording media are required.

The contemporary development of physical and coherent optics and the appropriate methods and technology [26–28] has provided us with the possibility of formulation and solving fundamental phase problems connected with detecting, processing, and analyzing the amplitude and phase characteristics of arbitrary optical signals varying in time. Thus, it is possible to solve the phase problem in optics as applied to time-dependent signals. In particular, to measure the amplitude and phase characteristics of arbitrary laser pulses as well as their spectra and the influence of an object on the probe signal, we use an approach based on registration of amplitude distributions of signals, which are specially formed by modulators, that is, the modulation-spectral method of analysis, and solving the phase problem in optics [29–31]. To solve problems of this kind, the interferometric methods are used as well, with frequency shift in one of the channels due to either the displacement of the mirror [32] or diffraction on the traveling acoustic wave [33] or modulation by an electrooptic crystal [34]. Further, a spectral device is used for the registration. The distinctive feature of the above-mentioned methods is connected with the fact that it is easier to obtain high spectral resolution than high time resolution in a wide spectral range.

In this paper, a scheme based on registration of two separate spectra of the investigated optical signal is considered. One spectrum corresponds directly to the signal. The other one is formed by two signal spectra displaced geometrically with respect to each other by a distance of the order of the spectral device resolution. Either the spectrum of the time-varying optical signal or the signal itself is analyzed.

The operation of the scheme is described within the framework of physical optics [35–37].

2. Spectral Intensity Distributions to Register

A schematic optical diagram for development, registration, and analysis of amplitude and phase spectrum characteristics of time-varying optical signals is shown in Fig. 1. The operation of the spectral device is based on formation and geometrical shift of two spectra of the studied signal. It contains the collimator objective **Ob1**, the camera objective **Ob2**, and the dispersion element **DS**, for example, a diffraction grating. The input slit **S** of the spectral device is of a specific structure, providing two displaced spectra, which is a distinctive feature of the scheme. In the simplest case, a slit doubled in height can be used. This leads to the formation of two spectra separated in height. Besides, one of the slits consists of two parts displaced in the direction of the device dispersion (see Fig. 1).

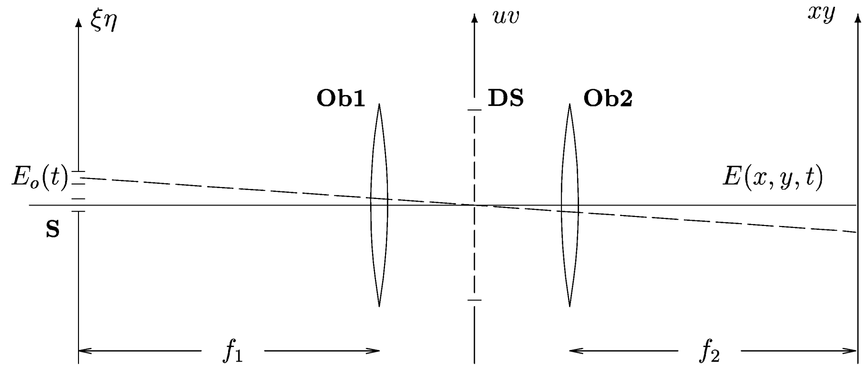

Fig. 1. Schematic optical diagram of the spectral device for analyzing the spectrum of a time-varying optical signal. **S** is the input slit, **Ob1** is the collimator, **Ob2** is the camera objective, and **DS** is the dispersion system.

The analyzed time-dependent optical signal (laser radiation)

$$E_o(t) \,=\, \mathcal{E}_o(t)\, \exp[-i\omega_o\, t] \tag{1}$$

is characterized by the time-dependent complex amplitude $\mathcal{E}_o(t)$, the duration T_o, and the average frequency ω_o. The spectrum of the analyzed signal is placed in the vicinity of the frequency ω_o. Its width is noticeably smaller than the carrier frequency, $\Delta\omega_s \,<\, \omega_o$. Then, the analyzed signal is directed onto the input slit **S** of the spectral device placed in the plane $\xi\eta$.

To simplify the calculations, it is assumed that the objectives **Ob1** and **Ob2** and the dispersion system **DS** are placed in the vicinity of the plane uv. The field in front of the collimator objective **Ob1** at distance f_1 from the input slit is described by the Kirchhoff integral [35–37]

$$E(u,v,t) \,=\, \mathcal{E}(u,v,t)\, \exp[-i\omega_o\, t] \,=\, -\,\frac{i}{\lambda\, f_1} \int \mathcal{E}_o(t_1)\, \exp[-i\,\omega_o\, t_1]\Big|_{t\,=\,t_1\,-r_1/c} d\xi\, d\eta. \tag{2}$$

The effect of the collimator and camera objectives (the focal distances f_1 and f_2, respectively) is described in the paraxial approximation by the quadratic phase factors [35–37],

$$O_1(u,v) \,=\, \exp\left[-\,i\,\frac{\omega_o}{c}\,\frac{u^2\,+\,v^2}{2\,f_1}\right], \quad O_2(u,v) \,=\, \exp\left[-\,i\,\frac{\omega_o}{c}\,\frac{u^2\,+\,v^2}{2\,f_2}\right]. \tag{3}$$

The effect of the dispersion system is described by the transition to the spectral representation of the complex amplitudes of the signal,

$$\mathcal{E}_o(t) = \frac{1}{2\,\pi} \int \mathcal{E}_o(\omega)\, \exp[-\,i\,\omega\, t]\, d\omega. \tag{4}$$

The angular dispersion is introduced from the phase relation describing the propagation of waves with different frequencies at different angles,

$$\frac{d\theta(\omega)}{d\omega} \,=\, \exp\left[i\,\frac{\omega_o}{c}\, u\, \cos[\theta(\omega)]\right], \tag{5}$$

where

$$\cos[\theta(\omega)] = \frac{x(\omega)}{f_2} = \frac{\omega_o}{f_2}\frac{dx}{d\omega} = \frac{\omega_o}{D\,f_2} \tag{6}$$

is the direction cosine for the component with frequency ω and $1\big/D = dx\big/d\omega$ is the inverse linear dispersion characterizing the spectral device. The effect of the dispersion element on the radiation is described by [see (5) and (6)]

$$D(u,\omega) = \exp\left[i\,\frac{\omega_o}{c}\,u\,\frac{x(\omega)}{f_2}\right]. \tag{7}$$

After the dispersion element and the camera objective, the field for the spectral component ω with account of (3) and (7) is

$$E_d(u,v,t) = \mathcal{E}_d(u,v,t)\,\exp[-i\omega_o\,t] = \mathcal{E}(u,v,t)\,O_1(u,v)\,O_2(u,v)\,D(u,\omega)\,\exp[-i\omega_o\,t].$$

The field in the registration plane xy at distance f_2 from the camera objective is described by a Kirchhoff integral similar to (2) [35– 37], the effects of the objectives [see (3)] and the dispersion element [see (7)] being taken into account,

$$E(x,y,t) = \mathcal{E}(x,y,t)\,\exp[-i\omega_o\,t]$$

$$= -\frac{i}{\lambda\,f_2}\int \mathcal{E}(u,v,t_2)\,O_1(u,v)\,O_2(u,v)\,D(u,\omega)\,\exp[-i\,\omega_o\,t_2]\Big|_{t_2\,=\,t\,-\,r_2/c}\,du\,dv. \tag{8}$$

One assumes that, in expressions (2) and (8), in the slow phase factors describing the signal structure (frequencies ω)

$$r_1 = f_1, \quad r_2 = f_2.$$

In high-frequency factors (the frequency ω_o), the Fresnel approximation is taken [35–37]

$$r_1 = f_1 + \frac{\xi^2 + \eta^2}{2\,f_1} + \frac{u^2 + v^2}{2\,f_1} - \frac{\xi u + \eta\,v}{f_1},$$

$$r_2 = f_2 + \frac{u^2 + v^2}{2\,f_2} + \frac{x^2 + y^2}{2\,f_2} - \frac{u\,x + v\,y}{f_2}.$$

Calculations with the introduction of all the above intermediate expressions [see (2)–(7)] into expression (8) that describes the field in the registration plane gives

$$E(x,y,t) = -\frac{1}{\lambda^2\,f_1\,f_2}\exp\left[-\,i\,\omega_o\,t + i\,\frac{\omega_o}{c}\left(f_1 + f_2 + \frac{x^2 + y^2}{2\,f_2}\right)\right]$$

$$\times\mathcal{E}_o\left(t - \frac{f_1 + f_2}{c}\right)\int \exp\left[i\,\frac{\omega_o}{c}\,\frac{\xi^2 + \eta^2}{2\,f_1}\right]$$

$$\times\left\{\int \exp\left[-\,i\,\frac{\omega_o}{c}\,u\left(\frac{\xi}{f_1} - \frac{x(\omega)}{f_2} + \frac{x}{f_2}\right) - i\,\frac{\omega_o}{c}\,v\left(\frac{\eta}{f_1} + \frac{y}{f_2}\right)\right]du\,dv\right\}d\xi\,d\eta. \tag{9}$$

Integration over u and v within $\pm u_o$ and $\pm v_o$ characterizing the size of the dispersion system $(2u_o \times 2v_o)$ gives the spread function

$$a(\xi,\eta,x,y) = \int \exp\left[-i\frac{\omega_o}{c}u\left(-\frac{x(\omega)}{f_2}+\frac{x}{f_2}+\frac{\xi}{f_1}\right)-i\frac{\omega_o}{c}v\left(\frac{y}{f_2}+\frac{\eta}{f_1}\right)\right]du\,dv$$

$$= 4\,u_o\,v_o\,\mathrm{sinc}\left[\frac{\omega_o}{c}u_o\left(-\frac{x(\omega)}{f_2}+\frac{x}{f_2}+\frac{\xi}{f_1}\right)\right]\mathrm{sinc}\left[\frac{\omega_o}{c}v_o\left(\frac{y}{f_2}+\frac{\eta}{f_1}\right)\right], \qquad (10)$$

which is caused by the diffraction on the aperture of the dispersion system. If the dispersion system is large, the spread function is similar to the δ–function. In the maximum of the spread function for the spectral component of the frequency ω,

$$\xi = -\Big(x - x(\omega)\Big)\frac{f_1}{f_2}, \quad \eta = -y\,\frac{f_1}{f_2}\,.$$

In the output plane xy, a component of the frequency ω forms the field determined by the field structure in the input slit. For a certain frequency ω in the output plane, $x = x(\omega)$. This corresponds to an unambiguous relation between the coordinate x and the frequency ω of the spectral component.

Omitting the constant phase delay $\Big(f_1,+f_2\Big)/c$, the field in the registration plane xy in the output of the spectral device [see (9)] with regard for (10) is

$$E(x,y,t) = -\frac{1}{\lambda^2 f_1 f_2}\exp\left[-i\,\omega_o\,t + i\,\frac{\omega_o}{c}\,\frac{x^2+y^2}{2\,f_2}\right]\exp\left[i\,\frac{\omega_o}{c}\,\frac{\Big(x-x(\omega)\Big)^2+y^2}{2\,f_2}\,\frac{f_1}{f_2}\right]\mathcal{E}_o(t).$$

Going to the spectral components, one obtains

$$E(x,y,\omega) = \int E(x,y,t)\,\exp[i\,\omega\,t]\,dt$$

$$= -\frac{1}{\lambda^2 f_1 f_2}\exp\left[-i\,\omega_o\,t + i\,\frac{\omega_o}{c}\,\frac{x^2+y^2}{2\,f_2} + i\,\frac{\omega_o}{c}\,\frac{\Big(x-x(\omega)\Big)^2+y^2}{2\,f_2}\,\frac{f_1}{f_2}\right]\mathcal{E}_o(\omega). \qquad (11)$$

In further calculations one can assume that, up to amplitude and phase factors inessential for the analysis, the field $E(x,y,\omega)$ at the output of the spectral device is determined by the value

$$\mathcal{E}_o(\omega)\,\exp[-i\omega_o\,t].$$

In the scheme under consideration, two spectra are formed in the output of the spectral device. The first one corresponds to the analyzed signal directly and is characterized for a certain coordinate x by the value

$$E_o(\omega,t) = \mathcal{E}_o(\omega)\,\exp[-i\omega_o\,t] = \int \mathcal{E}_o(t)\,\exp[i\,\omega\,t]\,dt\,\exp[-i\omega_o\,t]. \qquad (12)$$

The second spectrum is the sum of two components for a certain coordinate x. One of them coincides with the signal spectrum $E_o(\omega,t)$ [see (12)]. The other component is shifted geometrically, i.e., the frequency ω or ω_o of the analyzed signal in the first component gets into the frequency $\omega + \Delta\omega$ or $\omega_o + \Delta\omega$ of the second component. The geometrical shift of the spectrum does not change its structure. The second component of the spectrum is described similarly to (12) by

$$E_o(\omega, \Delta\omega, t) \,=\, \mathcal{E}_o(\omega \,+\, \Delta\omega)\, \exp[-i\,(\omega_o \,+\, \Delta\omega)\,t]$$

$$= \int \mathcal{E}_o(t)\, \exp[i\,(\omega \,+\, \Delta\omega)\,t]\,dt\, \exp[-i\,(\omega_o \,+\, \Delta\omega)\,t]. \tag{13}$$

For a small frequency shift ($\Delta\omega \ll \omega_o$) close to the spectral resolution $\delta\omega$ of the device ($\left|\Delta\omega\right| \sim \delta\omega$), this expression can be expanded in series. For the part containing the information about the signal structure [see (13)] this leads to

$$\exp[i\,(\omega \,+\, \Delta\omega)\,t] \,=\, \exp[i\,\omega\,t]\, \exp[i\,\Delta\omega\,t] \,\simeq\, \exp[i\,\omega\,t]\left[1 \,+\, i\,\Delta\omega\,t\right]$$

or

$$\mathcal{E}_o(\omega \,+\, \Delta\omega) \,=\, \int \mathcal{E}_o(t)\, \exp[i\,\omega\,t]\,dt \,+\, \Delta\omega \int (it)\,\mathcal{E}_o(t)\, \exp[i\,\omega\,t]\,dt$$

$$= \mathcal{E}_o(\omega) \,+\, \Delta\omega\,\frac{d}{d\omega}\mathcal{E}_o(\omega).$$

For the part containing the carrier frequency ω_o [see (13)] one can assume

$$\exp[-i\,(\omega_o \,+\, \Delta\omega)\,t] \,\simeq\, \exp[-i\omega_o\,t].$$

As a result, the second component of the spectrum is described by

$$E_o(\omega, \Delta\omega, t) \,=\, \left(\mathcal{E}_o(\omega) \,+\, \Delta\omega\,\frac{d}{d\omega}\mathcal{E}_o(\omega)\right) \exp[-i\omega_o\,t]. \tag{14}$$

The series expansion above definitely imposes a constraint on the total duration T_r of the analyzed signal:

$$T_r\,\Delta\omega \,\ll\, 2\,\pi \;\text{ or }\; T_r\,\Delta\nu \,\ll\, 1\,,\; T_r\,\Delta\lambda \,\ll\, \frac{\lambda_o^2}{c}$$

(see Sec. 4).

For further analysis, one has to take into consideration the amplitude and phase structure of the complex amplitude of the signal spectrum

$$\mathcal{E}_o(\omega) \,=\, A_o(\omega)\, \exp[i\,\Phi_o(\omega)]. \tag{15}$$

With account of possible differences in amplitudes (A_1 and A_2) and phases ($\exp[i\,\Phi_1]$ and $\exp[i\,\Phi_2]$) in the two formed spectra, the complex amplitude distributions of the spectra are described by the following expressions [see (12)–(15)]:

$$E_o(\omega, t) \,=\, A_1\, A_o(\omega)\, \exp\left[i\left(\Phi_1 \,+\, \Phi_o(\omega)\right)\right] \exp[-i\omega_o\,t],$$

$$E_o(\omega, \Delta\omega, t) \,=\, A_2\left[A_o(\omega) \,+\, \Delta\omega\,\frac{d}{d\omega}A_o(\omega) \,+\, i\,\Delta\omega\, A_o(\omega)\,\frac{d}{d\omega}\Phi_o(\omega)\right] \tag{16}$$

$$\times \exp\Big[i\Big(\Phi_2 + \Phi_o(\omega)\Big)\Big] \exp[-i\omega_o t].$$

The sum field distributions [see (16)]

$$E_o(\omega,t) \text{ and } E_s(\omega,\Delta\omega,t) = E_o(\omega,t) + E_o(\omega,\Delta\omega,t) \tag{17}$$

are registered in the spectral plane.

The recorded value, similarly to any optical measuring system, is the intensity distribution, which, for the two signal spectra [see (17)], are described, up to inessential factors and taking into account (15), by [see (16)]

$$\frac{I_o(\omega)}{A_o^2(\omega)} = A_1^2, \tag{18}$$

$$\frac{I_s(\omega,\Delta\omega)}{A_o^2(\omega)} = \Big[A_1^2 + A_2^2 + A_1 A_2 \cos\Big(\Phi_1 - \Phi_2\Big)\Big]$$
$$+ \frac{d}{d\omega}\Big(A_o(\omega)\Big)\frac{\Delta\omega}{A_o(\omega)}\, 2\, A_2 \Big[A_2 + A_1 \cos\Big(\Phi_1 - \Phi_2\Big)\Big]$$
$$+ \frac{d}{d\omega}\Big(\Phi_o(\omega)\Big)\, \Delta\omega\, 2\, A_1 A_2 \sin\Big(\Phi_1 - \Phi_2\Big). \tag{19}$$

The first summand in expression (19) (in square brackets) characterizes the intensity distribution of the two signal spectra in the absence of their mutual displacement. This summand coincides with the expression from [38, 39].

3. Processing the Information Registered

To process the information represented as the spectra intensity distributions [see (18) and (19)], one has to know the frequency shift $\Delta\omega$, the amplitudes in the two spectra A_1 and A_2, and the values of the constant phase delays Φ_1 and Φ_2 in the formed spectra. In correspondence with the measured intensity distribution $I_o(\omega)$ [see (18)] and with A_1 known, the amplitude distribution in the spectrum in this case (we take into account that the amplitude is positive and the sign of the signal is determined by the phase) is described by

$$A_o(\omega) = \frac{1}{A_1}\Big[I_o(\omega)\Big]^{1/2} \tag{20}$$

and further the derivative $dA_o(\omega)/d\omega$ distribution is calculated. From the measured intensity distribution $I_s(\omega)$ [see (19)] with account of the first-order terms in $\Delta\omega$, A_2, Φ_1, and Φ_2 being known, one determines the derivative of the phase in the spectrum

$$\frac{d}{d\omega}\Big(\Phi_o(\omega)\Big)\Delta\omega = \frac{1}{2\, A_1 A_2 \sin\Big(\Phi_1 - \Phi_2\Big)}\Bigg\{\frac{I_s(\omega,\Delta\omega)}{A_o^2(\omega)} - \Big[A_1^2 + A_2^2 + 2\cos\Big(\Phi_1 - \Phi_2\Big)\Big]$$
$$-\frac{d}{d\omega}\Big(A_o(\omega)\Big)\frac{\Delta\omega}{A_o(\omega)}\, 2A_1A_2\left[\frac{A_2}{A_1} + \cos\Big(\Phi_1 - \Phi_2\Big)\right]\Bigg\} \tag{21}$$

and calculates the phase distribution $\Phi_o(\omega)$ in the spectrum by integration. From the amplitude $A_o(\omega)$ and phase $\Phi_o(\omega)$ distributions found, the structure of the complex amplitude of the signal spectrum $\mathcal{E}_o(\omega)$

is determined [see (15)]. If necessary, the amplitude and phase structure of the time-varying optical signal $\mathcal{E}_o(t)$ or $E_o(t)$ [see (4)] is calculated by the inverse Fourier transformation.

When adjusting the set-up, it is convenient to choose certain phase delays Φ_1 and Φ_2 for the two spectra. For the phase difference $\Phi_1 - \Phi_2$, it is appropriate to take the value $\pi/2 \pm k\pi$, which corresponds to $\sin(\Phi_1 - \Phi_2) = 1$ and $\cos(\Phi_1 - \Phi_2) = 0$. In this case, expression (21) transforms to

$$\frac{d}{d\omega}\Big(\Phi_o(\omega)\Big)\,\Delta\omega \;=\; \frac{1}{2\,A_1\,A_2}\left\{\frac{I_s(\omega,\Delta\omega)}{A_o^2(\omega)} \;-\; \Big(A_1^2 \,+\, A_2^2\Big) \;-\; \frac{d}{d\omega}\Big(A_o(\omega)\Big)\,\frac{\Delta\omega}{A_o(\omega)}\,2\,A_2^2\right\}\;,$$

i.e., the derivative of the phase in the spectrum $d\Phi_o(\omega)/d\omega$ is determined more simply.

4. On Practical Realization of the Scheme

The total spectral range $\Delta\lambda_r$ or $\Delta\nu_r = \Delta\omega_r/2\,\pi$ registered determines the time resolution of the scheme

$$\delta t \;=\; \frac{1}{\Delta\nu_r} \;=\; \frac{2\pi}{\Delta\omega_r} \;=\; \frac{\lambda_o^2}{c\,\Delta\lambda_r}\,.$$

The total registration time T_r is characterized by the spectral range of the signal interference in the registration plane $\delta\omega_s$ determined by the spectrum shift $\Delta\omega$ and the spread function width of the device $\delta\omega$ (see Sec. 3) including the spread function of the dispersion element and the slit width [40]. The sum value of the spectral range is described in the generic case by the width of the convolution $s(\omega)$ of the spectral distributions for the spectrum shift $m(\omega')$ and the spread function $a(\omega - \omega')$

$$s(\omega) \;=\; \int m(\omega')\,a(\omega \,-\, \omega')\,d\omega'.$$

For estimations, one can use the approximate relationship

$$\delta\omega_s \;\simeq\; \Delta\omega \,+\, \delta\omega \quad \text{or} \quad \delta\nu_s \;=\; \frac{\delta\omega_s}{2\,\pi}\,, \qquad \delta\lambda_s \;=\; \delta\nu_s\,\frac{\lambda^2}{c}\,. \tag{22}$$

The full time T_r of the registration is described by (see Sec. 3)

$$T_r \;\ll\; \frac{1}{\delta\nu_s} \;=\; \frac{2\pi}{\delta\omega_s} \;=\; \frac{\lambda_o^2}{c\,\delta\lambda_s}\,.$$

For a spectrum localized in the vicinity of $\lambda \sim 1000$ nm and for a 150-mm reflective diffraction grating with 800 grooves per millimeter, the resolution is $R \sim 120000 - 240000$ to the first and second orders when using the narrow face of the groove [41]. This gives the spectral resolution $\delta\lambda \sim 0.008$ or 0.004 nm and, under the spectral range relation $\Delta\lambda \simeq 3\delta\lambda$, the sum spectral range is $\delta\lambda_s \simeq 0.03$ nm. As a result, the total time of registration $T_r \sim 10^{-11}$ or $3 \cdot 10^{-11}$ s. A time resolution of $\delta t \sim 10^{-14}$ s is provided for registration in the spectral range of $\Delta\lambda_r \sim 300$ nm.

5. Error Estimations in Field Structure Measurements

The estimation of the error of the measured amplitude and phase structures of the field is similar to that in [34].

The analysis is based on the expressions for the amplitude distribution $A_o(\omega)$ and the derivative of the phase $\Phi_o(\omega)$ in the spectrum [see (20) and (21)]. To simplify the expressions for making the error estimations, one assumes

$$A_1 = A_2 = 1 \ , \ \sin(\Phi_1 - \Phi_2) = 1 \ , \ \cos(\Phi_1 - \Phi_2) = 0.$$

In this case, expressions (20) and (21) are transformed to

$$A_o(\omega) = \Big[I_o(\omega)\Big]^{1/2}, \tag{23}$$

$$\frac{d}{d\omega}\Phi_o(\omega) = \frac{I_s(\omega,\Delta\omega)}{2\,\Delta\omega\,A_o^2(\omega)} - \frac{1}{\Delta\omega} - \frac{1}{A_o(\omega)}\,\frac{d}{d\omega}A_o(\omega). \tag{24}$$

The root-mean-square error of the field complex amplitude in the spectrum is described up to the first-order terms by [see (15)]

$$\Big\langle|\Delta\mathcal{E}_o(\omega)|^2\Big\rangle = \Big\langle|\Delta A_o(\omega)|^2\Big\rangle + A_o^2(\omega)\,\Big\langle|\Delta\Phi_o(\omega)|^2\Big\rangle. \tag{25}$$

The root-mean-square error of the amplitude is [see (23)]

$$\Big\langle|\Delta A_o(\omega)|^2\Big\rangle = \frac{1}{4}\,\frac{\Big\langle|\Delta I_o(\omega)|^2\Big\rangle}{I_o(\omega)}\,. \tag{26}$$

Taking into account the relation

$$\frac{d}{d\omega}A_o(\omega) = \frac{d}{d\omega}\Big[I_o(\omega)\Big]^{1/2} = \frac{1}{2}\,\frac{1}{\Big[I_o(\omega)\Big]^{1/2}}\,\frac{d}{d\omega}\Big[I_o(\omega)\Big], \tag{27}$$

one obtains for the phase [see (24)]

$$\frac{d}{d\omega}\Phi_o(\omega) = \frac{1}{2\,\Delta\omega}\left[\frac{I_s(\omega,\Delta\omega) - I_o(\omega) - \Delta\omega\,dI_o(\omega)/d\omega}{I_o(\omega)}\right] = \frac{1}{2\,\Delta\omega}\,\Psi(\omega) \tag{28}$$

and

$$\Phi_o(\omega) = \frac{1}{2\,\Delta\omega}\int_{\omega_o-\Delta\omega_s}^{\omega_o+\Delta\omega_s}\Psi(\omega')\,d\omega'.$$

The function $\Psi(\omega)$ is introduced to simplify the notation. The root-mean-square error of the phase is

$$\Big\langle|\Delta\Phi_o(\omega)|^2\Big\rangle = \left(\frac{1}{2\,\Delta\omega}\int_{\omega_o-\Delta\omega_s}^{\omega_o+\Delta\omega_s}\Delta\Psi(\omega')\,d\omega'\right)\left(\frac{1}{2\,\Delta\omega}\int_{\omega_o-\Delta\omega_s}^{\omega_o+\Delta\omega_s}\Delta\Psi(\omega'')\,d\omega''\right). \tag{29}$$

In this expression the integration can be replaced by summation. Partition of the spectrum $2\,\Delta\omega_s$ into the intervals $\delta\omega_s$ to sum over is determined by the value of the spectrum shift $\Delta\omega$ and by the resolution of the spectral device $\delta\omega$ [see (22)]. The total number of intervals is given by the relation

$$2\,n = 2\,m = \frac{2\,\Delta\omega_s}{\delta\omega_s}\,. \tag{30}$$

One can assume the function $\Delta\Psi(\omega)$ to be constant within a small interval $\delta\omega_s$. Then, expression (29) is transformed to

$$\left\langle |\Delta\Phi_o(\omega)|^2 \right\rangle = \left\langle \frac{1}{2\,\Delta\omega} \sum_{-n}^{+n} \delta\omega_s\,\Delta\Psi(\omega_n) \right\rangle \left\langle \frac{1}{2\,\Delta\omega} \sum_{-m}^{+m} \delta\omega_s\,\Delta\Psi(\omega_m) \right\rangle. \tag{31}$$

Since the errors for different intervals $\delta\omega_s$ are independent, only contributions from coinciding intervals in expression (31) survive

$$\left\langle |\Delta\Phi_o(\omega)|^2 \right\rangle = 2\,n \left(\frac{\delta\omega_s}{2\,\Delta\omega} \right)^2 \left\langle |\Delta\Psi(\omega)|^2 \right\rangle. \tag{32}$$

In correspondence with the general expressions for the root-mean-square error and the measured intensity distributions $I_o(\omega)$ [see (18)] and $I_s(\omega, \Delta\omega)$ [see (19)], one obtains

$$\left\langle |\Delta\Psi(\omega)|^2 \right\rangle = \left(\frac{\partial\Psi(\omega)}{\partial I_o(\omega)} \right)^2 \left\langle |\Delta I_o(\omega)|^2 \right\rangle + \left(\frac{\partial\Psi(\omega)}{\partial I_s(\omega, \Delta\omega)} \right)^2 \left\langle |\Delta I_s(\omega, \Delta\omega)|^2 \right\rangle. \tag{33}$$

For making further error estimations, one can make some simplifications. For sufficiently smooth spectra, one can assume

$$\frac{d}{d\omega}\Big[I_o(\omega) \Big] \sim 0$$

within the interval $\delta\omega_s$. In this case [see (28)],

$$\Psi(\omega) \simeq \frac{I_s(\omega, \Delta\omega) - I_o(\omega)}{I_o(\omega)}$$

and expression (33) is transformed to

$$\left\langle |\Delta\Psi(\omega)|^2 \right\rangle = \left(\frac{I_s(\omega, \Delta\omega)}{\Big[I_o(\omega) \Big]^2} \right)^2 \left\langle |\Delta I_o(\omega)|^2 \right\rangle + \left(\frac{1}{I_o(\omega)} \right)^2 \left\langle |\Delta I_s(\omega, \Delta\omega)|^2 \right\rangle. \tag{34}$$

Expression (34) is simplified noticeably if the relative errors of the measured intensities are assumed independent of the frequency, i.e.,

$$\frac{\Delta I_o(\omega)}{I_o(\omega)} \simeq \frac{\Delta I_s(\omega, \Delta\omega)}{I_s(\omega), \Delta\omega} \simeq \left(\frac{\Delta I}{I} \right).$$

In this case, the standard deviation is described by (26) for the amplitude and

$$\left\langle |\Delta\Phi_o(\omega)|^2 \right\rangle = 4\,n \left(\frac{\delta\omega_s}{2\,\Delta\omega} \right)^2 \left(\frac{I_s(\omega, \Delta\omega)}{I_o(\omega)} \right)^2 \left(\frac{\Delta I}{I} \right)^2$$

for the phase; or with account of $\delta\omega_s \simeq \Delta\omega$ [see also (30)]

$$\left\langle |\Delta\Phi_o(\omega)|^2 \right\rangle = \frac{\Delta\omega_s\,\delta\omega_s}{(\Delta\omega)^2} \left(\frac{I_s(\omega, \Delta\omega)}{I_o(\omega)} \right)^2 \left(\frac{\Delta I}{I} \right)^2 \simeq \frac{\Delta\omega_s}{\Delta\omega} \left(\frac{I_s(\omega, \Delta\omega)}{I_o(\omega)} \right)^2 \left(\frac{\Delta I}{I} \right)^2. \tag{35}$$

The estimations can be simplified further if one assumes the measured intensities in the spectra are reasonably close to each other,

$$I_s(\omega, \Delta\omega) \simeq I_o(\omega).$$

In this case, expression (35) is transformed to

$$\left\langle |\Delta\Phi_o(\omega)|^2 \right\rangle \simeq \frac{\Delta\omega_s\,\delta\omega_s}{(\Delta\omega)^2}\left(\frac{\Delta I}{I}\right)^2 \simeq \frac{\Delta\omega_s}{\Delta\omega}\left(\frac{\Delta I}{I}\right)^2. \tag{36}$$

As a result, the standard deviation of the complex amplitude of the field in the spectrum is described in correspondence with (25) and with account of the root-mean-square errors (26) and (36) by the following expression:

$$\left\langle |\Delta\mathcal{E}_s(\omega)|^2 \right\rangle \simeq \left[\frac{1}{4} + \frac{\Delta\omega_s\,\delta\omega_s}{(\Delta\omega)^2}\left(\frac{I_s(\omega,\Delta\omega)}{I_o(\omega)}\right)^2\right] I_o(\omega)\left(\frac{\Delta I}{I}\right)^2 \simeq \left[\frac{1}{4} + \frac{\Delta\omega_s}{\Delta\omega}\right] I_o(\omega)\left(\frac{\Delta I}{I}\right)^2.$$

Moreover, based on

$$\int \mathcal{E}(\omega)\,\mathcal{E}^*(\omega)\,d\omega = 2\pi \int E(t)\,E^*(t)\,dt$$

one can obtain

$$\left\langle \left|\Delta\mathcal{E}_s(\omega)\right|^2 \right\rangle = \left\langle \left|\Delta E_s(t)\right|^2 \right\rangle.$$

6. Conclusions

In this paper, we demonstrated a scheme for measuring the amplitude and phase structure of a time-varying optical signal. The method is based on a spectral device with registration of two spectra: the spectrum of the studied signal and the doubled spectrum with geometrical displacement in the direction of the device's dispersion. The estimations for a spectral device with a diffraction grating give a time resolution up to 10^{-14} s if the total duration of the analyzed signal is about 10^{-10} s, the spectral resolution is not lower than 0.01 nm, and the shift of the spectrum is about 0.04 nm.

Acknowledgments

This study was supported by the RF Minpromnauka Program "Fundamental Spectroscopy," by the RF Ministry of Education under Grant T02-02.4-998 in the field of Fundamental Investigations in Technical Sciences, and by the Division of Physical Sciences of the Russian Academy of Sciences under the Grant for Nonlinear Optics in Unique Laser Systems.

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